Section 1: Algebra

1.1 a,b,c

1.2 $p^{\frac{n(n-1)}{2}}$

1.3 A subgroup of order 125. Example: 3×3 upper triangular matrices with 1 on the diagonal.

1.4 9

1.5 a

1.6 $\frac{p(p-1)}{2}$

1.7 a,b

1.8 Any two linearly independent vectors of the form $(u, u, w, u + w)$. Example: $\{(1, 1, 0, 1), (1, 1, -1, 0)\}$.

1.9 6

1.10 a,b,c

Section 2: Analysis

2.1 $\frac{1}{e}$

2.2 a,b

2.3 a,c

2.4 a,c

2.5 a,b,c

2.6 a,b

2.7 a,c

2.8 b,c

2.9 a.
$$\sum_{n=0}^{\infty} \frac{1}{z^{n+1}} + \sum_{n=0}^{\infty} \frac{z^n}{2^{n+1}}$$

b. $2\pi i$

2.10 None

Section 3: Topology

3.1 b,c

3.2 a,c

3.3 c

3.4 a,c

3.5 b

3.6 a,b,c

3.7 b,c

3.8 a,b,c

3.9 a,b,c

3.10 a,b,c

Section 4: Calculus & Differential Equations

4.1 $\frac{1}{2}(1 - \log 2)$

4.2 $\frac{\pi}{6\sqrt{10}}$

4.3 $\frac{12\pi}{5}a^6$

4.4 $\frac{2}{3}(e^9 - e^{-9})$

4.5 $2 + \sqrt{2}$.

4.6 $\tan^{-1}(\frac{y}{x}) = \log\sqrt{x^2 + y^2} + c$

4.7
$$\begin{aligned} x(t) &= 2c_1 e^{4t} + c_2 e^{-t} \\ y(t) &= 3c_1 e^{4t} - c_2 e^{-t} \end{aligned}$$

4.8 $x - \sin x$

4.9 $(\lambda_n, u_n), \ n \in \mathbb{N} \cup \{0\}$, where
$$\lambda_n = (2n+1)^2 \frac{\pi^2}{4}, \ u_n(x) = \cos(2n+1)\frac{\pi}{2}x$$

4.10 $u(x, y) = \frac{R^2 - x^2 - y^2}{4}$

Section 5: Miscellaneous

5.1 $(n+1)\frac{(2n)!}{(n!)^2}$

5.2 $60k + 59, k \geq 0$

5.3 $\frac{e}{2} + \frac{2}{e}$

5.4 ± 5

5.5 $2x - 4y + 3z + 8 = 0$

5.6 $\frac{1}{6}(k+1)(4k^2 + 5k + 6)$

5.7 12 sq. units

5.8 1

5.9 a,b,c

5.10 120

Note: Please accept any correct equivalent form of the answers.