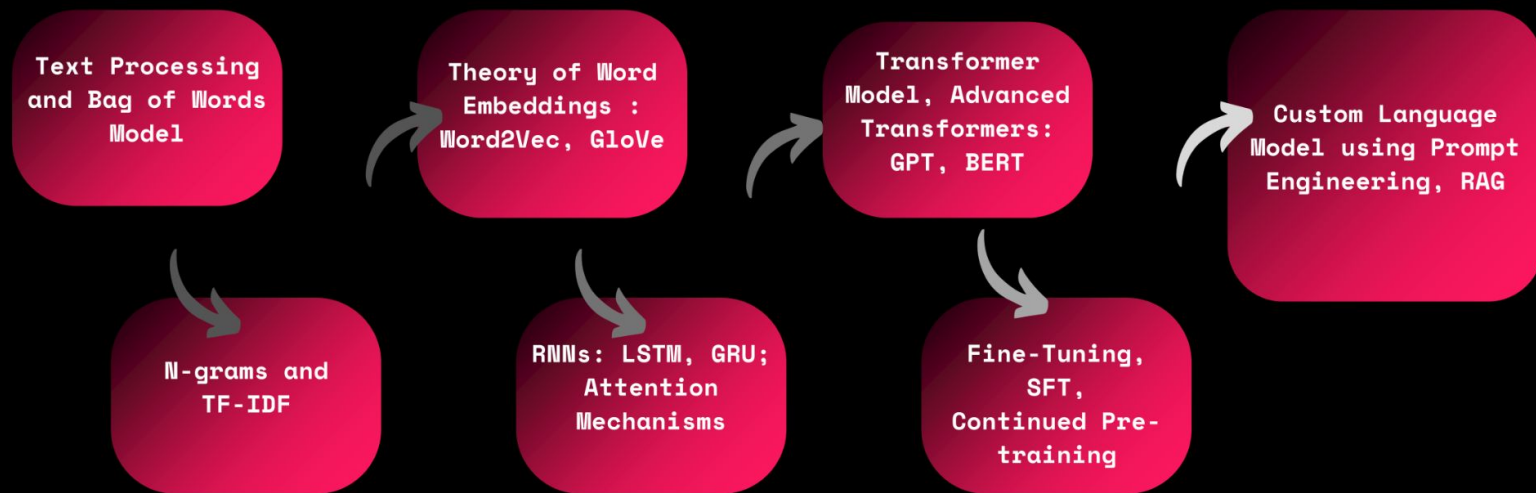


Sequence Modeling

Week #4 - 2 Oct '24

NLP Lab

NLP Lab Project : Build your own GPT



1. Language Models Recap

1.1 Language Models

- Models that assign a probability to each possible next word.
- Language models can also assign a probability to an entire sentence, telling us that the following sequence has a much higher probability of appearing in a text.
- **Goal** : Compute the probability of a sentence or sequence of words.

$$P(W) = P(w_1, w_2, w_3, w_4, w_5 \dots w_n)$$

1.2 Discriminative vs Generative Models

- **Discriminative model:** a model that calculates the probability of a latent trait given the data.

$$P(Y | X) \text{ (Conditional)}$$

- **Generative model:** a model that calculates the probability of the input data itself.

$$P(X)$$


stand-alone

$$P(X, Y)$$

joint

- Discriminative language models are designed to directly learn the boundary between different classes or outputs, typically in tasks involving classification or structured prediction.

1.3 Autoregressive Language Models

$$P(X) = \prod_{i=1}^I P(x_i \mid x_1, \dots, x_{i-1})$$


- The big problem : How do we predict

$$P(x_i \mid x_1, x_2, \dots, x_{i-1})$$

1.4 Examples of Language Models

- Count-based Language Model

- Independence assumption: • Count-based maximum-likelihood estimation:

$$P(x_i | x_1, \dots, x_{i-1}) \approx P(x_i)$$

- Count-based maximum-likelihood estimation.

$$P_{\text{MLE}}(x_i) = \frac{c_{\text{train}}(x_i)}{\sum_{\tilde{x}} c_{\text{train}}(\tilde{x})}$$

- Higher-order n-gram Models

$$P_{ML}(x_i | x_{i-n+1}, \dots, x_{i-1}) := \frac{c(x_{i-n+1}, \dots, x_i)}{c(x_{i-n+1}, \dots, x_{i-1})}$$

- Featurized Log-Linear Models

1.5 Problems & Solutions

- Cannot share strength among **similar words**

she bought a car she bought a bicycle
she purchased a car she purchased a bicycle

→ solution: class based language models

- Cannot condition on context with **intervening words**

Dr. Jane Smith Dr. Gertrude Smith

→ solution: skip-gram language models

- Cannot handle **long-distance dependencies**

for tennis class he wanted to buy his own racquet
for programming class he wanted to buy his own computer

→ solution: cache, trigger, topic, syntactic models, etc.

2. Sequence Modeling

2.1 NLP and Sequential Data

- NLP is full of sequential data :
 - Characters in words
 - Words in sentences
 - Sentences in discourse
 -

2.2 Long-distance Dependencies in Language

- Agreement in number, gender, etc.

He does not have very much confidence in **himself**.

She does not have very much confidence in **herself**.

- Selectional preference.

The **reign** has lasted as long as the life of the **queen**.

The **rain** has lasted as long as the life of the **clouds**.

2.3 Can be Complicated !

- What is the referent of “**it**”?

The trophy would not fit in the brown suitcase because it was too **big**.

Trophy

The trophy would not fit in the brown suitcase because it was too **small**.

Suitcase

2.4 Types of Sequential Prediction

Unconditioned vs Conditioned

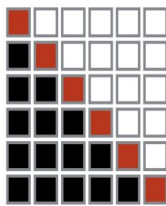
- **Unconditioned Prediction:** Predict the probability of a single variable $P(X)$.
- **Conditioned Prediction:** Predict the probability of an output variable given an input $P(Y | X)$.

2.4 Types of Unconditioned Prediction

Left-to-right Autoregressive Prediction

$$P(X) = \prod_{i=1}^{|X|} P(x_i | x_1, \dots, x_{i-1})$$

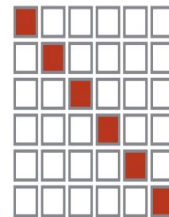
(e.g. RNN or Transformer LM)



Independent Prediction

$$P(X) = \prod_{i=1}^{|X|} P(x_i)$$

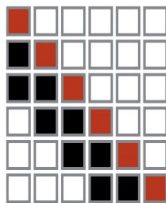
(e.g. unigram model)



Left-to-right Markov Chain (order n-1)

$$P(X) = \prod_{i=1}^{|X|} P(x_i | x_{i-n+1}, \dots, x_{i-1})$$

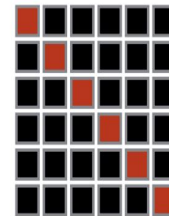
(e.g. n-gram LM, feed-forward LM)



Bidirectional Prediction

$$P(X) \neq \prod_{i=1}^{|X|} P(x_i | x_{\neq i})$$

(e.g. masked language model)



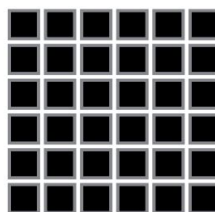
2.4 Types of Conditioned Prediction

Autoregressive Conditioned Prediction

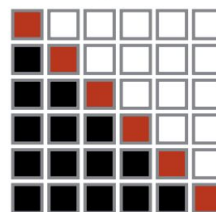
$$P(Y|X) = \prod_{i=1}^{|Y|} P(y_i|X, y_1, \dots, y_{i-1})$$

(e.g. seq2seq model)

Source X



Target Y

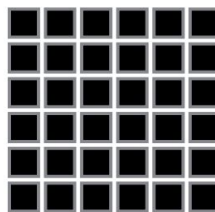


Non-autoregressive Conditioned Prediction

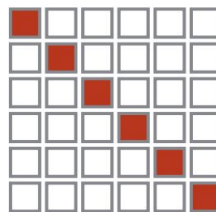
$$P(Y|X) = \prod_{i=1}^{|Y|} P(y_i|X)$$

(e.g. sequence labeling, non-autoregressive MT)

Source X

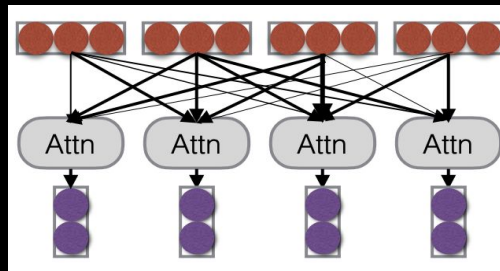
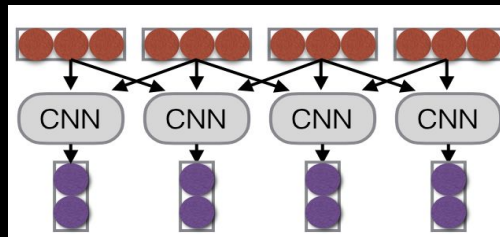
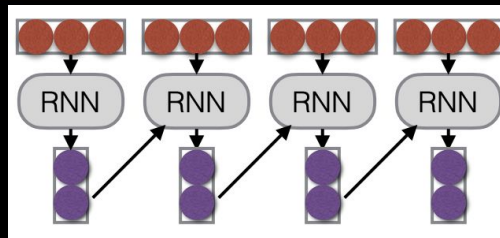


Target Y



2.5 Three Major Types of Sequence Models

- **Recurrence:** Condition representations on an encoding of the history.
- **Convolution:** Condition representations on local context.
- **Attention:** Condition representations on a weighted average of all tokens

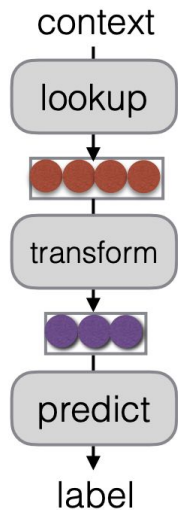


3. Recurrent Neural Networks

3.1 Recurrent Neural Networks

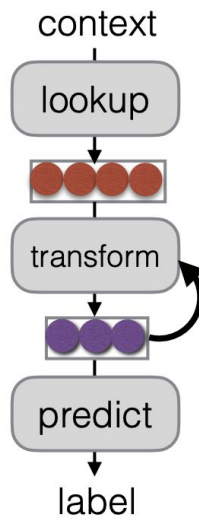
- Tools to “remember” information.

Feed-forward NN



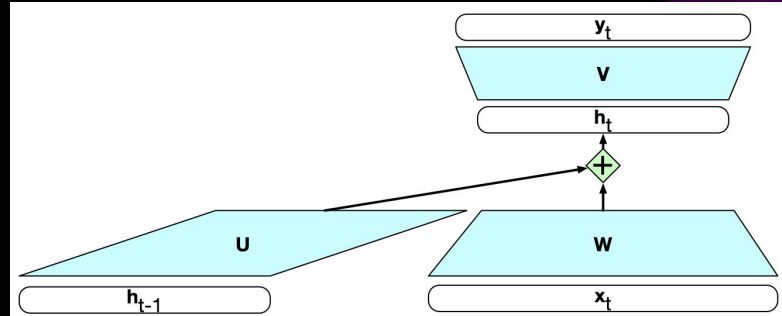
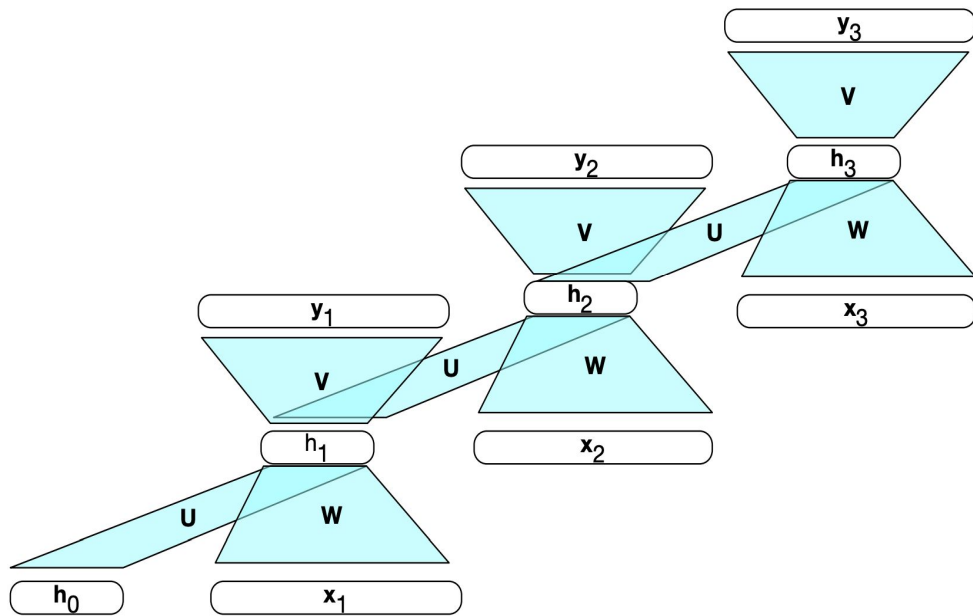
$$h_t = f(W_x x_t + b)$$

Recurrent NN



$$h_t = f(W_h h_{t-1} + W_x x_t + b)$$

3.2 Unrolling in Time

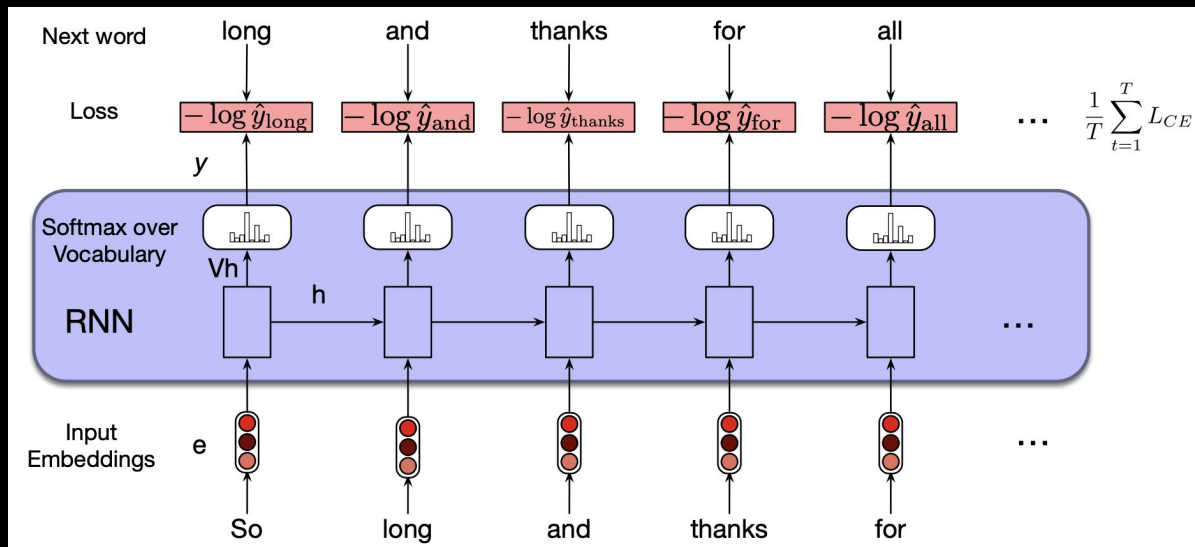


function FORWARDRNN(x , $network$) **returns** output sequence y

```

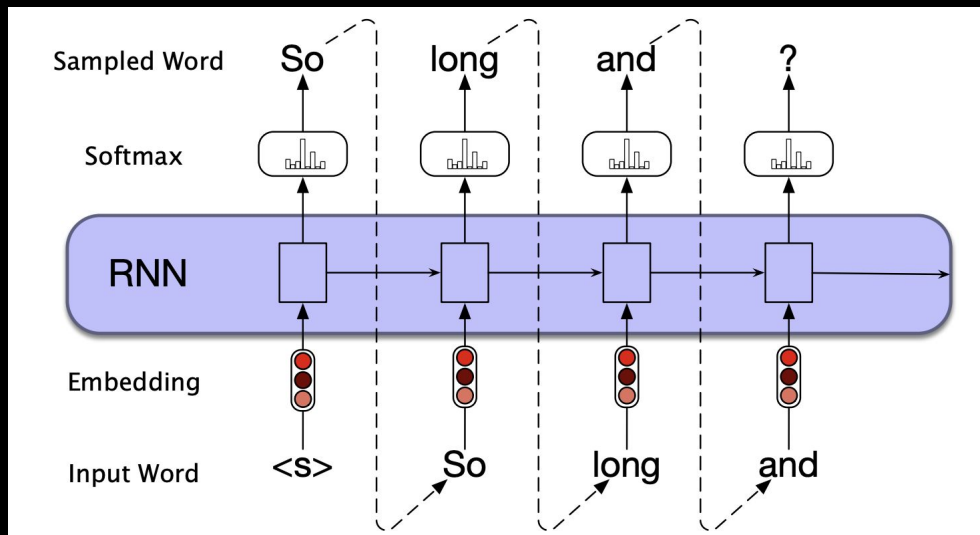
 $h_0 \leftarrow 0$ 
for  $i \leftarrow 1$  to LENGTH( $x$ ) do
     $h_i \leftarrow g(Uh_{i-1} + Wx_i)$ 
     $y_i \leftarrow f(Vh_i)$ 
return  $y$ 
    
```

3.3 Training RNN Language Model



- **Self-Supervision** : the natural sequence of words is its own supervision!
- **Teacher Forcing** : model is always given the correct history sequence to predict the next word (rather than feeding the model its best case from the previous time step).

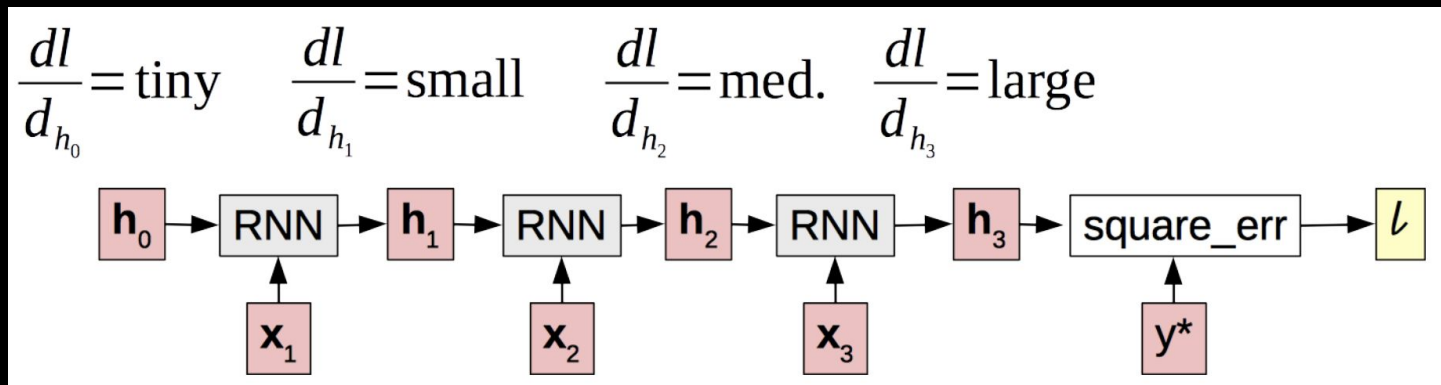
3.4 Generation with RNN-Based Language Models



- **Autoregressive Generation** : Sampling words conditioned on previous choices until a predetermined length is reached, or an end of sequence token is generated.
- The key to these approaches is to prime the generation component with an appropriate context. That is, instead of simply using <s> to get things started we can provide a richer task-appropriate context.

3.5 Vanishing Gradient

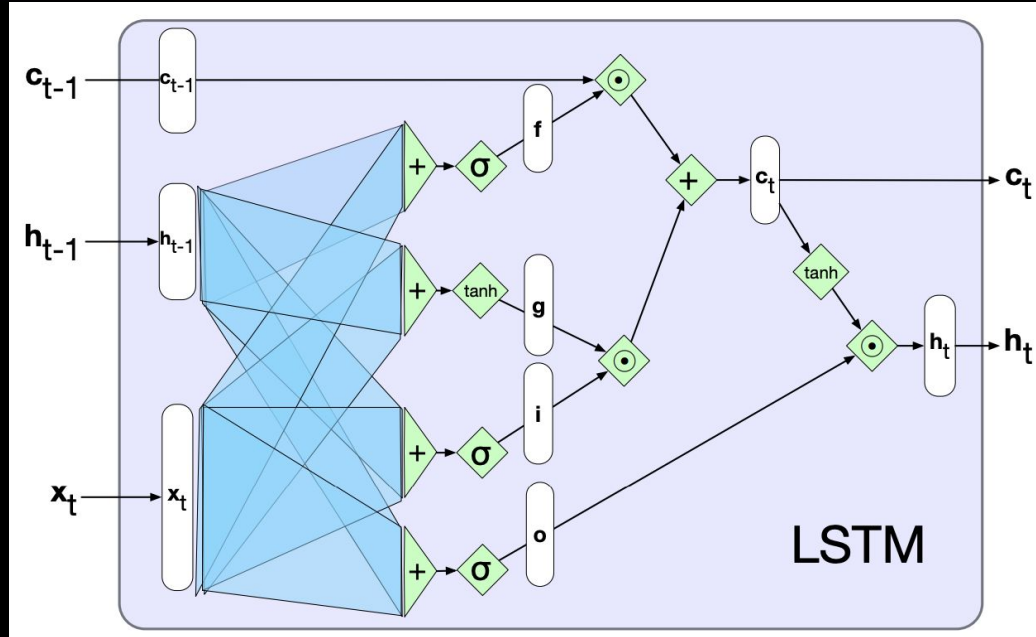
- Gradients decrease as they get pushed back.



- Why? “Squashed” by non-linearities or small weights in matrices.

4. Long Short Term Memory

4.1 Basic Idea of LSTM



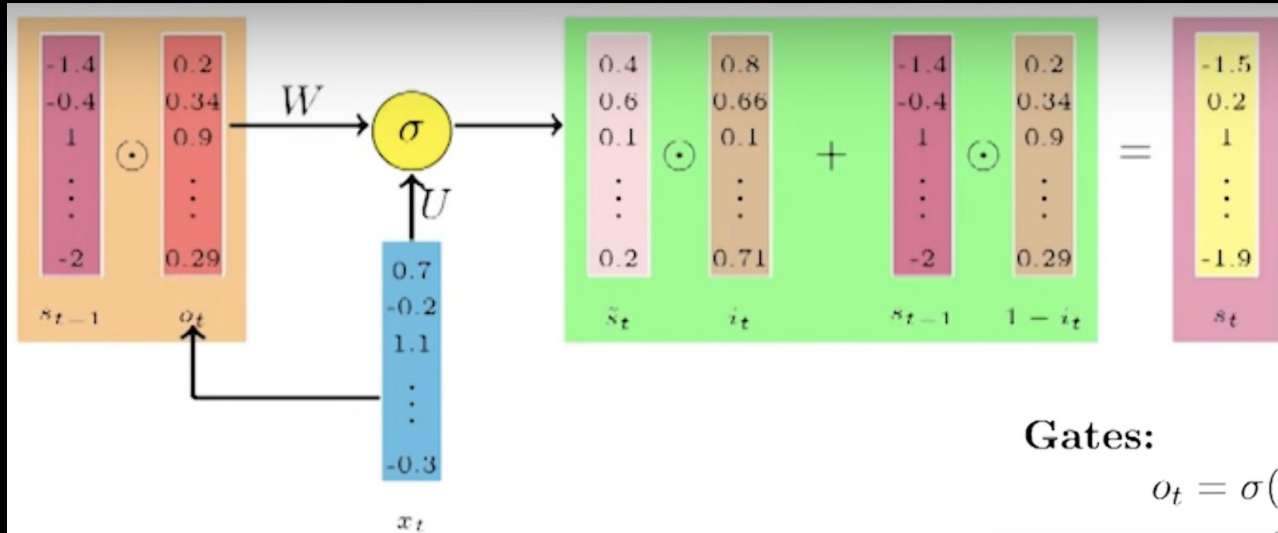
- **Basic idea:** make additive connections between time steps.
- Gates control the flow of the information and the gradients.

4.1 Equations of Gates in LSTM

- Forget Gate : $\mathbf{f}_t = \sigma(\mathbf{U}_f \mathbf{h}_{t-1} + \mathbf{W}_f \mathbf{x}_t)$
- Update Gate : $\mathbf{g}_t = \tanh(\mathbf{U}_g \mathbf{h}_{t-1} + \mathbf{W}_g \mathbf{x}_t)$
- Input/Add Gate : $\mathbf{i}_t = \sigma(\mathbf{U}_i \mathbf{h}_{t-1} + \mathbf{W}_i \mathbf{x}_t)$
- Output Gate : $\mathbf{o}_t = \sigma(\mathbf{U}_o \mathbf{h}_{t-1} + \mathbf{W}_o \mathbf{x}_t)$

5. Gated Recurrent Unit

4.1 GRU Architecture



Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s}_t = \sigma(W(s_t \odot s_{t-1}) + U x_t + b)$$

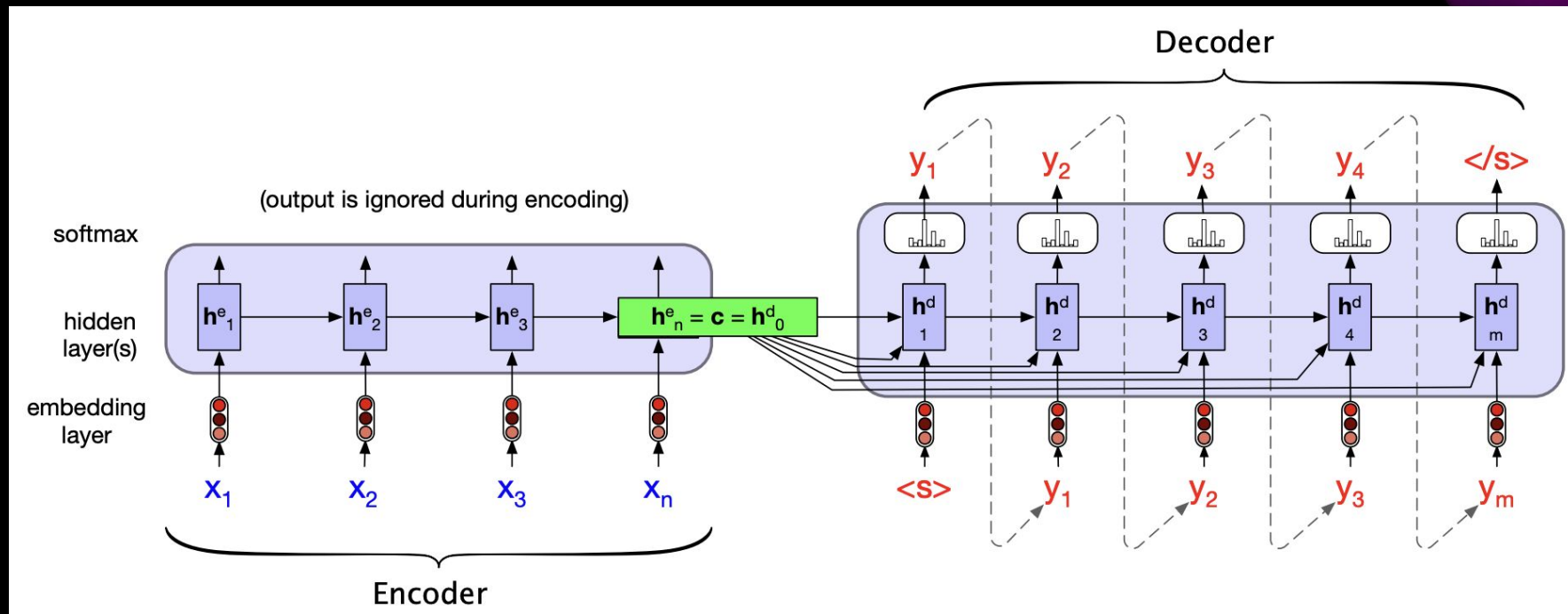
$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s}_t$$

6. Encoder-Decoder

6.1 What is Encoder-Decoder ?

- Encoder-decoder networks, sometimes called sequence-to-sequence networks, encoder-decoder are models capable of generating contextually appropriate, arbitrary length, output sequences given an input sequence.
- The key idea underlying these networks is the use of an encoder network that takes an input sequence and creates a contextualized representation of it, often called the context.
- This representation is then passed to a decoder which generates a task-specific output sequence.

6.2 Encoder-Decoder Models with RNN



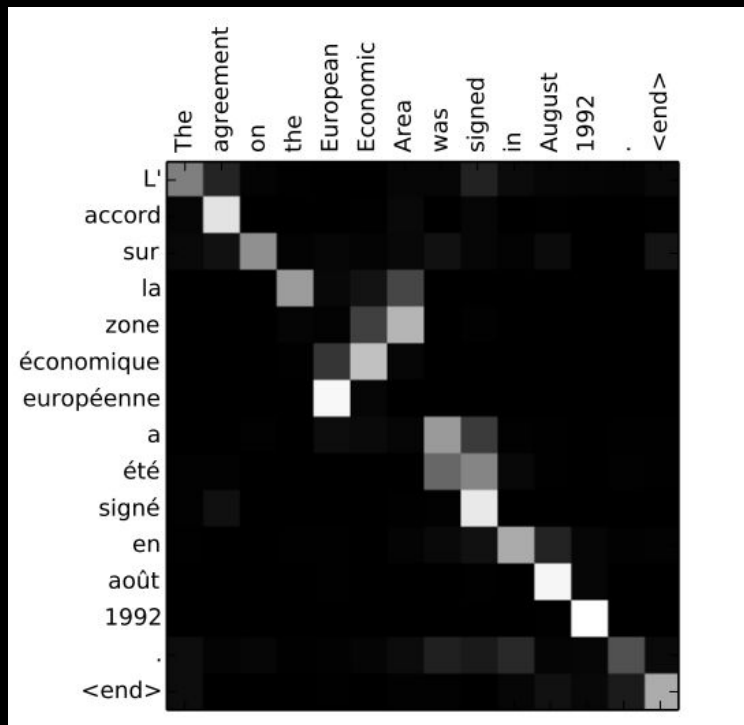
7. Attention

7.1 Basic Idea of Attention

- A mechanism that helps models focus on relevant parts of the input when making predictions.
- Overcomes limitations of traditional sequence models (like RNNs) that struggle with long dependencies.
- Two main types of Attention :
 - Cross-Attention
 - Self-Attention

7.2 Cross Attention

- Each element in a sequence attends to elements of another sequence.



7.3 Cross Attention : Bahdanau et al. 2015

- Encoder-Decoder Model:
 - The encoder processes the input sequence and generates hidden states.
 - The attention mechanism computes a weighted sum of these hidden states to form a context vector.
- Key Components:
 - Encoder Hidden States (h_1, h_2, \dots, h_n): Represent the input sequence.
 - Decoder Hidden State (s_t): Represents the decoder's state at time step t .
 - Alignment Score (e_{ti}): Measures the relevance between the decoder state at time t and the encoder hidden state h_i .
- Attention Weights (α_{ti}):
 - Computed using the alignment scores.
 - Determine how much "attention" should be paid to each input word.

7.3 Bahdanau Attention : Step-by-Step

- Alignment Scores:
 - Compute alignment scores between the current decoder hidden state and each encoder hidden state.
 - The alignment model is parametrized as a feedforward neural network and jointly trained with the remaining system components.

$$e_{ti} = v^T \tanh(W_1 s_{t-1} + W_2 h_i)$$

- Softmax Over Alignment Scores:
 - These attention weights tell how much the model should focus on each encoder hidden state.
- Context Vector (c_t):
 - Compute a weighted sum of the encoder hidden states based on the attention weights

$$c_t = \sum_{i=1}^n \alpha_{ti} h_i$$

7.4 Luong Attention Mechanism

- Encoder Hidden States (h_1, h_2, \dots, h_n): Represent the input sequence.
- Decoder Hidden State (s_t): Represents the decoder's state at time step t .
- Alignment Scores:
 - Compute alignment scores by taking the dot product of the decoder hidden state s_t and each encoder hidden state h_i .

$$e_{ti} = s_t \cdot h_i$$

- Attention Weights (α_i):
 - Apply softmax to the alignment scores to get attention weights
- Context Vector (c_t):
 - Compute a weighted sum of the encoder hidden states based on the attention weights.

$$c_t = \sum_{i=1}^n \alpha_{ti} h_i$$

Test you knowledge!

