Design and Analysis of Algorithm (Questions for Practice)

1. Given a sequence of numbers, design an algorithm to find the maximum number. Show your algorithm is correct and analyze the time complexity.
2. (a) Design an O (log n)-time algorithm to find the minimum item.
   (b) Show that your algorithm is correct.

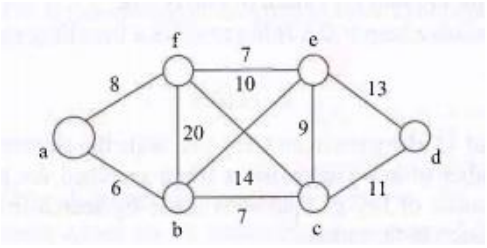
3. Give asymptotic upper bound for each T(n)
   a) $T\ (n)\ =\ 9T\ (n/\ 2\ )\ +\ n^3$
   b) $T(n)\ =\ 7T\ (n/\ 2\ )\ +\ n^3$
   c) $T\ (n)\ =\ T\ (\sqrt{n}\ )\ +\ log\ n$
   d) $T(n)\ =\ 0.5T\ (n/\ 2\ )\ +\ n$
   e) $T\ (n)\ =\ 3T\ (n/\ 3\ )\ +\ n/3$
   f) $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n$ (Using recursion tree)
   g) $T(n) = 2T\left(\frac{n}{2}\right) + 17$ (Show that the solution of given relation is $O(n \lg n)$)

4. State Euclid's Algorithm for finding GCD of two given numbers.
5. What is algorithm's optimality?
6. What do you mean by Worst-case Efficiency of an algorithm?
7. Prove that $100n + 5 \in O(n^2)$
8. Prove that $n^3 \in \Omega(n^2)$
9. Explain about Knapsack problem with example
10. Explain the method of comparing the order of the growth of two functions using limits. Compare order of growth of following functions
    (1) $\log_2 n$ and $\sqrt{n}$
    (2) $(\log_2 n)^2$ and $\log_2 n^2$
11. Write the merge sort algorithm and discuss its efficiency. Sort the list E,X,A,M,P,L,E in alphabetical order using merge sort.
12. What is divide and conquer technique? Apply this method to find multiplication of integers 2101 and 1130.
13. Consider the following recursive algorithm for computing the sum of the first n cubes.
    $s(n) = 1^3 + 2^3 + 3^3 + \cdots \ldots \ldots + n^3$

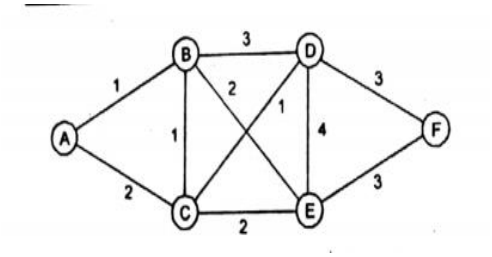
    Algorithm S(n)
        If (n==1) return 1
            Else return (S (n-1) + $n * n * n$)
    End Algorithm

Set up and solve a recurrence relation for the number of times the basic operation of the algorithm is executed.

14. Use kruskal's method to find min cost spanning tree for the following graph:



15. Write Huffman code construction algorithm.
16. Draw the decision tree for the 3-elements insertion sort.
17. Prove that $(n+a)^b$=$O(n^b)$
18. Find minimum spanning Tree for the following graph using Prim's Algorithm:



19. What is activity selection problem? Find out the set which contain maximum activity which are compatible to each other:

A_i	1	2	3	4	5	6	7	8	9
S_i	1	2	4	1	5	8	9	11	13
F_i	3	5	7	8	9	10	11	14	16

20. Symbols A, B, C, D, E, F are being produced by information source with probabilities (in percentage) 30, 40,6,10,15,4 respectively. Find the binary Huffman code for above symbols.
21. Apply Strassen's matrix multiplication algorithm to multiply the following matrices:

$$A = \begin{matrix} 1 & 3 \\ 5 & 7 \end{matrix} \qquad B = \begin{matrix} 8 & 4 \\ 6 & 2 \end{matrix}$$

22. Explain the algorithm of Maximum subarray sum with the help of Divide and Conquer method.