KARMASTAT

Infectious Disease Modelling System

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Section 1: Core Parameters and Variables

1.1 Population Parameters

Total Population (N):

- Definition: The total number of individuals in the study population
- Units: Number of individuals
- Typical Range: 1,000 - 10,000,000
- Considerations:
    - Must remain constant during simulation (closed population)
    - Should reflect the actual community size being modelled
    - Affects computation of contact rates

Initial Cases (I_0):

- Definition: Number of infectious individuals at the start of simulation
- Units: Number of individuals
- Typical Range: 1 - 1000
- Considerations:
    - Should be < 1% of total population
    - Based on surveillance data when available
    - Affects early epidemic dynamics

Susceptible Population (S):

- Definition: Number of individuals who can become infected
- Calculation: $S(t) = N - (E(t) + I(t) + R(t))$
- Units: Number of individuals
- Properties:
    - Initially: $S(0) = N - I_0$
    - Always non-negative
    - Monotonically decreasing in absence of births

1.2 Disease Transmission Parameters

Basic Transmission Rate (β):

- Definition: Rate at which susceptible individuals become infected upon contact with infectious individuals
- Units: Contacts per person per day
- Typical Range: 0.1 - 2.0
- Calculation Components:
    - Contact rate (c): Average number of contacts per person per day

  - Transmission probability (p): Probability of infection given contact
  - $\beta = c * p$

Effective Transmission Rate ($\beta_e ff$):

- Definition: Actual transmission rate after considering interventions
- Calculation: $\beta_e ff = \beta * (1 - \mu) * (1 - \delta)$ where:
  - μ = mask effectiveness (0-1)
  - δ = social distancing effectiveness (0-1)
- Properties:
  - Always less than or equal to β
  - Time-dependent based on interventions
  - Affected by environmental factors

1.3 Time-Related Parameters

Incubation Period ($1/\sigma$):

- Definition: Time between exposure and becoming infectious
- Units: Days
- Typical Range: 1-14 days
- Properties:
  - Varies by pathogen
  - Follows probability distribution
  - Affects epidemic timing
- Rate Parameter: $\sigma = 1/$ (incubation period)
  - σ represents rate of progression from exposed to infectious
  - Units: per day

Infectious Period ($1/\gamma$):

- Definition: Duration during which an individual can transmit disease
- Units: Days
- Typical Range: 3-21 days
- Components:
  - Pre-symptomatic period
  - Symptomatic period
  - Recovery period
- Rate Parameter: $\gamma = 1/$ (infectious period)
  - γ represents recovery rate

  - Units: per day

Generation Time (Tg):

- Definition: Average time between primary case infection and secondary case infection
- Units: Days
- Calculation: $Tg = 1/\sigma + 1/(2\gamma)$
- Significance:
  - Key for estimating epidemic growth rate
  - Used in R_0 calculations
  - Affects intervention timing

1.4 Disease Progression Parameters

Recovery Rate (γ):

- Definition: Rate at which infectious individuals recover
- Units: per day
- Calculation: $\gamma = 1/$ (infectious period)
- Properties:
  - Constant for given disease
  - Affects duration of infectiousness
  - Influences healthcare resource needs

Mortality Rate (μ):

- Definition: Proportion of infected individuals who die from the disease
- Units: Dimensionless (proportion)
- Typical Range: 0.001 - 0.20 (0.1% - 20%)
- Considerations:
  - Age-dependent
  - Healthcare capacity dependent
  - Affects total fatalities

Section 2: Mathematical Foundations & Calculations

2.1 Basic Disease Spread Equations

SEIR Model Differential Equations:

Susceptible Population Change: $dS/dt = -\beta SI/N$

- Represents rate of new infections
- Proportional to current susceptible and infectious populations
- Normalized by total population

Exposed Population Change: $dE/dt = \beta SI/N - \sigma E$

- First term: new exposures
- Second term: progression to infectious state
- Balance determines exposed population size

Infectious Population Change: $dI/dt = \sigma E - \gamma I$

- First term: new infectious cases
- Second term: recoveries
- Determines active case load

Recovered Population Change: $dR/dt = \gamma I$

- Represents cumulative recoveries
- Monotonically increasing
- Includes immune individuals

2.2 Reproduction Numbers

Basic Reproduction Number (R_0):

- Definition: Average number of secondary cases from one case in fully susceptible population
- Calculation: $R_0 = \beta/\gamma$
- Properties:
    - $R_0 > 1$: Epidemic growth
    - $R_0 = 1$: Endemic equilibrium
    - $R_0 < 1$: Epidemic decline

Effective Reproduction Number (Rt):

- Definition: Actual reproduction number at time t
- Calculation: $Rt = R_0 * (S(t)/N) * (1 - \text{intervention\_effectiveness})$
- Components:
    - Susceptible fraction (S(t)/N)
    - Intervention effects
    - Current transmission rate

2.3 Healthcare Capacity Metrics

Healthcare System Load:

- Definition: Proportion of healthcare capacity currently utilized
- Calculation: $H(t) = I(t)/Hmax$ where:
  - $H(t)$ is healthcare utilization at time t
  - $I(t)$ is number of infectious cases
  - Hmax is maximum healthcare capacity
- Critical Thresholds:
  - $H(t) < 0.7$: Normal operation
  - $0.7 \leq H(t) < 0.9$: Stressed system
  - $H(t) \geq 0.9$: Critical overload

Peak Case Load:

- Definition: Maximum number of simultaneous active cases
- Calculation: $Imax = max(I(t))$
- Significance:
  - Determines required healthcare capacity
  - Guides intervention timing
  - Indicates system stress points

Healthcare Resource Allocation:

- Definition: Distribution of available healthcare resources
- Components:
  - Basic care resources
  - Critical care capacity
  - Medical staff availability
- Utilization Rate: U(t) = (Active Cases)/ (Available Resources)

2.4 Intervention Effect Calculations

Non-Pharmaceutical Interventions (NPIs):

Social Distancing Effect:

- Definition: Reduction in contact rate due to physical distancing
- Calculation: $\beta eff = \beta (1 - \delta)$ where:
  - δ is social distancing effectiveness (0-1)
  - βeff is effective transmission rate
- Implementation Levels:
  - Mild: $\delta = 0.2\text{-}0.4$

  - Moderate: $\delta = 0.4\text{-}0.6$
  - Strict: $\delta = 0.6\text{-}0.8$

Mask Usage Effect:

- Definition: Reduction in transmission probability due to mask wearing
- Calculation: $\beta_{eff} = \beta (1 - \mu m)$ where:
  - μ is mask effectiveness (0-0.5)
  - m is mask compliance rate (0-1)
- Effectiveness Factors:
  - Mask type efficiency
  - Population compliance
  - Proper usage

Combined Intervention Effects:

- Definition: Total impact of multiple interventions
- Calculation: $\beta_{eff} = \beta \prod (1 - \varepsilon_i)$ where:
  - ε_i represents effectiveness of intervention i
- Interaction Considerations:
  - Multiplicative effects
  - Diminishing returns
  - Implementation feasibility

Section 3: Implementation Examples

3.1 Basic Disease Scenarios

Example 1: Respiratory Disease Outbreak Initial Conditions:

- Population: 100,000
- Initial Cases: 100
- R_0: 2.5
- Incubation Period: 5 days
- Infectious Period: 7 days

Calculations:

1. Basic Parameters:
    - $\beta = R_0\gamma = 2.5 * (1/7) = 0.357$
    - $\sigma = 1/5 = 0.2$
    - $\gamma = 1/7 = 0.143$
2. Early Growth Phase: Day 1:
    - S (1) = 99,900
    - E (1) = 100
    - I (1) = 0
    - R (1) = 0

Day 2:

- New Exposed = (0.357 * 99,900 * 0) / 100,000 = 0
- New Infectious = 0.2 * 100 = 20
- New Recovered = 0

3. Peak Estimation:
    - Expected Peak Time: $\sim\ln(N/I_0)/r$ where $r = \beta - \gamma$
    - Healthcare Load = I(t)/Hmax

Example 2: Intervention Timing Scenario:

- Base R_0: 2.5
- Social Distancing: 50% effective
- Mask Usage: 70% compliance, 50% effectiveness

Calculations:

1. Without Interventions:
    - $\beta = 0.357$
    - Peak Cases $\approx 0.3N$
2. With Interventions:

- β_{eff} = 0.357 * (1-0.5) * (1-0.35) = 0.116
- New R_0 = 0.116/0.143 = 0.81
- Expected Peak Reduction ≈ 65%

3.2 Advanced Implementation Scenarios

Example 3: Seasonal Variation Effects Scenario Parameters:

- Base Transmission Rate (β_0): 0.3
- Seasonal Amplitude (α): 0.2
- Annual Period: 365 days

Seasonal Transmission Calculation: $\beta(t) = \beta_0 [1 + \alpha \sin(2\pi t/365)]$

Sample Calculations: Summer (t = 182 days): β (182) = 0.3[1 + 0.2 sin(2π*182/365)] = 0.3[1 - 0.2] = 0.24

Winter (t = 365 days): β (365) = 0.3[1 + 0.2 sin(2π*365/365)] = 0.3[1 + 0] = 0.3

Example 4: Healthcare System Analysis Initial Conditions:

- Population: 500,000
- Healthcare Capacity: 1,000 beds
- Average Hospital Stay: 10 days
- Critical Care Requirement: 15% of cases

Healthcare Load Calculations:

1. Daily New Hospitalizations: H(t) = I(t) * hospitalization rate
2. Bed Occupancy: B(t) = Σ(H(t-n) for n = 0 to 10)
3. Critical Threshold Analysis: Alert Level 1: B(t) > 0.7 * capacity Alert Level 2: B(t) > 0.85 * capacity Alert Level 3: B(t) > capacity

Section 4: Practical Applications

4.1 Real-World Case Studies

Case Study 1: Urban Epidemic Control Setting:

- Metropolitan area
- Population: 2 million
- High population density
- Multiple intervention points

Analysis Steps:

1. Basic Reproduction Number Estimation
    - Contact tracing data analysis
    - Early growth rate calculation
    - $R_0 = r/\gamma + r/\sigma + 1$ where r is exponential growth rate
2. Intervention Timing Optimization
    - Critical threshold: I_c = Healthcare Capacity * 0.8
    - Time to threshold: $t_c = \ln(I_c/I_0)/r$
    - Recommended intervention start: $t = t_c - 14$ days
3. Resource Allocation
    - Daily resource need: R(t) = I(t) * resource_per_case
    - Peak resource calculation: $R_{max} = \max(R(t)) *$ buffer factor where buffer factor = 1.2

Case Study 2: School Reopening Analysis Parameters:

- Student population: 10,000
- Staff population: 1,000
- Contact patterns:
    - Student-Student: 15 contacts/day
    - Student-Staff: 4 contacts/day
    - Staff-Staff: 8 contacts/day

Risk Assessment:

1. Contact Matrix Construction: [15 4] [4 8]
2. Group-Specific R_0 Calculation: $R_{0,ss} = \beta_{ss} * C_s * D$ where:
    - β_{ss} is student-student transmission
    - C_s is average student contacts
    - D is infectious duration
3. Mitigation Strategy Evaluation:
    - Classroom size reduction: Contact reduction by 40%

- Hybrid scheduling: Population present reduced by 50%
- Ventilation improvements: Transmission reduction by 30%

4.2 Parameter Estimation from Data

Method 1: Growth Rate Analysis

1. Early Epidemic Growth: $N(t) = N_0 e^{rt}$ where:
   - N(t) is cumulative cases
   - r is growth rate
   - t is time in days
2. R_0 Estimation: $R_0 = 1 + rT_c$ where:
   - T_c is generation time
   - r is exponential growth rate

Method 2: Maximum Likelihood Estimation

1. Likelihood Function: $L(\beta, \gamma|data) = \Pi\ P(data|\beta, \gamma)$
2. Parameter Optimization: $\{\beta^*, \gamma^*\} = \text{argmax}\ L(\beta, \gamma|data)$

4.3 Results Interpretation

Key Metric Analysis:

1. Epidemic Peak Characteristics
   - Height: Maximum daily cases
   - Timing: Days to peak
   - Duration: Width at half-maximum
2. Intervention Impact Assessment
   - Relative reduction in peak height
   - Delay in peak timing
   - Change in total case count
3. Healthcare System Impact
   - Duration of capacity exceedance
   - Maximum capacity utilization
   - Resource consumption patterns

4.4 Model Limitations and Considerations

Model Assumptions:

1. Population Mixing Patterns
   - Homogeneous mixing assumed
   - No spatial clustering effects
   - Uniform susceptibility within groups

2. Parameter Stability
    - Constant transmission rates
    - Fixed incubation periods
    - Uniform recovery rates
3. Population Characteristics
    - Closed population (no births/deaths)
    - No age structure considered
    - No immigration/emigration

Practical Limitations:

1. Data Requirements
    - Quality of surveillance data
    - Reporting delays
    - Case definition changes
2. Parameter Uncertainty
    - Transmission rate variability
    - Intervention effectiveness
    - Healthcare capacity fluctuations

4.5 Best Practices for Model Application

Data Collection:

1. Essential Data Elements
    - Daily case counts
    - Testing numbers
    - Healthcare utilization
    - Intervention timing
2. Data Quality Assurance
    - Consistency checks
    - Missing data handling
    - Outlier identification

Model Validation:

1. Historical Data Comparison
    - Pattern matching
    - Peak timing accuracy
    - Magnitude alignment
2. Sensitivity Analysis

- Parameter range testing
- Scenario comparison
- Uncertainty quantification

Section 5: Bibliography

5.1 Core Epidemiological Theory

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- Result presentation
- Interactive visualization

Notes for Users:

1. **Regular updates to this guide will be provided as new research emerges**
2. **Additional case studies will be added based on real-world applications**
3. **Parameter updates will reflect latest epidemiological findings**
4. **New features will be documented as they are implemented**

- *Made with dedication by Karmayogi Last Updated: January 2025*