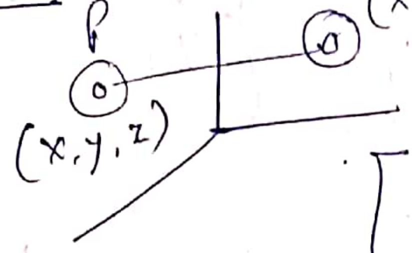
3 D Transformation

(x, y, z)

Translation –



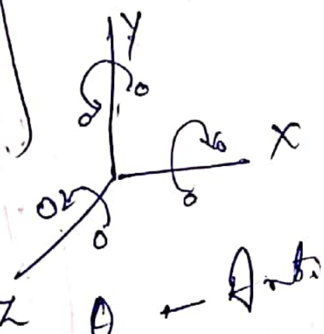
$x' = x + t_x$
$y' = y + t_y$
$z' = z + t_z$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation –

(1) Rotation about z axis

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



$z' = z$

$x = x\cos\theta - y\sin\theta$

$y = x\sin\theta + y\cos\theta$

θ – Anticlockwise

$-\theta$ – Clockwise

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

(2) About x axis

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$x' = x$

$y' = y\cos\theta - z\sin\theta$

$z' = y\sin\theta + z\cos\theta$

(3) About y axis –

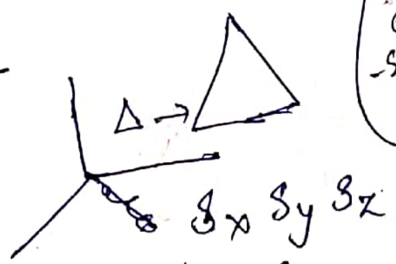
$$\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$y' = y$

$z' = z\cos\theta - x\sin\theta$

$x' = z\sin\theta + x\cos\theta$

Scaling –



$S_x \; S_y \; S_z$

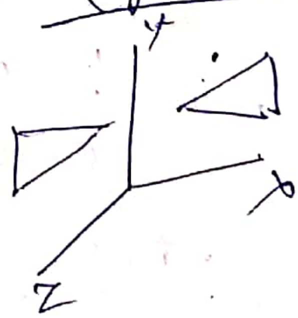
$x' = x \cdot S_x$

$y' = y \cdot S_y$

$z' = z \cdot S_z$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

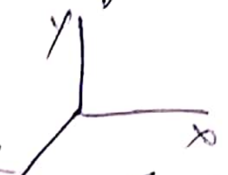
(4) Reflection –



(1) Reflection through (xy plane).

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

② Through YZ plane



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$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ y \\ -z \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} -x \\ y \\ z \end{bmatrix}$$

③ ZX plane —

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x \\ -y \\ z \end{bmatrix}$$

Shearing —

① Z axis Shearing

$z' = z$

$x' = x + z \cdot Sh_x$

$y' = y + z \cdot Sh_y$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & Sh_x & 0 \\ 0 & 1 & Sh_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

② x axis Shearing —

$x' = x$

$y' = y + x \cdot Sh_y$

$z = z + x \cdot Sh_z$

③ y axis shearing —

$y' = y$

$x' = x + y \cdot Sh_x$

$z' = z + y \cdot Sh_z$

① A point P(3, 2, 1) is translated in x, y, & z direction by -2, -2, & -2 followed by successive rotation of 60° about x axis. Find the final position of the point.

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix} = A$$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = B$$

$= T \cdot R$

$= C \times [3, 2, 1, 1]$

$= [1 \quad 0.864 \quad -0.5 \quad 1]$

(2) A cube of length 10 units having one of its corner at origin (0, 0, 0) & 3 edges along 3 principle axis. If the cube is to be rotated about Z axis by an angle of 45° in anticlockwise direction. After calculate new position of cube

Counter

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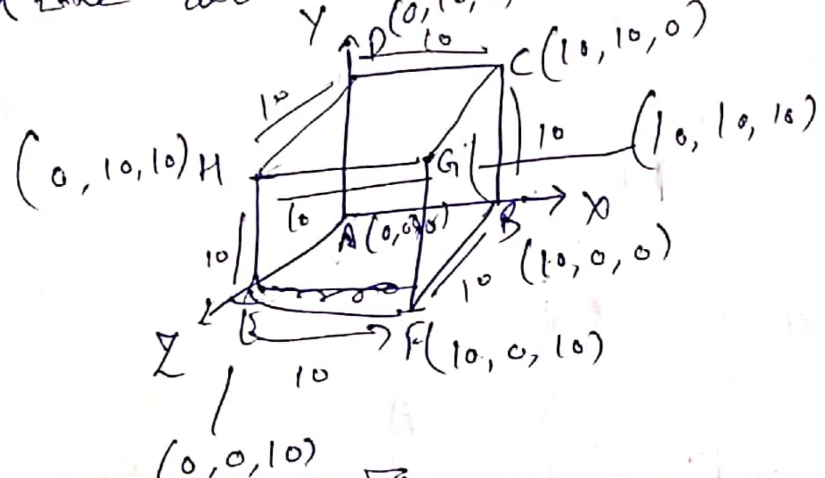
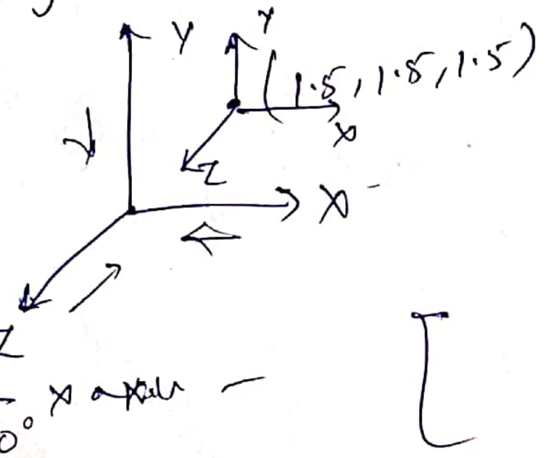


$$R = \left[\ \underline{\quad}_{45°}\ \right] \times \begin{bmatrix} A \\ B \\ C \\ D \\ \vdots \\ H \end{bmatrix}$$

Consider a region defined by position vector.

$$P = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix}$$

Relative to global axis XYZ axis system. It is rotated by +30° about X axis & passing through pt {1.5, 1.5, 1.5}. Find final position of region.



Translation —

$t_x = -1.5$

t_y —

t_z —

$$\left[\qquad\right] A$$

Rotation +30° X axis —

$$R = \left[\qquad\right] B$$

Re Translation

$t_x = 1.5$

$t_y =$ |

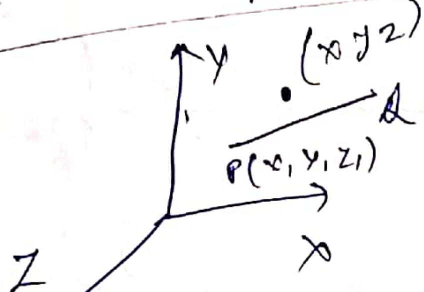
$t_z =$ |

$$\left[\qquad\right] C$$

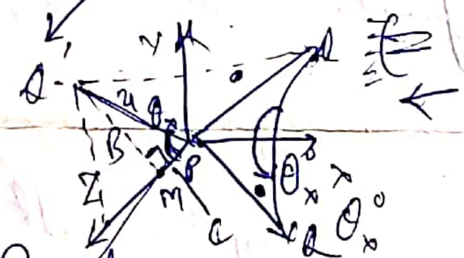
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.86 & 0.5 & 0 \\ 0 & -0.5 & 0.86 & 0 \\ 0 & 0.951 & -0.55 & 1 \end{bmatrix} \times \text{Initial}$$

$$\begin{bmatrix} 1 & 0.81 & 2.22 & 1 \\ 2 & 0.81 & 4.67 & 1 \\ 2 & 1.67 & 2.97 & 1 \\ 1 & 1.67 & 2.17 & 1 \end{bmatrix}$$

Rotate a point in 3D space about a given arbitrary 3D line



$A = \{x_2 - x_1\}$
$B = \{y_2 - y_1\}$
$C = \{z_2 - z_1\}$



1) Translation P to origin

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ t_x & t_y & t_z & 1 \end{bmatrix}$$
$-x_1 \quad -y_1 \quad -z_1$

$t_x = -x_1$
$t_y = -y_1$
$t_z = -z_1$

2) Rotation θ_x° x axis

$\Delta = QMP$

$u = \sqrt{B^2 + C^2}$

$\cos \theta_x = \frac{C}{u}$

$\sin \theta_x = \frac{B}{u}$

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C/u & B/u & 0 \\ 0 & -B/u & C/u & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T \times R_x \times R_y \times R_z \times R_y^{-1} \times R_x^{-1} \times T^{-1}$$