$$= \frac{3}{4}\left[\frac{1}{s^2+1^2}\right] - \frac{1}{4}\left[\frac{3}{s^2+3^2}\right]$$

$$= \frac{3}{4}\left[\frac{1}{s^2+1}\right] - \frac{3}{4}\left[\frac{1}{s^2+9}\right]$$

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$$\therefore L\{\sin^3 t\} = \frac{3}{4}\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]$$

$$\boxed{\sin^3 t = \frac{3}{4}\cdot\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]}$$

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Q1) Prove that $\Gamma_{n+1} = n\Gamma_n$

Soln:- Proof we know that

$$\Gamma_n = \int_0^\infty e^{-x}\, x^{n-1}\, dx \quad (\because \text{formula as written})$$

$$\Gamma_{n+1} = \int_0^\infty e^{-x}\, x^{n+1-1}\, dx$$

$$= \int_0^\infty e^{-x}\, x^n\, dx$$

$$= \int_0^\infty \underset{U}{x^n}\, \underset{V}{e^{-x}}\, dx$$

$$\left[\because \int U.V\,dx = U\int V\,dx - \int\left[\frac{d}{dx}U \int V\,dx\right] dx\right]$$

$$= x^n \int_0^\infty e^{-x} - \int_0^\infty \left[\frac{d}{dx} x^n \int_0^\infty e^{-x}\,dx\right] dx$$

$$= x^n \left[\frac{e^{-n}}{-1}\right]_0^\infty - \int_0^\infty n\,x^{n-1}\, \frac{e^{-x}}{-1}\, dx$$

$$= x^n \left[\frac{e^{-\infty}}{-1} - \frac{e^{-0}}{-1}\right] + n\underline{\int_0^\infty e^{-x}\, x^{n-1}\, dx}$$

$$\Gamma_n$$

$= 0 + n\Gamma(n)$

$\therefore \Gamma(n+1) = n\Gamma(n)$

Hence Proved

L [Cos

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Q2) $\int_0^\infty e^{-x^2} \cdot x^2 \, dx$

Soln:- $I = \int_0^\infty e^{-x^2} \cdot x^2 \, dx$

put,

$x^2 = t$

$x = t^{1/2}$

$dx = \frac{1}{2\sqrt{t}} dt \qquad \left(\because \frac{d}{dx}\sqrt{y} = \frac{1}{2\sqrt{x}}\right)$

when $x \to 0,\ t \to 0$

$x \to \infty,\ t \to \infty$

$I = \int_0^\infty e^{-t}\, t \, \frac{1}{2} t^{-1/2} \, dt$

$= \frac{1}{2} \int_0^\infty e^{-t} \cdot t^1 t^{-1/2} \, dt \qquad \boxed{t^1 \cdot t^{-1/2} = t^{1-1/2} = t^{1/2}}$

$= \frac{1}{2} \int_0^\infty e^{-t} \cdot t^{1/2} \, dt$

$= \frac{1}{2} \int_0^\infty e^{-t}\, t^{3/2 - 1} \, dt$

$$\because \left[\Gamma n = \int_0^\infty e^{-x}\, x^{n-1}\, dx\right]$$

$$I = \frac{1}{2}\Gamma\frac{3}{2}$$

L [Cos

$$= \frac{1}{2}\Gamma\left(\frac{1}{2}+1\right)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2} \qquad \left(\because \Gamma\frac{1}{2} = \sqrt{\pi}\right)$$

$$I = \frac{1}{4}\sqrt{\pi}$$

$$\boxed{\int_0^\infty e^{-x^2}\cdot x^2\, dx = \frac{\sqrt{\pi}}{4}}$$

Q3) $\int_0^\infty x^{1/4}\cdot e^{-\sqrt{x}}\, dx$

Soln:- $I = \int_0^\infty x^{1/4}\cdot e^{-\sqrt{x}}\, dx$

Put $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt \qquad \left[\because \frac{d}{dx} x^n = n x^{n-1}\right]$

when $x \to 0,\ t \to 0$

$x \to \infty,\ t \to \infty$

$$I = \int_0^\infty (t^2)^{1/4}\cdot e^{-t}\cdot 2t\,dt$$

$$= 2\int_0^\infty t^{2\times 1/4}\cdot e^{-t}\cdot t\,dt$$

$$= 2\int_0^\infty e^{-t}\, t^{1/2}\, t\,dt \qquad \left(t^{1/2}\cdot t^1 = t^{1/2+1}\right)$$

$$= 2\int_0^\infty e^{-t}\, t^{1/2+1}\, dt$$

$$= 2\int_0^\infty e^{-t}\, t^{3/2}\, dt$$

$$= 2\int_0^\infty e^{-t} \cdot t^{5/2 - 1} \cdot dt$$

$$\left[\because \Gamma n = \int_0^\infty e^{-x} \, x^{n-1} \, dx \right]$$

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$$\therefore \; I = 2 \, \Gamma\left(\frac{5}{2}\right)$$

$$= 2 \, \Gamma\left(\frac{3}{2} + 1\right)$$

$$= \cancel{2} \cdot \frac{3}{\cancel{2}} \, \Gamma\left(\frac{3}{2}\right)$$

$$= 3 \, \Gamma\left(\frac{1}{2} + 1\right)$$

$$= \frac{3}{\cancel{2}} \times \frac{1}{2} \, \Gamma\left(\frac{1}{2}\right)$$

$$= \frac{3}{2} \sqrt{\pi}$$

$$\boxed{\int_0^\infty x^{1/4} \, e^{-\sqrt{x}} \, dx = \frac{3}{2} \sqrt{\pi}}$$

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Q4) If $\beta(n, 3) = \frac{1}{3}$ & n is a positive integer. find n.

Solu:- $\beta(n,3) = \frac{1}{3}$

$$\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma(m+n)} = \frac{1}{3}$$

$$\beta(n,3) = \frac{\Gamma n \, \Gamma 3}{\Gamma(n+3)} = \frac{1}{3}$$

$$= \frac{\Gamma n \, \Gamma(2+1)}{\Gamma(n+2+1)} = \frac{1}{3}$$

$$= \frac{\Gamma n \; 2!}{(n+2) \, \Gamma(n+2)} = \frac{1}{3}$$

$$= \frac{2 \, \Gamma n}{(n+2) \, \Gamma((n+1)+1)} = \frac{1}{3} \quad \left(\because \Gamma(n+1) = n \Gamma n \right)$$

$$= \frac{2 \, \Gamma n}{(n+2)(n+1) \, \Gamma(n+1)} = \frac{1}{3}$$

$$\frac{2 \, \cancel{\Gamma n}}{(n+2)(n+1) \, n \, \cancel{\Gamma n}} = \frac{1}{3}$$

$$\frac{6}{n(n+2)(n+1)} = 1$$

$n(n+2)(n+1) = 6$

$n^2 + 2n(n+1) = 6$

$n^3 + 2n^2 + n^2 + 2n + n$

$n - 1 = 0$

$n = 1$

$n^3 + 3n^2 + 2n - 6 = 0$

$\boxed{n(n^2 + 4n + 6) = 0}$

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Q5) Prove that $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$

→ By definition of beta function

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

put $x = \sin^2\theta$ and $(1-x) = \cos^2\theta$

$dx = 2\sin\theta\cos\theta\,d\theta$

where $x \to 0,\ \theta \to 0$

$x \to 1,\ \theta \to (\pi/2)$

$$\beta(m,n) = \int_0^{\pi/2} (\sin^2\theta)^{m-1}(\cos^2\theta)^{n-1}\, 2\sin\theta\cos\theta\,d\theta$$

$$= 2\int_0^{\pi/2} (\sin\theta)^{2m-2} \cdot (\cos\theta)^{2n-2} \sin\theta \cdot \cos\theta\,d\theta$$

$$= 2\int_0^{\pi/2} \sin\theta^{2m-2+1} \cos\theta^{2n-2+1}\,d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\,d\theta$$

$$\boxed{\therefore\ \beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cos^{2n-1}\theta\,d\theta}$$

Q6) Prove that

$\beta(m,n) = \beta(n,m)$

Soln:- $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

put $1-x = t$

$x = 1-t$

$dx = -dt$

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where $x=0,\ t=1$

$x=1,\ t=0$

$$\beta = \int_1^0 (1-t)^{m-1}\cdot(t)^{n-1} - dt$$

$$= \int_1^0 (1-t)^{m-1}\cdot(t)^{n-1} - dt$$

$$= -\int_1^0 (1-t)^{m-1}\cdot t^{n-1}\cdot dt$$

To remove minus the limits will change

$$\beta = \int_0^1 (1-t)^{m-1}\cdot t^{n-1}\,dt$$

$$\beta = \int_0^1 t^{n-1}\cdot(1-t)^{m-1}\cdot dt$$

$$\beta(n,m) = \int_0^1 t^{n-1}\cdot(1-t)^{m-1}\,dt$$

$\beta(m,n) = \beta(n,m)$ Hence Prov

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Q1) Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr.d\theta$

Soln:- Let $I = \int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$

$$= \int_0^{\pi}\left[\frac{r^2}{2}\right]_0^{a\sin\theta} d\theta$$

$$= \frac{1}{2}\int_0^{\pi}\left[r^2\right]_0^{a\sin\theta} d\theta$$

$$= \frac{1}{2}\int_0^{\pi}\left[a^2\sin^2\theta - 0\right] d\theta$$

$$= \frac{1}{2}\int_0^{\pi} a^2\sin^2\theta\, d\theta$$

$$= \frac{a^2}{2}\int_0^{\pi}\sin^2\theta\, d\theta$$

$$\left(\because \cos 2\theta = 1 - 2\sin^2\theta \quad 2\sin^2\theta = 1-\cos 2\theta \quad \sin^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)\right)$$

$$= \frac{a^2}{2}\int_0^{\pi}\frac{(1-\cos 2\theta)}{2}\,d\theta$$

$$= \frac{a^2}{4}\int_0^{\pi}(1-\cos 2\theta)\,d\theta$$

$$\left(\because \int \cos 2\theta = \frac{\sin 2\theta}{2}\right)$$

$$= \frac{a^2}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{a^2}{4}\left[\pi - \frac{\sin 2\theta}{2} - 0\right]$$

$$= \frac{a^2}{4}\left[\pi - 0\right] \qquad \left(\because \sin \pi/2 = 0\right)$$

$$= \frac{a^2\pi}{4}$$

$$\boxed{\therefore \int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta = \frac{a^2\pi}{4}}$$

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Q2) Evaluate :- $\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2 \sin\theta \, dr\, d\theta$

Soln:- Let $I = \int_0^{\pi} \sin\theta \left[\frac{r^3}{3}\right]_0^{a(1-\cos\theta)} d\theta$

$$= \frac{a^3}{3}\int_0^{\pi} \sin\theta \,(1-\cos\theta)^3 \, d\theta$$

Let $1-\cos\theta = t$

Derivation on both sides

$0-(-\sin\theta) = dt$

$\sin\theta \, d\theta = dt$ $\qquad \left(\because \frac{d\cos\theta}{d\theta} = -\sin\theta\right)$

where $\theta \longrightarrow 0,\ t \longrightarrow 0$

$\theta \longrightarrow \pi,\ t \longrightarrow 2$

$$\therefore I = \frac{a^3}{3}\int_0^{2} t^3\,dt$$

$$= \frac{a^3}{3}\left[\frac{t^4}{4}\right]_0^{2}$$

$$= \frac{a^3}{3}\left[\frac{16}{4} - 0\right]$$

$$= \frac{a^3}{3}\left[\frac{\cancel{16}^{4}}{\cancel{4}}\right]$$

$$\therefore I = \frac{4a^3}{3}$$

$$\boxed{\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2\sin\theta\,dr\,d\theta = \frac{4a^3}{3}}$$

3) Evaluate by changing order of

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} dy \cdot dx$$

Soln:- Linear Limit of Integration

$x = 0$ to $x = a$

$y = 0$ to $y = \sqrt{a^2-x^2}$

$y^2 = a^2 - x^2$

$x^2 + y^2 = a^2$

Now by changing the order of Integration

$y = 0$ & $y = a$

$x = 0$ & $x = \sqrt{a^2-x^2}$

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} dx \cdot dy = \int_{y=0}^{a} \int_{x=0}^{\sqrt{a^2-x^2}} dx \cdot dy$$

$$= \int_{y=0}^{a} [x]_0^{\sqrt{a^2-x^2}} dy$$

$$= \left[\frac{1}{2} y \sqrt{a^2-x^2} + \frac{1}{2} a^2 \sin^{-1} y/a\right]$$

$$= \frac{1}{2} a^2 \sin^{-1}(1)$$

$$= \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{a^2\pi}{4}$$

$$\boxed{\therefore \int_0^a \int_0^{\sqrt{a^2-x^2}} dy \cdot dx = \frac{a^2\pi}{4}}$$

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Q4) Find the double integration the area included between the Curves $y^2 = 4ax$ & $x^2 = 4ay$

Soln:- Given equation of Curve are $x^2 = 4ay$, $y^2 = 4ax$

The point of intersection are

$y^2 = 4ax$

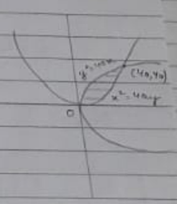
$\therefore y^4 = 16a^2x^2$ ($\because$ Squaring on both sides)

$\therefore y^4 = 16a^2(4ay)$ (place $x^2 = 4ay$ as given above)

$\therefore y^4 = \cancel{16a}\ 64a^3 y$

$\therefore y^3 = 64a^3$

$\therefore y = 4a$



$\therefore (4a, 4a)$ is the point of intersection

we know that, Area $= \iint dx \cdot dy$

$\therefore$ So, $y^2 = 4ax$ $x^2 = 4ay$

$y = \sqrt{4ax}$ $y = \frac{x^2}{4a}$

$y = 2\sqrt{ax}$

$$\therefore \text{Required area is } A = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dx\,dy$$

$$= \int_0^{4a} dx \, [y]_{x^2/4a}^{2\sqrt{ax}}$$

$$A = \int_0^{4a} \left(2\sqrt{ax} - \frac{x^2}{4a}\right) dx$$

$$= 2\sqrt{a} \cdot \left[\frac{x^{3/2}}{3/2}\right]_0^{4a} - \frac{1}{4a}\left(\frac{x^3}{3}\right)_0^{4a}$$

$$= 2\sqrt{a} \cdot \frac{2}{3} \cdot 4^{3/2} \cdot a^{3/2} - \frac{1}{12a} \, \underline{64a^3}$$

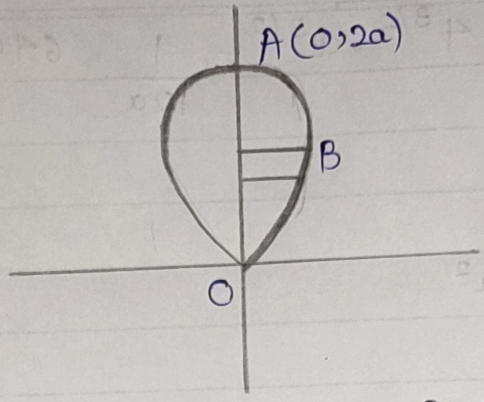
$$= \frac{16}{3} a^2$$

$\therefore$ Area $= A = \frac{16}{3} a^2$

$\boxed{\text{Area included between the curves is } \frac{16}{3} a^2}$

Q- Find the area of the curve $a^2x^2 = y^3(2a-y)$

$\longrightarrow$



As, it contains only even powers of x, hence, it is symmetrical about y-axis

Total Area = 2 × area OAB

$$\text{Area } OAB = \iint dy\,dx$$

$$= \int_{y=0}^{2a} \int_{x=0}^{f(y)} dx\,dy$$

$$= \int_{y=0}^{2a} \int_{x=0}^{x=\frac{y^{3/2}\sqrt{2a-y}}{a}} dx\,dy$$

$$= \int_{y=0}^{2a} [x]_0^{\frac{y^{3/2}\sqrt{2a-y}}{a}} dy$$

$$= \frac{1}{a}\int_0^{2a} y^{3/2}\sqrt{2a-y}\,dy$$

Putting $y = 2a\sin^2\theta$

$dy = 4a\sin\theta\cos\theta\,d\theta$

$$= \frac{1}{a}\int_0^{\pi/2} (2a\sin^2\theta)^{3/2}\sqrt{2a-2a\sin^2\theta}\cdot 4a\sin\theta\cos\theta\,d\theta$$

$$= 16a^2\int_0^{\pi/2} \sin^4\theta\cos^2\theta\,d\theta$$

$$= \frac{16a^2\,\Gamma(5/2)\,\Gamma(3/2)}{2\,\Gamma(4)}$$

$$= 16a^2\cdot\frac{3}{2}\cdot\frac{1}{2}\cdot\frac{\sqrt{\pi}}{2\cdot 3}\cdot\frac{1}{2}\cdot\frac{\sqrt{\pi}}{1}\cdot\frac{1}{2} = \frac{\pi a^2}{2}$$

$$\text{Total Area} = \cancel{2}\cdot\frac{1}{\cancel{2}}\pi a^2 = \pi a^2$$

Value of $\lambda = 3.142$

The Correct answer is = 3.142