1. **A unit square is transformed by 2 x 2 transformation matrix. The resulting position vector are :-**

$$\begin{pmatrix} 0 & 2 & 8 & 6 \\ 0 & 3 & 4 & 1 \end{pmatrix}$$

what is the transformation matrix?

2. **Translate the square ABCD whose co-ordinate are A(0,0), b(3,0), C(3,3), D(0,3) by 2 units in both direction and then scale it by 1.5 units in x direction and 0.5 units in y direction.**

3. **Find the transformation matrix that transforms the square ABCD whose center is at (2,2) is reduced to half of its size, with center still remaining at (2,2). The coordinate of square ABCD are A(0,0), B(0,4), C(4,4) and D(4,0). Find the co-ordinate of new square.**

(HINT:- After scaling the square to half of its size, the new translated square will have center at (1,1) so, translate again the new square by (1,1), so that center again reach to (2,2).)

Ans: Suppose the unit square have coordinates

(x , y)
(x+1, y)
(x+1, y+1)
(x , y+1)

and let the transformation matrix be $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

So, $$\begin{pmatrix} \mathbf{0} & \mathbf{2} & \mathbf{8} & \mathbf{6} \\ \mathbf{0} & \mathbf{3} & \mathbf{4} & \mathbf{1} \end{pmatrix} = \begin{pmatrix} \mathbf{a} & \mathbf{c} \\ \mathbf{b} & \mathbf{d} \end{pmatrix} \begin{pmatrix} \mathbf{x} & \mathbf{x+1} & \mathbf{x+1} & \mathbf{x} \\ \mathbf{y} & \mathbf{y} & \mathbf{y+1} & \mathbf{y+1} \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{ax+cy} & \mathbf{a(x+1)+cy} & \mathbf{a(x+1)+c(y+1)} & \mathbf{ax+c(y+1)} \\ \mathbf{bx+by} & \mathbf{b(x+1)+dy} & \mathbf{b(x+1)+d(y+1)} & \mathbf{bx+d(y+1)} \end{pmatrix}$$

Now, $ax+cy=0$ and $bx+cy=0$

$a(x+1)+cy=2$ and $b(x+1)+dy=3$

$a(x+1)+c(y+1)= 8$ and $b(x+1)+d(y+1) = 4$

$ax+c(y+1) = 6$ and $bx+d(y+1)=1$

from this we get,

$a=2$, $b=3$, $c=6$, $d=1$

Thus, the transformation matrix is $\begin{pmatrix} \mathbf{2} & \mathbf{6} \\ \mathbf{3} & \mathbf{1} \end{pmatrix}$