"Computer Graphics" Assignment

Ques 1. Given a set of lines and rectangular area of interest, the is to remove lines which are outside the area of interest lines are clip which are partially inside.

Cohen-Sutherland Algorithm divides a 2-D space into 9 regions and then efficiently determines the lines and portions of lines that are inside the given rectangular area.

The algo. can be outlined as follows -

for a given line extreme points (x, y), we can quickly find it's region's 4-bit code.

- If "x" is less than X_{min} than bit no. 1 is set
- If "x" is greater than X_{max} than bit no. 2 is set.
- If "y" is less than Y_{min} than bit no. 3 is set.
- If "y" is greater than Y_{max} than bit no. 4 is set.

Top

	1001	1000	1010
Left	0001	0000	0010	Right
	0101	0100	0110

Bottom

1. Completely inside the given rectangle -

Bitwise OR of region of two end points of line is
(both points are inside the rectangle) 0

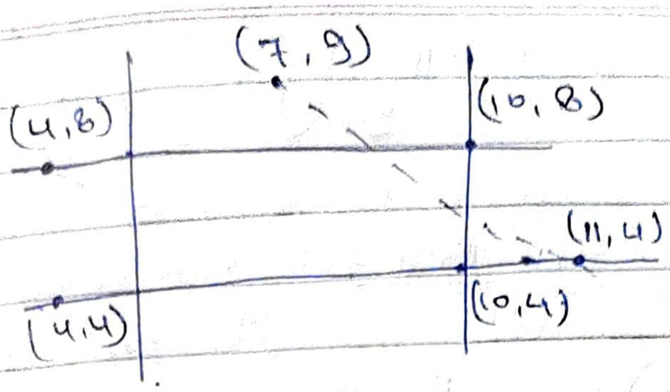
2. Completely outside the given rectangle -

Both end points share at least one outside region
which implies that the lines does not cross
the visible region (bitwise AND != 0) otherwise
the lines are partially inside the view window.

Let us take an example line - (7, 9) and (11, 4)
and the view window $(x_{min}, y_{min}) = (4,4)$
$(x_{max}, y_{max}) = (10, 8)$

Follow the below mentioned steps

1. Region code of point [7, 9] = 1000
Region code of point [11, 4] = 0010.



2. Both points are not completely in 0000 region code which shows the line is not completely inside.

3. Perform AND b/w both region code

$(7, 9) \rightarrow 1000$

$(11, 4) \rightarrow 0010$

0000

This represent line is partially inside.

4. find the intersection point of line with view window

eq^n of line $\rightarrow (y-4) = \left[\dfrac{9-4}{7-11}\right](x-11)$

$$y-4 = \frac{-5}{4}(x-11)$$

$$4y-16 = -5(x-11)$$

$$4y+5x = 55+16$$

$$4y+5x = 71$$

$$y = \frac{71 - 5x}{4} \quad , \quad x = \frac{71 - 4y}{5}$$

put $x = 10$ in $y = \frac{71 - 5x}{4}$.

$$y = \frac{71 - 5(10)}{4} = 5.25.$$

put $y = 8$ in $x = \frac{71 - 4y}{5}$

$$x = \frac{71 - 32}{5} = 7.8$$

Ans The accepted line is $= (7.8, 8)$ to $(10, 5.25)$

Ques General Terms →

- **World co-ordinate** → it is cartesian co-ordinate w.r.t which we define the diagram like X_{wmin}, X_{wmax}, Y_{wmin}, Y_{wmax}.

- **Device co-ordinate** → it is the cartesian coordinate where the object is to be displayed, like X_{vmin}, X_{vmax}, Y_{vmin}, Y_{vmax}

- **Window** → it is the area on world co-ordinate selected for display.

- **View port** → It is the area on device co-ordinate where graphics is to be displayed

(X_w, Y_w) : A point on window

(X_v, Y_v) : corresponding point on viewpoint.

* we have to calculate the point (x_v, Y_v)

Normalized point on window $\rightarrow \left[\dfrac{X_w - Xw_{min}}{Xw_{max} - Xw_{min}}, \dfrac{Y_w - Yw_{min}}{Yw_{max} - Yw_{min}}\right]$

Normalized point on viewpoint $\rightarrow \left[\dfrac{X_v - Xv_{min}}{Xv_{max} - Xv_{min}}, \dfrac{Y_v - Yv_{min}}{Yv_{max} - Yv_{min}}\right]$

for x- co-ordinate

$$\frac{X_w - Xw_{min}}{Xw_{max} - Xw_{min}} = \frac{X_v - Xv_{min}}{Xv_{max} - Xv_{min}}$$

for Y - co- ordinate

$$\frac{Y_w - Yw_{min}}{Yw_{max} - Yw_{min}} = \frac{Y_v - Yv_{min}}{Yv_{max} - Yv_{min}}$$

$$\left.\begin{aligned} &* \; X_v = Xv_{min} + (X_w - X_{w\,min}) S_x \\ &* \; Y_v = Yv_{min} + (Y_w - Yw_{min}) S_y \end{aligned}\right] - \text{final results}$$

Ques 3

DDA stands for digital Differential Analyzer. It is a incremental method of scan conversion of line. In this method calculation is perform at each step but by using results of previous steps.

Let us take an general point (x_0, y_0) and (x_1, y_1) are the initial point and final point which form a line lies between 45 to -45 slop

follow the points mentioned below →

- starting co-ordinate $= (x_0, y_0)$
- Ending co-ordinate $= (x_n, y_n)$

1. Calculate Δx, Δy and m (slop) from the given input.

These parameters are calculate are -

$$\Delta x = x_1 - x_0$$
$$\Delta y = y_1 - y_0$$

m(slop) as given in the ques.

$$\tan(-45°) < m < \tan(45°)$$
$$-1 < m < 1$$

2. find the no. of steps or points in b/w the starting and ending co-ordinate.

if (absolute (Δx)) > absolute (Δy))
steps = absolute (Δx);

else
steps = absolute (Δy);

3. Suppose the current point is (x_p, y_p) and the next point is (x_{p+1}, y_{p+1})

Case I → if $m < 1$

$$x_{p+1} = \text{round off } (1 + x_p)$$

$$y_{p+1} = \text{round off } (m + y_p)$$

Case II → if $m = 1$

$$x_{p+1} = \text{round off } (1 + x_p)$$

$$y_{p+1} = \text{round off } (1 + y_p)$$

Case III → if $m > 1$

$$x_{p+1} = \text{round off } (1/m + x_p)$$

$$y_{p+1} = \text{round off } (1 + y_p)$$

4. Keep repeating step-3 until the end point is reached or the no. of generated new points (including start and end points) equal to step count.

Ques 4 8

Transformation means changing some graphics into something else by applying rules. We can have various types of transformation such as translation, scaling up or down, rotation, shearing etc. When transformation take place on a 2D-plane, it is called 2D-Transformation.

Homogeneous co-ordinates

To perform a seq. of transformation such as translation followed by rotation and scaling, we need to follow a seq. process.
- Translate the co-ordinates
- Rotate the translated co-ordinate and than
- scale the rotated co-ordinates to complete the composite transformation.

Any cartesian point $P(x, y)$ can be converted to homogenals co-ordinate by $P'(X_h, Y_h, h)$

Translation

$$X' = (x) + t_x$$
$$Y' = (y) + t_y$$

where pair (t_x, t_y) is called translation or shift vector.

Rotation

The rotation of the following / giving point about the origin represented as

$$[x' \; y'] = [x \; y] \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

OR

$$[P' = P \cdot R]$$

Rotational matrix $R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$$x' = x\cos\theta - y\sin\theta$$

$$y' = x\sin\theta + y\cos\theta$$

Scaling

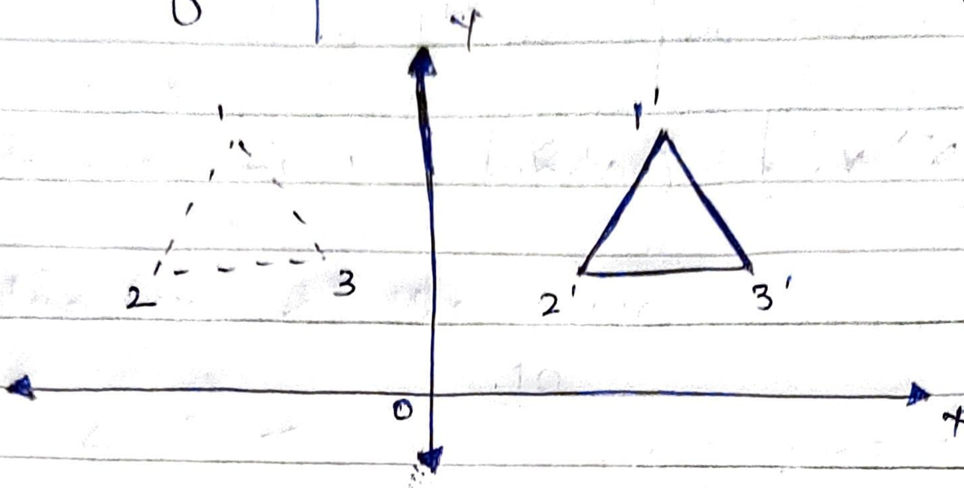
Let us assume that the original co-ordinate are x, y the scaling factor are (S_x, S_y) and the produced coordinates are x', y'. This can be mathematically represented as shown below -

$$x' = x \cdot S_x \qquad \begin{matrix} x' \\ y' \end{matrix} = \begin{pmatrix} x \\ y \end{pmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

$$y' = y' \cdot S_y$$

Reflection

Reflection is the mirror image of original. In other words we can say that it is a rotational operation with 180°. In reflection Transformation the size of objection does not change.



Shear

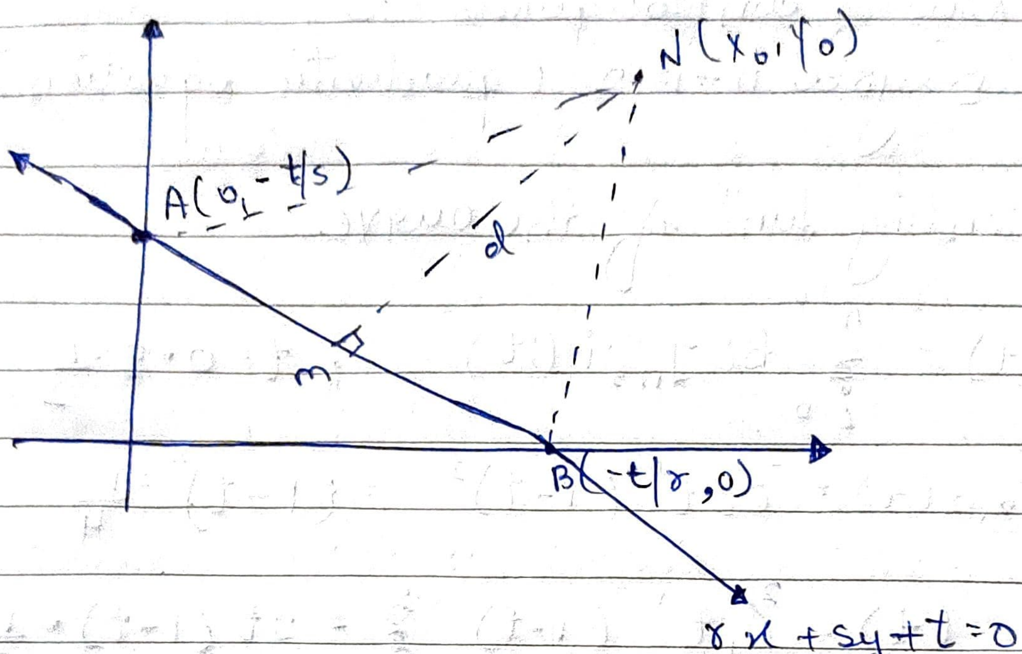
A transformation that slants the shape of an object is called the shear transformation. There are 2 shear transformation - X - shear
- Y - shear

$$X_{sh} = \begin{bmatrix} 1 & Sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad Y_{sh} = \begin{bmatrix} 1 & 0 & 0 \\ Sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} Y = Y + Sh_y \cdot X \\ Y' = X \end{bmatrix}, \qquad \begin{bmatrix} X' = X + Sh_x \cdot Y \\ Y' = Y \end{bmatrix}$$

Ques 5. Given point → (x_0, y_0)

Eq. of line = $rx + sy + t = 0$



area $(\Delta NAB) = \frac{1}{2} \times d \times M$

$$d = \frac{2 \times \text{area}(\Delta NAB)}{M}$$

where

(ΔNAB) area $= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3[y_1 - y_2]\right]$

$= \frac{1}{2}\left[x_0\left(0 + \frac{t}{3}\right) + (-t/r)(-t/s - y_0) + 0\right]$

$= \frac{1}{2}\left[\frac{t}{rs}\right] \cdot \left[rx_0 + sy_0 + t\right]$

therefore,

$$d = \frac{2 \times \frac{1}{2}[t/rs] \cdot [rx_0 + sy_0 + t]}{\sqrt{(t^2/s^2) + (t^2/r^2)}}$$

$$d = \frac{[t/rs] \cdot [rx_0 + sy_0 + t]}{t/rs\sqrt{r^2 + s^2}}$$

Ans $\left[d = \frac{[r x_0 + s y_0 + t]}{\sqrt{r^2 + s^2}} \right]$

Ques 8 (i) No. of control points = 3

$\therefore$ Degree $n-1 = 2$ (quadratic equation)

blending fun^n of the curve.

$P(t) = \sum_{i=0}^{n} B_i J_{n,i}(t)$; $t = 0.5 = \frac{1}{2}$

$J_{0,2}(t) = {}^2C_0 \, t^0 (1-t)^2 = (1-t)^2 = \frac{1}{4}$

$J_{1,2}(t) = {}^2C_1 \, t^1 (1-t) = 2t(1-t) = \frac{1}{2}$.

$J_{2,2}(t) = {}^2C_2 \, t^2 (1-t) = t^2 = \frac{1}{4}$

Bezier curve eq. for the given control point is

$P(t) = B_0 J_{0,2}(t) + B_1 J_{1,2}(t) + B_2 J_{2,2}(t)$

$x(t) = 3t^2 + 10t + 3$

$y(t) = 2t^2 + 2$.

find $x(t)$ and $y(t)$ at $t = 0.5$

$x(t) = 3 \times \frac{1}{4} + 8 \times \frac{1}{2} + 16 \times \frac{1}{4}$.

$\frac{3}{4} + 4 + 4 = \frac{3}{4} + 8 = \frac{35}{4}$.

$y(t) = 2 \times \frac{1}{4} + 2 \times \frac{1}{2} + 4 \times \frac{1}{4} = \frac{1}{2} + 1 + 1 = 5/2$.

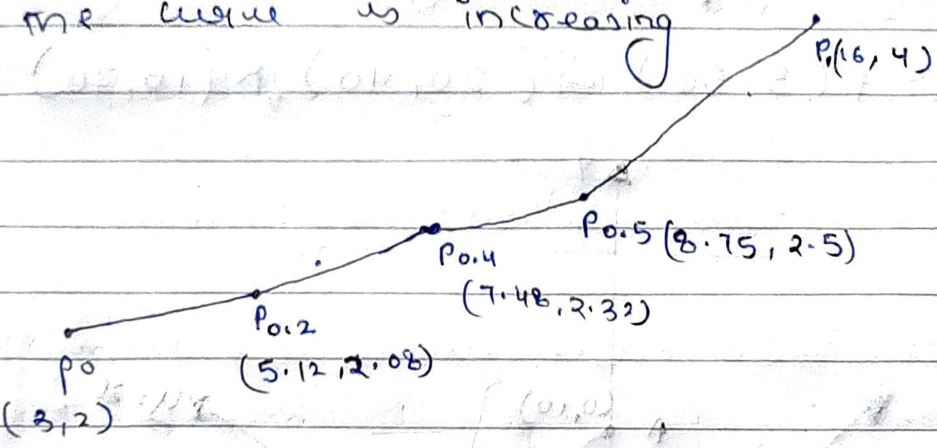
$x(0.5) = \frac{35}{4}$

Ans $y(0.5) = \frac{5}{2}$

To draw the sketch $(0 < t < 1)$

$t = ?$	x	y
$t=0$	3	2
$t=0.2$	5.12	2.08
$t=0.4$	7.48	2.32
$t=0.5$	8.75	2.5
$t=1$	16	4

The curve is increasing



Ques 8. Square A(1,0) B(0,0) C(0,1) D(1,1).

Shifting the origin at point A(1,0) → origin.
New co-ordinates are A' (0,0)
B' (−1,0)
C' (−1,1)
D (0,1)

B'(−1,0) Rotate $(\theta = -45°)$ w.r.t. A'(0,0).

$$\begin{pmatrix} x'' \\ y'' \end{pmatrix} = \begin{bmatrix} x'(1/\sqrt{2}) - y'(-1/\sqrt{2}) \\ x'(-1/\sqrt{2}) + y'(1/\sqrt{2}) \end{bmatrix}$$

$B''(-1/\sqrt{2}, +1/\sqrt{2})$ New coordinate

similarly $\Rightarrow$ $C''(0, \sqrt{2})$ and $D''(+1/\sqrt{2}, 1/\sqrt{2})$

Now again shift the $A'(0,0)$ to original position $A(1,0)$

New final co-ordinate after rotation

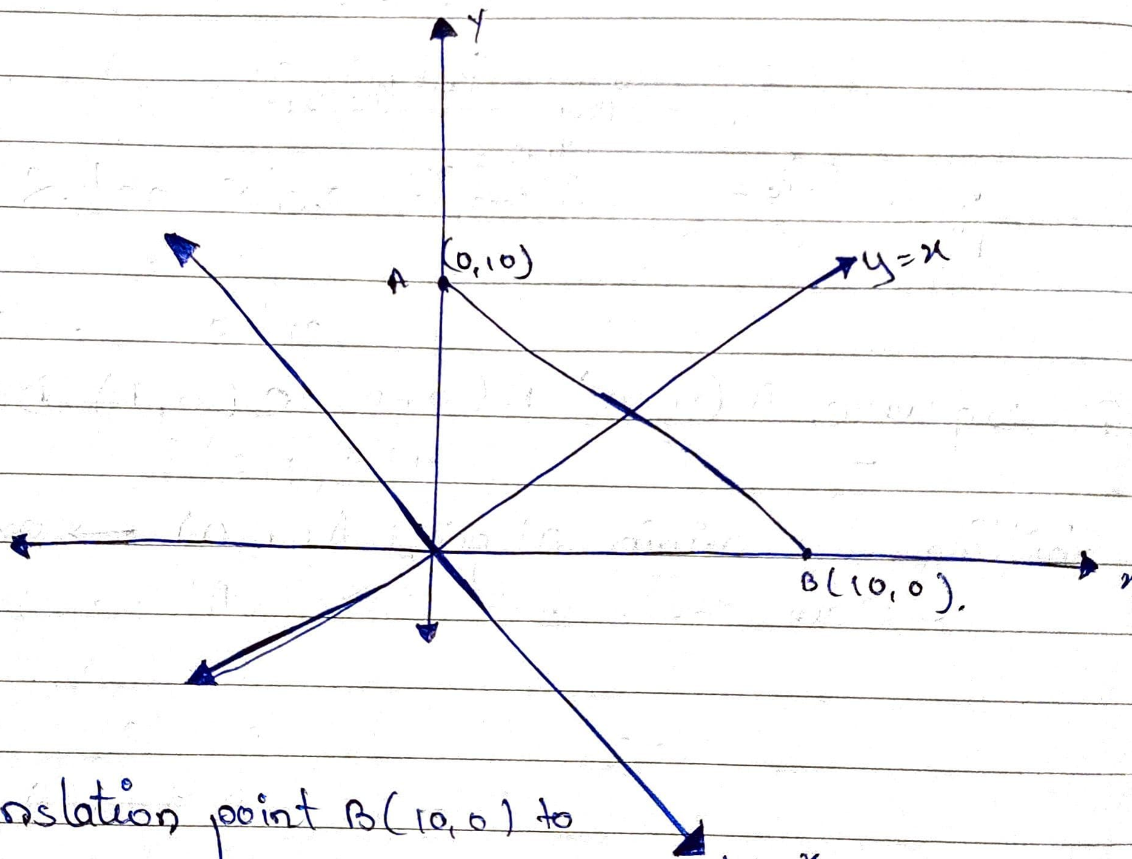
$A(1,0)$

$B\left(\frac{\sqrt{2}-1}{\sqrt{2}}, +1/\sqrt{2}\right)$

$C = (1, \sqrt{2})$

$D = (\sqrt{2}+1/\sqrt{2}, 1/\sqrt{2})$

Ques 9 $\Delta = P(5, 50)$ $Q(20, 40)$, $R(10, 70)$



Translation point $B(10,0)$ to origin =

$A'(-10, 10)$
$B'(0,0)$ origin.

new co-ordinates after translation

$P'(-5, 50)$ $Q'(10, 40)$ $R'(0, 70)$

Rotation of $A'(-10, 10)$ by $45°$ clockwise w.r.t. to origin $B'(0,0)$

$$A''\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{bmatrix} x'(1/\sqrt{2}) + y'(+1/\sqrt{2}) \\ x'(-1/\sqrt{2}) + y'(1/\sqrt{2}) \end{bmatrix}$$

$A''(0, 10\sqrt{2})$

$B''(0, 0)$ origin

triangle co-ordinates are after rotation →

$P''(45/\sqrt{2}, 55/\sqrt{2})$

$Q''(50/\sqrt{2}, 30/\sqrt{2})$

$R''(70/\sqrt{2}, 70/\sqrt{2})$

Reflection about y-axis = y

$$P'''(x, y) = (45/\sqrt{2}, 55/\sqrt{2}) \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$P'''(x, y) = (-45/\sqrt{2}, 55/\sqrt{2})$,

$Q'''(x, y) = (-50/\sqrt{2}, 30/\sqrt{2})$

$R'''(x, y) = (-70/\sqrt{2}, 70/\sqrt{2})$

Rotate line $A''(0, 10\sqrt{2})$ and $B''(0,0)$ origin. By $45°$ anticlockwise w.r.t. $B''(0,0)$

$A'(-10, 10)$ $B'(0,0)$

$$P^{IV}\begin{pmatrix} X^{IV} \\ Y^{IV}\end{pmatrix} = \begin{bmatrix} x'''(1/\sqrt{2}) - y'''(1/\sqrt{2}) \\ y'''(1/\sqrt{2}) + y'''(1/\sqrt{2}) \end{bmatrix}$$

$$P^{IV} = \frac{-100}{(\sqrt{2})^2}, \frac{10}{(\sqrt{2})^2} = -50, 5$$

$Q^{IV} = (-40, -10)$

$R^{IV} = (-70, 0)$

Translation of point $B'(0,0)$ to $B(10,0)$

The final co-ordinate of the triangle

$P_R = (-40, 5)$
$Q_R = (-30, -10)$
$R_R = (-60, 0)$

Question Edge table mentioned below of polygon

S.No.	Edge	1/m	Ymin	x	Ymax
0.	A(2,7), B(4,12)	2/5	7	2	12
1.	B(4,12), C(8,15)	4/3	12	4	15
2.	C(8,15), D(16,9)	-8/6	9	16	15
3.	D(16,9), E(11,5)	5/4	5	11	9
4.	E(11,5), F(8,7)	-3/2	5	11	7

5.	F(8,7) , G(5,5)	3/2	5	5	7
6.	G(5,5) , A(2,7).	-3/2	5	5	7