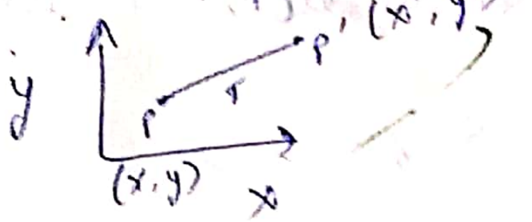


2 D Transformation

$x' = x + t_x$

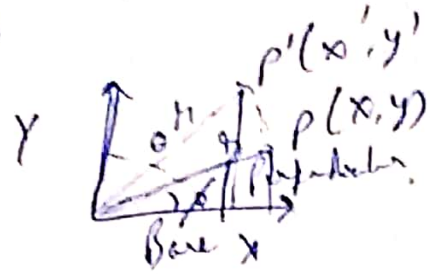
$y' = y + t_y$

$P' = P + T$

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$

$\cos(\theta + \phi) = \frac{x'}{r}$

23



$x' = r\cos(\theta + \phi)$ — ①

$y' = r\sin(\theta + \phi)$ — ②

$x = r\cos\phi$ — ③ $\quad y = r\sin\phi$ — ④

$x' = r\cos(\phi + \theta)$

$= r(\cos\phi\cos\theta - \sin\phi\sin\theta)$

$x' = r\cos\phi\cos\theta - r\sin\phi\sin\theta$ — 5

$\sin A\cos B + \sin B\cos A$

$y' = r[\sin\phi\cos\theta + \sin\theta\cos\phi]$

From

$x' = x\cos\theta - y\sin\theta$

$y' = y\cos\theta + x\sin\theta$

$P' = RP$

$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$

$P = \begin{bmatrix} x \\ y \end{bmatrix}$

Scaling

Scaling factor

$x' = x \cdot S_x$

$y' = y \cdot S_y$

scaling matrix

$P' = S \cdot P$

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

① Translate a $\triangle ABC$ with the coordinates $A(0,0)$, $B(5,0)$, $C(5,5)$ by 2 units in x direction & 3 units in y direction.

$x' = x + t_x \qquad y' = y + t_y$

$P' = T \cdot P$

$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ 1 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 2 & 7 & 7 \\ 3 & 3 & 8 \\ 1 & 1 & 1 \end{bmatrix}$

Perform a 45° rotation of $\triangle$ A(0,0) B(1,1) C(5,2) about the origin

	0°	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

$\tan\theta = \frac{\sin}{\cos}$

(34)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$[1\ 1\ 1]$

(0, 0) $(3/\sqrt{2}, 7/\sqrt{2})$

$P' = R(\theta)\, P$ $(0, \sqrt{2})$

$$[45^\circ] \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Anti Clockwise Rotation

$$\begin{bmatrix} 0 & 0 & 3/\sqrt{2} \\ 0 & \sqrt{2} & 7/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix}$$

Scaling

A(0,0) B(1,1) C(5,2)

Magnified to twice its size

$S_x = 2$
$S_y = 2$

$$P = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

A (0,0)
(2,2)
(10,4)

Homogeneous ~~Coordinate~~ Representation & Matrix Transformations (Matrix form)

$P' = PM_1 + M_2$

Multiplicative matrix

$P' = PM_1$

$(X, Y) = (x, y, h)$

$X = \frac{x}{h}$ $h = 1$

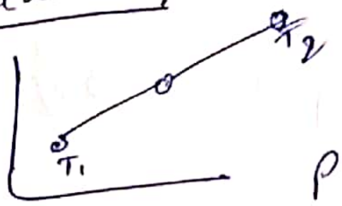
$Y = \frac{y}{h}$

Composite Transformation

Multiple → More than 2 transformation

Translation — Suppose $P(x, y)$ — Initial

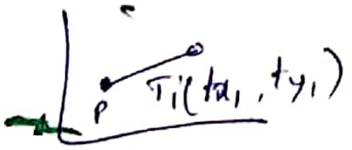
$P'(x', y')$ — After 2 translation



$$P' = T_2(x_2, y_2)\,[T_1(x_1, y_1) \cdot P]$$

Matrix Mult — Commutative & Associative

A.B.C
B.A.C



$$= [T_2(x_2, y_2) \cdot T_1(x_1, y_1)] \cdot P]$$

$T_2(x_2, y_2)$

$$\begin{bmatrix} 1 & 0 & x_2 \\ 0 & 1 & y_2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & x_1 \\ 0 & 1 & y_1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_1 + x_2 \\ 0 & 1 & y_1 + y_2 \\ 0 & 0 & 1 \end{bmatrix} \cdot P$$

$$P' = T(x_1 + x_2, y_1 + y_2) \cdot P$$

Rotation — 2 times

$\theta_1 \quad \theta_2$

$$P' = R(\theta_2)\,[R(\theta_1) \cdot P]$$

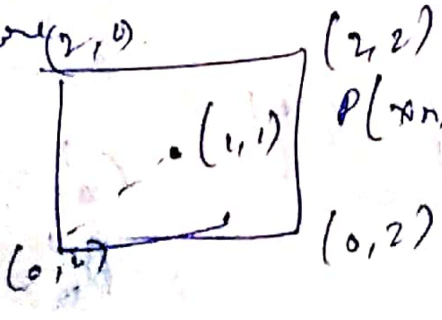
$$= [R(\theta_2) \cdot R(\theta_1)]\,P$$

$$P' = R(\theta_1 + \theta_2) \cdot P$$

Scaling —

$$P' = S(S_{x_1} \cdot S_{x_2},\ S_{y_1} \cdot S_{y_2})\,P$$

A Square with vertices (0, 0) (2,0) (2, 2) & (0, 2) is scaled 2 units in x or y direction about the fixed point which is the center of square (1,1) find coordinates of new square



$P(x_n, y_n)$

1. Translate the fixed point at the origin
2. Scaling the object
3. Inverse translation of object

$$S' = T_v \, S_c(s_x, s_y) \, T_{-v}$$

$$= \begin{bmatrix} 1 & 0 & x_r \\ 0 & 1 & y_r \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -x_r \\ 0 & 1 & -y_r \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

(36)

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P = S'_{(x,y)} \cdot P'_{(x,y)} \text{ (Square)}$$

$$= \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 & 0 \\ 0 & 0 & 2 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{array}{c} \begin{bmatrix} -1 & 3 & 3 & -1 \\ -1 & -1 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{l} - X \\ - Y \\ \end{array}$$

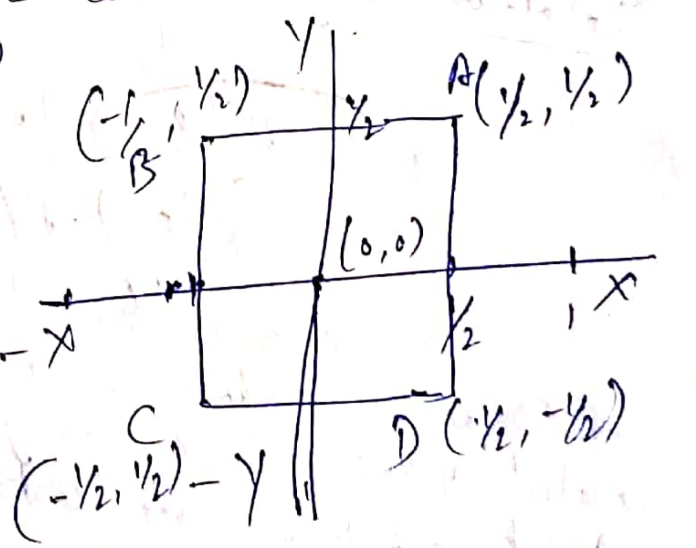
Consider a unit square centered at origin the coordinates of square are translated by a factor $(1/2, 1)$ and rotated by angle of $90°$. What will be coordinates of new square

$A(-1/2, 0)$ $(1/2, 1)$ $(-3/2, 1)$ $(-3/2, 0)$

$B(-1/2, 0)$ $(1/2, 1)$ $(3/2, 1)$ $(3/2, 0)$

$C(-1/2, 0)$ $(1/2, 0)$ $(-3/2, 1)$ $(-3/2, 0)$

$D(-1/2, 0)$ $(1/2, 1)$ $(3/2, 1)$ $(3/2, 0)$



$t_x = 1/2 \quad t_y = 1$

$x' = x + t_x$

$y' = y + t_y$

$Ax' = 1/2 + 1/2 \qquad Ay' = 1/2 + 1$

$A' = (1, 3/2)$

$Bx' = (0, 3/2)$

$Cx' = (0, 1/2)$

$Dx' = (1, 1/2)$

(37)

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$x = 1\cos 90 - 3/2\sin 90$

$A' = (-3/2, 1)$ $B(-3/2, 0)$ $C(-1/2, 0)$ $D(-1/2, 1)$

A point P(5, 1) is rotated by 90° about a pivot point (2,2). What is the coordinate of new transformed point P'.

Resultant Matrix $= T_v R(\theta) T_{-v}$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

Consider a Δ A(0, 0), B(1,1) C(5,2). The Δ has to be rotated by 45° angle about pivot pt. P(-1,-1). Find coordinates of new triangle.

$t_x = -1$

$t_y = -1$

$T_m = T_v R(\theta) T_{-v}$

$$\begin{bmatrix} & & \end{bmatrix} * \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Shearing —

x – Shear

y – Shear

x axis $y' = y$

$x' = x + sh_x \cdot y$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & sh_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

y axis $x' = x$

$y' = y + sh_y \cdot x$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ sh_y & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

~~X axis~~

about both axis —

$x' = x + sh_x \cdot y$

$y' = y + sh_y \cdot x$

Reflection along x axis —

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

y

$$\begin{bmatrix} x' \\ y' \\ h \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Along origin

$$\begin{bmatrix} \\ \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Scanned with CamScanner

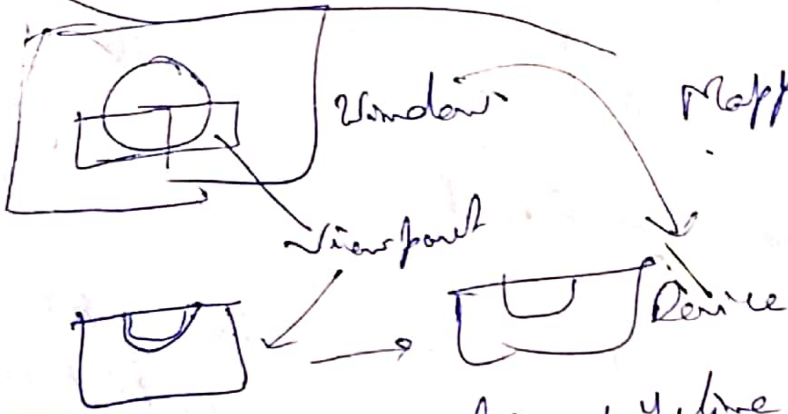
(2,2) (4,2) (3,4) — △

Viewing Transformation

Camera → Click → forming Coordinate matrix

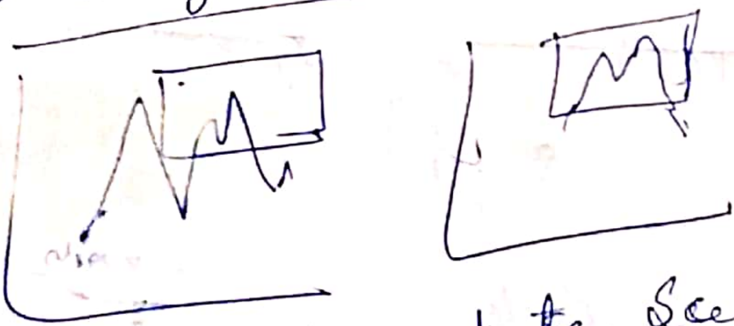
Window Coordinate → Viewport Coordinate

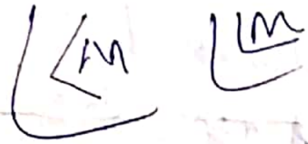
Device



Mapping 38

2D viewing Transformation pipeline —





Construct world coordinate Scene using modelling coordinate System

→ World Coordinate to viewing coordinate (VC)

Convert world coordinate to normalized VC using window viewport specification

Maps view coordinate to

Map normalized viewport to Device Coordinates