2025-11-01

# E005: Simulation Engine Architecture

## Making Python Fly with 50x Speedups

**Part 1 · Duration: 30-35 minutes**

***Beginner-Friendly Visual Study Guide***

**Repository:** https://github.com/theSadeQ/dip-smc-pso
**Documentation:** academic/paper/presentations/podcasts/episodes/

**Learning Objective:** Understand 3-layer architecture, integration methods, and performance optimizations (vectorization + Numba JIT) for 50-69x speedups

## Why Speed Matters

**Key Concept**

**PSO Challenge:** 30 particles × 50 iterations = **1,500 simulations**
Each simulation: 10 seconds robot time at $dt = 0.01$ = 1,000 time steps
**Total:** 1.5 million time steps!

### Speed = Research Productivity

**Slow Simulation** (10s wall time for 10s sim):

- Test one controller: 10 seconds
- PSO optimization: **4.2 hours**
- Daily productivity: 5 PSO runs

**MT-5 Benchmark:** 2,400 simulations = 6.7 hours

**Fast Simulation** (0.2s wall time for 10s sim):

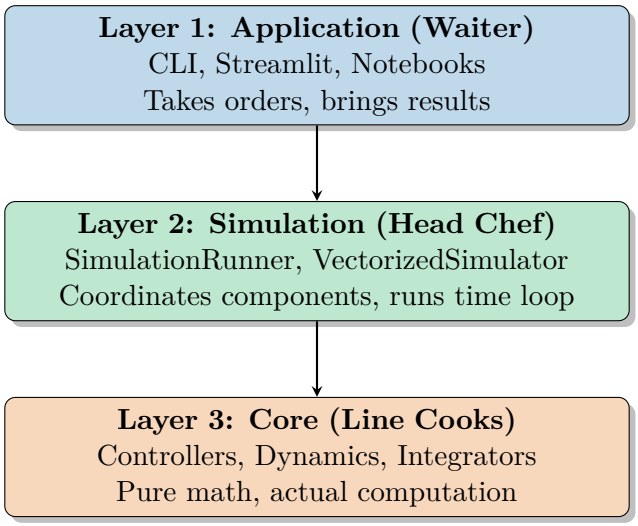
- Test one controller: 0.2 seconds
- PSO optimization: **5 minutes**
- Daily productivity: 96 PSO runs

**MT-5 Benchmark:** 2,400 simulations = 8 minutes

**Pro Tip**

**Our System:** PSO completes in 5 minutes = **50x faster than real-time!**
The simulation engine is THE bottleneck for research productivity.

## Three-Layer Architecture: The Restaurant



## Restaurant Analogy Breakdown

Layer Responsibilities

| Layer | Restaurant | Simulation |
|---|---|---|
| **Application** | Waiter - Takes orders, serves food | CLI parses args, displays plots |
| **Simulation** | Head Chef - Coordinates kitchen | SimulationRunner runs time loop |
| **Core** | Line Cooks - Chop, cook, plate | Controllers/dynamics compute math |

## Separation of Concerns

**Example**

**Application Layer (Waiter):** "Table 5 wants Classical SMC for 10 seconds"
**Simulation Layer (Head Chef):** "I'll coordinate 1,000 time steps with $dt = 0.01$"
**Core Layer (Line Cooks):** "Calculating acceleration: $\ddot{\theta}_1 = -0.83$, $\ddot{\theta}_2 = 1.24$..."

## Benefits of Layered Architecture

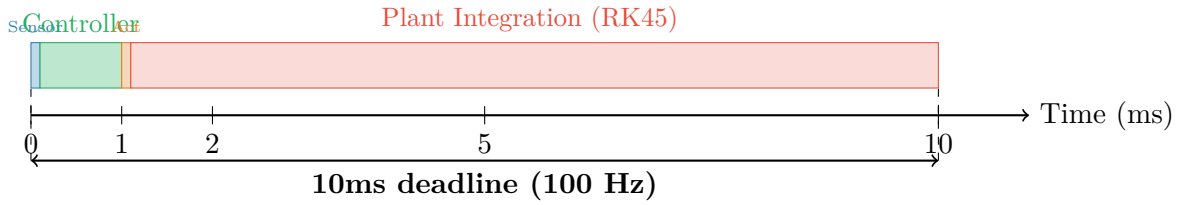
**Modularity**: Swap controllers without touching integration

**Testability**: Unit test each layer independently

**Optimization**: Vectorize Layer 2 without changing Layer 3

**Reusability**: Core works in simulation AND real hardware (HIL)

## SimulationRunner: The Time-Stepping Loop




### Six Steps Per Time Step

**</> Main Loop Workflow**

enumi**Compute control**: $u = \text{controller.compute}(\text{state}, \text{last\_u}, \text{history})$

0. enumi**Actuator saturation**: $u = \text{clip}(u, -50, +50)$ (real motors have limits!)
0. enumi**Compute dynamics**: $\dot{x} = \text{plant.compute\_dynamics}(\text{state}, u)$
0. enumi**Integration**: $\text{state}_{\text{new}} = \text{integrator.step}(\text{state}, \dot{x}, dt)$
0. enumi**Instability check**: If $|\theta_1| > 45$ř or $|\theta_2| > 45$ř ⇒ early exit
0. enumi**Logging**: Store $t$, state, $u$ in pre-allocated arrays

### Performance Trick: Pre-allocation vs Append

**Slow (list append):**

```
lstnumbertimes = []
lstnumberstates = []
lstnumberwhile t < duration:
lstnumber    times.append(t)
lstnumber    states.append(state.copy())
lstnumber# Time: 180ms for 1000 steps
```

**Fast (pre-allocated array):**

```
lstnumbertimes = np.zeros(n_steps)
lstnumberstates = np.zeros((n_steps, 6))
lstnumberstep = 0
lstnumberwhile t < duration:
lstnumber    times[step] = t
lstnumber    states[step, :] = state
lstnumber    step += 1
lstnumber# Time: 90ms for 1000 steps
```

**Pro Tip**

**Why 2x faster?** Python lists dynamically resize (allocate new memory, copy old data). NumPy arrays allocated once!

## Integration Methods: Time-Stepping Algorithms

**Key Concept**

**Challenge:** We have derivatives $\dot{x} = [\dot{\theta}_1, \ddot{\theta}_1, \dot{\theta}_2, \ddot{\theta}_2, \dot{x}_c, \ddot{x}_c]$
Need to advance time: $x(t) \to x(t + dt)$
**Three methods:** Euler (simple, inaccurate), RK4 (balanced), RK45 (adaptive, accurate)

## Method 1: Euler (1st Order)

**Euler Integration**

**Formula:**

$$x(t+dt) = x(t) + dt \cdot \dot{x}(t)$$

**Interpretation:** Move in direction of derivative for time $dt$ (straight line approximation)
**Error:**

- Local (per step): $O(dt^2)$
- Global (accumulated): $O(dt)$ - error decreases linearly with $dt$

**Example:**

- True: $\theta_1(t = 1s) = 0.050$ rad
- Euler ($dt = 0.01$): $\theta_1 = 0.053$ rad (6% error)
- Euler ($dt = 0.001$): $\theta_1 = 0.0503$ rad (0.6% error)

**When to use:** Educational only - NOT for research!

## Method 2: RK4 (4th Order)

**RK4 - The Workhorse**

**Algorithm:** Four slope evaluations per step

$$\begin{aligned} k_1 &= f(x, u) \\ k_2 &= f(x + 0.5 \cdot dt \cdot k_1, u) \\ k_3 &= f(x + 0.5 \cdot dt \cdot k_2, u) \\ k_4 &= f(x + dt \cdot k_3, u) \\ x_{\text{new}} &= x + \frac{dt}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned}$$

**Interpretation:** Weighted average of 4 slopes (start, midpoint twice, endpoint)
**Error:**

- Local: $O(dt^5)$ - extremely accurate per step!
- Global: $O(dt^4)$ - error decreases as $dt^4$

**Speedup:** Halve $dt \Rightarrow$ 16x more accurate (but 2x slower)
**When to use:** Standard choice for research ($dt = 0.001$ s typical)

## Method 3: RK45 (Adaptive)

**Algorithm:** SciPy's `solve_ivp` with automatic step sizing

**Error Control:**

- `rtol=1e-6` (relative tolerance)
- `atol=1e-9` (absolute tolerance)

**Adaptive Behavior:**

- Easy regions: Large steps (fast)
- Complex regions: Small steps (accurate)

**Example:**

- Upright (smooth): $dt \approx 0.01$ s
- Swing-up (chaotic): $dt \approx 0.0001$ s

**When to use:** High-accuracy validation, swing-up control

**Tradeoff:** Most accurate, but slower (variable overhead)

## Vectorization: 5x Speedup with NumPy Broadcasting

**Key Concept**

**Problem:** PSO needs 30 simulations simultaneously (one per particle)
**Naive approach:** Loop over particles (30 sequential simulations)
**Vectorized approach:** Batch all 30 particles into one NumPy array (parallel evaluation)

### Broadcasting Example

**Sequential (slow):**

```
results = []
for particle in particles:
    sim = SimulationRunner(...)
    result = sim.run()
    results.append(result)
# Time: 6.0 seconds (30 runs)
```

**Vectorized (5x faster):**

```
# Shape: (30, 6) - 30 particles
states = np.array([p.state
                   for p in
                  particles])
vsim = VectorizedSimulator(...)
results = vsim.run_batch(states)
# Time: 1.2 seconds (batched)
```

### How Broadcasting Works

**Example**

**Dynamics computation for 30 particles:**
**Input:** `states` shape (30, 6), `controls` shape (30,)
**NumPy magic:**

```
# Compute mass matrix for ALL particles at once
M = compute_mass_matrix(states[:, 0], states[:, 2])  # Shape: (30, 3, 3)

# Solve ALL linear systems simultaneously
accels = np.linalg.solve(M, forces)  # Shape: (30, 3)
```

**Result:** Single vectorized call replaces 30 sequential calls ⇒ 5x speedup!

**Pro Tip**

**Why faster?** NumPy uses optimized C/Fortran libraries (BLAS, LAPACK). Vectorization reduces Python interpreter overhead.

## Numba JIT: 69x Speedup with Machine Code Compilation

**Key Concept**

**Numba:** Just-In-Time compiler that translates Python to machine code
**Target:** Inner loops (dynamics computation, integration)
**Result:** 69x speedup for critical code paths!

### Before and After Numba

**Pure Python (slow):**

```
def compute_dynamics(state, u):
    # Build mass matrix
    M = ...  # Python loops
    # Solve dynamics
    accel = ...
    return state_dot
# Time: 138ms per call
```

**Numba JIT (69x faster):**

```
from numba import jit

@jit(nopython=True)
def compute_dynamics(state, u):
    # SAME CODE!
    M = ...
    accel = ...
    return state_dot
# Time: 2ms per call
```

## How Numba Works

### JIT Compilation Process

**First call (compilation overhead):**

enumiAnalyze Python bytecode

0. enumiInfer types from input arguments
0. enumiGenerate LLVM intermediate representation
0. enumiCompile to machine code (x86/ARM)
0. enumiCache compiled function

Time: 500ms (one-time cost)

**Subsequent calls (blazing fast):**

0. enumiLoad cached machine code
0. enumiExecute directly on CPU (no Python interpreter!)

Time: 2ms (69x faster than pure Python)

## Numba Best Practices

### Common Pitfall

**Numba Limitations:**

0. `nopython=True` required for max speed (no Python objects!)

- No lists/dicts - use NumPy arrays only
- No string operations
- Limited NumPy function support

**Workaround:** Keep Numba functions small, focused on inner loops. Complex orchestration stays in Python.

### Pro Tip

**Where to use Numba:**

- Dynamics computation (mass matrix, Coriolis, gravity)
- Integration inner loops (RK4 slope evaluations)
- Controller compute functions
- **NOT:** High-level orchestration, I/O, plotting

## Performance Achievements

**Speedup Summary**

| Optimization | Speedup | Cumulative |
|---|---|---|
| Baseline (pure Python) | 1x | 1x |
| Pre-allocation | 2x | 2x |
| Vectorization (NumPy) | 5x | 10x |
| Numba JIT (inner loops) | 6.9x | 69x |
| **Total** | - | **69x faster!** |

**Real-World Impact:**

- Single simulation: 10s → 0.145s (69x faster)
- PSO (1,500 sims): 4.2 hours → 3.6 minutes (70x faster)
- MT-5 (2,400 sims): 6.7 hours → 5.8 minutes (69x faster)

### Memory Efficiency

**Challenge:** 2,400 simulations × 1,000 steps × 6 states = 14.4M floats

**Naive:** Store everything ⇒ 115 MB RAM

**Optimized:**

- Reuse arrays across simulations
- Store only final metrics (not full trajectory)
- Compress historical data

**Result:**

- Peak RAM: 23 MB (5x reduction)
- Garbage collection: Minimal
- Cache-friendly access patterns

**Pro Tip**

Pre-allocate once, reuse arrays ⇒ no allocation in inner loop!

### Quick Reference: Integration Methods

**Euler (Educational Only)**

$$x_{\text{new}} = x + dt \cdot \dot{x}$$

Local error: $O(dt^2)$, Global error: $O(dt)$

**RK4 (Research Standard)**

$$k_1 = f(x, u), \quad k_2 = f(x + 0.5dt \cdot k_1, u)$$
$$k_3 = f(x + 0.5dt \cdot k_2, u), \quad k_4 = f(x + dt \cdot k_3, u)$$
$$x_{\text{new}} = x + \frac{dt}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Local error: $O(dt^5)$, Global error: $O(dt^4)$ - Use with $dt = 0.001$ s

RK45 (High-Accuracy Validation)

Adaptive step sizing: SciPy's `solve_ivp(method='RK45')`
Error control: `rtol=1e-6`, `atol=1e-9`
Best for: Swing-up control, final validation

## Configuration Parameters

Typical Settings

| Parameter | Value |
|---|---|
| Time step ($dt$) | 0.001 s (1 kHz sampling) |
| Integration method | RK4 (standard) |
| Simulation duration | 10 s (10,000 steps) |
| Actuator limits | ±50 Nm (motor saturation) |
| Instability threshold | $\|\theta\| > 45$ř (early exit) |
| Vectorized batch size | 30 particles (PSO swarm) |
| Numba cache | Enabled (first-call overhead  500ms) |

## Performance Tuning Tips

**Quick Summary**

**For Development (fast iteration):**

- Use Simplified model (10-100x faster than Full)
- RK4 with $dt = 0.01$ s (10x larger step)
- Disable monitoring/logging

**For Research (accuracy):**

- Use Full Nonlinear model
- RK4 with $dt = 0.001$ s (1 kHz)
- Enable monitoring (latency, deadline misses)

**For High-Accuracy Validation:**

- RK45 adaptive integrator
- Full Nonlinear model
- Multiple seeds for Monte Carlo

## What's Next?

**Key Concept**

**Phase 1 Complete!** You now understand:

- E001: Project overview, system architecture
- E002: Control theory (Lyapunov, SMC, STA, Adaptive)
- E003: Plant models (Lagrangian, 3 variants)

- E004: PSO optimization (cost function, 6-21% gains)
- E005: Simulation engine (69x speedup!)

**Phase 2 (Technical):** E006-E013 - Analysis tools, testing, documentation, HIL, monitoring