AP Calculus | Name: ____________________

Review Unit 1: Limits and Continuity

SHOW ALL WORK!

On the test, you will be required to show all of your work. **This review is not comprehensive**. Please look back over your notes, your homework, and your quizzes to help you study for the test. ***This review will count as part of your grade!***

Topics to study:

1. Limits graphically and numerically (using a table)
2. Limit Properties
3. Techniques for finding limits
    a. Trigonometry special limits
    b. factoring
    c. conjugate
    d. least common denominator
4. Infinite limits and vertical asymptotes
5. Limits at infinity and horizontal asymptotes
6. Continuity
    a. definition
    b. problems using continuity
7. Intermediate Value Theorem

Use the graphs to find each limit, if it exists for problems 1-10.

1. $\lim_{x\to 2}(f(x)+g(x))$

2. $\lim_{x\to -1}\dfrac{f(x)}{g(x)}$

3. $\lim_{x\to 1}\sqrt{3+f(x)}$

4. $\lim_{x\to 2}x^3 f(x)$

5. $\lim_{x\to 1}g(x)$

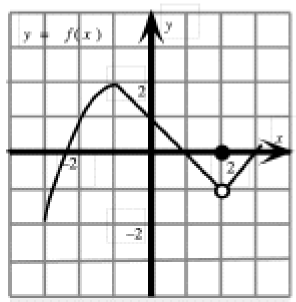
6. $\lim_{x\to 1^-}g(x)$

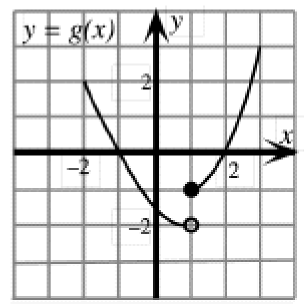
7. $\lim_{x\to 2^+}\dfrac{f(x)}{g(x)}$

8. $\lim_{x\to 2}f(g(x))$

9. $\lim_{x\to -1}2f(x)g(x)$

10. $\lim_{x\to 2}f(x)$





Find each limit, if it exists.

Add a basic substitution

11. $\lim_{x\to 27}\sqrt[3]{x^2}$

12. $\lim_{x\to 0}f(x)$ if $f(x)=\begin{cases}\dfrac{2x^2-3x}{x}, & x<0\\ x-3, & x\geq 0\end{cases}$

13. $\lim_{x\to 0}\dfrac{\sin(3x)}{x}$

14. $\lim_{x\to 2}\sec\left(\dfrac{\pi x}{3}\right)$

15. $\lim_{x\to 0}\dfrac{2x+\sin 3x}{x}$

16. $\lim_{x\to -4^-}\dfrac{3}{(x+4)^2}$

17. $\lim_{x\to -2}\dfrac{x^2-4x-12}{x^2-4}$

18. $\lim_{x\to 0}\dfrac{\sin x-\sin x\cos x}{3x^2}$

19. $\lim_{x\to -2}(x-6)^{2/3}$

20. Sketch a graph for each function and find all vertical asymptotes. Using the calculus definition, how do you know these are the vertical asymptotes?

(A) $g(x)=\dfrac{3}{3x-6}$ (B) $g(x)=\dfrac{-1}{(x+2)^2}$ (C) $g(x)=\dfrac{x^2+5x-24}{2x^2-5x-3}$

(no graph needed for part c)

21. Calculator allowed. Sketch the graph from your calculator.

(A) $\lim\limits_{x\to\pi^+}\cot x=$ (B) $\lim\limits_{x\to\frac{\pi}{2}^-}\sec x=$

22. (A) $\lim\limits_{x\to\infty}\dfrac{3x^2-x+5}{x^2-4}$ (B) $\lim\limits_{x\to-\infty}\dfrac{3x^2-x+5}{x^2-4}$ (C) identify all horizontal asymptotes, if any for $f(x)=\dfrac{3x^2-x+5}{x^2-4}$

23. (A) $\lim\limits_{x\to\infty}\dfrac{6x^3-4x+8}{x^2-5}$ (B) $\lim\limits_{x\to-\infty}\dfrac{6x^3-4x+8}{x^2-5}$ (C) identify all horizontal asymptotes, if any for $g(x)=\dfrac{6x^3-4x+8}{x^2-5}$

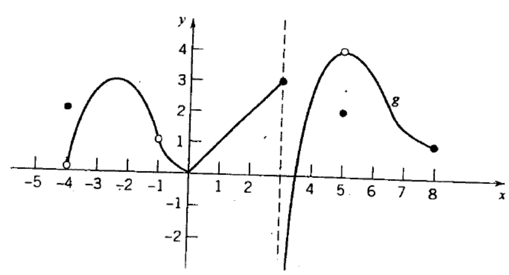
24. (A) $\lim\limits_{x\to\infty}\dfrac{4x^2+6x+1}{x^3-4}$ (B) $\lim\limits_{x\to-\infty}\dfrac{4x^2+6x+1}{x^3-4}$ (C) identify all horizontal asymptotes, if any for $g(x)=\dfrac{4x^2+6x+1}{x^3-4}$

25. (A) Explain what it means to say that

$$\lim_{x\to1^-}f(x)=3 \text{ and } \lim_{x\to1^+}f(x)=7.$$

(B) In this situation, it is possible that $\lim\limits_{x\to1}f(x)$ exists?

Use the graph of g(x) to answer the following questions.



26. (A) $\lim\limits_{x\to-1^-}g(x)=$ (B) $\lim\limits_{x\to-1^+}g(x)=$ (C) $g(-1)=$

(D) Is g continuous at x = -1? Explain why or why not.

27. (A) $\lim\limits_{x\to3^-}g(x)=$ (B) $\lim\limits_{x\to3^+}g(x)=$ (C) $g(3)=$

(D) Is g continuous at x = 3? Explain why or why not.

28. (A) $\lim\limits_{x\to5^-}g(x)=$ (B) $\lim\limits_{x\to5^+}g(x)=$ (C) $g(5)=$

(D) Is g continuous at x = 5? Explain why or why not.

For problems 29-31, determine the value of *a, b* or *c* such that each function is continuous on $(-\infty,\infty)$

29. $f(x)=\begin{cases}3x-2, x<5\\ x^2+c, x\ge5\end{cases}$

30. $f(x)=\begin{cases}x^2-1, x<3\\ 2cx, 3\le x\le6\\ a(x-1)^2-4, x>6\end{cases}$

31. $f(x)=\begin{cases}ax+3, x>5\\ 8, x=5\\ x^2+bx+1, x<5\end{cases}$

32. Find all horizontal asymptotes for the function $f(x)=\dfrac{4-7^x}{1+7^x}$.

33. If the function f is continuous for all real numbers and if $f(x)=\dfrac{x^2-11x-42}{x+3}$ when $x \neq -3$, then find the value of $f(-3)$.

x	4.75	4.9	4.99	4.999	5	5.001	5.01	5.1	5.25
f(x)	-2.5	-2.75	-2.92	-2.98	Undefined	-3.02	-3.03	-3.25	-3.5

34. Using the table , find $\lim_{x \to 5} f(x)$.

35. Let f be a function defined by $f(x)=\begin{cases} x^2-2x, \text{ for } x \leq 6 \\ 2x+12, \text{ for } x > 6 \end{cases}$. Show that f is continuous at $x=6$.

36. A function f is continuous on $-2 \leq x \leq 2$ and some of the values of f are shown in the table.

x	-2	0	2
f(x)	3	b	4

If f has only one zero, z , on the closed interval $-2 \leq x \leq 2$ and $z \neq 0$, then a possible value of b is

(A) -3 (B) -2 (C) -1 (D) 0 (E) 1

37. The rate, in calories per minute, at which a person using an exercise machine burns calories is modeled by the function f , where t is in minutes and $f(t)$ is given in calories per minute. Selected values of f are given in the table.

t (minutes)	0	4	12	16	20	24
$f(t)$ Calories per minute	0	9	7	15	15	3

Is there a time t , $12 \leq t \leq 16$, at which $f(t)=10$? Justify your answer.