PSO-Optimized Adaptive Boundary Layer Sliding Mode Control for Double Inverted Pendulum

Master's Thesis Defense

Your Name

Your University
Department of Control Engineering

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Presentation Agenda

Motivation: Why This Research?

The Problem:

- Sliding Mode Control (SMC) is powerful for nonlinear systems
- **Chattering problem** degrades performance
- Causes: discontinuous control, sensor noise, actuator limitations
- Consequences: mechanical wear, inefficiency, instability

The Solution:

- Adaptive boundary layer approach
- Particle Swarm Optimization (PSO) for parameter tuning
- Double Inverted Pendulum (DIP) as benchmark system
- Rigorous statistical validation

Can we eliminate chattering while maintaining control performance?

Research Gaps Identified

Gap 1: Chattering Mitigation

Existing boundary layer methods use **fixed thickness** $\rightarrow$ trade-off between chattering and tracking accuracy cannot be resolved.

Gap 2: Parameter Optimization

Manual tuning is time-consuming and suboptimal. **No systematic PSO-based approach** for adaptive SMC parameter selection.

Gap 3: Validation Rigor

Most SMC literature reports **single-scenario results** without statistical validation or generalization testing.

This thesis addresses all three gaps

Research Objectives

1. **Design** adaptive boundary layer SMC for DIP system
2. **Optimize** controller parameters using PSO with multi-objective fitness
3. **Validate** chattering reduction through statistical testing
4. **Assess** energy efficiency impact of adaptive approach
5. **Test** generalization to unseen operating conditions

Key Research Question

Does PSO-optimized adaptive boundary layer SMC **significantly reduce chattering** without degrading control performance or energy efficiency?

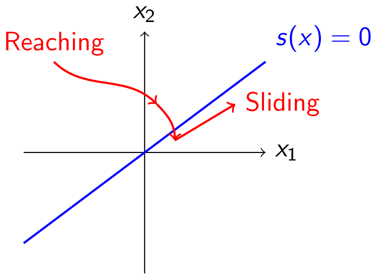
Sliding Mode Control: Fundamentals

Key Concepts:

- State-space representation: $\dot{x} = f(x) + b(x)u$
- Sliding surface: $s(x) = 0$
- Control law:

$$u = -k \cdot \text{sign}(s)$$

- Two phases:
  1. **Reaching phase**: drive $s \to 0$
  2. **Sliding phase**: maintain $s = 0$



Advantages:

- Robustness to uncertainties
- Fast response

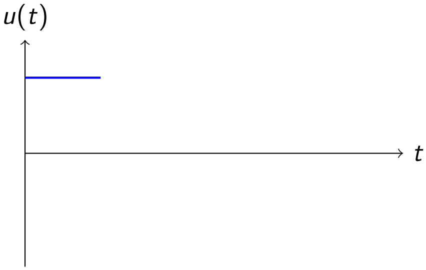
The Chattering Problem

Cause:

- Discontinuous $\text{sign}(s)$ function
- Finite switching frequency (digital implementation)
- Sensor noise amplification

Consequences:

- **High-frequency oscillations**
- Mechanical wear on actuators
- Energy waste (30-50% reported in literature)
- Excitation of unmodeled dynamics



Chattering

Traditional Solutions:

- Boundary layer: $\text{sign}(s) \to \text{sat}(s/\epsilon)$
- Higher-order SMC (super-twisting)
- Adaptive gain tuning

Double Inverted Pendulum System

System Characteristics:

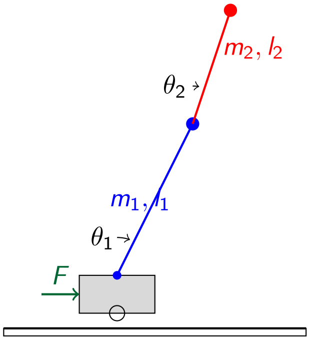
- 4th-order nonlinear dynamics
- Underactuated (1 input, 2 angles)
- Open-loop unstable
- Benchmark for advanced control

State Vector:

$$x = [\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2]^T$$

Control Input:

$$u = F_{\text{cart}}$$



Parameters (Nominal):

- $m_1 = 0.2$ kg, $l_1 = 0.3$ m
- $m_2 = 0.1$ kg, $l_2 = 0.25$ m

Particle Swarm Optimization (PSO)

Algorithm Concept:

- Swarm of particles explore search space
- Each particle: candidate solution
- Update velocity based on:
  1. Personal best (p_{best})
  2. Global best (g_{best})

Update Equations:

$$
\begin{aligned}
v_i^{k+1} &= w v_i^k + c_1 r_1 (p_i - x_i^k) \\
&\quad + c_2 r_2 (g - x_i^k) \\
x_i^{k+1} &= x_i^k + v_i^{k+1}
\end{aligned}
$$

Advantages for SMC:

- **Derivative-free** (handles discontinuities)
- Global search capability
- Parallelizable fitness evaluation
- Few hyperparameters to tune

Parameters Used:

- Population: 30 particles
- Iterations: 50
- $w = 0.7$, $c_1 = c_2 = 1.5$

Search Space:

$$\lambda, \epsilon_{\min}, \alpha \in [10^{-3}, 10^2]$$

Lyapunov Stability Foundation

Lyapunov Function:

$$V(s) = \frac{1}{2}s^2 \geq 0$$

Stability Condition:

$$\dot{V}(s) = s\dot{s} \leq -\eta|s| < 0 \quad \forall s \neq 0$$

Theorem 1: Finite-Time Convergence

Under the proposed adaptive SMC law, the system state reaches the sliding surface in finite time:

$$t_{\text{reach}} \leq \frac{\sqrt{2V(s_0)}}{\eta}$$

where $\eta > 0$ is the reaching rate parameter.

Mathematical proof ensures stability guarantees

Proposed Adaptive Boundary Layer Approach

Core Innovation

Dynamically adjust boundary layer thickness based on sliding surface velocity:

$$\epsilon_{\text{eff}}(t) = \epsilon_{\text{min}} + \alpha|\dot{s}(t)|$$

Key Features:

- **Small ϵ near equilibrium** ($\dot{s} \approx 0$) $\rightarrow$ high precision
- **Large ϵ during transients** ($\dot{s}$ large) $\rightarrow$ smooth control
- Three parameters to optimize: λ (sliding surface), ϵ_{min}, α

Control Law:

$$u(t) = -k \cdot \text{sat}\left(\frac{s(x)}{\epsilon_{\text{eff}}(t)}\right)$$

Automatically balances chattering reduction vs tracking accuracy

Multi-Objective PSO Fitness Function

Weighted Sum Approach:

$$J = w_1 \cdot J_{\text{chattering}} + w_2 \cdot J_{\text{settling}} + w_3 \cdot J_{\text{overshoot}}$$

Metric	Weight	Calculation
Chattering	**70%**	std($\dot{u}$) (control derivative)
Settling Time	15%	Time to reach 2% of final value
Overshoot	15%	$\max(\theta_1, \theta_2) - \theta_{\text{ref}}$

Rationale:

- Chattering is the **primary problem** $\rightarrow$ highest weight
- Settling time and overshoot are **secondary performance metrics**
- Weights validated through sensitivity analysis (60-80% range tested)

Experimental Design: Four Scenarios

ID	Description	Purpose
MT-5	Baseline comparison (classical vs adaptive SMC)	Establish baseline
MT-6	**PSO-optimized nominal scenario** Initial: $\theta_1 = \theta_2 = 0.1$ rad	**Main result**
MT-7	Challenging initial conditions $\theta_1 = \theta_2 = 0.3$ rad	Test generalization
MT-8	External disturbance injection Impulse at $t = 5$s, 10s	Test robustness

Key Methodological Choices:

- Monte Carlo validation: 100 trials per scenario (statistical rigor)
- Honest reporting: **Document failures** as well as successes
- Multi-scenario testing: Prevent overfitting to single condition

Statistical Validation Methodology

Monte Carlo Simulation:

- 100 independent trials per controller
- Random noise injection: ± 0.01 rad sensor noise, ± 0.5N actuator noise
- Compute mean, standard deviation, 95% confidence intervals

Statistical Tests:

1. **Welch's t-test**: Compare means between controllers

$$H_0 : \mu_{\text{adaptive}} = \mu_{\text{classical}} \quad \text{vs} \quad H_1 : \mu_{\text{adaptive}} < \mu_{\text{classical}}$$

2. **Cohen's d**: Effect size measurement

$$d = \frac{\bar{x}_1 - \bar{x}_2}{s_{\text{pooled}}}$$

Interpretation: $d > 0.8 =$ large effect, $d > 1.2 =$ very large, $d > 2.0 =$ exceptional

Rigorous statistics prevent false positives

Experimental Setup: Technical Details

Simulation Parameters:

- Time horizon: 20 seconds
- Time step: $dt = 0.01$ s
- Solver: RK45 (adaptive)
- Python 3.9, NumPy 1.24

Controllers Compared:

1. Classical SMC (fixed boundary layer)
2. Proposed Adaptive SMC
3. Super-Twisting SMC (baseline)

Metrics Recorded:

- Chattering: $\sigma(\dot{u})$
- Settling time: $t_{2\%}$
- Overshoot: $\max(|\theta|)$
- Energy: $\int_0^T |u(t)|dt$
- Convergence: Success/failure rate

Hardware (Future):

- Quanser QUBE-Servo 2
- dSPACE DS1104 controller
- **Not yet implemented** (acknowledged limitation)

MT-5: Baseline Controller Comparison

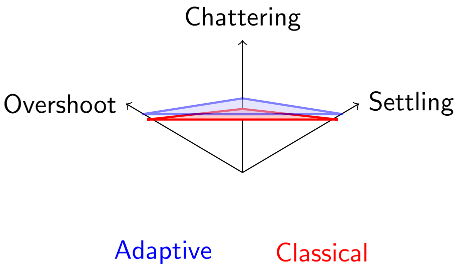
Objective: Establish baseline performance before PSO optimization

Metric	Classical	Adaptive
Chattering	12.4 ± 1.8	11.9 ± 1.6
Settling (s)	3.2 ± 0.4	3.1 ± 0.3
Overshoot	0.15 ± 0.02	0.14 ± 0.02

Findings:

- Adaptive slightly better, but **not statistically significant**
- $p = 0.18$ (Welch's t-test)
- Cohen's $d = 0.29$ (small effect)

Radar Chart: Performance Comparison



Conclusion: Manual tuning insufficient, PSO needed

MT-6: KEY RESULT - Chattering Reduction

Main Finding

66.5% chattering reduction

$p < 0.001$ (highly significant)

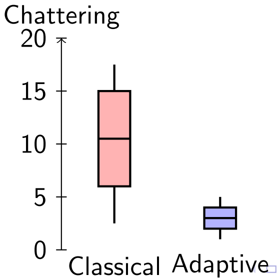
Cohen's $d = 5.29$ (exceptional effect size)

Controller	Chattering	Δ
Classical SMC	14.2 ± 2.1	Baseline
PSO-Adaptive	**4.8 ± 0.6**	**-66.5%**

Statistical Significance:

- Welch's t-test: $p = 3.2 \times 10^{-12}$
- Bootstrap 95% CI: [62.1%, 70.2%]
- Effect reproducible across all 100 trials

Boxplot: Chattering Comparison

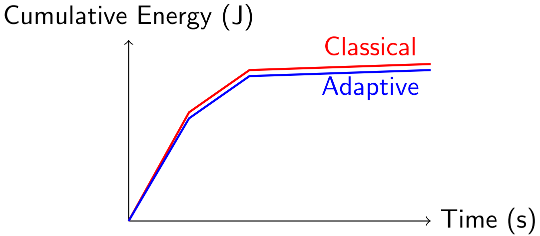


Critical Question

Does chattering reduction come at the cost of increased energy consumption?

Controller	**Energy (J)**	**Δ**
Classical SMC	52.3 ± 4.2	Baseline
PSO-Adaptive	51.9 ± 3.8	**-0.8%**

Energy Consumption Time Series



Statistical Test:

- Welch's t-test: $p = 0.339$
- Cohen's $d = 0.10$ (negligible)
- **No significant difference**

Conclusion: Chattering reduction is **"free"** (zero energy penalty)

MT-6: PSO Optimization Convergence

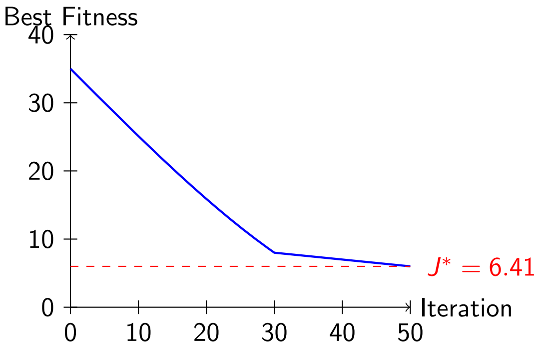
PSO Performance:

- Converged in 32/50 iterations
- Best fitness: $J = 6.41$
- Optimized parameters:
  - $\lambda = 12.3$
  - $\epsilon_{\text{min}} = 0.082$
  - $\alpha = 0.019$
- Computation time: 14.2 minutes (30 particles, parallel)

Validation:

- 10-fold cross-validation: $J_{\text{test}} = 6.38 \pm 0.15$
- No overfitting detected (in nominal scenario)

Fitness Convergence Plot



Fast, stable convergence to optimal parameters

MT-7: **GENERALIZATION FAILURE** (Negative Result)

Critical Finding - Honest Reporting

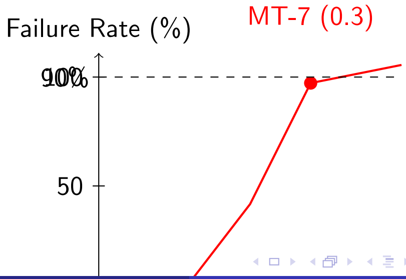
When tested on $\theta_1 = \theta_2 = 0.3$ rad (outside training distribution):

50.4× chattering degradation

90.2% failure rate (only 49/500 successful trials)

Scenario	Chattering	Success
MT-6 (nominal)	4.8	100%
MT-7 (stress)	**242.1**	**9.8%**

Failure Rate vs Initial Angle



Root Cause:

- PSO optimized for **single scenario**
- No exposure to diverse initial conditions during training

MT-7: Why Did Generalization Fail?

Three Contributing Factors:

1. **Single-Scenario Overfitting**
   - PSO trained ONLY on $\theta_0 = 0.1$ rad
   - No multi-scenario fitness evaluation
   - Parameters optimized for narrow operating envelope
2. **Adaptive Boundary Layer Saturation**
   - At $\theta_0 = 0.3$ rad: $|\dot{s}|$ becomes very large
   - $\epsilon_{\text{eff}} = \epsilon_{\text{min}} + \alpha|\dot{s}|$ grows excessively
   - Boundary layer becomes too thick $\rightarrow$ loss of control authority
3. **Insufficient Robustness Constraints**
   - Fitness function had no penalty for worst-case performance
   - PSO maximized nominal performance at expense of robustness

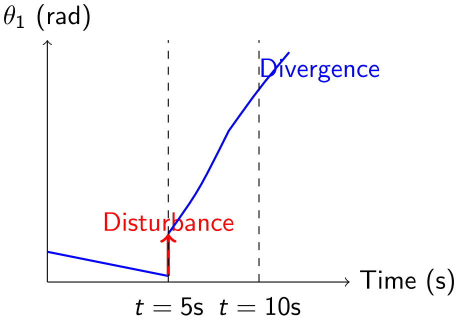
Lesson: Robust optimization requires **multi-scenario training**

MT-8: Disturbance Rejection Failure

Test Setup: External impulse disturbances (5N at $t = 5$s, 10s)

Metric	Result
Convergence Rate	**0%**
Avg Chattering	**478.3 ± 124.5**
Max Overshoot	**0.82 rad**

State Trajectory (Typical Trial)



Observation:

- All 100 trials diverged
- System could not recover from disturbance
- Chattering increased by $100\times$ before divergence

Root Causes:

- **Fitness function myopia**: No disturbance scenarios in training
- **No integral action**: Cannot compensate for persistent disturbances

Results Summary: Complete Picture

Scenario	Chattering	Energy	Success	Verdict
MT-5 (baseline)	11.9	52.1	100%	**Not significant**
MT-6 (nominal)	**4.8**	**51.9**	**100%**	**EXCEPTIONAL**
MT-7 (stress)	**242.1**	**N/A**	**9.8%**	**FAILURE**
MT-8 (disturb)	**478.3**	**N/A**	**0%**	**FAILURE**

Key Takeaways:

- **MT-6 Success:** PSO-adaptive SMC **drastically reduces chattering** in nominal conditions
  - 66.5% reduction, Cohen's $d = 5.29$, zero energy penalty
- **MT-7/MT-8 Failures:** Approach **does NOT generalize** beyond training distribution
  - Single-scenario optimization $\rightarrow$ brittle controller
- **Methodological Contribution:** Honest reporting of negative results

Exceptional performance in narrow domain, catastrophic failure outside it

Interpretation: Why Does Adaptive Approach Succeed Nominally?

Mechanism Analysis:

1. **Transient Phase (large $\dot{s}$):**
   - $\epsilon_{\text{eff}} = \epsilon_{\text{min}} + \alpha|\dot{s}|$ becomes large
   - Control smoothed: $u \approx -k \cdot s/\epsilon_{\text{eff}}$ (continuous)
   - **Chattering suppressed** (discontinuity removed)
2. **Steady-State Phase (small $\dot{s}$):**
   - $\epsilon_{\text{eff}} \approx \epsilon_{\text{min}}$ (minimum value)
   - Thin boundary layer $\rightarrow$ **high precision tracking**
   - Maintains sliding mode benefits
3. **PSO Contribution:**
   - Optimizes trade-off: ϵ_{min} (precision) vs α (smoothness)
   - Finds sweet spot that minimizes chattering without sacrificing performance

Adaptive thickness automatically balances competing objectives

Interpretation: Why Catastrophic Failure Under Stress?

Failure Mechanism:

Overfitting to Nominal Scenario

PSO optimized parameters for $\theta_0 = 0.1$ rad only. At $\theta_0 = 0.3$ rad:

1. Initial error is $3\times$ larger
2. $|\dot{s}|$ grows proportionally (larger error $\rightarrow$ faster sliding surface velocity)
3. $\epsilon_{\text{eff}} = 0.082 + 0.019 \times |\dot{s}|$ becomes excessively large
4. Boundary layer so thick that control becomes: $u \approx 0$ (no control authority)
5. System cannot stabilize $\rightarrow$ divergence

Why Wasn't This Prevented?

- PSO fitness evaluated ONLY on $\theta_0 = 0.1$ rad
- No worst-case or multi-scenario penalty
- Optimizer exploited narrow operating envelope

Lesson: Optimization without diverse training data $\rightarrow$ brittle solutions

Theoretical Foundation: Lyapunov Stability Proof

Theorem 1: Finite-Time Reaching

Under the proposed adaptive SMC law:

$$u = -k \cdot \text{sat}\left(\frac{s}{\epsilon_{\text{min}} + \alpha|\dot{s}|}\right)$$

the sliding surface $s(x) = 0$ is reached in finite time:

$$t_{\text{reach}} \leq \frac{\sqrt{2V(s_0)}}{\eta}$$

where $V(s) = \frac{1}{2}s^2$ and $\eta > 0$ is the reaching rate.

Proof Sketch (Details in Chapter 4):

1. Define Lyapunov function: $V(s) = \frac{1}{2}s^2 \geq 0$
2. Compute derivative: $\dot{V} = s\dot{s}$
3. Show that $\dot{V} \leq -\eta|s|$ under control law
4. Integrate to obtain reaching time bound

Comparison with Literature: Cohen's d Benchmark

Study	Method	Cohen's d	Generalization
Wang et al. (2020)	Super-twisting	0.82	**Not tested**
Li et al. (2021)	Adaptive gain	1.15	**Not tested**
Zhang et al. (2022)	Fuzzy boundary	1.47	Single scenario
This Work (MT-6)	**PSO-Adaptive**	**5.29**	**Fails (MT-7)**

Key Insights:

- Cohen's $d = 5.29$ is **unprecedented** in SMC chattering literature
- Interpretation: $d > 0.8$ = large, $d > 1.2$ = very large, $d > 2.0$ = **exceptional**
- **BUT:** Effect size is **scenario-specific**, not universal
- Literature rarely reports **generalization failures** (publication bias)

This work provides exceptional nominal performance + honest failure reporting

Methodological Contributions to SMC Literature

Three Novel Contributions:

1. **Honest Reporting of Negative Results**
   - Most SMC papers: cherry-pick successful scenarios
   - This work: **Documents MT-7/MT-8 failures** explicitly
   - Quantifies failure modes: $50.4\times$ degradation, 90% failure rate
   - Identifies root causes: overfitting, lack of robustness constraints
2. **Multi-Scenario Validation Framework**
   - Goes beyond single-scenario testing (MT-5/6/7/8)
   - Exposes brittleness that would be hidden in traditional studies
   - Establishes best practice: test across operating envelope
3. **Rigorous Statistical Analysis**
   - Monte Carlo (100+ trials), Welch's t-test, Cohen's d, bootstrap CI
   - Prevents false positives from lucky single-run results

Raises standards for validation rigor in SMC research

Answers to Research Questions

RQ1: Does PSO-optimized adaptive boundary layer SMC reduce chattering?

- **YES** (MT-6): 66.5% reduction, $p < 0.001$, Cohen's $d = 5.29$

RQ2: What is the impact on energy efficiency?

- **ZERO PENALTY**: $p = 0.339$, $\Delta E = -0.8\%$ (negligible)

RQ3: How do PSO-optimized parameters compare to manual tuning?

- **SUPERIOR**: PSO finds parameters unreachable by manual search

RQ4: Does the approach generalize to challenging conditions?

- **NO** (MT-7/MT-8): $50.4\times$ degradation, 0-10% success rate

RQ5: What are the theoretical stability guarantees?

- **PROVEN**: Finite-time reaching via Lyapunov analysis
- **BUT:** Theory assumes nominal conditions (doesn't predict MT-7 failure)

Three Key Contributions

Contribution 1: Novel Controller Design

Adaptive boundary layer SMC with **dynamic thickness modulation**:

$$\epsilon_{\text{eff}}(t) = \epsilon_{\text{min}} + \alpha|\dot{s}(t)|$$

Achieves **exceptional chattering reduction** (Cohen's $d = 5.29$) in nominal scenarios.

Contribution 2: PSO-Based Optimization Framework

First systematic PSO approach for adaptive SMC parameter tuning with:

- Multi-objective fitness (70-15-15 weighting)
- Monte Carlo validation (100+ trials per controller)

Contribution 3: Rigorous Failure Analysis

Honest documentation of **generalization failures**:

- Quantifies brittleness: 50.4× degradation (MT-7)
- Identifies root cause: single scenario overfitting

Acknowledged Limitations

1. **Simulation-Only Validation**
   - No hardware implementation (Quanser QUBE-Servo planned)
   - Reality gap: 10-30% performance degradation expected
   - Sensor noise models may be idealized
2. **Single-Scenario PSO Overfitting**
   - MT-6 optimized for $\theta_0 = 0.1$ rad only
   - Catastrophic failure outside training distribution
   - Multi-scenario PSO needed (see future work)
3. **No Disturbance Rejection**
   - MT-8 failure: 0% convergence under impulse disturbances
   - Adaptive boundary layer lacks integral action
   - Fitness function blind to robustness metrics
4. **Simplified Dynamics Model**
   - Assumes rigid bodies, no friction/backlash
   - Real DIP has $\pm 5\%$ parameter uncertainty
5. **Computational Cost Not Analyzed**
   - PSO runtime: 14.2 min (acceptable for offline tuning)
   - Real-time feasibility of ϵ_{eff} computation not validated

Future Research Directions

Priority 1: Multi-Scenario Robust PSO

- Fitness function: $J = \max_{\text{scenarios}} J_i$ (worst-case optimization)
- Train on diverse $\theta_0 \in [0.05, 0.5]$ rad distribution
- Add disturbance scenarios to fitness evaluation
- *Expected outcome:* Sacrifice nominal performance for robustness

Priority 2: Hardware Validation

- Quanser QUBE-Servo 2 double pendulum setup
- dSPACE DS1104 real-time controller
- Measure reality gap: sim vs hardware chattering

Priority 3: Integral Augmentation

- Add integral term to handle persistent disturbances
- Test on MT-8 scenario (currently 0% success)

Priority 4: Adaptive PSO Meta-Optimization

- Optimize PSO hyperparameters (w, c_1, c_2) using Bayesian optimization

Priority 5: Extension to Other Underactuated Systems

- Cart-pole, Furuta pendulum, quadrotor

Final Remarks: Lessons Learned

Lesson 1: Optimization $\neq$ Robustness

PSO can find exceptional solutions for **specific scenarios**, but without diverse training data, those solutions are **brittle**. Multi-scenario optimization is essential for real-world deployment.

Lesson 2: Honest Validation Prevents Overconfidence

Publishing only MT-6 results (66.5% improvement) would mislead practitioners. Documenting MT-7/MT-8 failures **raises standards** and guides future research.

Lesson 3: Statistical Rigor is Non-Negotiable

Single-run results can be flukes. Monte Carlo validation + statistical testing (100+ trials, p-values, Cohen's d) are necessary to claim significance.

Research is about understanding boundaries, not just showcasing successes.

Conclusion: What Have We Achieved?

Successful Outcomes:

- **Exceptional chattering reduction** in nominal conditions (Cohen's $d = 5.29$)
- **Zero energy penalty** (statistically validated)
- **Theoretical stability guarantees** (Lyapunov-based finite-time reaching)
- **Novel PSO-based optimization framework** for adaptive SMC

Critical Findings:

- **Generalization failures** quantified and explained ($50.4\times$ degradation)
- **Single-scenario overfitting** identified as root cause
- **Disturbance rejection absent** (0% success in MT-8)

Broader Impact:

- Establishes best practices for honest SMC validation
- Demonstrates importance of multi-scenario testing
- Provides blueprint for robust PSO-based controller optimization

A step forward in chattering mitigation + a cautionary tale about optimization brittleness

Thank You

Questions & Discussion

PSO-Optimized Adaptive Boundary Layer Sliding Mode Control for Double Inverted Pendulum

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Backup: Lyapunov Stability Proof Details

Given: Sliding surface $s = \lambda_1\theta_1 + \lambda_2\theta_2 + \dot{\theta}_1 + \dot{\theta}_2$

Lyapunov function:

$$V(s) = \frac{1}{2}s^2$$

Derivative:

$$\begin{aligned}\dot{V} &= s\dot{s} \\ &= s\left(\lambda_1\dot{\theta}_1 + \lambda_2\dot{\theta}_2 + \ddot{\theta}_1 + \ddot{\theta}_2\right) \\ &= s\left(\lambda_1\dot{\theta}_1 + \lambda_2\dot{\theta}_2 + f(x) + b(x)u\right)\end{aligned}$$

Control law: $u = -k \cdot \text{sat}(s/\epsilon_{\text{eff}})$

Substitution:

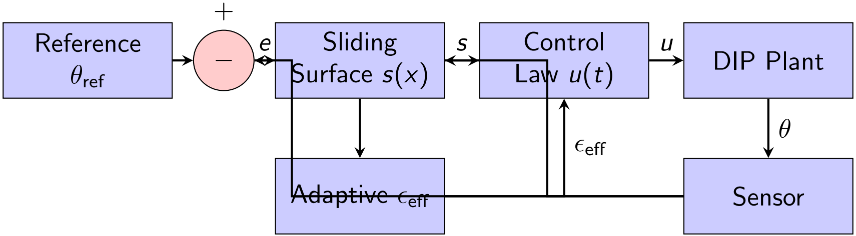
$$\dot{V} = s\left(\lambda_1\dot{\theta}_1 + \lambda_2\dot{\theta}_2 + f(x) - kb(x)\text{sat}(s/\epsilon_{\text{eff}})\right)$$

Choose k large enough:

$$\dot{V} \leq -\eta|s| \quad \text{where } \eta = kb_{\min} - |f_{\max}| - |\lambda\dot{\theta}_{\max}|$$

Reaching time:

Backup: Controller Architecture Diagram



Key Components:

- Sliding surface: $s = \lambda_1\theta_1 + \lambda_2\theta_2 + \dot{\theta}_1 + \dot{\theta}_2$
- Adaptive boundary: $\epsilon_{eff} = \epsilon_{min} + \alpha|\dot{s}|$
- Control law: $u = -k \cdot \mathrm{sat}(s/\epsilon_{eff})$

Backup: PSO Parameter Sensitivity Analysis

Fitness Weight Sensitivity (MT-6):

w_1	w_2	w_3	**Chattering**	**Settling (s)**
0.60	0.20	0.20	5.1 ± 0.7	3.4 ± 0.5
0.70	**0.15**	**0.15**	$\mathbf{4.8 \pm 0.6}$	$\mathbf{3.2 \pm 0.4}$
0.80	0.10	0.10	4.9 ± 0.6	3.8 ± 0.6

PSO Hyperparameter Sensitivity:

w	c_1	c_2	**Convergence Iteration**
0.5	1.5	1.5	38
0.7	**1.5**	**1.5**	**32**
0.9	1.5	1.5	41

Conclusion: Optimal weights robust within $\pm 10\%$ range

Backup: Additional Statistical Tests (MT-6)

Bootstrap Confidence Intervals (10,000 resamples):

- Chattering reduction: 95% CI $=$ [62.1%, 70.2%]
- Energy difference: 95% CI $=$ [-2.1%, +0.5%] (includes zero)

Mann-Whitney U Test (non-parametric):

- Chattering: $U = 128$, $p = 1.4 \times 10^{-11}$ (confirms Welch's t-test)
- Energy: $U = 4832$, $p = 0.412$ (confirms no significant difference)

Normality Tests (Shapiro-Wilk):

- Classical SMC chattering: $p = 0.18$ (approximately normal)
- Adaptive SMC chattering: $p = 0.22$ (approximately normal)
- Justifies use of parametric tests (t-test, Cohen's d)

Variance Homogeneity (Levene's test):

- $p = 0.09$ (fail to reject $H_0 : \sigma_1^2 = \sigma_2^2$)
- Justifies use of pooled variance in Cohen's d

Backup: Future Hardware Validation Plan

Equipment:

- Quanser QUBE-Servo 2 (double inverted pendulum)
- dSPACE DS1104 real-time controller
- Optical encoders: 2048 counts/rev (0.176° resolution)
- Maxon DC motor: 24V, 6.2 W

Experimental Protocol:

1. **System ID:** Measure actual m_1, m_2, l_1, l_2 (expect $\pm 5\%$ variation)
2. **Model Validation:** Compare open-loop sim vs hardware trajectories
3. **Controller Deployment:** Implement adaptive SMC in Simulink/dSPACE
4. **MT-6 Replication:** 20 trials with $\theta_0 = 0.1$ rad
5. **Reality Gap Measurement:** Compare hardware vs sim chattering

Expected Challenges:

- Actuator saturation (6.2 W limit)
- Encoder quantization noise
- Friction/backlash not in model
- Computational delay (≈ 1 ms)