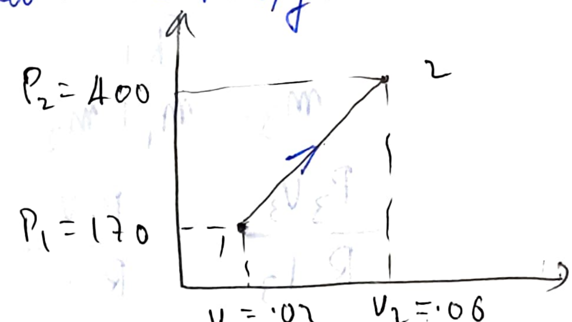
$PV^{\gamma} = C$ is valid for an ideal gas undergoing reversible adiabatic process.

Conventional Problems

Q=> A fluid is contained in a cylinder by a spring loaded frictionless piston so that the pressure in fluid is a linear function of volume i.e $P = a + bV$, where a & b are constants. internal energy of the fluid is given by $U = 34 + 3.15PV$ where U is in KJ, P in K Pa & V in m^3 if the fluid changes from initial state of $P_1 = 170$ KPa, $V_1 = .03 m^3$ to a final state of $P_2 = 400$ KPa, $V_2 = .06 m^3$ find the magnitude & direction of heat & work t/y.

Sol^n => $W = \frac{1}{2}(170 + 400)(.06 - .03)$



$W = 8.55 KJ$

$đQ = dU + đW$

$U_2 = \cancel{34} + 3.15 P_2 V_2$

$U_1 = \cancel{34} + 3.15 P_1 V_1$

$U_2 - U_1 = 3.15(P_2 V_2 - P_1 V_1)$

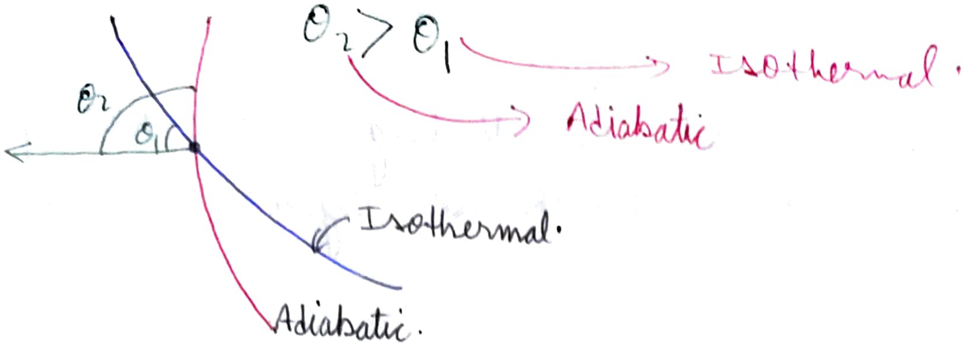
$dU = 3.15[400 \times .06 - 170 \times .03]$

$dU = 59.535 KJ$

$đQ = 59.535 + 8.55$

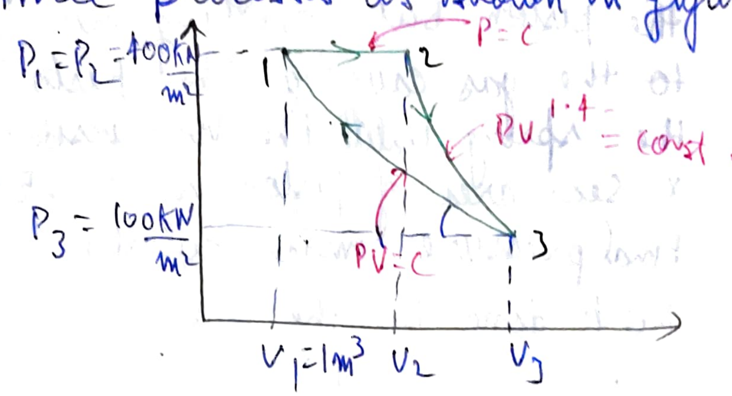
$đQ = 68.085 KJ$

Q=> An insulated rigid pressure vessel is divided into two portions by a partition the first part of the vessel is occupied by an ideal gas at a pressure P_1, Vol V_1 & Temperature T_1. The other part is occupied by the same ideal gas but at pressure P_2, Vol V_2, Temp. T_2 suddenly the partition is removed & two portions mix with each other show that P_3 & T_3 are given by



Conventional Problems

A system undergoes three processes as shown in figure
Find (i) V_2.
(ii) Net work t/f.



Sol^n ⇒ Process (3-1) $T = C$

$$P_3 V_3 = P_1 V_1 \Rightarrow \boxed{V_3 = 4\,m^3}$$

Process (2-3) $PV^{1.4} = C$

$$P_2 V_2^{1.4} = P_3 \cdot V_3^{1.4} \Rightarrow \boxed{V_2 = 1.486\,m^3}$$

WD for 1-2

$$W_{1-2} = P(V_2 - V_1) = 400 \times (1.486 - 1) = 194.4\,KJ.$$

WD for 2-3

$$W_{2-3} = \frac{P_2 V_2 - P_3 V_3}{\gamma - 1} = \frac{400 \times 1.486 - 100 \times 4}{1.4 - 1} = 486\,KJ$$

WD for 3-1

$$W_{3-1} = P_3 V_3 \ln\left(\frac{V_1}{V_3}\right) = -\cancel{554.5465\,KJ} \quad -554.5\,KJ.$$

Net work $= W_{1-2} + W_{2-3} + W_{3-1} = 125.86\,KJ = 125.86\,KJ$

Concluding Remarks ⇒ Net work in a cycle = area of closed region.

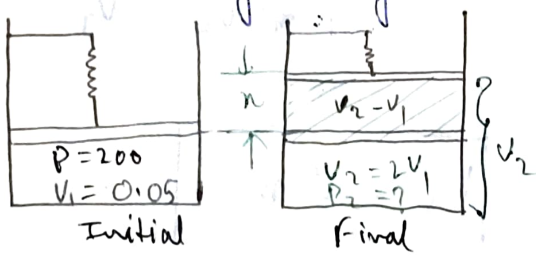
All clockwise cycle on P-V diagram are work producing cycles & all anticlockwise cycles are work absorbing cycles.

Q2 ⇒ A piston cylinder device contains $0.05\ m^3$ of a gas initially at 200 KPa at this state a linear spring which has a spring const. of $150\ KN/m^2$ is just touching the piston but exerting no force on it. Heat is transferred to the gas causing the piston to rise & to compress the spring until the vol. inside the cylinder doubles if the X-sec. area of piston is $0.25\ m^2$ find–

(i) Final pressure inside cylinder

(ii) Work done by the gas.

Sol^n ⇒



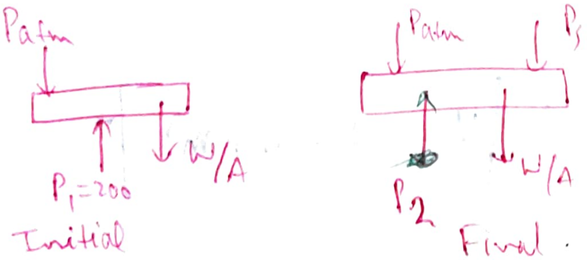
$V_2 - V_1 = Ax$

$2V_1 - V_1 = Ax$

$V_1 = Ax$

$x = \frac{V_1}{A} \Rightarrow \frac{.05}{.25}$

$x = .2\ m$



From initial FBD

$P_{atm} + \frac{W}{A} = 200$

From Final FBD

$P_{atm} + P_s + \frac{W}{A} = P_2 \Rightarrow P_{atm} + P_s + \frac{W}{A} = P_2$

$P_s + 200 = P_2$

$P_s = \frac{F_s}{A} = \frac{K \times x}{A} = 120\ KPa$

$\Rightarrow P_2 = 320\ KPa$

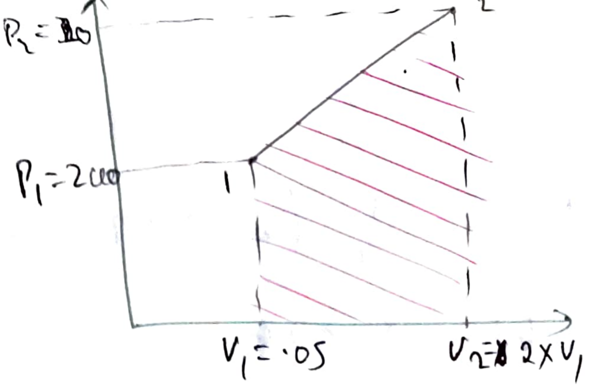
$P_2 = 200 + P_s$

$P_s = \frac{F_s}{A} = \frac{Kx}{A}$

$V_2 - V_1 = Ax$

$\Delta V = Ax$

$x = \frac{\Delta V}{A}$



$P_s = \frac{K}{A} \frac{\Delta V}{A}$

$P_2 = 200 + \frac{K \Delta V}{A^2}$ K, A = const ⇒ Linear.

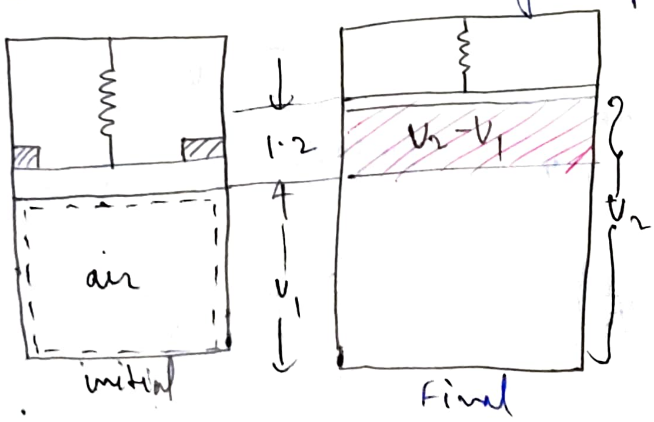
$W = \frac{1}{2}(200 + 320)(0.1 - .05)$

$W = 13 KJ$

★★★★★

Q ⇒ A closed cylinder of 0.25 m dia fitted with a light frictionless piston. The piston is retained in posn by stops in cylinder wall. The vol. on one side of piston contains air at a pressure of 750 KN/m². The vol. on the other side of the piston is evacuated. A spring is mounted in this evacuated space to give a force of 120 N on piston in this posn. The stops are removed & the piston travels along the cylinder untill it comes to rest after a stroke of 1.2 meter. The piston is then held in this posn. The spring force increases linearly to final value of 5 KN. Calculate the work done by compressed air on piston.

Soln ⇒



$P_1 = \frac{F_1}{A} = \frac{120 N}{\frac{\pi}{4}(.25^2)}$

$= 2444.6 N/m^2$

$= 2.444 KN/m^2$

$P_2 = \frac{F_2}{A_2} = \frac{5 KN}{\frac{\pi}{4} .25^2}$

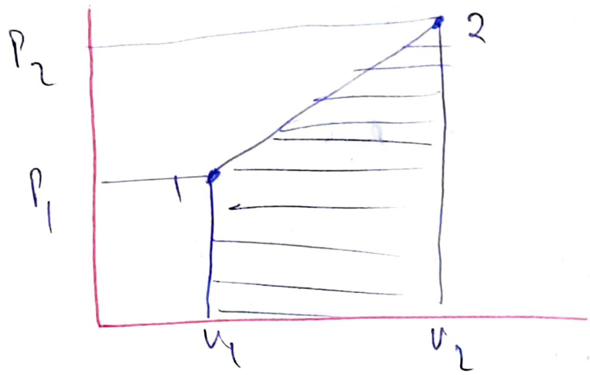
$P_2 = 101.85 KN.$

$V_2 - V_1 = A \times h$

$V_2 - V_1 = \frac{\pi}{4} \times (.25)^2 \times 1.2 \Rightarrow V_2 - V_1 = 0.0589 m^3$

$W = \frac{1}{2}(2.444 + 101.85) \times 0.0589$

$W = 3.07 K$

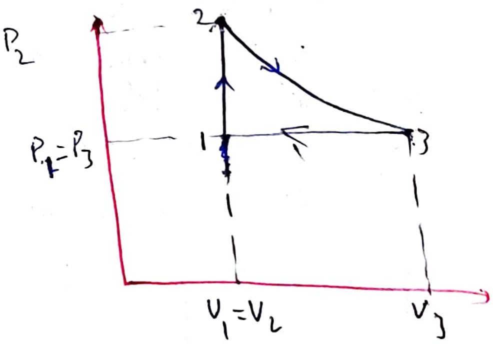


Q ⇒ An ideal gas is heated at const. vol. untill its temp. is three times the original temperature it is then expanded isothermaly ~~&~~ till it reaches original pressure the gas is than cooled at const pressure till it is restored to the original stated. Determine the net work done by the gas / Kg. the initial temp is 350 K. Express your ans in terms of gas constant R.

Ans ⇒ 453.5R

Sol^n ⇒ $T_1 = 350 K$

$T_3 = T_2 = 3T_1 = 1050 K$



WD for 31 $= P_1(V_3 - V_2)$

$PV = mRT$

$PV = RT$

$P_1V_1 = RT_1 = ~~P_1V_1~~\ 350R$

$P_2V_2 = RT_2 = 1050R \Rightarrow V_2 = \frac{1050R}{P_2}$

$P_2V_2 = mRT_2$

$P_3V_3 = RT_3 = 1050R \Rightarrow V_3 = \frac{1050R}{P_3}$

$\Rightarrow \frac{P_3}{P_2} = \frac{V_2}{V_3}$

WD for 23 $= P_2V_2 \ln\left(\frac{V_3}{V_2}\right) = P_2V_2 \ln\left(\frac{P_2}{P_3}\right) = 1153.54R$.

$= ~~1050R~~$

WD 13 $= P_3(V_3 - V_2) = P_3\, 1050R\left(\frac{1}{P_3} - \frac{1}{P_2}\right) = 1050R\left(1 - \frac{P_3}{P_2}\right)$

$= 700R$

$W_{net} = 1153.54R - 700R$

$= 453.54R.$