## 2.7 Length of Curves

Name:________________________

The arclength of a function $y = f(x)$ from $x = a$ to $x = b$ is

$$\text{Arclength} = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_a^b \sqrt{1+(f'(x))^2}\,dx.$$

**9-16** Find the arclength of $y = f(x)$ from $x = a$ to $x = b$.

1. $y = \frac{1}{3}x^{\frac{3}{2}} - \sqrt{x}$ from $x = 1$ to $x = 4$.

SOLUTION: We need to compute $\int_1^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$, but it will take a little work outside of the integral first.

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}.$$

$$\begin{aligned} 1+\left(\frac{dy}{dx}\right)^2 &= 1+\left(\frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}\right)^2 \\ &= 1+\frac{1}{4}x - \frac{1}{2} + \frac{1}{4}x^{-1} \\ &= \frac{1}{4}x + \frac{1}{2} + \frac{1}{4}x^{-1}. \end{aligned}$$

Since we just computed $\left(\frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}\right)^2 = \frac{1}{4}x - \frac{1}{2} + \frac{1}{4}x^{-1}$, we can see that $\frac{1}{4}x + \frac{1}{2} + \frac{1}{4}x^{-1}$ is just $\left(\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)^2$. So,

$$\begin{aligned} \int_1^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx &= \int_1^4 \sqrt{\left(\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\right)^2}\,dx \\ &= \int_1^4 \frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}\,dx \\ &= \frac{1}{3}x^{\frac{3}{2}} + x^{\frac{1}{2}}\Big|_1^4 = \frac{8}{3} + 2 - \frac{1}{3} - 1 = \frac{10}{3} \quad \square. \end{aligned}$$

2. $y = \frac{1}{3}(x-2)^{\frac{3}{2}}$ from $x = 2$ to $x = 6$.

3. $y = \sqrt{9 - x^2}$ from $x = 0$ to $x = 3$. (Set up and simplify, but do not evaluate the integral.)

4. $y = \frac{x^4}{8} - \frac{x^2}{4}$ from $x = 1$ to $x = 2$. (Hint: All of the stuff under the square-root will be something squared again.)