2025-11-01

E002: Control Theory Fundamentals

DIP-SMC-PSO Educational Series

January 25, 2026

Overview

This episode covers control theory fundamentals from the DIP-SMC-PSO project.

Part: Part1 Foundations
Duration: 15-20 minutes
Source: Comprehensive Presentation Materials

section0   Seven Controller Types: Overview

- **Classical SMC** – Boundary layer for chattering reduction
    - Simplest implementation, robust to uncertainties - 'src/controllers/classical_smc.py'
    - **Super-Twisting Algorithm (STA)** – Continuous higher-order SMC
    - Second-order sliding mode, finite-time convergence - 'src/controllers/sta_smc.py'
    - **Adaptive SMC** – Online parameter estimation
    - Adapts to unknown system parameters - 'src/controllers/adaptive_smc.py'
    - **Hybrid Adaptive STA-SMC** – Combines adaptive + super-twisting
    - Best of both approaches - 'src/controllers/hybrid_adaptive_sta_smc.py'

section0   Seven Controller Types: Advanced & Experimental

- **Swing-Up SMC** – Large-angle stabilization
    - Energy-based swing-up + SMC balance - 'src/controllers/swing_up_smc.py'
    - **Model Predictive Control (MPC)** – Experimental optimization-based
    - Predicts future states, optimizes control sequence - 'src/controllers/mpc.py'
    - **Factory Pattern** – Thread-safe controller registry
    - Unified interface for all controllers - 'src/controllers/factory.py'
    [OK] All 7 controllers validated with:
    - Lyapunov stability proofs (LT-4) - 100 Monte Carlo runs (MT-5) - Model uncertainty analysis (LT-6) - Disturbance rejection testing (MT-8)

section0   Sliding Mode Control: Fundamental Concept

Core Idea: Design a sliding surface = 0 such that:
    - System trajectories converge to the surface (reaching phase) - System slides along the surface to equilibrium (sliding phase)
    Sliding Surface Design for DIP:

$$equation= k_1\theta_1 + k_2\dot{\theta}_1 + \lambda_1\theta_2 + \lambda_2\dot{\theta}_2 \tag{0}$$

    where:
    - θ_1, θ_2 – Angular positions of poles 1 and 2 - $\dot{\theta}_1, \dot{\theta}_2$ – Angular velocities - $k_1, k_2, \lambda_1, \lambda_2$ – Design gains (tuned by PSO)
    - **Robustness:** Insensitive to matched uncertainties - **Finite-time convergence:** Reaches = 0 in finite time - **Invariance:** Dynamics on surface independent of disturbances

section0   Classical SMC: Control Law

Control Law with Boundary Layer:

$$equationu = -K \cdot \tanh\left(\frac{-}{\epsilon}\right) \tag{0}$$

    where:
    - K – Control gain (determines reaching speed) - ϵ – Boundary layer thickness (chattering reduction) - $\tanh(\cdot)$ – Smooth approximation of $(\cdot)$
    Chattering Phenomenon:
    - **Cause:** Discontinuous control switching across = 0 - **Effect:** High-frequency oscillations, actuator wear - **Solution:** Boundary layer ϵ trades precision for smoothness
    Adaptive boundary layer: $\epsilon(t) = \epsilon_0 + \alpha$
Result: Marginal 3.7

section0   Super-Twisting Algorithm (STA)

Second-Order Sliding Mode: u = $u_1 + u_2$

$u_1 = -\alpha^{1/2}()$

$\dot{u}_2 = -\beta()$

where:

- α, β – STA gains (positive constants) - u_1 – Continuous proportional term - u_2 – Integral term (eliminates steady-state error)

Key Advantages:

- **Continuous control:** $u(t)$ is continuous (no chattering) - **Finite-time convergence:** Both and ˙ reach zero - **Robustness:** Handles Lipschitz disturbances

STA achieves **lowest chattering frequency** among all 7 controllers

section0   Adaptive SMC: Parameter Estimation

Motivation: System parameters (m, ℓ, g) may be unknown or time-varying

Adaptive Law: u = $-\hat{\theta} \cdot \Phi() - K()$

$\dot{\hat{\theta}} = \gamma \Phi()$

where:

- $\hat{\theta}$ – Estimated parameter vector - $\Phi()$ – Regressor vector (known functions of state) - $\gamma > 0$ – Adaptation rate

Lyapunov-Based Stability:

$$equation= \frac{1}{2}{}^2 + \frac{1}{2\gamma}\tilde{\theta}^T \tilde{\theta} \tag{0}$$

where $\tilde{\theta} = \theta^* - \hat{\theta}$ (parameter error)

˙ $\leq -\eta$ ensures asymptotic convergence

section0   Hybrid Adaptive STA-SMC

Combines:

- Adaptive parameter estimation (handles uncertainties) - Super-twisting algorithm (continuous control, no chattering)

Control Law: u = $-\hat{\theta} \cdot \Phi() + u_{STA}$

$u_{STA} = -\alpha^{1/2}() + u_2$

$\dot{u}_2 = -\beta()$

$\dot{\hat{\theta}} = \gamma \Phi()$

Performance Characteristics:

- **Best robustness** – Adapts to parameter variations - **Low chattering** – STA provides continuous control - **Fast convergence** – Second-order sliding mode - **Complexity tradeoff** – More states, higher computational cost

Hybrid controller shows **smallest performance degradation** under ±20

section0   Swing-Up SMC: Energy-Based Control

Two-Phase Strategy:

Phase 1: Energy-Based Swing-Up (large angles)

$$equation u = k_e(E^* - E)(\dot{\theta}_1 \cos \theta_1) \tag{0}$$

where:
- $E = \frac{1}{2}m\ell^2\dot{\theta}_1^2 + mg\ell(1 - \cos\theta_1)$ – Total energy - E^* – Target energy (upright equilibrium)
- k_e – Energy gain

Phase 2: SMC Balance (small angles)

$$equation\, u = -K\tanh\left(\frac{-}{\epsilon}\right) \tag{0}$$

Switching Condition:

$$equation\, Switch\, to\, SMC\, when\, \theta_1 < \theta_{threshold}\, and\, \dot{\theta}_1 < \dot{\theta}_{threshold} \tag{0}$$

section0 Model Predictive Control (MPC): Experimental

Optimization-Based Control:

At each time step, solve: $\min \quad J = \sum_{k=0}^{N-1}\left(k - {}^{*2}_Q + u_k{}^2_R\right)$
$subject\, to: \quad k+1 = f(k, u_k)$
$u_{min} \le u_k \le u_{max}$

where:
- N – Prediction horizon - Q, R – State and control weighting matrices - $f(\cdot)$ – Nonlinear dynamics model

Status:
- Experimental implementation - Computational cost limits real-time performance - Suitable for offline trajectory planning

section0 Lyapunov Stability Theory

Fundamental Tool: Prove controller stability mathematically

Lyapunov Function Candidate:

$$equation\, () = \frac{1}{2}{}^2 \tag{0}$$

Stability Condition:

$$equation\, \dot{} = \dot{} < 0 \quad \forall \ne 0 \tag{0}$$

For Classical SMC: $\dot{} = \dot{}$
$= (f(, u))$
$\le -\eta \quad (with\, appropriate\, u)$

where $\eta > 0$ (reaching rate)

Complete proofs for all 7 controllers (1,000 lines)
'docs/theory/lyapunov_proofs_existing.md'

section0 Chattering Analysis: Frequency Domain

Definition: High-frequency oscillations in control signal

Metrics (QW-4 Task):
- **Zero-Crossing Rate:** Count sign changes in $u(t)$

$$equation\, ZCR = \frac{1}{T}\sum_{k=1}^{N-1} I[(u_{k+1}) \ne (u_k)] \tag{0}$$

- **FFT Analysis:** Identify dominant frequencies

$$equation U(f) = \mathcal{F}\{u(t)\}, \quad P(f) = U(f)^2 \tag{0}$$

- **High-Frequency Energy:**

$$equation E_{HF} = \int_{f_{cutoff}}^{f_{Nyquist}} P(f) df \tag{0}$$

Lowest chattering: STA-SMC
*Highest chattering:** Classical SMC (without boundary layer)

- **FFT Analysis:** Identify dominant frequencies

$$equation U(f) = \mathcal{F}\{u(t)\}, \quad P(f) = U(f)^2$$

Resources

- **Repository:** `https://github.com/theSadeQ/dip-smc-pso.git`

- **Documentation:** See `docs/` directory

- **Getting Started:** `docs/guides/getting-started.md`

Educational podcast episode generated from comprehensive presentation materials