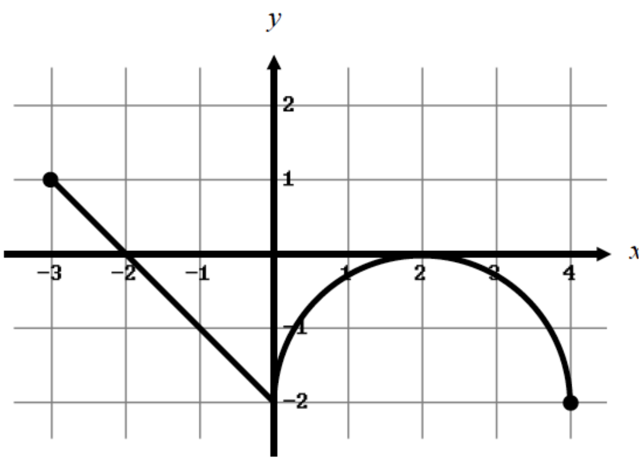
Homework

Complete the following questions. #3 and #12 are FRQ questions from previous years. Give them a go! You SHOULD try them out first and then go over the scoring guidelines.

1. Let f be a function defined on the closed interval $3 < x < 4$ with $f(0) = 3$. The graph of f', the derivative of f , consists of one line segment and a semicircle, as shown below.



Graph of f'

   (a) On what open intervals, if any, is f increasing? Decreasing? Justify your answer.

   (b) Find all values of x for which f assumes a relative maximum. Justify your answer.

   (c) Where is the graph of f concave up? concave down? Justify your answers.

   (d) Find the x-coordinate of each point of inflection of the graph of f on the open interval $3 < x < 4$. Justify your answer.

   (e) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (f) Sketch a possible graph of f.

2. Find the following using:

$$f(x) = 2x(x-1)^2$$

   (a) Find $f'(x)$ and $f''(x)$.

   (b) Intervals over which f is increasing and decreasing. Justify your response.

   (c) Intervals over which f is concave up and concave down. Justify your response.

   (d) State the x-values at which the function has any local extrema. Justify your response.

   (e) Identify any points of inflection. Justify your response.

3. **2001 AB 4** Scoring Guidelines
   Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \dfrac{x^2 - 2}{2}$ for all $x \neq 0$.
   (a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
   (b) On what intervals, if any, is the graph of h concave up? Justify your answer.
   (c) Write an equation for the line tangent to the graph of h at x = 4.
   (d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

4. An object moving along the $x-$axis with initial position $x(0) = 2$ is described by the following velocity function:
$$v(t) = \sin\left(\frac{\pi}{3}\, t\right)$$
   (a) Find the acceleration at $t = 4$.
   (b) Determine the direction(s) the particle is moving from $t = 0$ to $t = 7$. Show your analysis and justify your answer.
   (c) Is the speed of the particle increasing or decreasing at $t = 4$? Justify your response.
   (d) Determine the where the particle is speeding up from $t = 0$ to $t = 7$. Justify your response.

5. Approximate $\sqrt[3]{70}$. Determine if your approximation is an over/under approximation. Justify your response.

6. Approximate $\tan^{-1}(1.4)$. Determine if your approximation is an over/under approximation. Justify your response.

7. Approximate $\arctan(-1.4)$. Determine if your approximation is an over/under approximation. Justify your response.

8. Approximate $\sin^{-1}(0.7)$. Determine if your approximation is an over/under approximation. Justify your response.

9. Approximate $e^{0.1}$. Determine if your approximation is an over/under approximation. Justify your response.

10. Solve the following integrals:

(a) $\displaystyle\int \frac{7x}{x^2-7x-8}\ dx$

(b) $\displaystyle\int \frac{-x\csc^2(2x^2)}{\cot(2x^2)}\ dx$

(c) $\displaystyle\int \frac{5x-1}{x^2-2x-8}\ dx$

(d) $\displaystyle\int \frac{4x^2}{\sqrt{2x-7}}\ dx$

(e) $\displaystyle\int 3^{\cos(2x)}\sin(2x)\ dx$

(f) $\displaystyle\int \frac{3x-12}{x^2-5x-50}\ dx$

(g) $\displaystyle\int\ dx$

(h) $\displaystyle\int 2x\sqrt{2x^2-5}\ dx$

(i) $\displaystyle\int \frac{3x^2-4\sqrt{x}}{\sqrt[3]{x}}\ dx$

(j) $\displaystyle\int \frac{e^{3x}}{e^{3x}-7}\ dx$

(k) $\displaystyle\int \frac{e^{4x-2}}{\pi}\ dx$

(l) $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\ dx$

(m) $\displaystyle\int \frac{1}{1+x^2}\ dx$

(n) $\displaystyle\int \frac{1}{25+9x^2}\ dx$

(o) $\displaystyle\int e^{x+e^x}\ dx$

(p) $\displaystyle\int 0.125x\sin^4(8x^2+7)\cos(8x^2+7)\ dx$

11. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfying $\frac{d^2y}{dx^2} = y^3(1+3x^2y^2)$ with the given initial condition of $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

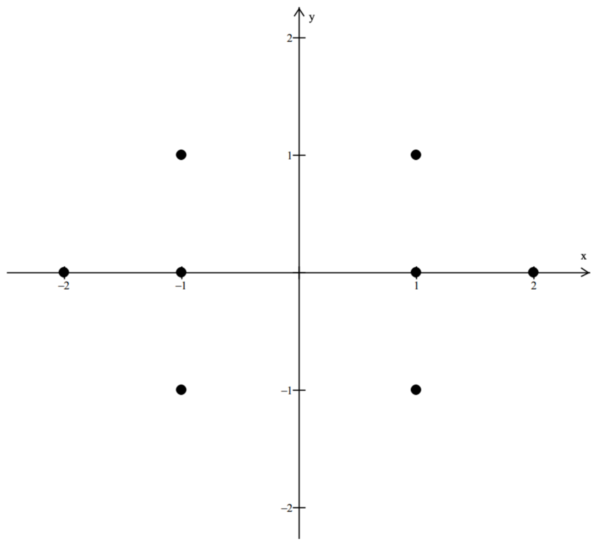
(b) Use the tangent line from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

12. **2006 AB 5** Scoring Guidelines

Consider the differential equation $\dfrac{dy}{dx} = \dfrac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the indicated points.



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.