Cointegration-Based Strategies in Forex Pair Trading

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Abstract

Pair trading involves exploiting pricing differentials between correlated assets. Cointegration is a statistical technique that can be used to identify long-term equilibrium relationships between non-stationary time series data, providing a potential basis for pair trading strategies. In this paper, we review existing approaches to pair trading and cointegration, and propose a methodology for developing cointegration-based pair trading strategies in the Forex market. Our methodology involves a comprehensive analysis of trading strategies, incorporating a range of parameters such as training/testing windows and z-score thresholds. We apply this methodology to a dataset of historical daily adjusted closing prices for seven major currency pairs, spanning a period of 17 years. Our results seem to demonstrate the potential for cointegration-based pair trading strategies to generate Sharpe ratios of up to 0.9. In our conclusion, we discuss the implications of our findings for traders and researchers and suggest avenues for future research in this area.

1 Introduction

In the landscape of financial markets, traders are constantly seeking strategies to capitalize on market inefficiencies and exploit relative price movements. Among the diverse array of trading methodologies, pair trading has emerged as a potentially profitable strategy. The interest in the pair trading theme arises from its nuanced approach to leveraging pricing differentials among correlated assets — a pursuit made feasible by the dynamics within financial markets. At the heart of the strategy presented in our study lies the concept of cointegration, a notion in econometrics that highlights the long-term equilibrium connecting non-stationary time series data. Serving as a cornerstone, this principle elucidates how deviations from established relationships can be exploited for profitable opportunities.

Cointegration permits the quantification of long-term equilibrium relationships between different assets. This equilibrium underscores a stable co-movement over time, and any divergence from this state signifies a temporary imbalance. Such deviations, in the context of cointegration, are not merely random fluctuations but rather indicators for potential trading opportunities. A critical facet of this research revolves around the practical implications of the findings. If cointegration is conclusively established among selected pairs, it appears to pave the way for the application of a trading strategy rooted in the mean-reversion theory.

The pair trading strategy involves simultaneously taking both long and short positions in two cointegrated assets, with the goal of exploiting the likely convergence of their price movements to generate profits. This methodology is founded on the principle that deviations from historical price synchronization are expected to revert to a standard condition over time.

2 Existing Approaches

Krauss et al. [1] identify five streams of literature analyzing pair trading using different approaches. These approaches include the Distance approach, Cointegration approach, Time-series approach, Stochastic control approach, and other approaches.

2.1 Distance Approach

Gatev et al. [2] introduce the Distance approach, focusing on two periods—formation and trading. This method identifies security pairings that historically move prices in the same direction, utilizing distance metrics. Traders then create signals based on observable distance measures, initiating trades when the spread between two securities deviates significantly from its historical norm. While the Distance approach is straightforward and resistant to data snoops, the use of Euclidean squared distance as a measure has analytical limitations.

2.2 Cointegration Approach

Vidyamurthy's [3] framework forms the basis for the Cointegration approach, involving three phases: preselection of pairs, tradability analysis using a modified Engle-Granger cointegration test [4], and determination of entry and exit criteria through nonparametric approaches. Rad et al. [5] practically implement the cointegration technique, selecting stocks with the smallest sum of Euclidean squared distance (SSD) and using the Engle-Granger method to assess cointegration. The return behavior between the cointegration and distance methods is observed to be similar. Galenko et al. [6] introduce a statistical arbitrage technique based on a multivariate cointegration framework, where the return process is a weighted sum of asset returns determined by the cointegration vector. This approach demonstrates mean-reversion and outperforms the benchmark in empirical applications with index exchange-traded funds (ETFs). However, it involves a significant amount of data mining.

2.3 Time-Series Approach

Elliott et al. [7] proposed a time-series approach, utilizing a Gaussian Markov chain settling back to its mean value. This approach, based on a state space model, initiates pairs trades under specific conditions. The advantages of this model include traceability, time model forecasting, and reliance on mean-reversion theory.

2.4 Stochastic Control Approach

Jurek and Yang's [8] study in the stochastic control approach considers two investor scenarios and offers arbitrageurs the choice to invest in a mean-reversion spread or a risk-free product. Their model, incorporating uncertainty through the OU-process, provides closed form solutions for both value and policy functions.

2.5 Other Approaches

Huck [9, 10] introduces studies involving three main steps: outranking, forecasting, and trading. Elman neural networks are used for forecasting, and a multicriteria decision procedure is employed for outranking. Trading decisions are based on rankings rather than equilibrium theory.

3 Research Objectives

While existing literature has provided a rich array of methodologies for pair trading, there remains a notable gap in the application of these strategies within the context of the Forex market.

Cointegration permits the quantification of long-term equilibrium relationships between different assets. This equilibrium underscores a stable co-movement over time, and any divergence from this state signifies a temporary imbalance. Such deviations, in the context of cointegration, are not merely random fluctuations but rather indicators for trading opportunities.

The proposed research focuses on the application of a cointegration-based pair trading strategy specifically targeting the pairs that include different combinations of the most liquid currencies such as Euro to US Dollar (EUR/USD), British Pound to US Dollar (GBP/USD), US Dollar to Japanese Yen (USD/JPY), US Dollar to Swiss Franc (USD/CHF), US Dollar to Canadian Dollar (USD/CAD), Australian Dollar to US Dollar (AUD/USD), New Zealand Dollar to US Dollar (NZD/USD).

Furthermore, we conducted a comprehensive study on trading strategies, which encompassed a range of parameters that included various combinations of training/testing windows and z-score thresholds. Such variation aims to optimize pair trading strategies by identifying the most effective combinations of parameters and the best-performing pairs. Additionally, the study seeks to assess the robustness of these strategies across various market conditions, conduct sensitivity analyses to understand parameter impacts, compare different approaches for performance, and validate the strategies against historical data. Ultimately, the objective is to enhance understanding and improve the profitability and risk management capabilities of pair trading strategies for traders.

Overall, this research aims to identify and capitalize on trading strategies and statistical arbitrage possibilities by determining the existence of cointegration on selected pairs.

4 Methodology

4.1 Data Collection

We have collected historical daily adjusted closing prices of 7 most liquid currencies from Yahoo Finance [11]: Euro/US Dollar (EUR/USD), British Pound/US Dollar (GBP/USD), Japanese Yen/US Dollar (JPY/USD), Swiss Franc/ US Dollar (CHF/USD), Canadian Dollar/ US Dollar (CAD/USD), Australian Dollar/US Dollar (AUD/USD), New Zealand Dollar/US Dollar (NZD/USD). In this study, we transformed the pairs to have a common denominator of USD. The overall study period spans 17 years from January 1, 2007, to January 1, 2024, which is the longest period over which continuous data is available for the entire set of selected currencies.

4.2 Cointegration Analysis

We conducted cointegration tests to determine whether a long-term relationship exists between the prices of the paired assets. There are two main approaches to test for cointegration in time series: the Engle-Granger test and the Johansen test. The Engel-Granger test is a widely used method for assessing the presence of cointegration between two non-stationary time series variables. The Johansen test is a statistical method used to determine the number of cointegrating relationships among multiple non-stationary time series variables.

The Engle-Granger cointegration test offers several advantages over the Johansen test, particularly in terms of simplicity and practical application. Firstly, the Engle-Granger test is easier to implement as it involves a two-step approach: testing for unit roots in the individual series and then testing for a unit root in the residuals of the long-run equilibrium regression. This simplicity makes it more accessible for practitioners and those with less advanced econometric training. Additionally, the Engle-Granger test is less computationally intensive, requiring fewer parameters to estimate. Moreover, it is straightforward to interpret, providing clear insights into whether a cointegrating relationship exists between the variables. While the Johansen test can handle multiple cointegrating relationships and offers a more sophisticated analysis, the Engle-Granger test's ease of use and interpretability make it a practical choice for many applied econometric analyses.

Within the limited scope of this study, we chose to start our investigations with the use the Engle-Granger test due to its advantages over the Johansen test. The Engle-Granger test consists of the following steps:

- Individual Augmented Dickey-Fuller (ADF) Tests: Begin by conducting Augmented Dickey-Fuller tests on each of the two time series variables individually. The objective is to determine whether each variable is individually non-stationary. The null hypothesis for the ADF test is that the variable has a unit root, indicating non-stationarity. If the null hypothesis cannot be rejected, the variable is considered non-stationary.

- Regression Analysis: If both variables are found to be individually non-stationary, proceed to the next step by regressing one variable on the other. This regression should be conducted in levels, not in first differences.

- Residual Augmented Dickey-Fuller Test: Perform an Augmented Dickey-Fuller test on the residuals from the regression obtained in the previous step. The null hypothesis for this test is that the residuals contain a unit root, indicating non-stationarity. If the null hypothesis is rejected, it is assumed that the residuals may be stationary.

- Cointegration Inference: If the residuals are assumed to be stationary, the results imply that there is evidence of cointegration between the two variables. Cointegration signifies a long-term equilibrium relationship, presuming that any deviations from this equilibrium will eventually be corrected.

- Interpretation: The rejection of the null hypothesis in the residual ADF test suggests the presence of cointegration. Conversely, if the null hypothesis cannot be rejected, it implies a lack of evidence for cointegration.

4.3 Mean-Reversion Strategy Development

Based on the cointegration analysis, we developed a mean-reversion trading strategy. This involves establishing a threshold for the spread between the paired assets. The proposed strategy compares the logarithmic transformation of the prices of two assets represented by series1 and series2. The core steps of the strategy can be summarized as follows:

4.3.1 Spread Calculation

The process described in 4.2 can be summarized in a simple mathematical representation. In the context of cointegration, if we have two time series, $\mathbf{y}_t$ and $\mathbf{x}_t$, we often regress one time series (e.g., $\mathbf{y}_t$) on another (e.g., $\mathbf{x}_t$) to identify a long-term equilibrium relationship. The regression equation is typically formulated as:

$$\mathbf{y}_t = \beta \cdot \mathbf{x}_t + \mathbf{r}_t$$

where β is the coefficient estimating the relationship between $\mathbf{x}_t$ and $\mathbf{y}_t$, and $\mathbf{r}_t$ denotes the residuals. The residuals, if found to be stationary through appropriate statistical tests (like the Augmented Dickey-Fuller test), suggest cointegration, implying that deviations from this equilibrium relationship are temporary and expected to revert to the mean.

Now we can express the residuals time series as:

$$\mathbf{r}_t = \mathbf{y}_t - \beta \cdot \mathbf{x}_t$$

If $\mathbf{y}_t$ and $\mathbf{x}_t$ are not stationary and $\mathbf{r}_t$ is stationary, then $\mathbf{y}_t$ and $\mathbf{x}_t$ are presumed to be cointegrated. This means that although the individual time series may have trends, there exists a stable long-term relationship between them. Here, the expression $\mathbf{y}_t$-β·$\mathbf{x}_t$ represents the spread, and β is a cointegration coefficient that measures the strength of the relationship between the two time series.

In this study, $\mathbf{y}_t$ and $\mathbf{x}_t$ are the logarithmic transformation of the prices of the considered pair of currencies. The logarithmic transformation of the prices helps normalize the data, making it easier to compare assets with significantly different price levels (e.g. the Euro versus the Japanese Yen). This is particularly useful in pair trading, where the strategy is based on the relative movements between two assets.

Thus, in this study, we calculated the price spread by the following formula:

$$\text{Spread} = \log(\text{price\_currency1}) - \beta \cdot \log(\text{price\_currency2})$$

4.3.2 Mean and Standard Deviation Calculation

We computed the mean and standard deviation of the prices spread over a specified training period preceding the corresponding testing period.

4.3.3 Z-Score Calculation

We calculated the z-score of the spread, representing the number of standard deviations the current spread is from its historical mean. We calculated the z-score for each day of the testing period by the following formula:

$$\text{Z-Score(t)} = (\text{Spread(t)} - \text{Mean Spread}) \ / \ \text{Standard Deviation of Spread}$$

Here Spread(t) is the spread in currency prices obtained for the current day of the testing period, Mean Spread and Standard Deviation of Spread are the mean and the standard deviation of the spread in currency prices obtained for the training period preceding the testing period.

4.3.4 Trading Signals

We generated trading signals based on the z-score. Therefore, enter a long position (buy) if the z-score is below the negative threshold and enter a short position (sell) if the z-score is above the positive threshold.

A z-score below the negative threshold indicates that the spread is below its historical average, suggesting a potential long trading opportunity. A z-score above the positive threshold suggests that the spread is above its historical average, indicating a potential short trading opportunity.

In essence, the strategy identifies times when the spread between the two assets deviates significantly from its historical mean and generates trading signals to take advantage of the anticipated mean-reverting behavior. The chosen threshold serves as a sensitivity parameter, determining the level of deviation from the mean required to trigger a trading signal.

4.3.5 Signal Application

To compute the daily returns of the strategy correctly, the signal should be applied with a shift of two days. Thus, when the signal occurs at time t, the trade is executed at time $t + 1$. This approach ensures a more robust and realistic simulation of trading conditions, providing a better reflection of potential real-world trading outcomes.

To calculate the daily returns of a trading strategy, one should multiply the daily price change by the trading signal from the previous day. This method assumes that the action to buy or sell is executed at the closing price of the signal day, thereby affecting the return of the next day. The formula is as follows:

$$\text{Daily Return}(t) = \text{Signal}(t - 1) \times \text{Daily Price Change}(t)$$

For example, if the signal on Monday evening is -1 (indicating a sell), the position is taken immediately at the closing price of Monday. Consequently, the return for Tuesday is computed by multiplying Tuesday's price change by the signal from Monday.

The above calculation assumes the ability to execute the position exactly at the close of the day when the signal is generated. However, in practical trading scenarios, executing trades precisely at the close can be challenging due to various factors.

To address this issue and make the back-testing more robust, it is prudent to assume that the position is entered at the close of the day following the signal generation. This means that if a signal is generated on Monday evening, the position is taken at the close of Tuesday, not Monday. Therefore, the signal from Monday affects the return of Wednesday, not Tuesday. This necessitates a shift of two days instead of one when applying the signal in the calculation of daily returns. Thus, the formula calculating daily returns takes the following form:

$$\text{Daily Return}(t) = \text{Signal}(t-2) \times \text{Daily Price Change}(t)$$

4.4 Trading Strategy Parameters

4.4.1 Train-Test Split

The data were split into two distinct sets: a training set and a testing set. The primary purpose of the training set is to assess whether the time series exhibit cointegration. Upon confirming cointegration, we proceeded to apply the proposed trading strategy.

Our approach involves utilizing various training and testing time windows shown in Figure 1. We implemented the trading strategy using different combinations of these time windows for both training and testing periods throughout the entire study period, ensuring continuity in the testing data. For instance, we designated a train period of n days and a test period of m days. During the initial n days, we assessed cointegration, followed by the application of the trading strategy over the subsequent m days of the test period. Once the test period concludes at the Nth day (where N = m + n), we transition to a new test period of m days starting from the (N+1)th day and ending at the (N + m)th day, while the preceding n days serve as a new train period, starting from the (N-n)th day and ending at the Nth day. This iterative process ensures a continuous evaluation of the trading strategy's performance.

Training / Testing windows	1	5	21	63	128
63	+	+	+		
128	+	+	+	+	
257	+	+	+	+	+

Figure 1: Different combinations of time windows for training and testing periods implemented in this study

4.4.2 Z-Score Threshold

Exploring different z-score thresholds allows us to understand how sensitivity to deviations from the mean affects the performance and effectiveness of the trading strategy. Lower z-score thresholds, such as ± 1, imply a more aggressive approach, where trading signals are triggered by smaller deviations from the mean, potentially leading to more frequent trades. On the other hand, higher z-score thresholds, like ± 2 or ± 3, indicate a more conservative strategy, where trades are initiated only when significant deviations occur, possibly resulting in fewer trades. We have studied the implementation of the trading strategy with different z-score thresholds of ± 1, ± 2, and ± 3.

For simplicity, we are going to refer to the considered z-score thresholds as 1, 2, and 3.

4.5 Strategy Implementation

In this study, we examined 7 distinct currencies, as outlined in Section 4.1. Considering that cointegration is not symmetrical, we generated a total of 42 unique pair combinations. Subsequently, we applied the trading strategy to all 42 pairs and aggregated the returns obtained from each pair over the entire study period with different parameter settings discussed in Section 4.4. The business metrics were then calculated based on the total returns accrued throughout the study period. We analyzed the trading performances of portfolios based on different pairs/windows/z-score thresholds cross-sections. Then we introduced the trading strategy based on the filtering parameters defined by the cross-sectional analysis.

To evaluate the performance of the proposed strategy, we calculated the aggregate business metrics. First, we computed the total returns series by summarizing the individual returns series. Then, we calculated the standardized returns series by dividing the total returns series by the number of selected combinations. This yields small average returns and volatility. A practitioner who wants to implement a similar strategy may scale their positions by taking on some leverage to ensure their position sizing complies with their risk objectives, whether in terms of volatility, maximum drawdown, value at risk, conditional value at risk, or any other measure of the risk of their choice. For example, if the historical performance metrics show an annualized return of 1% and an annualized volatility of 1%, the trader could adjust the number of lots accordingly based on their risk objective. In our study, we assumed an arbitrary scaling such that the trader would use a leverage of 10.

4.6 Performance Metrics

We used the Sharpe ratio as an evaluating criterion for the trading performances of portfolios based on different pairs/windows/z-score thresholds cross-sections. We evaluated the performance of the proposed strategy by using the following metrics as suggested in [12, 13]: annualized return, annualized volatility, Sharpe ratio, Sortino ratio, Calmar ratio, and maximum drawdown. To plot the cumulative returns graph with more representative drawdowns, we used the non-exponential form for calculating the cumulative returns series ($\sum_{i=1}^{n} \text{returns}_i$) instead of the exponential form of this equation ($\exp(\sum_{i=1}^{n} \text{returns}_i)$). In this case, when plotting the resulting cumulative returns series, we did not subtract 1.

5 Results

5.1 Cross-Sectional Analysis

Tables 1, 2, and 3 show the Sharpe ratios calculated for the aggregate trading performances of all 42 currency pairs at different parameter settings.

Table 1: Sharpe ratios calculated for the portfolios based on all the windows combinations across all pairs and all z-score thresholds

Training / Testing windows	**1**	**5**	**21**	**63**	**128**
63	-0.32	-0.14	-0.26	NA	NA
128	0.11	0.21	0.05	-0.03	NA
257	0.17	0.15	0.54	0.11	0.08

Table 2: Sharpe ratios calculated for portfolios based on all the windows combinations across all pairs and all z-score thresholds, listed in descending order of the Sharpe ratio

Training window	Testing window	Sharpe ratio
257	21	0.54
128	5	0.21
257	1	0.17
257	5	0.15
128	1	0.11
257	63	0.11
257	128	0.08
128	21	0.05
128	63	-0.03
63	5	-0.14
63	21	-0.26
63	1	-0.32

From the results presented by Tables 1 and 2, one can notice that across all pairs and z-score thresholds, the highest Sharpe ratio (0.54) was obtained for the trading strategy executed with a training window of 257 days and a testing window of 21 days. The lowest Sharpe ratio (-0.32) was obtained for the trading strategy executed with a training window of 63 days and a testing window of 1 day.

Table 3: Sharpe ratios calculated for the portfolios based on all the z-score thresholds across all pairs and all windows combinations, listed in descending order of the Sharpe ratio

z-score threshold	Sharpe ratio
1	0.10
3	0.03
2	-0.05

The results presented in Table 3 show that across all pairs and windows combinations, the highest Sharpe ratio was observed at the z-score threshold of 1, while the lowest Sharpe ratio was obtained at the z-score threshold of 2. Tables 4 and 5 show the Sharpe ratios calculated for the trading performances of each of the 42 currency pairs across all the windows combinations and z-score thresholds.

Table 4: Sharpe ratios of all pairs across all windows combinations and all z-score thresholds

	AUDUSD	EURUSD	GBPUSD	NZDUSD	JPYUSD	CHFUSD	CADUSD
AUDUSD	NA	-0.18	-0.44	-0.42	0.10	-0.02	-0.08
EURUSD	0.06	NA	0.08	-0.09	-0.16	0.13	-0.36
GBPUSD	-0.38	0.05	NA	0.05	0.24	0.29	0.35
NZDUSD	-0.24	-0.03	0.45	NA	0.05	0.31	-0.15
JPYUSD	0.04	0.05	-0.36	-0.09	NA	0.07	-0.32
CHFUSD	-0.04	0.17	0.10	0.09	0.13	NA	0.19
CADUSD	-0.10	-0.05	0.19	-0.003	-0.15	0.04	NA

Table 5: Sharpe ratios calculated for all pairs across all windows combinations and all z-score thresholds, listed in descending order of the Sharpe ratio

Currency 1	Currency 2	Sharpe ratio
NZDUSD	GBPUSD	0.45
GBPUSD	CADUSD	0.35
NZDUSD	CHFUSD	0.31
GBPUSD	CHFUSD	0.29
GBPUSD	JPYUSD	0.24
CADUSD	GBPUSD	0.19
CHFUSD	CADUSD	0.19
CHFUSD	EURUSD	0.17
CHFUSD	JPYUSD	0.13
EURUSD	CHFUSD	0.13
AUDUSD	JPYUSD	0.10
CHFUSD	GBPUSD	0.10
CHFUSD	NZDUSD	0.09
EURUSD	GBPUSD	0.08
JPYUSD	CHFUSD	0.07
EURUSD	AUDUSD	0.06
GBPUSD	EURUSD	0.05
GBPUSD	NZDUSD	0.05
JPYUSD	EURUSD	0.05
NZDUSD	JPYUSD	0.05
JPYUSD	AUDUSD	0.04
CADUSD	CHFUSD	0.04
CADUSD	NZDUSD	-0.003
AUDUSD	CHFUSD	-0.02
NZDUSD	EURUSD	-0.03
CHFUSD	AUDUSD	-0.04
CADUSD	EURUSD	-0.05
AUDUSD	CADUSD	-0.08
EURUSD	NZDUSD	-0.09
JPYUSD	NZDUSD	-0.09
CADUSD	AUDUSD	-0.10
CADUSD	JPYUSD	-0.15
NZDUSD	CADUSD	-0.15
EURUSD	JPYUSD	-0.16
AUDUSD	EURUSD	-0.18
NZDUSD	AUDUSD	-0.24
JPYUSD	CADUSD	-0.32
JPYUSD	GBPUSD	-0.36
EURUSD	CADUSD	-0.36
GBPUSD	AUDUSD	-0.38
AUDUSD	NZDUSD	-0.42
AUDUSD	GBPUSD	-0.44

It was mentioned above that across all pairs and z-score thresholds the highest Sharpe ratio was obtained for the strategy executed at the training window of 257 days and the testing window of 21 days. This choice of windows is frequently used in pair trading. Thus, we fixed this parameter and calculated the performance of the strategies for the individual pairs across all z-score thresholds at the training/testing windows of 257/21 (see Table 6). In addition, we calculated the performance of the strategies executed at each z-score threshold at fixed training/testing windows of 257/21 across all pairs (see Table 7).

Table 6: Sharpe ratios calculated for the trading of all pairs executed at the training window of 257 days and the testing window of 21 days across all z-score thresholds, listed in descending order of the Sharpe ratio

Currency 1	**Currency 2**	**Sharpe ratio**
EURUSD	JPYUSD	0.46
GBPUSD	JPYUSD	0.46
NZDUSD	GBPUSD	0.39
EURUSD	GBPUSD	0.38
JPYUSD	EURUSD	0.35
GBPUSD	CADUSD	0.32
CHFUSD	NZDUSD	0.32
EURUSD	CADUSD	0.31
NZDUSD	CHFUSD	0.31
GBPUSD	EURUSD	0.29
NZDUSD	JPYUSD	0.27
JPYUSD	AUDUSD	0.23
NZDUSD	EURUSD	0.23
JPYUSD	NZDUSD	0.23
GBPUSD	CHFUSD	0.21
AUDUSD	JPYUSD	0.20
EURUSD	CHFUSD	0.20
CADUSD	GBPUSD	0.20
CHFUSD	GBPUSD	0.19
GBPUSD	NZDUSD	0.17
CHFUSD	JPYUSD	0.17
CHFUSD	EURUSD	0.17
JPYUSD	CHFUSD	0.16
AUDUSD	CADUSD	0.15
CADUSD	NZDUSD	0.10
NZDUSD	AUDUSD	0.09
AUDUSD	NZDUSD	0.08
NZDUSD	CADUSD	0.07
EURUSD	AUDUSD	0.05
AUDUSD	CHFUSD	0.03
CADUSD	CHFUSD	0.01
GBPUSD	AUDUSD	-0.002
JPYUSD	GBPUSD	-0.03
CHFUSD	CADUSD	-0.04
EURUSD	NZDUSD	-0.10
CADUSD	AUDUSD	-0.11
CADUSD	EURUSD	-0.12
CADUSD	JPYUSD	-0.13
AUDUSD	EURUSD	-0.22
CHFUSD	AUDUSD	-0.23
AUDUSD	GBPUSD	-0.27
JPYUSD	CADUSD	-0.43

The results presented in Table 6 show that across all windows combinations and all z-score thresholds, the Sharpe ratios range from -0.43 to 0.46. The highest Sharpe ratios were observed for the following pairs: EURUSD/JPYUSD, GBPUSD/JPYUSD, NZDUSD/GBPUSD, EURUSD/GBPUSD, and JPYUSD/ EURUSD.

Table 7: Sharpe ratios calculated for the portfolios based on all the z-score thresholds across all pairs and all windows combinations, listed in descending order of the Sharpe ratio

z-score threshold	**Sharpe ratio**
1	0.55
3	0.44
2	0.41

The results presented in Table 7 show that the values of the Sharpe ratios obtained for the strategies executed at different z-score thresholds and the fixed windows of 257/21 across all pairs do not vary significantly. The highest Sharpe ratio of 0.55 was observed at the z-score threshold of 1, while the lowest Sharpe ratio of 0.41 was obtained at the z-score threshold of 2.

5.2 Proposed Strategy

The optimal parameters identified in Section 5.1 can be employed as filtering criteria to develop a robust and efficient trading strategy. Thus, we proposed a strategy based on the following filters:

1. We fixed the training and testing windows at 257 and 21 days respectively.
2. We fixed the z-score threshold at 1.
3. We selected 20 best-performing pairs at this windows/z-score threshold setting.

Table 8 shows the performance of the resulting strategy.

Table 8: Aggregate business metrics for the strategy based on the pre-defined filtering parameters

Metrics	Value
Annualized return (%)	8.50
Annualized volatility (%)	9.40
Sharpe ratio	0.90
Sortino ratio	1.02
Calmar ratio	0.51
Maximum drawdown (%)	-16.57

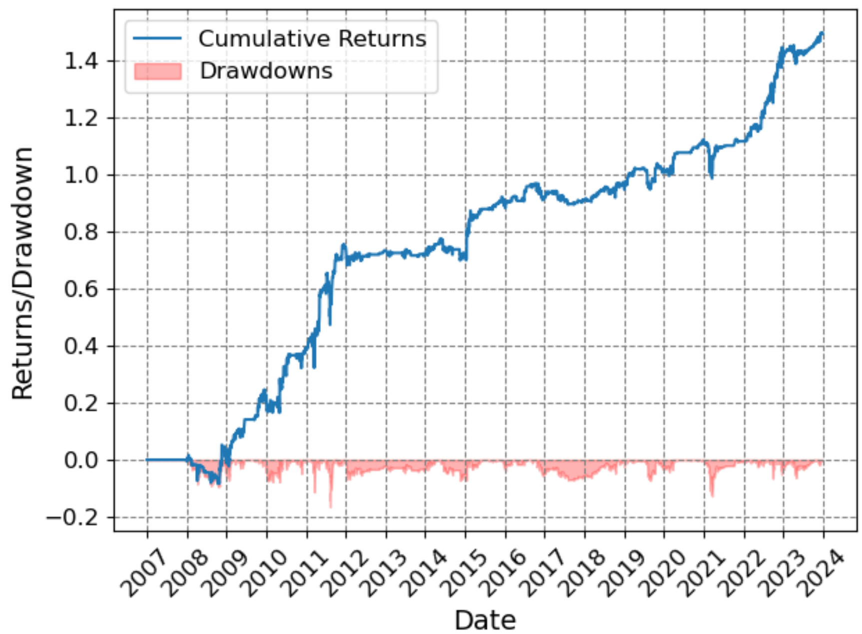

Figure 2: Cumulative returns obtained for the strategy based on the pre-defined filtering parameters

From the results presented in Table 8 and Figure 2, one can estimate the performance of the strategy based on the filtering parameters pre-defined by the cross-sectional analysis. The cumulative return curve exhibits a clear upward trend, indicating viable performance of the proposed strategy. The annualized return stands at 8.50%, accompanied by a slightly higher annualized volatility of 9.40%. The Sharpe and Sortino ratios are reported at viable values of 0.90 and 1.02, respectively, indicating viable risk-adjusted performance. Nevertheless, this strategy results in a Calmar ratio of 0.51, due to a maximum drawdown of -16.57%. Overall, the strategy based on the filtering parameters pre-defined by the cross-sectional analysis shows viable performance.

6 Conclusions

In summary, this study delves into the integration of the cointegration-based pair trading strategies for various currency pairs. By leveraging the stable co-movement indicated by cointegration, the research identifies trading opportunities based on deviations from long-term equilibrium relationships. Through a comprehensive analysis of trading strategies, encompassing different parameters such as testing/training windows and z-score thresholds, the study seeks to optimize pair trading strategies. Subsequently, we developed a robust and thus reliable strategy based on the filtering parameters pre-defined by the cross-sectional analysis. This strategy showed overall good performance with decent annualized returns at a reasonable level of risk. The proposed strategy resulted in values of Sharpe and Sortino ratios of 0.90 and 1.02, respectively.

7 References

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