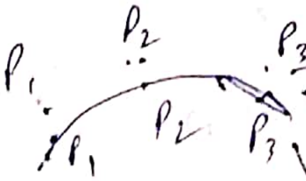
Bezier Curves — Smooth Curves (Spline) — 1. perspective



Control points → Near About → Approximation Curve (1)

→ Interpolation curve

(2) Parametric Curve

16
29
41
42
49
32
22

$u=0$ $u.25$ $u=.5$ $u=1$

P_0 P_1 $0 \le u \le 1$

(3) We can get n degree polynomial with $n+1$ Control point

$P(x) = a_0 x^n + a_1 x^{n-1} + \ldots$ Degree = 3

$\cancel{P_1(x)} = x^3 + x + 1$

3 Cont. pt. — x^2 poly. — Quad.

Cubic polynomial

(a) Easy to implement

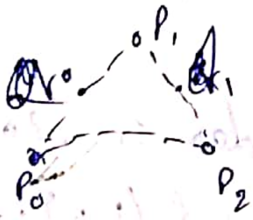
(b) CAD, Drawing etc.

Quadratic Bezier Curve

Parametric eq.

Let the parameter is u

$0 \le u \le 1$



Q_0 & Q_1 pts. are line between $P_0 \to P_1$ & $P_1 \to P_2$

$P_0 \to P_1$ $Q_0 = (1-u)P_0 + uP_1$ — (1)

$P_1 \to P_2$ $Q_1 = (1-u)P_1 + uP_2$ — (2)

Suppose $C(u)$ is the pt. betn $Q_0 \to Q_1$

$C(u) = (1-u)Q_0 + uQ_1$

$$C(u) = (1-u)[(1-u)P_0 + uP_1] + u[(1-u)P_1 + uP_2]$$

$$= (1-u)^2 P_0 + 2u(1-u)P_1 + u^2 P_2$$

Blending function specification for Bézier Curve

→ Combine Geometric Constraints

→ Position specify

Suppose we have n+1 control points positions $P_k(x_k, y_k, z_k)$ then the positional vector $P(u)$ where $0 \le u \le 1$ is given by —

$$P(u) = \sum_{k=0}^{n} P_k \underbrace{BEZ_{k,n}(u)}_{\text{Blending fn}} \qquad 0 \le u \le 1 \qquad (1)$$

$$BEZ_{k,n} = C(n,k)\, u^k (1-u)^{n-k}$$

where $C(n,k) = \dfrac{\lfloor n}{\lfloor k \lfloor n-k}$

$$x(u) = \sum_{k=0}^{n} x_k BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^{n} y_k \text{ —}$$

z_k —

(1) Construct the Bezier Curve of order 3 with four vertices of control polygon $P_0(0,0)$, $P_1(1,2)$; $P_2(3,2)$ and $P_3(2,0)$. Generate at least 5 points on the curve.

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$n = 3$ (Order)

$$\boxed{B(t) = \sum_{i=0}^{n} P_i B_{n,i}(t)} \qquad 0 \le t \le 1$$

$$B_{n,i}(t) = \binom{n}{i} t^i (1-t)^{n-i}$$

$i = 0, 1, 2, 3$

$^nC_r = \dfrac{\lfloor n}{\lfloor r \lfloor n-r}$

$$B_{3,0}(t) = \binom{3}{0} t^0 (1-t)^{3-0}$$

$$= \frac{\lfloor 3}{\lfloor 0 \lfloor 3-0} \times 1 (1-t)^3$$

$$= \frac{6}{6} \times (1-t)^3 = (1-t)^3$$

$n = 3,\ i = 1$

$$B_{3,1}(t) = \binom{3}{1} t^1 (1-t)^{3-1}$$

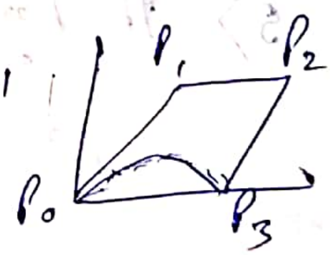
$$= 3t(1-t)^2$$

$$B_{3,2}(t) = 3t^2(1-t)$$

$$B_{3,3}(t) = t^3$$

$$B(t) = P_0 B_{3,0}(t) + P_1 B_{3,1}(t) + P_2 B_{3,2}(t) + P_3 B_{3,3}(t)$$

$$= P_0(1-t)^3 + P_1 3t(1-t)^2 + P_2 3t^2(1-t) + P_3 t^3$$



Taking $t = 0, 0.15, 0.35, 0.5, 0.65, 0.85, 1$

$B(0) = (0, 0) \rightarrow P_0$

$B(0.15) = P_0(\ \ - t = 0.15)$ $+ (0.325) P_1 + (0.058) P_2 + (0.003) P_3$

$= (0.614) P_0 + (0.325) P_1 + (0.058) P_2 + (0.003) P_3$

$= (0.50, 0.76)$

$B(0.35) = (1.24, 1.36)$

$B(0.5) = (1.75, 1.50)$

$B(0.65) = (2.12, 1.36)$

$B(0.85) = (2.14, 0.76)$

$B(1) = (2, 0)$

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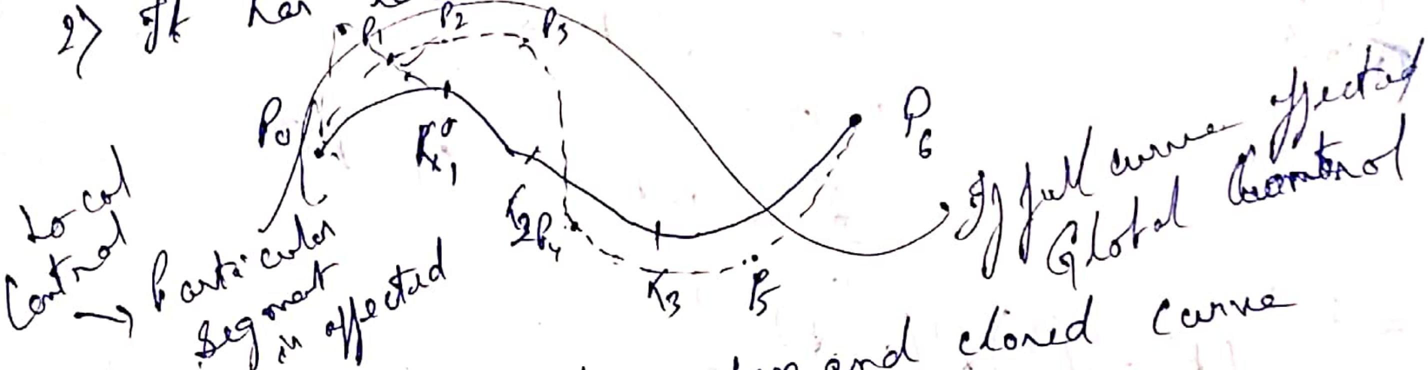
B - Spline Curve —

Control pt → Decide the degree → Quad.

Segment → Poly. degree control

1) It is made of n+1 Control points & order of curve (K).

2) It has local control over the curve.



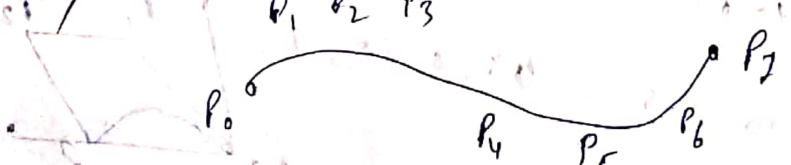
3) It is used to draw open and closed curve

4) It gives us polynomial of degree $K-1$

$K=3$ $\quad P(x) = x^2$

$$2 \leq K \leq n+1$$

5) It has $n-K+2$ segments.



$n=6$

Control pt. $n+1=7$

Let $K=3$ $\quad 6-3+2 = 5$

6) Cont. pt $=5$, $K=3$ $\quad u \to 0$ to 5

$P(u) = u^2$ $\quad$ 0-1, 1-2, 2-3, 3-4, 4-5

It allows us to change the no. of Control points without changing the degree of the polynomial.

$$P(u) = \sum_{i=0}^{n} B_{ik}(u) P_k$$

Positional vector

$P_k (x_k\ y_k\ z_k)$

K = Responsible for Segment

$$x(u) = \sum_{i=0}^{n} B_{ik}(u)\, x_k$$

$n = 6,\ k = 3$

$0 \le u \le n - k + 2$

$u = 0 \to 5$ $(n - k + 2)$

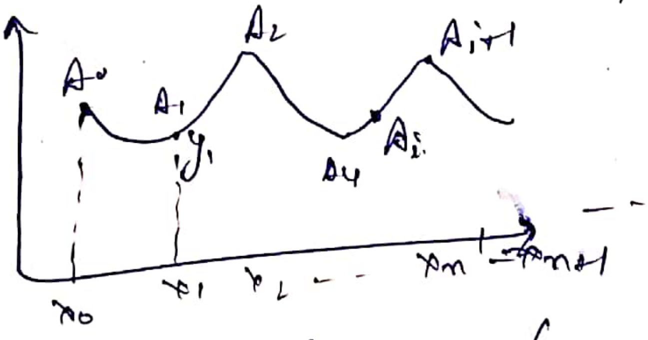
$2 \le k \le n + 1$

$$B_{ik}(u) = \frac{(u - t_i)\, B_{i,k-1}(u)}{t_{i+k-1} - t_i} + \frac{(t_{i+k} - u)\, B_{i+1,k-1}(u)}{t_{i+k} - t_{i+1}}$$

t = Knot vector (Set of Subinterval end pts)

$t_i \to (0 \le i \le n + k)$

Cubic Spline



More precise value b/w curve

$$f(x) = \frac{(x_{i+1} - x_i)^3 M_i}{6h} + \frac{(x - x_i)^3 M_{i+1}}{6h}$$