2.2 Working with Symmetric Integrals + AVG(f)

Name: ______________________________

Recall that $f(x)$ is even if $f(-x) = f(x)$, and $f(x)$ is odd if $f(-x) = -f(x)$. For instance x^2 and $\cos(x)$ are even, while x^3 and $\sin(x)$ are odd.

> If $f(x)$ is even, then
> $$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx.$$
> If $f(x)$ is odd, then
> $$\int_{-a}^{a} f(x)\,dx = 0.$$

The second formula is especially useful, because sometimes it is the only way to evaluate such integrals.

11-22 Check if the integrand $f(x)$ is even or odd. Then use symmetry to evaluate the integral.

1. $$\int_{-2}^{2} 1 + |x|\,dx$$

2. $$\int_{-\pi}^{\pi} \sin(x^3)\,dx$$

3.

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2(x)\,dx$$

4.

$$\int_{-1}^{1} x\sqrt{x^4+1}\,dx$$

The other important formula in this section is the definition of the average value of a function.

> For $f : [a,b] \to \mathbb{R}$, the average value of $f(x)$ on the interval $[a,b]$ is
>
> $$\mathrm{Avg}(f) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

The Mean Value Theorem for Integrals is an important variation of the Mean Value Theorem from the previous chapter. It says that if f is continuous on $[a,b]$ then there is $c \in (a,b)$ for which $f(c)$ is the average of f on the interval.

> For $f : [a,b] \to \mathbb{R}$ continuous, there is $c \in (a,b)$ with
>
> $$f(c) = \frac{1}{b-a}\int_a^b f(x)\,dx.$$

25-32 Find the average value of the function on the interval.

1. $f(x) = x(2+3x)$ on $[-1,3]a$.

2. $f(x) = \cos(\pi x)$ on $[0, .5]$.

$\boxed{\text{39-44}}$ Verify the Mean Value Theorem for integrals, by finding $c \in (a, b)$ with $f(c)$ the average of f on $[a, b]$.

1. $f(x) = 3x^2 + 1$ on $[-3, 0]$.

2. $f(x) = |x|$ on $[-2, 2]$.

3. $f(x) = \frac{1}{\sqrt{x}}$ on $[1, 4]$.