Average Value Problems

1. **Average Temperature** A rod 3 meters long is heated to $25x$ °C, where x is the distance in meters from one end of the rod. Find the average temperature of the rod.

2. **Average Daily Rainfall** The rainfall per day, x days after the beginning of the year, is modeled by the function $r(x) = 0.00002(6511 + 366x - x^2)$, measured in centimeters. Find the average daily rainfall for the first 180 days of the year.

3. **Structural Engineering** A structural engineer designing a member of a structure must consider the forces that will act on that member. Most often, natural forces like snow, wind, or rain distribute force over the entire member. For practical purposes, however, an engineer determines the distributed force as a single resultant force acting at one point on the member. If the distributed force is given by the function $W = W(x)$, in newtons per meter (N/m), then the magnitude F_R of the resultant force is

$$F_R = \int_a^b W(x)\,dx$$

The position $\bar{x}$ of the resultant force measured in meters from the origin is given by

$$\bar{x} = \frac{\int_a^b x\,W(x)\,dx}{\int_a^b W(x)\,dx}$$

If the distributed force is $W(x) = 0.75x^3$, $0 \le x \le 5$, find:

(a) The magnitude of the resultant force.

(b) The position from the origin of the resultant force.

Source: Problem contributed by the students at Trine University, Avalon, IN.

4. **Chemistry: Enthalpy** In chemistry, **enthalpy** is a measure of the total energy of a system. For a nonreactive process with no phase change, the change in enthalpy ΔH is given by $\Delta H = \int_{T_1}^{T_2} C_p\,dT$, where C_p is the specific heat of the system in question. The specific heat per mol of the chemical benzene is

$$C_p = 0.126 + (2.34 \times 10^{-6})T,$$

where C_p is in kJ/(mol °C), and T is in degrees Celsius.

(a) What are the units of the change in enthalpy ΔH?

(b) What is the change in enthalpy ΔH associated with increasing the temperature of 1.0 mol of benzene from 20 °C to 40 °C?

(c) What is the change in enthalpy ΔH associated with increasing the temperature of 1.0 mol of benzene from 20 °C to 60 °C?

(d) Does the enthalpy of benzene increase, decrease, or remain constant as the temperature increases?

Source: Problem contributed by the students at Trine University, Avalon, IN.

5. **Average Mass Density** The mass density of a metal bar of length 3 meters is given by $\rho(x) = 1000 + x - \sqrt{x}$ kilograms per cubic meter, where x is the distance in meters from one end of the bar. What is the average mass density over the length of the entire bar?

6. **Average Velocity** The acceleration at time t of an object in rectilinear motion is given by $a(t) = 4\pi \cos t$. If the object's velocity is 0 at $t = 0$, what is the average velocity of the object over the interval $0 \le t \le \pi$?

7. **Average Area** What is the average area of all circles whose radii are between 1 and 3 m?

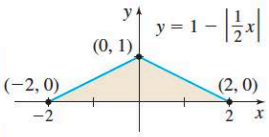
8. **Area**

(a) Use properties of integrals and the Fundamental Theorem of Calculus to find the area under the graph of $y = 3 - |x|$ from -3 to 3.

(b) Check your answer by using elementary geometry.

9. **Area**

(a) Use properties of integrals and the Fundamental Theorem of Calculus to find the area under the graph of $y = 1 - \left|\frac{1}{2}x\right|$ from -2 to 2.



(b) Check your answer by using elementary geometry. See the figure.