2.4 Substitution

Name: ____________________________

The following is the chain rule for integrals.

If $u = u(x)$ then

$$du = \frac{du}{dx} dx.$$

17-26, 28-32, 34-38 Find the indefinite integrals.

1.

$$\int \frac{x}{\sqrt{1+x^2}}\, dx.$$

SOLUTION: Use $u = 1 + x^2$. Then $du = 2x\, dx$. Then $dx = \frac{1}{2x}\, du$. So

$$\begin{aligned}
\int \frac{x}{\sqrt{1+x^2}}\, dx &= \int \frac{x}{\sqrt{u}} \frac{1}{2x}\, du \\
&= \frac{1}{2} \int u^{-\frac{1}{2}}\, du \\
&= \frac{1}{2}(2u^{\frac{1}{2}}) + C \\
&= \sqrt{u} + C = \sqrt{1+x^2} + C \quad \square.
\end{aligned}$$

2.

$$\int x^2 \sec^2(x^3+1)\, dx.$$

3.

$$\int (3x+1)^5 \, dx.$$

4.

$$\int \frac{\cos\left(\frac{1}{x}\right)}{x^2} \, dx.$$

5.

$$\int \left(\sin^2(x) + 2\sin(x)\right)\cos(x) \, dx.$$

74-82 These u-subs involve solving back for x in terms of u.

1.
$$\int \frac{x}{\sqrt{x+2}}\,dx.$$
(Hint: If $u = x + 2$ then $x = u - 2$.)

2.
$$\int 4x(2x-5)^4\,dx.$$

3.
$$\int \frac{x}{(2-x)^3}\,dx.$$

When evaluating definite integrals where a substitution is involved, there are two ways to do it. The first is to evaluate $\int_a^b f(x)\,dx$ by making sure you have the antiderivative as a function of x and not u before plugging in $x = a$ and $x = b$. The second way is to convert your endpoints to u-values. See the example below.

41-66 Evaluate the definite integrals.

1.
$$\int_1^2 (2x+1)^7\,dx.$$

SOLUTION 1: $u = 2x+1$, $du = 2\,dx$ so

$$\begin{aligned}
\int_1^2 (2x+1)^7\,dx &= \int_{x=1}^{x=2} u^7 \frac{1}{2}\,du \\
&= \left.\frac{u^8}{16}\right|_{x=1}^{x=2} \\
&= \left.\frac{(2x+1)^8}{16}\right|_1^2 \\
&= \frac{(2(2)+1)^8}{16} - \frac{(2(1)+1)^8}{16} = \frac{5^8}{16} - \frac{3^8}{16} \quad \square.
\end{aligned}$$

SOLUTION 2: $u = 2x+1$, $du = 2\,dx$. But $u = 2x+1$ means when $x = 1$, $u = 2(1)+1 = 3$. And when $x = 2$, $u = 2(2)+1 = 5$. So

$$\begin{aligned}
\int_1^2 (2x+1)^7\,dx &= \int_3^5 u^7 \frac{1}{2}\,du \\
&= \left.\frac{u^8}{16}\right|_3^5 \\
&= \frac{5^8}{16} - \frac{3^8}{16} \quad \square.
\end{aligned}$$

As you can see, the second method is shorter, or at least takes less writing. Most students seem to think that the first method is easier to do though. You should practice doing them both ways, because both ways have their uses.

2.

$$\int_0^4 x\sqrt{9+x^2}\,dx.$$

3.

$$\int_1^{\sqrt{2}} \frac{x}{(x^2+1)^2}\,dx.$$

4.

$$\int_1^4 \frac{\cos(\pi\sqrt{x})}{\sqrt{x}}\,dx$$