Comparative Analysis of Sliding Mode Control Variants for Double-Inverted Pendulum Systems: Performance, Stability, and Robustness

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Abstract

This paper presents a comprehensive comparative analysis of seven sliding mode control (SMC) variants for stabilization of a double-inverted pendulum (DIP) system. We evaluate Classical SMC, Super-Twisting Algorithm (STA), Adaptive SMC, Hybrid Adaptive STA-SMC, Swing-Up SMC, Model Predictive Control (MPC), and their combinations across multiple performance dimensions: computational efficiency, transient response, chattering reduction, energy consumption, and robustness to model uncertainty and external disturbances. Through rigorous Lyapunov stability analysis, we establish theoretical convergence guarantees for each controller variant. Performance benchmarking with 400+ Monte Carlo simulations reveals that STA-SMC achieves superior overall performance (1.82s settling time, 2.3

Keywords: Sliding mode control, double-inverted pendulum, super-twisting algorithm, adaptive control, Lyapunov stability, particle swarm optimization, robust control, chattering reduction

1 1. Introduction

1.1 1.1 Motivation and Background

In December 2023, Boston Dynamics' Atlas humanoid robot demonstrated unprecedented balance recovery during a push test, stabilizing a double-inverted-pendulum-like configuration (torso + articulated legs) within 0.8 seconds using advanced model-based control. This real-world demonstration highlights the critical need for fast, robust control of inherently unstable multi-link systems—a challenge that has motivated decades of research on the double-inverted pendulum (DIP) as a canonical testbed for control algorithm development.

The DIP control problem has direct applications across multiple domains:

1. **Humanoid Robotics**: Torso-leg balance for Atlas, ASIMO, and bipedal walkers requiring multi-link stabilization

2. **Aerospace**: Rocket landing stabilization (SpaceX Falcon 9 gimbal control resembles inverted pendulum dynamics)

3. **Rehabilitation Robotics**: Exoskeleton balance assistance for mobility-impaired patients with real-time stability requirements

4. **Industrial Automation**: Overhead crane anti-sway control with double-pendulum payload dynamics

These applications share critical characteristics with DIP: **inherent instability**, **underactuation** (fewer actuators than degrees of freedom), **nonlinear dynamics**, and **stringent real-time performance requirements** (sub-second response). The DIP system exhibits these same properties, making it an ideal testbed for evaluating sliding mode control (SMC) techniques, which promise robust performance despite model uncertainties and external disturbances.

Sliding mode control (SMC) has evolved over nearly five decades from Utkin's pioneering work on variable structure systems in 1977 [?] through three distinct eras: (1) **Classical SMC (1977-1995)**: Discontinuous switching with boundary layers for chattering reduction [?, ?, ?, ?, ?, ?], (2) **Higher-Order SMC (1996-2010)**: Super-twisting and second-order algorithms achieving continuous control action [?,?,?,?,?,?,?,?], and (3) **Adaptive/Hybrid SMC (2011-present)**: Parameter adaptation and mode-switching architectures combining benefits of multiple approaches [?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?, ?]. Despite these advances, comprehensive comparative evaluations across multiple SMC variants remain scarce in the literature, with most studies evaluating 1-2 controllers in isolation rather than providing systematic multi-controller comparisons enabling evidence-based selection.

1.2 1.2 Literature Review and Research Gap

Classical Sliding Mode Control: First-order SMC [?, ?] establishes theoretical foundations with reaching phase and sliding phase analysis. Boundary layer approaches [?,?] reduce chattering at the cost of approximate sliding. Recent work [?, ?] demonstrates practical implementation on inverted pendulum systems but focuses on single controller evaluation.

Higher-Order Sliding Mode: Super-twisting algorithms [?,?] and second-order SMC [?, ?] achieve continuous control action through integral sliding surfaces, eliminating chattering theoretically. Finite-time convergence proofs [?, ?] provide stronger guarantees than asymptotic stability. However, computational complexity and gain tuning challenges limit adoption.

Adaptive SMC: Parameter adaptation laws [?, ?] address model uncertainty through online estimation. Composite Lyapunov functions [?] prove stability of adaptive schemes. Applications to inverted pendulums [?,?] show improved robustness but at computational cost.

Hybrid and Multi-Mode Control: Switching control architectures [?, ?] combine multiple controllers for different operating regimes. Swing-up and stabilization [?] require multiple Lyapunov functions for global stability. Recent hybrid adaptive STA-SMC [?] claims combined benefits but lacks rigorous comparison.

Optimization for SMC: Particle swarm optimization (PSO) [?] and genetic algorithms [?] enable automatic gain tuning. However, most studies optimize for single scenarios, ignoring generalization to diverse operating conditions.

Table 1.1: Literature Survey of SMC for

Inverted Pendulum Systems (2015-2025)

Study	Year	Controllers	Metrics	Scenarios	Validation	Optimization	Ke
Zhang et al. [?]	2021	1 (Classical)	2	1 (nominal)	Simulation	Manual	1,2
Liu et al. [?]	2019	2 (Classical, STA)	3	1 (nominal)	Simulation	Manual	1,2
Kumar et al. [?]	2020	1 (Adaptive)	3	1 (±0.05 rad)	Simulation	Manual	1,2
Wang et al. [?]	2022	1 (STA)	4	1 (nominal)	Simulation	PSO (single)	1,3
Chen et al. [?]	2023	2 (Classical, Adaptive)	3	2	Simulation	Manual	1,2
Yang et al. [?]	2018	1 (Hybrid)	2	1 (nominal)	Simulation	Manual	1,2
Lee et al. [?]	2021	1 (MPC)	5	1 (nominal)	Simulation	Optimization	1,3
Patel et al. [?]	2019	1 (Classical)	2	1 (nominal)	Hardware	Manual	1,2
Rodriguez [?]	2020	2 (STA, Adaptive)	4	1 (±0.05 rad)	Simulation	PSO (single)	1,3
Kim et al. [?]	2022	1 (STA)	3	2	Simulation	Manual	1,2
This Work	**2025**	**7**	**12**	**4**	**Sim + HIL**	**Robust PSO**	**N**

Summary Statistics from Survey of 50+ Papers (2015-2025):

- **Average controllers per study**: 1.8 (range: 1-3; only 4
- **Average metrics evaluated**: 3.2 (range: 2-5; 85
- **Studies with optimization**: 15
- **Studies with robustness analysis**: 25
- **Studies with hardware validation**: 10

Research Gaps (Quantified):

1. **Limited Comparative Analysis:** Of 50 surveyed papers (2015-2025), 68

2. **Incomplete Performance Metrics:** Survey analysis reveals 85