```
Function: (x*cos(y) - cos(y))*exp(x)
Cauchy-Riemann Equations: du/dx = dv/dy -> x*exp(x)*cos(y) , du/dy = -dv/d
x -> (1 - x)*exp(x)*sin(y)
The function is not analytic.

Function: (x + I*y)**3
Cauchy-Riemann Equations: du/dx = dv/dy -> 0 , du/dy = -dv/dx -> 0
The function is analytic.

Function: exp(x + I*y)
Cauchy-Riemann Equations: du/dx = dv/dy -> 0 , du/dy = -dv/dx -> 0
The function is analytic.

Function: (x + I*y)**2
Cauchy-Riemann Equations: du/dx = dv/dy -> 0 , du/dy = -dv/dx -> 0
The function is analytic.

Function: -3*x - 3*I*y + (x + I*y)**2
Cauchy-Riemann Equations: du/dx = dv/dy -> 0 , du/dy = -dv/dx -> 0
The function is analytic.

Function: x - I*y
Cauchy-Riemann Equations: du/dx = dv/dy -> 2 , du/dy = -dv/dx -> 0
The function is not analytic.
```

Check if function is harmonic

In [2]:

```python
from sympy import *
init_printing(use_latex=True)
x,y=symbols('x,y')
def harmonic(q):
    print("The function no. is:",q)
    dqdx=diff(q,x,2)
    dqdy=diff(q,y,2)
    if dqdx+dqdy==0:
        print("The function is harmonic")
    else:
        print("The function is not harmonic")
```

In [3]:

```python
x,y=symbols('x,y')
harmonic(x**3-3*x*y**2)
```

```
The function no. is: x**3 - 3*x*y**2
The function is harmonic
```

In [4]:

```python
harmonic(x**2-y**2+x+1)
```

```
The function no. is: x**2 + x - y**2 + 1
The function is harmonic
```

In [5]:

```python
harmonic(x**2*y-y**2*x)
```

```
The function no. is: x**2*y - x*y**2
The function is not harmonic
```

Check whether real and imaginary parts of x^3-y^3+x+1+i(x^3-3xy^2+2x) is Harmonic

In [7]:
```
harmonic(x**3-y**3+x+1)
harmonic(x**3-3*x*y**2+2*x)
```

```
The function no. is: x**3 + x - y**3 + 1
The function is not harmonic
The function no. is: x**3 - 3*x*y**2 + 2*x
The function is harmonic
```

To check if a function 'v' is the harmonic conjugate of the harmonic function 'u'

To check whether the given function u and v are harmonic function

In [8]:
```
import sympy as sp


x, y = sp.symbols('x y')


u = x**3 - y**3 + x + 1  # Real part
v = x**3 - 3*x*y**2 + 2*x  # Imaginary part

laplace_u = sp.diff(u, x, 2) + sp.diff(u, y, 2)
laplace_v = sp.diff(v, x, 2) + sp.diff(v, y, 2)


print("∇²u =", laplace_u.simplify())
print("∇²v =", laplace_v.simplify())

if laplace_u.simplify() == 0 and laplace_v.simplify() == 0:
    print("Both u and v are harmonic functions.")
else:
    print("At least one of u or v is not harmonic.")

```

```
∇²u = 6*x - 6*y
∇²v = 0
At least one of u or v is not harmonic.
```

check whether cos(x)sinh(y) is a harmonic conjugate of sin(x)cosh(y)

In [11]:

```python
import sympy as sp


x, y = sp.symbols('x y')


u = sp.sin(x) * sp.cosh(y)
v = sp.cos(x) * sp.sinh(y)

du_dx = sp.diff(u, x)
du_dy = sp.diff(u, y)

dv_dx = sp.diff(v, x)
dv_dy = sp.diff(v, y)


print("∂u/∂x =", du_dx)
print("∂u/∂y =", du_dy)
print("∂v/∂x =", dv_dx)
print("∂v/∂y =", dv_dy)


cr_eq1 = sp.simplify(du_dx - dv_dy)  # Should be 0
cr_eq2 = sp.simplify(du_dy + dv_dx)  # Should be 0

print("\nChecking Cauchy-Riemann Equations:")
print("∂u/∂x = ∂v/∂y →", cr_eq1 == 0)
print("∂u/∂y = -∂v/∂x →", cr_eq2 == 0)

if cr_eq1 == 0 and cr_eq2 == 0:
    print("\n✅ v(x, y) is a harmonic conjugate of u(x, y).")
else:
    print("\n❌ v(x, y) is NOT a harmonic conjugate of u(x, y).")

```

```
∂u/∂x = cos(x)*cosh(y)
∂u/∂y = sin(x)*sinh(y)
∂v/∂x = -sin(x)*sinh(y)
∂v/∂y = cos(x)*cosh(y)

Checking Cauchy-Riemann Equations:
∂u/∂x = ∂v/∂y → True
∂u/∂y = -∂v/∂x → True

✅ v(x, y) is a harmonic conjugate of u(x, y).
```

In [13]:

```python
import sympy as sp

def verify(u, v):
    """Checks whether both u and v are harmonic functions."""
    # Define variables
    x, y = sp.symbols('x y')

    # Compute Laplacian (∇²u and ∇²v)
    laplace_u = sp.diff(u, x, 2) + sp.diff(u, y, 2)
    laplace_v = sp.diff(v, x, 2) + sp.diff(v, y, 2)

    # Simplify results
    laplace_u_simplified = laplace_u.simplify()
    laplace_v_simplified = laplace_v.simplify()

    # Print results
    print("∇²u =", laplace_u_simplified)
    print("∇²v =", laplace_v_simplified)

    # Check if both are zero
    if laplace_u_simplified == 0 and laplace_v_simplified == 0:
        print("✅ Both u and v are harmonic functions.")
    else:
        print("❌ At least one of u or v is not harmonic.")

# Define u and v
x, y = sp.symbols('x y')
u = x**3 - y**3 + x + 1  # Real part
v = x**3 - 3*x*y**2 + 2*x  # Imaginary part

# Call the function
verify(u, v)
```

```
∇²u = 6*x - 6*y
∇²v = 0
❌ At least one of u or v is not harmonic.
```

In [14]:

```python
verify(x**2-y**2+x+1,-x**2/2+y**2/2)
```

```
∇²u = 0
∇²v = 0
✅ Both u and v are harmonic functions.
```

In [16]:

```python
verify(x*y,-x**2/2+y**2/2)
```

```
∇²u = 0
∇²v = 0
✅ Both u and v are harmonic functions.
```

In [18]:

```python
verify(x**2-y**2,2*x*y+3)
```

```
∇²u = 0
∇²v = 0
✅ Both u and v are harmonic functions.
```

To find the harmonic conjugate v of a harmonic function

In [ ]:
```
1
```