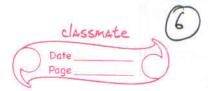
Divide and-Conquer - 1



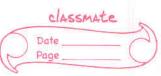
Liver Line	In divide and longuer we solve a problem recursively, applying
	time steps at each level of recursion.
	1. Divide the problem into a no. of Subproblem that are
	Snaker instances of the same problem.
	The Charles of the Control of the State of State
	2. Conquer the Subjoublans by Solving them recyr sirely. When the
	Subproblem Sizes are Small, Solve them Straightforward.
	3. Combine the Solutions to the Subproblems into the solution
	for the original problem. books metality and the
all 1	the first part patent age to the man the first the design
-	Relarsine case - when Subproblems are large enough to Some
to the	recursively.
4131	The thing the same of the same
: - .	Base Case - When Supproblems become song! shough that
B 7.8	no longer recurse (recursion bottom out ").
	and the second of the second o
-	Solve Subproblems that are not some as original problem
hend	in Combine Steps.
0	
	nces - met lent les - met lent les
	It is an equation or inequality that describe a function in
2.20	terms of Its value on Smaller inputs.
	area mattered the all that benefit had been presented by
	eg Merge Sort
	(OLI) 4 N=1
7	$T(n) = \frac{1}{2} 2T(n/2) + O(n)$ if $n > 1$
(1-1	all the sample of a state make the same and
	$= \Theta(n\log n)$
	NEIN >
-	Subproblems are not necessarily constrainted to being a

(Agay RAWAT)

goodstale.	Coreate one Subproblem Containing only one element fewer
	-than the original toroplan.
S. Joseff	than the original problem. $T(n) = T(n-1) + O(1)$
	an area with the contract of the property of the contract of t
-	There are 3 metrods for solving recurrences
	- Substitution meltrad
	- Rely Sion - tree melyod
	- Master theorn meland.
No-AM	in the state of th
D Subs	titution metrod
-	The substitution method for solving occurrences comprises 2 Steps
a-17 a	1. Guess the form of the Solution.
	R. Use mathematical induction to find Constants and Show
	that the Solution works
المراح	College And American And College Colle
_	It is powerful but Can be applied only in Cases when it is
	lasy to guess the form of the answer.
-341	Lance and the second that the second
-	Com be used to established either upper or loner bound.
eg	$1 T(n) = 2T(\lfloor n/2 \rfloor) + n$
9.34	we guess the sol T(n) = O(n logn)
	Substitution meltiod require to prove that T(n) < (nlogn
We	Assume that bound holds for all positive m <n m="[1/2]</th" posticular=""></n>
	yrelding.
	$T(n) \leq 2(c \lfloor n/2 \rfloor \log \lfloor n/2 \rfloor) + n$
	5 Cn (og (n/2) +n
	= Cnlogn - Cnlop +n = Cnlogn - n(elog1)
	= Cologn
	< onlyn.
J. J.	when c>1

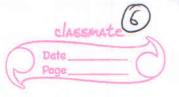
0g 2	Consider the returnance TCn) = 2T ([11/2] +18) +n
	Show that it is asymptotically bound by O(nlopn)
political	Show that it is asymptotically bound by O(nlogn) T(n) = O(nlogn) we have to show that for Constant (
- 100 AL	T(n) & cn logn
	Put tuis in given returnance equation
Janes L	T(m) < 2[c([n/2]+16) log([n/2]+16)]+n
	$= c_n \log(n_b) + 32 + n$
	= Cnlogn - Cnlog 2 +32+n
	= Cnlign - Cn + 32 +n
	= Cnlogn - (C-1)n + 32
	5 Cologn for (CZI)
	thuy fon = O(nlogn)
293	$T(n) = \int \int d^n x d^n x d^n x d^n x$
	2T([7/2])+n n>1 find asymptotic bound on T
	The same of the sa
14.1	we guess the solution is O (nlogn). Thus for a constanst C
	T(n) & Cnlogn
	T(n) < 2 c[n/n] log[n/n] + n
	< conlogn - conlogs + n
	= (n/ogn - n(c/og2 -1)
	5 cnlogn + c21
Mak	ing a good guess
1.	If a recurrence is similar to one we have seen before tany
	guessing a Somelar Solution is reasonable.
2.	Another way is to prove loose upper and lower bounds on the
	recurrence and thou reduce the range of uncestainty.
3.	In a great mater deem't been to work out in face industrien, we

revise the guess by Subbacking alower order tesm than



	matus often goes twongh.
	The same of the sa
4.	Changing Variables Sometimes, a little algebraic manipulation
	Can make a unknown delumente Similar to one we have seen
	before.
	Example T(n) = 2+ (In) + logy . Sohe it by changing variable.
	Suppore m = log n = 2m
	Suppose $m = \log n \Rightarrow n = 2^m$ $n^{1/2} = 2^{m/2} \Rightarrow \ln \Rightarrow 2^{m/2}$
	Put value we get
103 C L	$T(2^m) = 2T(2^{m/2}) + m$
	Again $S(m) = T(2^m)$
	$S(m) = 2S(\frac{m}{2}) + m$
	the state of the s
	Wekness this relimence has Solution
Transmission .	S(m) = O(mlog m)
	Substitute the value of m.
- P - Luce	T(n) = S(m) = 0 (Nogm) = 0 (m) (og log m)
-	= O(mlogm) = O(lognloglogn)
(1	The second secon
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	the entrance are the Cartie parties A
	Washing to grant from
100/4-10	The seal was an an about the seal of the s
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	The state of the s
-	

Divide and Congner



2.	Recursion	Tree

- Each node represente the cost of a Single Sub problem.

- Sum the Costs within lacy level of the tree to obtain a set of per-level Costs.

- Then Sum all the per-level Cost to determine the total Cost of all level of the recursion.

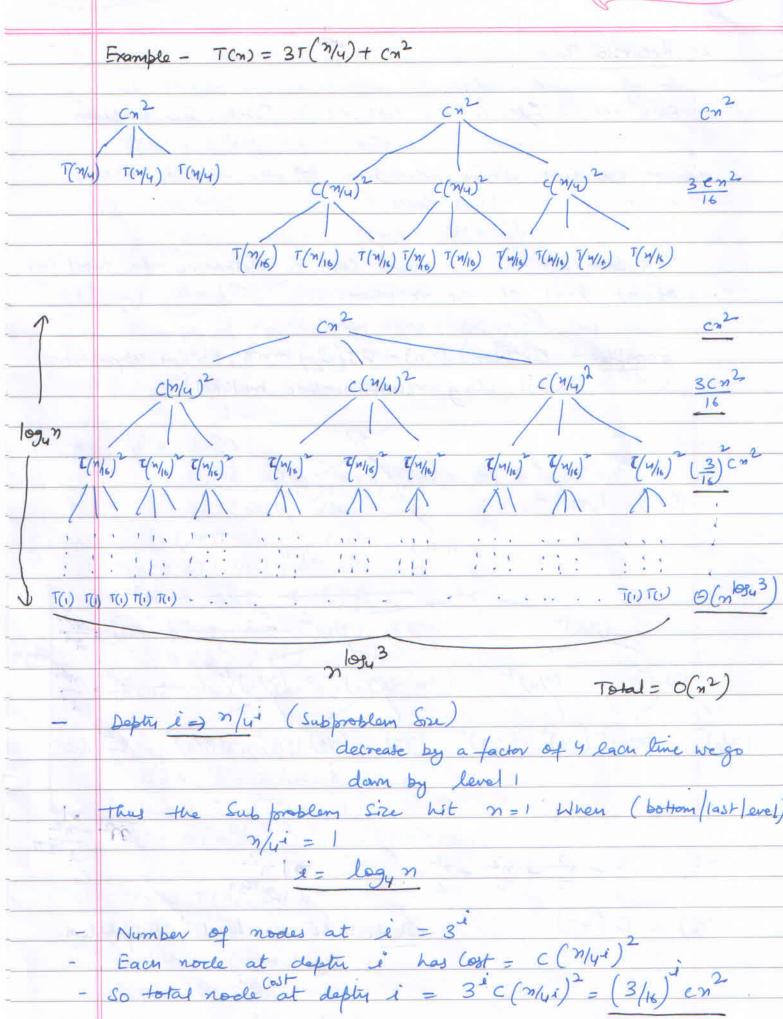
Example - Consider $T(n) = 2T(\frac{n}{2}) + n^2$, obtain asymptotic bound using recursion tree melitod.

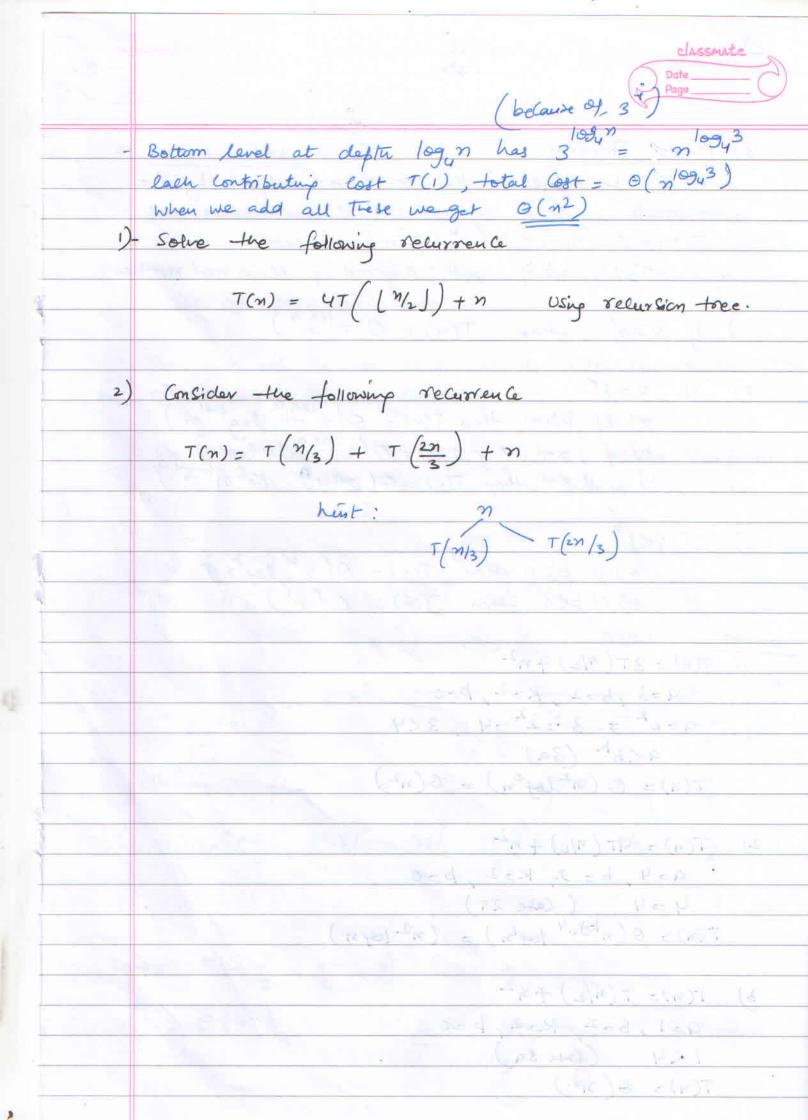
 $T(n|2) \quad T(n|2) \quad T(n|4) \quad T$

 $T(n) = n^2 + \frac{n^2}{2} + \frac{n^2}{4} + \frac{n^2}{8} - \cdots + \frac{n^2}{2^{\log n}}$

Ton) = 9 (n2) Depth - At each level i Subproblem







```
3) Master Theorm
     let T(n) be defined on the non-negative integers by the
      T(n) = aT(n/b) + O(nklogpn) Where
       a>1, b>1, k>0 and b is a real number
    1) if a>bk then T(n) = O(n/09,a)
   2) if a=b^{K}
          a) if $>-1 + may T(m) = O(n log a log ++1 m)
          b) If p=-1 +any T(m) = O(m logs a log log n)
          c) If p <-1 than T(n) = 0(n 108,9
   3) if a < bk
           a) if $ > 0 they T(n) = O(nk lopkn)
           b) if p(0 +non T(n) = 0 (nk)
Examples
   1) T(n) = 3T(n/2) + n^2
       9=3, b=2, K=2, b=0
       a = b^{k} = 3 = 2^{2} = 4, 3<4
       96K (3a)
      T(n) = () (n2 logon) = O(n2)
  2) T(n) = 4T(n/2) + n^2
      a=4, b= 2, K=2, b=0
     4=4 ( Case 29)
     T(n) = 0 (n/0924 log/n) = (n2 logn)
  3) T(n)= T(n/2) +n2
     a=1, b=2, K=2, b=0
     1 K4 (Case 3a)
```

T(n)= 0(n2)

	Trage
4)	$T(n) = 2^n T(n/2) + n^n - n - (2/2) + 2 = (n) T (11)$
	. a = 2" it Swould be Constant
	So Cannot be Solved with Master Theorm.
2 10	THE SHIP OF THE STATE OF THE ST
(2	T(n) = 16T(n/4) +n 0=4 3=4 2=d 5=8
190	a=16, b=4, K=1, b=0 (se = 10)
	16 >4 (Case -1) (-1 10 = (10)T
	$T(n) = O(n^{\log_4 16}) = O(n^2)$
	(4.9-2) a (1.10) TO - 1.30 (4.10)
6)	$T(n) = 2T(n/2) + n \log n$
	422,622, R21, D211- ME
	2 = 2 ((age 2a)) = (1)
	$T(n) = O(n^{\log_5 q} \log^{b+1} n) = O(n' \log^2 n) = O(n \log^2 n)$
	14) T/W = 15 T (20/12) + 18+ 20
7)	T(n) = 2T(n/2) + n/logn
	a=2, b=2, k=1, b=-40
	$a = 2, b = 2, k = 1, (b = -1+a)$ $2 = 2 \qquad (Case 2b) = -1+a$
	$T(n) = O(n' \log \log n) = O(n \log^2 n)$
8)	$T(n) = 2T(n/4) + n^{0.51}$ $a = 2$, $b = 4$, $k = 0.51$, $b = 0$, $2 < 4^{0.51}$ (Case 39) T(n) = 0 (no.51)
(6)	a=2, b=4 K=0.51, b=0, 2 < 40.51 (Care 39)
	T(n) = 0 (m051) = (m) = (m)
9)	T(n) = 0.5T(n/2) + 1/n at (dis) the = (m)T (d)
	a = 0.5, a should be 21
	So Cannot be solved by Master theory
	(2 mm) = (m) T
10)	$T(n) = 6T(n/3) + n^2 \log n$
	a=6, b=3, K=2, b=(b+(8/11) T8=(N)T (F)
<u> </u>	$6 ext{ (Case 3a)}$ $T(n) = 0 ext{ (n2 logn)}$
	$T(n) = O(n^2 \log n) = O(n^2 \log n)$

PE

11) T(n) = 64T (n/8) (-) n2/ogn (1) T'S = (10) T (1) because of -re sign Camet be solved. 12) $T(n) = 7T(n/3) + n^2$ a=7, b=3, K=2, P=0 m+ (AP)(A) = (M)T (2 7<9 (Case 3a) 0=4 1=4 4=4 Alex $T(n) = O(n^2)$ (1-30) P(3) $(-1n) = (-1)^{n} = (-1)^{n} = (-1)^{n}$ 13) T(n) = 4T (n/2) + logn a=4, b=2, K=0, b=1 n+(aN) T=a(n) T=a A=0 ATONDE (PETER) a (METER) = 6 MILLION) = 6 MILLION) 14) T(n) = 52 T (n/2) + logn a=52, b=2, k=0, b=1/1 + (a) TS=(A)T (F $\frac{J_2 > 2^0}{T(n) = \Theta(n^{\log_2 J_2}) = \Theta(J_n)}$ T(x) = 5 (x 187 189 x) = 6 (3 189 x) 15) T(n) = 2T (n/2) + In a=2, b=2, k=1/2, b=0+ (p) 75 = (0) $\frac{2 \cdot 2^{1/2}}{\Gamma(n) = O(n^{\log_0 2})} = O(n^{\log_0 2}) = O(n) = O(n^{\log_0 2})$ 16) T(n) = 3T (n/2) +n + (11) 120 = (10) T (F a=3, b=2, k=1, b=0 3 D 2 to votal (Case + I) and terms of $T(n) = O\left(n^{\log_2 3}\right)$ 10) T(11) = 6T (11/3) + 22-107 (0) 17) $T(n) = 3T(n/3) + J_n$ q = 3, b = 3, k = 1/2, b = 0 $3 > 3^{1/2}$ (Case - I) $T(n) = O(n^{\log_3}) = O(n)$

18)	T(n) = 4T (n/2) + (n
	a=4, $b=2$, $K=1$, $b=0$
i	a=4, $b=2$, $k=1$, $b=04>2^{l} (Care-I)T(n) = \Theta(n^{lOf_2 4}) = \Theta(n^2)$
	$T(n) = \Theta(n^{109_29}) = \Theta(n^2)$
	a=3, $b=4$, $K=1$, $P=1$
	a=3, $b=4$, $k=1$, $p=13 < u'$ (Case 3a) T(n)=b(n') = b(n') = 0 (nlogn)
-	1(x) =0(x) 20g x) = G (x(sqx)
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