Aliasing — Jaggies

Ladder (Aliasing effect)

Scan Conversion → (Low frequency Sampling)

Antialiasing — 6

Smoothing effect → Not Happen (lines)

1) Super Sampling / Post filtering

2) Area Sampling / Pre filtering

Area define

finer grid assumption

Intensity Level →

Average Intensity

(High Resolution Sampling → Low Resolution view)

More pixel image 90% (Intensity)

18%

Less image (Intensity)

Scan Converting a Circle —

Circle — Set of points that all are at a same distance from a common point

Center of circle; radius

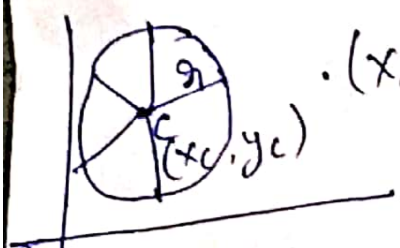
Polynomial method / Direct method

Trigonometric

Incremental

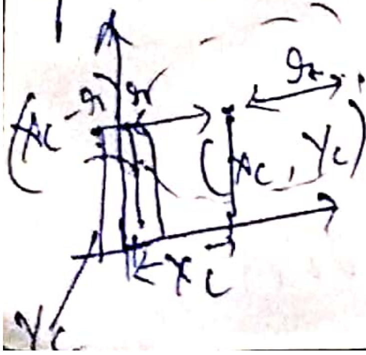
Mid point Circle drawing

Bresenham Circle drawing



$$(x-x_c)^2+(y-y_c)^2=r^2$$

2 way Symmetry



$$(y-y_c)^2=r^2-(x-x_c)^2$$

$$y-y_c=\pm\sqrt{r^2-(x-x_c)^2}$$

$$\boxed{y=y_c\pm\sqrt{r^2-(x-x_c)^2}}$$

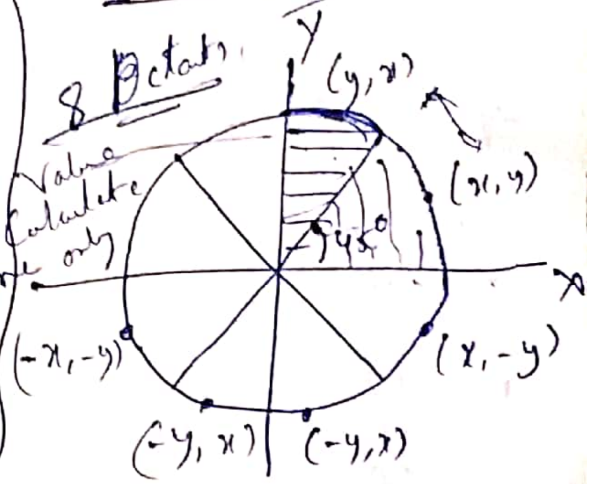
Polynomial —

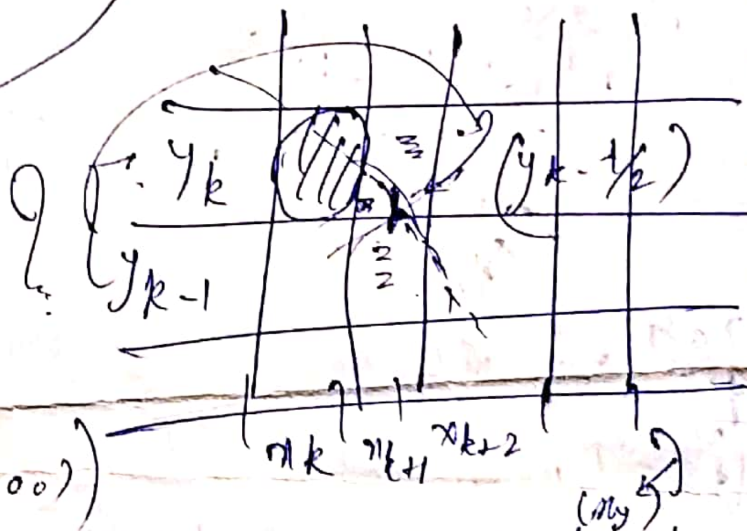
1. I/P Center (x_c, y_c) & radius r
2. Set initial values $x = x_c - r$, $y = y_c$
3. Plot pixel (x, y)
4. while $(x \le x_c + r)$
   increment $x = x + 1$
   Compute $y = y_c \pm \sqrt{r^2 - (x - x_c)^2}$
   round off values of y
   Plot pixel (x, y)
   End while
5. Stop

Mid point Circle Algorithm —

8 Octets

for one only Value Calculate





$x^2 + y^2 = r^2$ — Circle Equation (Center (0,0)) — radius

$f_{circle}(x, y) = x^2 + y^2 - r^2$

$f_{circle}(x,y) = \begin{cases} < 0 & \text{if } (x,y) \text{ inside the circle boundary} \\ = 0 & \text{if on circle boundary} \\ > 0 & \text{outside the circle boundary} \end{cases}$

$P_k = f_{circle}(x_k + 1, y_k - \tfrac{1}{2})$

Decision parameter $P_k = (x_k + 1)^2 + (y_k - \tfrac{1}{2})^2 - r^2$ —— (2)

If $P_k < 0$ midpoint is inside the circle. then next $y = y_k$

otherwise ($P_k > 0$) $y = y_k - 1$

From (2) $x_{k+1} = x_{k+1} + 1$ and $y_k = y_{k+1}$

$P_{k+1} \overset{f_{circle}}{=} (x_{k+1} + 1, y_{k+1} - 1/2)$

$= [(x_k + 1) + 1]^2 + (y_{k+1} - 1/2)^2 - r^2$

(8)

Now $P_{k+1} - P_k$

$= (x_k + 1)^2 + 1^2 + 2(x_k + 1) \cdot 1 + y_{k+1}^2 + 1/4 - 2(1/2)(y_{k+1})$

$- r^2 - [(x_k + 1)^2 + (y_k - 1/2)^2 - r^2]$

$= \cancel{(x_k + 1)^2 + 1 - 2(x_k + 1) + y_{k+1}^2 + 1/4 - y_{k+1}}$

$= \underbrace{y_{k+1}^2 - y_k^2 - y_{k+1} + y_k + 2(x_k + 1) + 1}$

$P_{k+1} = P_k + \underbrace{\quad}$

$P_k < 0 \quad y_{k+1} = y_k$

$P_k > 0 \quad y_{k+1} = y_{k-1}$

Case I - $P_k < 0$

$P_{k+1} = P_k + \cancel{y_k^2} - \cancel{y_k^2} - \cancel{y_k} + \cancel{y_k} + 2(x_k + 1) + 1$

$P_{k+1} = P_k + 2(x_k + 1) + 1 \longrightarrow$ Plotting pts (x_{k+1}, y_k)

Case II - $P_k > 0$

$P_{k+1} = P_k + (y_{k-1})^2 - y_k^2 - (y_{k-1}) + y_k + 2(x_k + 1) + 1$

$= P_k + 2x_{k+1} + 1 - 2y_{k+1}$

$P_0 = f_{circle}(1, r - 1/2)$ $\quad (x_0, y_0) = (0, r)$

(Initial Decision Parameter) $\quad f(x_{k+1}, y_k - \tfrac{1}{2})$

$$P_k = (0+1) + (r - 1/2)^2 - r^2$$

$$= 1 + r^2 + 1/4 - r - \cancel{r^2}$$

(9)

$$\boxed{P_0 = 5/4 - r}$$

If $r \to$ integer, $5/4 \to 1$

$$\boxed{P_0 = 1 - r}$$

Algo – (1) I/P radius r and circle center (x_c, y_c). Obtain the first point on the circumference of a circle centered on origin as $(x_0, y_0) = (0, r)$

(2) Calculate initial decision parameter $P_0 = 5/4 - r$ or $1 - r$

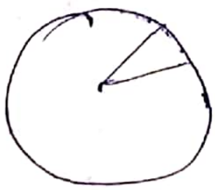
(3) At each x_k position starting $k=0$ perform the following test
if $P_k < 0$ then next plotting point (x_{k+1}, y_k)
& $P_{k+1} = P_k + 2x_{k+1} + 1$
else next point along the circle is $(x_k + 1, y_{k-1})$
& $P_{k+1} = P_k + 2x_{k+1} + 1 - 2y_{k+1}$

(4) Determine symmetry points in other seven octets

(5) Move each point to given center by
$x = x + x_c$
$y = y + y_c$

(6) Repeat 3 – 5 until $x \geq y$.

Mid point Circle Drawing —



(10)

Example – Given = $r = 10$

Center = $(0, 0)$

(x_c, y_c) $(0, r)$ $(0, 10)$

K	P_K	Next point x_{K+1}, y_{K+1}
0	−9	(1, 10)
1	−6	(2, 10)
2	−1	(3, 10)
3	6	(4, 9)
4	−3	(5, 9)
5	8	(6, 8)
6	5	(7, 7)

$x \geq y$

First Octant

$P_0 = 1 - r$
$= 1 - 10$
$P_0 = -9$

Case I —
$P_K < 0$
Next (x_{K+1}, y_K)
$P_{K+1} = P_K + 2x_{K+1} + 1$

$P_K > 0$
Next $(x_{K+1}, y_K - 1)$
$P_{K+1} = P_K + 2x_{K+1} + 1 - 2y_{K+1}$

$P_{K+1} = P_K + 2x_{K+1} + 1$
$= -9 + 2(1) + 1$
$= -6$

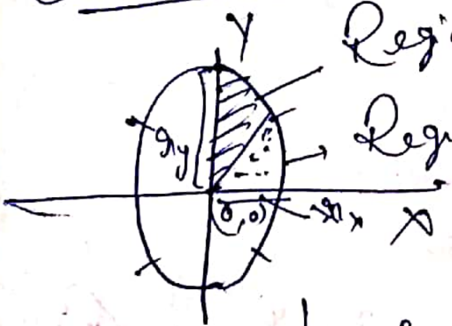
$P_{K+1} = -6 + 2(2) + 1$
$= -1$

$P_{K+1} = -1 + 2(3) + 1$
$= 6$

$P_{K+1} = 6 + 2(4) + 1 - 2(9)$
$= -3$

Mid point ellipse drawing Algo

Derivation –



Region 1 (x direction sampling) P_{K_1}

Region 2 (y direction) P_{K_2}

P_{K_1}

$$\frac{x^2}{r_x^2} + \frac{y^2}{r_y^2} = 1 \quad \text{Equation of Ellipse}$$

$$x^2 r_y^2 + y^2 r_x^2 = r_x^2 r_y^2$$

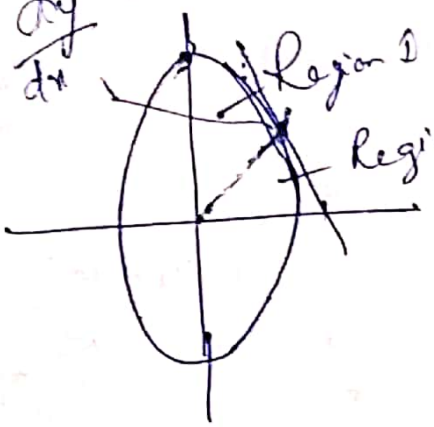
$$f(x, y) = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2 \quad \text{—— (1)}$$

$f(x,y) < 0$ (x,y) Inside the boundary

$= 0$ On the boundary

> 0 Outside of "

$\frac{dy}{dx} = -1$



when region 1 Connecting to Reg. 2 we calculate Slope

$$f(x,y) = r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2$$

11

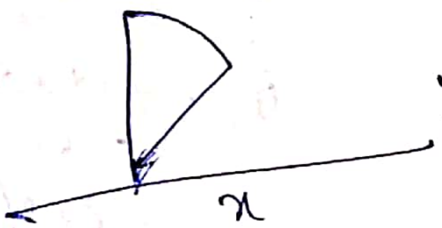
$$\frac{dy}{dx} = -\frac{2r_y^2 x}{2r_x^2 y}$$

$$-1 = -\frac{2r_y^2 x}{2r_x^2 y}$$

$$2r_y^2 x = 2r_x^2 y$$

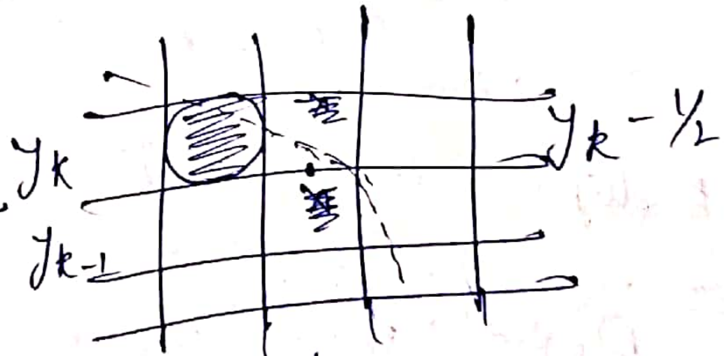
$$\boxed{2r_y^2 x \geq 2r_x^2 y}$$ from Reg. 1 to 2

P_{k_1} for Region 1 —



$x = x+1$

$y = ?$



$$P_{k_1} = f\left(x_k+1,\ y_k - \tfrac{1}{2}\right)$$

$$P_{k_1} = f(x,y) = r_y^2 x^2 + r_x^2 r_y^2 - r_x^2 r_y^2$$

$$= r_y^2 (x_k+1)^2 + r_x^2\left(y_k - \tfrac{1}{2}\right)^2 - r_x^2 r_y^2 \quad \text{—(2)}$$

Now —

$$P_{k_1+1} = f(x_k+1+1,\ y_{k+1} - \tfrac{1}{2})$$

$$= r_y^2\left[((x_k+1)+1)\right]^2 + r_x^2\left[(y_{k+1} - \tfrac{1}{2})\right]^2 - r_x^2 r_y^2$$

$$P_{k_1+1} = r_y^2\left[(x_k+1)^2 + 1^2 + 2\times(x_k+1)\cdot 1\right] + r_x^2\left[y_{k+1}^2 + \tfrac{1}{4} - 2\times\tfrac{1}{2}y_{k+1}\right] - r_x^2 r_y^2 \quad (12)$$

$$P_{k_1+1} - P_{k_1} = r_y^2 + 2r_y^2(x_k+1) + r_x^2\left[(y_{k+1} - \tfrac{1}{2})^2 - (y_k - \tfrac{1}{2})^2\right] \quad (4)$$

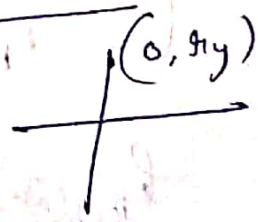
$$P_{k_1+1} = P_{k_1} + r_y^2 + 2r_y^2 \quad (5)$$

If (1) $P_{k_1} < 0$ $\quad y_{k+1} = y_k$

(2) $P_{k_1} > 0$ $\quad y_{k+1} = y_k - 1$

(1) → $P_{k_1+1} = P_{k_1} + r_y^2 + 2r_y^2(x_k+1)$ — (6)

(2) — $P_{k_1+1} = P_{k_1} + r_y^2 + 2r_y^2(x_k+1) - 2r_x^2 y_{k+1}$



$$P_{k_1}(0) = f(1,\ r_y - \tfrac{1}{2})$$

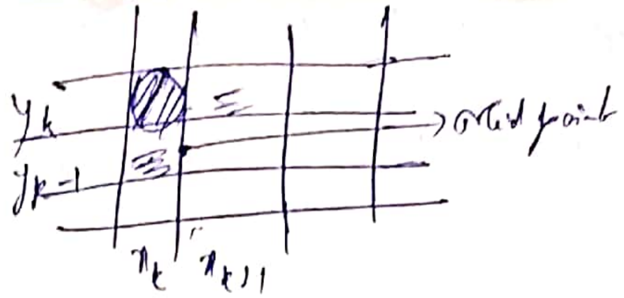
$x_k+1 \quad (y - \tfrac{1}{2})$

$$= r_y^2 \cdot x_k^2 + r_x^2 y_k^2 - r_x^2 r_y^2$$

$$= r_y^2 \cdot 1^2 + r_x^2(r_y - \tfrac{1}{2})^2 - r_x^2 r_y^2$$

$$\boxed{P_{k(0)} = r_y^2 + \tfrac{1}{4} r_x^2 - r_x^2 r_y}$$ — Initial Decision parameter for Region 1

Region 2 —



(13)

$$P_{k_2} = f(x_k + 1/2, y_k - 1)$$

$$P_{k_2} = r_y^2(x_k + 1/2)^2 + r_x^2(y_k - 1)^2 - r_x^2 r_y^2$$

$$P_{k_2+1} = f(x_{k+1} + 1/2, y_{k+1} - 1)$$
$$= r_y^2(x_{k+1} + 1/2)^2 + r_x^2((y_k - 1) - 1)^2 - r_x^2 r_y^2$$

$P_{k_2+1} - P_{k_2}$

$$P_{k_2+1} = P_{k_2} - 2r_x^2(y_k - 1) + r_x^2 + r_y^2[(x_{k+1} + 1/2)^2 - (x_k + 1/2)^2 \cdot 2]$$

$P_{k_2} < 0 \quad x_{k+1} = x_k$

$P_{k_2} > 0 \quad x_{k+1} = x_k + 1$

$$P_{k_2+1} = P_{k_2} - 2r_x^2 y_{k+1} + x_k^2$$

$$P_{k_2+1} = P_{k_2} + 2r_y^2 x_{k+1} - 2r_x^2 y_{k+1} + r_x^2$$

$$P_{k_2+1} = P_{k_2}$$

$$P_{k_2k(0)} = f(x_0 + 1/2, y_0 - 1)$$
$$= r_y^2(x_0 + 1/2)^2 + r_x^2(y_0 - 1)^2 - r_x^2 r_y^2$$

$r_x = 8$ $r_y = 6$ start $(0, r_y)$ $(0, 6)$

Reg 1

	(x_k, y_k)	Decision par. P_1 or P_2	(x_{k+1}, y_{k+1})	dx $2x_{k+1} r_y^2$	dy $2y_{k+1} r_x^2$
0	$(0,6)$	$p_1(0) = -332$	$(1, 6)$	$2(1)(6)^2 = 72$	$2(6)(8)^2 = 768$
1	$(1,6)$	$p_1(1) = -332 + 72 + (6)^2 = -224$	$(2,6)$	$4(36) = 144$	768
	$(2,6)$	$p_1(2) = -44$	$(3,6)$	$6(36) = 216$	768
	$(3,6)$	$p_1(3) = \underline{208}$	$(4,5)$	$8(36) = 288$	640
	$(4,5)$	$p_{1,4} = -108$	$(5,5)$	360	640
	$(5,5)$	$p_{1,5} = 288$	$(6,4)$	432	512
	$(6,4)$	$p_{1,6} = 244$	$(7,3)$	504	384

14

$dx > dy$

R2 — $\boxed{2r_y^2 x_{k+1} \geq 2r_x^2 y_{k+1}}$ $(8,2)$ 576 256

$(7,3)$ -23

Reg 1 —

$P_{1k} = r_y^2 + \frac{r_x^2}{4} - r_y r_x^2$

$P_{k+1} =$ $P_{1k} \geq 0$ (x_k+1, y_k-1)

< 0 (x_k+1, y_k)

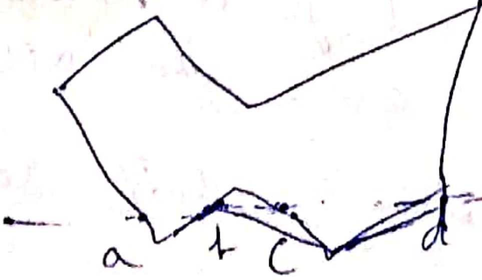
$P_{2k} \geq 0$ (x_k, y_k-1)

< 0 (x_k+1, y_k-1)

Polygon fill —

Many edges & vertices

Color such that for boundary



- For each scan line
- Locate the intersection of scan line with the edges
- Sort the intersection point from left to right.
- Draw the interior intersection points pairs $(a-b)$ $(c-d)$ pixels fill colour