機器學習概論 作業 11

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P1

Mean Square Error 不保證大於或等於 Relative Squared Error。因為 Relative Squared Error 可以被轉換成下列型式：

$$\frac{(p_1-a_1)^2+\cdots+(p_n-a_n)^2}{(a_1-\bar{a})^2+\cdots+(a_n-\bar{a})^2}=\frac{(p_1-a_1)^2+\cdots+(p_n-a_n)^2}{n*\delta^2}$$

其中 δ 為資料集 $\{a_1,\ldots,a_n\}$ 的標準差。如果 $\delta<1$，則 Relative Squared Error 會大於 Mean Square Error。

P2

Bernolli trial：p
實驗數量：N
信心水準：α
$z=z_{\alpha/2}$

$$p=\frac{f+\frac{z^2}{N}\pm z\sqrt{\frac{f}{N}-\frac{f^2}{N}+\frac{z^2}{4N^2}}}{1+\frac{z^2}{N}}$$

假設使用一個二分類分類器為300個測試案例分類，其中180個分類正確

$N = 300, f = 0.6$

- $1 - \alpha = 0.6$

$$\begin{gathered}\alpha = 0.4 \\ z = z_{\alpha/2} = 0.842 \\ p = \frac{0.6 + \frac{0.842^2}{300} \pm 0.842\sqrt{\frac{0.6}{300} - \frac{0.6^2}{300} + \frac{0.842^2}{4 \cdot 300^2}}}{1 + \frac{0.842^2}{300}} = 0.6009 \pm 0.024\end{gathered}$$

- $1 - \alpha = 0.8$

$$\begin{gathered}\alpha = 0.2 \\ z = z_{\alpha/2} = 1.282 \\ p = \frac{0.6 + \frac{1.282^2}{300} \pm 1.282\sqrt{\frac{0.6}{300} - \frac{0.6^2}{300} + \frac{1.282^2}{4 \cdot 300^2}}}{1 + \frac{1.282^2}{300}} = 0.602 \pm 0.036\end{gathered}$$

假設使用一個二分類分類器為300個測試案例分類，其中170個分類正確

$N = 300, f \approx 0.567$

- $1 - \alpha = 0.6$

$$\begin{gathered}\alpha = 0.4 \\ z = z_{\alpha/2} = 0.842 \\ p = \frac{0.567 + \frac{0.842^2}{300} \pm 0.842\sqrt{\frac{0.567}{300} - \frac{0.567^2}{300} + \frac{0.842^2}{4 \cdot 300^2}}}{1 + \frac{0.842^2}{300}} = 0.568 \pm 0.024\end{gathered}$$

- $1 - \alpha = 0.8$

$$\alpha = 0.2$$
$$z = z_{\alpha/2} = 1.282$$
$$p = \frac{0.567 + \frac{1.282^2}{300} \pm 1.282\sqrt{\frac{0.567}{300} - \frac{0.567^2}{300} + \frac{1.282^2}{4 \cdot 300^2}}}{1 + \frac{1.282^2}{300}} = 0.569 \pm 0.037$$

假設使用甲乙兩個二分類分類器為1000個測試案例分類

$N = 1000, f_{甲} = 0.5, 1 - \alpha = 0.1, z = z_{\alpha/2} = 1.645$

$$p_{甲} < p_{乙}$$
$$\frac{f_{甲} + \frac{z^2}{N} \pm z\sqrt{\frac{f_{甲}}{N} - \frac{f_{甲}^2}{N} + \frac{z^2}{4N^2}}}{1 + \frac{z^2}{N}} < \frac{f_{乙} + \frac{z^2}{N} \pm z\sqrt{\frac{f_{乙}}{N} - \frac{f_{乙}^2}{N} + \frac{z^2}{4N^2}}}{1 + \frac{z^2}{N}}$$
$$f_{甲} + \frac{z^2}{N} + z\sqrt{\frac{f_{甲}}{N} - \frac{f_{甲}^2}{N} + \frac{z^2}{4N^2}} < f_{乙} + \frac{z^2}{N} - z\sqrt{\frac{f_{乙}}{N} - \frac{f_{乙}^2}{N} + \frac{z^2}{4N^2}}$$
$$f_{甲} + z\sqrt{\frac{f_{甲}}{N} - \frac{f_{甲}^2}{N} + \frac{z^2}{4N^2}} < f_{乙} - z\sqrt{\frac{f_{乙}}{N} - \frac{f_{乙}^2}{N} + \frac{z^2}{4N^2}}$$
$$0.5 + 1.645\sqrt{\frac{0.5}{1000} - \frac{0.5^2}{1000} + \frac{1.645^2}{4 \cdot 1000^2}} < f_{乙} - 1.645\sqrt{\frac{f_{乙}}{1000} - \frac{f_{乙}^2}{1000} + \frac{1.645^2}{4 \cdot 1000^2}}$$
$$0.526 < f_{乙} - 1.645\sqrt{\frac{f_{乙}}{1000} - \frac{f_{乙}^2}{1000} + \frac{1.645^2}{4 \cdot 1000^2}}$$
$$f_{乙} > 0.5519$$

乙分類器至少要猜對 552 個案例才行