Applications of Integration

2.5 Regions Between Curves

Name:______________________

If $g(x) < f(x)$ on (a, b), the area between f and g from a to b is

$$\text{Area} = \int_a^b f(x) - g(x)\,dx.$$

For typical problems in this section, you have two intersecting curves f and g. Then you need to find where the curves intersect to get the endpoints of integration a and b. You also need to determine which function is bigger. If $g > f$ then you need to do $\int_a^b g(x) - f(x)\,dx$ to get the area.

Sketch the region between the two curves and find its area .

1. $f(x) = x^2 + 1$ and $g(x) = 1 + 2x - x^2$.

   SOLUTION: To find the endpoints of the region,

   $$x^2 + 1 = 1 + 2x - x^2 \qquad \Rightarrow \qquad 2x^2 - 2x = 0 \qquad \Rightarrow \qquad 2x(x-1) = 0.$$

   So $x = 0, 1$. Also, $1 + 2x - x^2$ is a parabola pointing down, and sits above $x^2 + 1$ between 0 and 1. So the area is

   $$\begin{aligned} \text{Area} &= \int_0^1 (1 + 2x - x^2) - (x^2 + 1)\,dx \\ &= \int_0^1 2x - 2x^2\,dx \\ &= x^2 - \frac{2}{3}x^3\Big|_0^1 = \frac{1}{3} \quad \square. \end{aligned}$$

2. $f(x) = x^2 - 2x$ and $g(x) = 2 - x$.

3. $y = \sqrt{5x - 1}$ and $y = x + 1$.

4. $x = 2y^2 - 2$ and $x = y^2 - y$.

Find the area between the two curves by adding two integrals. Draw a picture.

1. $y = x^3 - x$ and $y = 3x$.

2. $y = 4 - x^2$ and $y = 2 + |x|$.