PRACTICAL - 1

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Q1) Find the Adjoint of the given matrix & find inverse

$$\begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 2 \\ 5 & 1 & 8 \end{bmatrix}$$

Soln:- Minors of Matrix $M_{11} = \begin{vmatrix} 6 & 2 \\ 1 & 8 \end{vmatrix} = 48 - 2 = 46$

$M_{12} = \begin{vmatrix} 4 & 2 \\ 5 & 8 \end{vmatrix} = 32 - 10 = 22$

$M_{13} = \begin{vmatrix} 4 & 6 \\ 5 & 1 \end{vmatrix} = 4 - 30 = -26$

$M_{21} = \begin{vmatrix} -1 & 3 \\ 1 & 8 \end{vmatrix} = -8 - 3 = -11$

$M_{22} = \begin{vmatrix} 2 & 3 \\ 5 & 8 \end{vmatrix} = 16 - 15 = 1$

$M_{23} = \begin{vmatrix} 2 & -1 \\ 5 & 1 \end{vmatrix} = 2 - (-5) = 7$

$M_{31} = \begin{vmatrix} -1 & 3 \\ 6 & 2 \end{vmatrix} = -2 - 18 = -20$

$M_{32} = \begin{vmatrix} 2 & 3 \\ 4 & 2 \end{vmatrix} = 4 - 12 = -8$

$M_{33} = \begin{vmatrix} 2 & -1 \\ 4 & 6 \end{vmatrix} = 12 - (-4) = 16$

Now, Co-factor of matrix $\Rightarrow c_{ij} = (-1)^{i+j} \cdot (M_{ij})$

$C_{11} = (-1)^{1+1} \cdot (46) = 46$

$C_{12} = (-1)^{1+2} \cdot (22) = -22$

$C_{13} = (-1)^{1+3} \cdot (-26) = -26$

$C_{21} = (-1)^{2+1} \cdot (-11) = 11$

$C_{22} = (-1)^{2+2} \cdot (1) = 1$

$C_{23} = (-1)^{2+3} \cdot (7) = -7$
$C_{31} = (-1)^{3+2} \cdot (-20) = -20$
$C_{32} = (-1)^{3+2} \cdot (-8) = 8$
$C_{33} = (-1)^{3+3} \cdot (16) = 16$

The Co-factor matrix is = $\begin{bmatrix} 46 & -22 & -26 \\ 11 & 1 & -7 \\ -20 & 8 & 16 \end{bmatrix}$

The Adjoint Matrix is = $\begin{bmatrix} 46 & 11 & -20 \\ -22 & 1 & 8 \\ -26 & -7 & 16 \end{bmatrix}$

The inverse of Matrix is

formula = $A^{-1} = \frac{1}{|A|} \cdot (\text{Adj}(A))$

$$A^{-1} = \frac{1}{\begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & 2 \\ 5 & 1 & 8 \end{bmatrix}} \cdot \begin{bmatrix} 46 & 11 & -20 \\ -22 & 1 & 8 \\ -26 & -7 & 16 \end{bmatrix}$$

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Q2. Find the rank of Matrix by echolen form

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

→ Given Matrix is, $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

Operate $R_2 - 2R_1$; $R_3 - 3R_1$; $R_4 - 6R_1$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$

Operate $R_{23} \to R_3$ $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & 11 & 5 \end{bmatrix}$

Operate $R_4 - R_2$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

Operate $R_4 - R_3$ $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Operate $\frac{R_2}{-4}$; $\frac{R_3}{-3}$ $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

which is Echelon Form of the matrix A.

Rank of matrix = Total number of rows − Number of rows containing all zeros.

$\rho(A) = 4-1$
$\rho(A) = 3$

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Q3) Verify Cayley-Hamilton Theorem for the given matrix

$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

→ The characteristic equation is $\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$

$\therefore (2-\lambda)\left[(2-\lambda)^2 - 1\right] + 1\left[-1(2-\lambda)+1\right] + 1\left[1-(2-\lambda)\right] =$

$\therefore \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

Cayley-Hamilton theorem states that this equation is satisfied by A

i.e $A^3 - 6A^2 + 9A - 4I = 0$ — (1)

Now

$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

$$A^3 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

Now replace all matrix with A in the eqn 1

$$A^3 - 6A^2 + 9A - 4I = 0$$

$$\begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} + 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} -$$

$$4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 22-36+18-4 & -21+30-9-0 & 21-30+9-0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9-0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{bmatrix} = 0$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

Hence Proved Cayley - Hamilton Theorem

Q.4 Express in the polar form $(-1+i)$

→ By comparing the complex number with standard form $z = x+iy$, we get $x=-1$, $y=1$ then.

Modulus $= |z| = \sqrt{x^2+y^2}$

$= \sqrt{(-1)^2+1^2}$

$= \sqrt{2}$

Amplitude $= \theta = 2\pi - \alpha$

$\alpha = \tan^{-1}\left(\frac{-1}{1}\right)$

$= \tan^{-1}(1)$

$\alpha = \frac{\pi}{4}$

Amplitude $= \theta = 2\pi - \alpha$

$= 2\pi - \frac{\pi}{4}$

$= \frac{7\pi}{4}$

$z = r(\cos\theta + i\sin\theta)$

$z = \sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

Q5) find the principle value of $\log(2+3i)$

$\rightarrow \log(x+iy) = \frac{1}{2}\log(x^2+y^2) + i\left[2n\pi + \tan^{-1}\left(\frac{y}{x}\right)\right]$

So, by comparing with Complex Number $x+iy$, we get
$x=2,\ y=3$

$\therefore \log(2+3i) = \frac{1}{2}\log(2^2+3^2) + i\left[2n\pi + \tan^{-1}\left(\frac{3}{2}\right)\right]$

$= \frac{1}{2}\log(4+9) + i\left[2n\pi + \tan^{-1}\left(\frac{3}{2}\right)\right]$

$= \frac{1}{2}\log 13 + i\left[2n\pi + \tan^{-1}\left(\frac{3}{2}\right)\right]$

$$\boxed{\therefore \log(2+3i) = \frac{1}{2}\log 13 + i\left[2n\pi + \tan^{-1}\left(\frac{3}{2}\right)\right]}$$

Q.6 Prove that $(1+i)^8 + (1-i)^8 = 32$

Soln:- $(1+i)^8 + (1-i)^8 = 32$ Given equation

$[(1+i)^2]^4 + [(1-i)^2]^4 = 32$

$[1^2+2i+i^2]^4 + [1^2-2i+i^2]^4 = 32$

$[\cancel{1}+2i-\cancel{1}]^4 + [\cancel{1}-2i-\cancel{1}]^4 = 32$

$[2i]^4 + [-2i]^4 = 32$

$16i^4 + 16i^4 = 32$

$16+16 = 32$

$32 = 32$

Here L.H.S is equal to R.H.S Hence proved.

PRACTICAL - 2

Q1) $3e^x \tan y\, dx + (1-e^x)\sec^2 y\, dy = 0$

Soln:- Consider the given differential equation

$$3e^x \tan y\, dx + (1-e^x)\sec^2 y\, dy = 0$$
$$3e^x \tan y\, dx = -(1-e^x)\sec^2 y\, dy$$

$$\frac{-3e^x}{1-e^x}\,dx = \frac{\sec^2 y}{\tan y}\,dy$$

Integrating on both sides we get

$$3\int \frac{-e^x}{1-e^x}\,dx = \int \frac{\sec^2 y}{\tan y}\,dy$$

Let $1-e^x = t$

Differentiate on both sides

$$0 - e^x dx = dt$$
$$e^x dx = -dt$$

$$-3\log(1-e^x) + \log(\tan y) = \log C$$

$$\log(1-e^x)^{-3} + \log(\tan y) = \log C$$

$$\log(1-e^x)^{-3}\cdot \tan y = \log C \qquad (\because \log m + \log n = \log m\cdot n)$$

$$\frac{\tan y}{(1-e^x)^3} = C$$

$\boxed{(1-e^x)^3 = \tan y \cdot C}$ General Soln

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Q2) $(x^2-y^2)\,dx + 2xy\,dy = 0$

Soln:- $(x^2-y^2)\,dx = -2xy\,dy$

$$-(x^2-y^2)\,dx = 2xy\,dy$$
$$(y^2-x^2)\,dx = 2xy\,dy$$

$$\therefore \frac{dy}{dx} = \frac{(y^2-x^2)}{2xy} \quad \text{--- (1)}$$

Is the homogenous differential equation

Put $y = vx$

Differentiate on both sides with respect to x

$$\therefore \frac{dy}{dx} = v\frac{dx}{dx} + x\frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = v + x\cdot\frac{dv}{dx}$$

$\therefore$ The differential equation becomes So put $y = vx$ in eqn (1)

$$v + x\frac{dv}{dx} = \frac{(vx)^2 - x^2}{2x\,vx}$$

$$\therefore x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x^2 v} - v$$

$$\therefore x\frac{dv}{dx} = \frac{(v^2-1)\cancel{x^2}}{2v\cancel{x^2}} - v$$

$$\therefore\ x\frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$\therefore\ x\frac{dv}{dx} = \frac{-v^2-1}{2v}$$

$$\therefore\ \frac{2v\,dv}{v^2+1} = -\frac{dx}{x}$$

Integrating on both sides

$$\int \frac{2v}{1+v^2}\,dv = -\int \frac{dx}{x}$$

$$\log|1+v^2| = -\log|x| + \log|c|$$

$$\log|1+v^2| + \log|x| = \log|c|$$

Now put $v = \frac{y}{x}$

$$\log\left|1+\frac{y^2}{x^2}\right| + \log|x| = \log|c|$$

$$\log\left(\frac{x^2+y^2}{x^2}\right) + \log x = \log c$$

$[\therefore \log m + \log n = \log m\cdot n]$

$$\log\left(\frac{x^2+y^2}{x^{\cancel{2}}}\right)\cdot \cancel{x} = \log c$$

$$\cancel{\log}\left(\frac{x^2+y^2}{x}\right) = \cancel{\log}\, c \qquad \boxed{\therefore\ x^2+y^2 = c\cdot x \quad \text{general Solution}}$$

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(3) $\frac{dy}{dx} + 2y\tan x = \sin x$

Soln:- $\frac{dy}{dx} + 2y\tan x = \sin x$

Comparing with $\frac{dy}{dx} + Py = Q$

$P = 2\tan x$ and $Q = \sin x$

Now using Integrating Factor formula

Integral Factor $= e^{\int P dx}$

$$= e^{\int 2\tan x\, dx}$$

$$= e^{\int 2\frac{\sin x}{\cos x}\, dx}.$$

$$I.F = e^{2\log \sec x}$$

$$= e^{\log_e \sec^2 x} \qquad (\therefore \text{ Here log can be writ[ten] as } \log_e)$$

$$I.F = \sec^2 x$$

The general Solution is

$$y \times I.F = \int Q \times I.F\, dx + c$$

$$y \times \sec^2 x = \int \sin x \times \sec^2 x\, dx + c$$

$$= \int \sin x \frac{1}{\cos^2 x} dx + c$$

$$= \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} dx + c$$

$$= \int \sec x \cdot \tan x \; dx + c$$

$\left(\because \text{So, } \int \sec x \cdot \tan x \, dx = \log \sec x \right)$

$$y \times \sec^2 x = \log \sec x + c$$

The General Solution is

$$\boxed{y \times \sec^2 x = \log \sec x + C}$$

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Q 4) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x$

Soln:- The given Differential equation is

$$\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} - \frac{dy}{dx} - y = \cos 2x \quad \left(\because \frac{dy}{dx} = D \text{ can also be written} \right)$$

$$D^3 + D^2 - D - y = \cos 2x$$

$$(D^3 + D^2 - D - 1) y = \cos 2x$$

The complete solution of this differential equation is

$y = y_c + y_p$ = Complementary function (C.F) + Particular In (P.I)

Step 1:- $D^3 + D^2 - D - 1 = 0$

$D^2 (D+1) - 1 (D+1) = 0$

$(D+1)(D^2 - 1) = 0$

$(D+1)(D-1)(D+1) = 0 \quad \left(\because (D^2 - 1) = D^2 - 1^2 = (D-1)^2 = (D-1)(D+1) \right)$

So, $D = 1, -1, -1$

$\therefore y_c = C_1 e^{x} + (C_1 + C_3 x) e^{-x} \quad (C.F = C_1 e^{m_1 x} + C_2 e^{m_2}$

Step 2:- Particular Integral $= \frac{1}{(D+1)(D^2-1)} \cos 2x \quad (\because P.I =$

Replace D^2 by -4 ($\because$ Because $\cos 2x$ coefficient so we took D^2 as -4)

$$= \frac{1}{(D+1)(-4-1)} \cos 2x$$

$$= \frac{1}{-5} \frac{1}{(D+1)} \cos 2x$$

$$= \frac{1}{-5} \frac{\cos 2x}{(D+1)} \times \frac{(D-1)}{(D-1)}$$ ($\because$ Conjugate $D+1$ as $D-1$ and multiple on numerator or Denominator)

$$= \frac{1}{-5} \frac{\cos 2x (D-1)}{D^2 - 1^2}$$

$$= \frac{1}{-5} \frac{\cos 2x (D-1)}{-4-1}$$

$$= \frac{1}{-5} \frac{\cos 2x (D-1)}{-5}$$

$$= \frac{1}{25} \cos 2x (D-1)$$

$$= \frac{1}{25} [D \cos 2x - D]$$

$$= \frac{1}{25} [-2\sin 2x - \cos 2x]$$ ($\because$ D is Differentiation so $\frac{d \cos 2x}{dx} = -2\sin 2x$)

$$= -\frac{1}{25} [2\sin 2x + \cos 2x]$$

$y = C.F + P.I$

$= C_1 e^x + (C_2 + C_3 x) e^{-x} - \frac{1}{25} [2\sin 2x + \cos 2x]$ is a General solution.

Q5) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x$

The given differential equation is

$D^2 - 3D + 2y = e^x$ ($\because \frac{dy}{dx} = D$ can also be written as)

$(D^2 - 3D + 2) y = 0$

Step 1:- $D^2 - 3D + 2 = 0$

$D^2 - 2D - D + 2 = 0$

$D(D-2) - 1(D-2) = 0$

$(D-2)(D-1) = 0$

$(D-2)(D-1) = 0$

$D = 2$ or $D = 1$

Complementary function $= C_1 e^x + C_2 e^{2x}$

Step 2:- Particular Integral $= \frac{1}{f(D)} e^{ax}$

$$= \frac{1}{(D^2 - 3D + 2)} e^x$$

$$= \frac{1}{(D-1)(D-2)} e^x$$

$\therefore$ So if by put $D=1$ or $D=2$ we get the value 0 then $P.I = \frac{1}{(D-a)^r} e^{ax}$

$$= \frac{x^r}{r!} e^{ax}$$

where r is $1, 2, 3 \ldots\ldots$

$$= \frac{-1}{(D-1)} e^x$$

$$= \frac{-1}{1!} x e^x$$

Particular Integral $= -xe^x$

$$\boxed{\begin{aligned} \text{Complete Solution} &= \text{Complementary Function} + \text{Particular Integral} \\ &= c_1 e^x + c_2 e^{2x} - xe^x \end{aligned}}$$

Q6) $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} = \sinh 2x$

Soln :- $D^3 - 4D = \sinh 2x$ $\left(\because \frac{dy}{dx} = D \text{ can be also written as}\right)$

$D^3 - 4D = 0$

Step 1 :- $D^3 - 4D = 0$

$D(D^2 - 4) = 0$

$D(D+2)(D-2) = 0$ $\left(\because (D^2 - 4) = (D-2)^2 = (D+2)(D-2)\right)$

$D = 0, -2, 2$

Complementary function $= C_1 e^0 + C_2 e^{-2x} + C_3 e^{2x}$

$= C_1 + C_2 e^{-2x} + C_3 e^{2x}$

Step 2 :- Particular Integral $= \frac{1}{D^3 - 4D} \sinh 2x$

$\left(\because \text{Particular Integral} = \frac{x}{f'(D)} \sin(ax+b)\right)$

$$= \frac{1}{3D^2 - 4} x \sinh 2x$$

$$= \frac{x \sinh 2x}{3D^2 - 4}$$

($\therefore$ As we have seen in $P.I = \frac{1}{f(D)} \cos ax$ or $\frac{1}{f(D)} \sin ax$

the coefficient becomes the value of D if positive then negative)

$$= \frac{x \sinh 2x}{3(-2)^2 - 4}$$

$$= \frac{x \sinh 2x}{12 - 4}$$

Particular Integral $= \frac{x \sinh 2x}{8}$

Complete Solution = Complementary function + Particular Integral
$= C_1 + C_2 e^{2x} + C_3 e^{-2x} + \frac{x \sinh 2x}{8}$

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Q1) $L[\cos at] = \frac{s}{s^2 + a^2}$

Soln:- $\therefore \cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}$

$\therefore \cos at = \frac{e^{i(at)} + e^{-i(at)}}{2}$

By Laplace transformation formula

$$L\{\cos at\} = \int_0^\infty e^{-st} \cos at \, dt$$

$$= \int_0^\infty e^{-st} \left(\frac{e^{i(at)} + e^{-i(at)}}{2} \right) dt$$

$$= \frac{1}{2} \int_0^\infty e^{-st} \left(e^{(ia)t} + e^{(-ia)t} \right) dt$$

$$= \frac{1}{2} \left[\int_0^\infty e^{-st} e^{(ia)t} \, dt + \int_0^\infty e^{-st} (e^{(-ia)t} \, dt \right]$$

$$= \frac{1}{2} \left[\frac{1}{s - ai} + \frac{1}{s - (-ia)} \right]$$

$\left(\therefore \int_0^\infty e^{-st} e^{(ia)} \, dt = \frac{1}{s-\ldots} \right.$

$= \frac{1}{2}\left[\frac{s+\cancel{a} + s-\cancel{a}}{s^2+a^2}\right]$

$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}s}{s^2+a^2}\right]$

$L[\cos at] = \frac{s}{s^2+a^2}$

Hence Proved

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Q2) $L\{\cos^3 t\} = ?$

Soln:- Since, $\cos 3t = 4\cos^3 t - 3\cos t$

$\cos 3t + 3\cos t = 4\cos^3 t$

$\cos^3 t = \frac{\cos 3t}{4} + \frac{3\cos t}{4}$

$\therefore L\{\cos^3 t\} = \frac{1}{4} L\{\cos 3t\} + \frac{3}{4} L\{\cos t\}$

$\therefore \boxed{\therefore L\{\cos at\} = \frac{s}{s^2+a^2}}$ Here a is a Coefficient

$= \frac{1}{4}\frac{s}{s^2+a^2} + \frac{3}{4}\frac{s}{s^2+a^2}$

$= \frac{1}{4}\frac{s}{s^2+3^2} + \frac{3}{4}\frac{s}{s^2+1^2}$

$= \frac{1}{4}\left[\frac{s}{s^2+9} + \frac{3s}{s^2+1}\right]$

$= \frac{s}{4}\left[\frac{1}{s^2+9} + \frac{3}{s^2+1}\right]$

$= \frac{s}{4}\left[\frac{s^2+1 + 3(s^2+9)}{(s^2+9)(s^2+1)}\right]$

$$= \frac{s}{4}\left[\frac{s^2+1+3s^2+27}{(s^2+9)(s^2+1)}\right]$$

$$= \frac{s}{4}\left[\frac{4s^2+28}{(s^2+9)(s^2+1)}\right]$$

$$= \frac{s}{\cancel{4}}\left[\frac{\cancel{4}(s^2+7)}{(s^2+9)(s^2+1)}\right]$$

L[

$$L\{\cos^3 t\} = \frac{s(s^2+7)}{(s^2+9)(s^2+1)}$$

$$\boxed{\therefore L\{\cos^3 t\} = \frac{s(s^2+7)}{(s^2+9)(s^2+1)}}$$

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Q3) Prove that $L\{e^{at}\} = \frac{1}{s-a}$

Soln:- $L\{f(t)\} = \int_0^\infty e^{-st}.f(t)\,dt$

$$L\{e^{at}\} = \int_0^\infty e^{-st}.e^{at}\,dt$$

[$\therefore$ when base is same power are added]

$$= \int_0^\infty e^{-st+at}.dt$$

$$= \int_0^\infty e^{-(s-a)t}.dt$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty \quad \left(\because \int e^x dx = \frac{e^x}{x}\right)$$

$$= \left[\frac{e^{-(s-a)\infty}}{-(s-a)} - \frac{e^{-(s-a)0}}{-(s-a)}\right]$$

($\therefore$ Multiplication of anything with ∞ is ∞ and with 0 is 0)

$$= \frac{e^\infty}{-(s-a)} - \frac{e^0}{-(s-a)}$$

$$= 0 + \frac{1}{s-a}$$

[$\therefore e^\infty$ is 0 and e^0 by is 1]

$$L\{e^{at}\} = \frac{1}{s-a}$$ Hence Proved

Q4) Find Laplace transformation of $f(t) = \left(\frac{\sin 4t}{t}\right)$

Soln:- By Defination of Laplace transformation

$$L\{\sin at\} = \int_0^\infty e^{-st} \cdot f(t)\,dt$$

$$e^{iat} = \cos at + i\sin at$$

L[Taking Laplace on both sides

$$L\{e^{iat}\} = L\{\cos at + i\sin at\}$$

$$\frac{1}{s-ia} = L\{\cos at\} + i\,L\{\sin at\} \qquad \left(\because L\{e^{iat}\} = \frac{1}{s-ia}\right)$$

$$\frac{1}{s-ia} \times \frac{s+ia}{s+ia} = L\{\cos at\} + i\,L\{\sin at\}$$

$$\frac{s+ia}{s^2-i^2a^2} = L\{\cos at\} + i\,L\{\sin at\}$$

$$\frac{s+ia}{s^2+a^2} = L\{\cos at\} + i\,L\{\sin at\}$$

$$\frac{s}{s^2+a^2} + \frac{ia}{s^2+a^2} = L\{\cos at\} + i\,L\{\sin at\}$$

Equating real and imaginary part.

$$L\{\cos at\} = \frac{s}{s^2+a^2}, \qquad L\{\sin at\} = \frac{a}{s^2+a^2}$$

So by using above formula

$$L(\sin 4t) = \frac{4}{s^2+16}$$

$$\therefore L\left(\frac{\sin 4t}{t}\right) = \int_0^\infty \frac{4}{s^2+16}\,ds$$

$$\boxed{\therefore L\left[\frac{f(t)}{t}\right] = \int_0^\infty f(s)\,ds}$$

$$= \left[\tan^{-1}\left(\frac{s}{4}\right)\right]_0^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{4}\right)$$

$$= \cot^{-1}\left(\frac{s}{4}\right)$$

$$\boxed{\therefore \left(\frac{\sin 4t}{t}\right) = \cot^{-1}\left(\frac{s}{4}\right)}$$

Q5) Find Laplace transformation of $f(t) = e^{2t}\sinh 3t$

Soln:- $\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

where $\theta = at$

$L\{\sinh(at)\} = L\left\{\frac{e^{at} - e^{-at}}{2}\right\}$

$= \frac{1}{2} L\{e^{at} - e^{-at}\}$

$= \frac{1}{2} L\{e^{at}\} - L\{e^{-at}\}$ $\left(\because L\{e^{at}\} = \frac{1}{s-a}\right)$

$= \frac{1}{2}\left\{\frac{1}{s-a}\right\} - \left\{\frac{1}{s+a}\right\}$

$= \frac{1}{2}\left\{\frac{1}{s-a} - \frac{1}{s+a}\right\}$

$= \frac{1}{2}\left[\frac{\cancel{s}+a - \cancel{s}+a}{s^2-a^2}\right]$

$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}a}{s^2-a^2}\right]$

$\therefore L\{\sinh(at)\} = \left[\frac{a}{s^2-a^2}\right]$

$f(t) = e^{2t}\sinh 3t$

So, from above equation we have,

$L[\sinh 3t] = \frac{3}{s^2-9}$

$\therefore L[f(t)] = \frac{3}{(s-2)^2-9}$ $\left[\because L\{e^{at} f(t)\} = f(s-a)\right]$

$\therefore L[f(t)] = \frac{3}{s^2-4s-5}$

$$\boxed{\therefore L[f(t)] = e^{2t}\sinh 3t = \frac{3}{s^2-4s-5}}$$

Q6) Find Laplace transformation of $\sin^3 t$?

Soln:- $\sin^3 t = \frac{(3\sin t - \sin 3t)}{4}$

$= \frac{3}{4}\sin t - \frac{\sin 3t}{4}$

$\therefore L\{\sin^3 t\} = \frac{3}{4} L\{\sin t\} - \frac{1}{4} L\{\sin 3t\}$

$\therefore \boxed{L\{\sin at\} = \frac{a}{s^2+a^2}}$

$= \frac{3}{4}\left[\frac{a}{s^2+a^2}\right] - \frac{1}{4}\left[\frac{a}{s^2+a^2}\right]$

$= \frac{3}{4}\left[\frac{1}{s^2+1^2}\right] - \frac{1}{4}\left[\frac{3}{s^2+3^2}\right]$

$= \frac{3}{4}\left[\frac{1}{s^2+1}\right] - \frac{3}{4}\left[\frac{1}{s^2+9}\right]$

$\therefore L\{\sin^3 t\} = \frac{3}{4}\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]$

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$$\boxed{\therefore \sin^3 t = \frac{3}{4}\cdot\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]}$$

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Q1) Prove that $\Gamma n+1 = n\Gamma n$

Soln:- Proof we know that

$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\, dx$ ($\because$ formula as written)

$\Gamma n+1 = \int_0^\infty e^{-x} x^{n+1-1}\, dx$

$= \int_0^\infty e^{-x} x^n\, dx$

$= \int_0^\infty \underset{U}{x^n}\, \underset{V}{e^{-x}}\, dx$

$\left[\because \int U.V\,dx = U\int V dx - \int\left[\frac{d}{dx} U \int v dx\right] dx\right]$

$= x^n \int_0^\infty e^{-x} - \int_0^\infty \left[\frac{d}{dx} x^n \int_0^\infty e^{-x} dx\right] dx$

$= x^n \left[\frac{e^{-n}}{-1}\right]_0 - \int_0^\infty n x^{n-1} \frac{e^{-x}}{-1}\, dx$

$= x^n \left[\frac{e^{-\infty}}{-1} - \frac{e^{-0}}{-1}\right] + n \underbrace{\int_0^\infty e^{-x} x^{n-1}\, dx}_{\Gamma n}$

$= 0 + n\Gamma n$

$\therefore \Gamma n+1 = n\Gamma n$

Hence Proved

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Q2) $\int_0^\infty e^{-x^2} \cdot x^2 dx$

Soln:- $I = \int_0^\infty e^{-x^2} \cdot x^2 dx$

put,

$x^2 = t$

$x = t^{1/2}$

$dx = \frac{1}{2\sqrt{t}} dt$ $\quad \left(\because \frac{d}{dx} \sqrt{y} = \frac{1}{2\sqrt{x}} \right)$

when $x \to 0, \; t \to 0$

$x \to \infty, \; t \to \infty$

$I = \int_0^\infty e^{-t} \cdot t \frac{1}{2} t^{-1/2} dt$

$= \frac{1}{2} \int_0^\infty e^{-t} \cdot t^1 t^{-1/2} dt$ $\qquad \boxed{t^1 \cdot t^{-1/2} = t^{1-1/2} = t^{1/2}}$

$= \frac{1}{2} \int_0^\infty e^{-t} \cdot t^{1/2} dt$

$= \frac{1}{2} \int_0^\infty e^{-t} t^{3/2-1} dt$

$$\therefore \left[\Gamma n = \int_0^\infty e^{-x} x^{n-1} dx\right]$$

$$I = \frac{1}{2}\Gamma\frac{3}{2}$$

$$= \frac{1}{2}\Gamma\left(\frac{1}{2}+1\right)$$

$$= \frac{1}{2}\cdot\frac{1}{2}\Gamma\frac{1}{2} \qquad \left(\because \Gamma\frac{1}{2} = \sqrt{\pi}\right)$$

$$I = \frac{1}{4}\sqrt{\pi}$$

$$\boxed{\int_0^\infty e^{-x^2}\cdot x^2\, dx = \frac{\sqrt{\pi}}{4}}$$

Q3) $\int_0^\infty x^{1/4}\cdot e^{-\sqrt{x}}\, dx$

Soln:- $I = \int_0^\infty x^{1/4}\cdot e^{-\sqrt{x}}\, dx$

Put $\sqrt{x} = t$

$x = t^2$

$dx = 2t\,dt$

$$\left[\therefore \frac{d}{dx} x^n = n x^{n-1}\right]$$

when $x \to 0,\ t \to 0$

$x \to \infty,\ t \to \infty$

$$I = \int_0^\infty (t^2)^{1/4}\cdot e^{-t}\cdot 2t\,dt$$

$$= 2\int_0^\infty t^{2\times 1/4}\cdot e^{-t}\cdot t\,dt$$

$$= 2\int_0^\infty e^{-t}\, t^{1/2}\, t\,dt \qquad \left(t^{1/2}\cdot t^1 = t^{1/2+1}\right)$$

$$= 2\int_0^\infty e^{-t}\, t^{1/2+1}\, dt$$

$$= 2\int_0^\infty e^{-t}\, t^{3/2}\, dt$$

$$= 2\int_0^\infty e^{-t} \cdot t^{5/2 - 1} \cdot dt$$

$$\left[\because \Gamma n = \int_0^\infty e^{-x} x^{n-1} dx \right]$$

$$\therefore I = 2\Gamma\frac{5}{2}$$

L

$$= 2\Gamma\left(\frac{3}{2}+1\right)$$

$$= \cancel{2} \cdot \frac{3}{\cancel{2}} \Gamma\frac{3}{2}$$

$$= 3\Gamma\left(\frac{1}{2}+1\right)$$

$$= \frac{3}{\cancel{2}} \times \frac{1}{2} \Gamma\frac{1}{2}$$

$$= \frac{3}{2}\sqrt{\pi}$$

$$\boxed{\int_0^\infty x^{1/4} e^{-\sqrt{x}} dx = \frac{3}{2}\sqrt{\pi}}$$

Q4) If $\beta(n,3) = \frac{1}{3}$ & n is a positive integer. find n.

Soln:- $\beta(n,3) = \frac{1}{3}$

$$\beta(m,n) = \frac{\Gamma m \, \Gamma n}{\Gamma m+n} = \frac{1}{3}$$

$$\beta(n,3) = \frac{\Gamma n \, \Gamma 3}{\Gamma n+3} = \frac{1}{3}$$

$$= \frac{\Gamma n \, \Gamma 2+1}{\Gamma n+2+1} = \frac{1}{3}$$

$$= \frac{\Gamma n \, 2!}{(n+2)\Gamma n+2} = \frac{1}{3}$$

$$= \frac{2\Gamma n}{(n+2)\Gamma(n+1)+1} = \frac{1}{3} \quad (\because \Gamma n+1 = n\Gamma n)$$

$$= \frac{2\Gamma n}{(n+2)(n+1)\Gamma n+1} = \frac{1}{3}$$

$$\frac{2\cancel{\Gamma n}}{(n+2)(n+1)n\cancel{\Gamma n}} = \frac{1}{3}$$

$$\frac{6}{n(n+2)(n+1)} = 1$$

$n(n+2)(n+1) = 6$

$n^2+2n(n+1) = 6$

$n^3+2n^2+n^2+2n+n$

$n-1=0$

$n=1$

$n^3+3n^2+2n-6=0$

$\boxed{n(n^2+4n+6)=0}$

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Q5) Prove that $\beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$

$\rightarrow$ By definition of beta function

$$\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$$

put $x = \sin^2\theta$ and $(1-x) = \cos^2\theta$

$dx = 2\sin\theta\cos\theta\,d\theta$

where $x \rightarrow 0,\ \theta \rightarrow 0$

$x \rightarrow 1,\ \theta \rightarrow (\pi/2)$

$$\beta(m,n) = \int_0^{\pi/2} (\sin^2\theta)^{m-1}(\cos^2\theta)^{n-1}\,2\sin\theta\cos\theta\,d\theta$$

$$= 2\int_0^{\pi/2} (\sin\theta)^{2m-2}\cdot(\cos\theta)^{2n-2}\sin\theta\cdot\cos\theta\,d\theta$$

$$= 2\int_0^{\pi/2} \sin\theta^{2m-2+1}\ \cos^{2n-2+1}\theta\,d\theta$$

$$= 2\int_0^{\pi/2} \sin^{2m-1}\theta\ \cos^{2n-1}\theta\,d\theta$$

$$\boxed{\therefore \beta(m,n) = 2\int_0^{\pi/2} \sin^{2m-1}\theta\ \cos^{2n-1}\theta\,d\theta}$$

Q6) Prove that

$\beta(m,n) = \beta(n,m)$

Soln:- $\beta(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$

put $1-x = t$

$x = 1-t$

$dx = -dt$

where $x=0,\ t=1$

$x=1,\ t=0$

$\beta = \int_1^0 (1-t)^{m-1}.(t)^{n-1}\,-dt$

$= \int_1^0 (1-t)^{m-1}.(t)^{n-1}\,-dt$

$= -\int_1^0 (1-t)^{m-1}.t^{n-1}.dt$

To remove minus the limits will change

$\beta = \int_0^1 (1-t)^{m-1}.t^{n-1}\,dt$

$\beta = \int_0^1 t^{n-1}.(1-t)^{m-1}.dt$

$\beta(n,m) = \int_0^1 t^{n-1}.(1-t)^{m-1}\,dt$

$\beta(m,n) = \beta(n,m)$ Hence Proved.

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Q1) Evaluate $\int_0^{\pi}\int_0^{a\sin\theta} r\,dr.d\theta$

Soln:- Let $I = \int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta$

$= \int_0^{\pi}\left[\frac{r^2}{2}\right]_0^{a\sin\theta} d\theta$

$= \frac{1}{2}\int_0^{\pi}\left[r^2\right]_0^{a\sin\theta} d\theta$

$= \frac{1}{2}\int_0^{\pi}\left[a^2\sin^2\theta - 0\right] d\theta$

$= \frac{1}{2}\int_0^{\pi} a^2\sin^2\theta\,d\theta$

$= \frac{a^2}{2}\int_0^{\pi} \sin^2\theta\,d\theta$

$\therefore \cos 2\theta = 1 - 2\sin^2$

$2\sin^2\theta = 1 - \cos$

$\sin^2\theta = (1-$

$= \frac{a^2}{2}\int_0^{\pi}\frac{(1-\cos 2\theta)}{2}\,d\theta$

$= \frac{a^2}{4}\int_0^{\pi}(1-\cos 2\theta)\,d\theta$

$$= \frac{a^2}{4}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$\left(\because \int 1\, d\theta = \theta,\ \because \int \cos 2\theta = \frac{\sin 2\theta}{2}\right)$

$$= \frac{a^2}{4}\left[\pi - \frac{\sin 2\theta}{2} - 0\right]$$

$$= \frac{a^2}{4}\left[\pi - 0\right] \qquad \left(\because \sin \pi/2 = 0\right)$$

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$$= \frac{a^2\pi}{4}$$

$$\boxed{\therefore \int_0^{\pi}\int_0^{a\sin\theta} r\,dr\,d\theta = \frac{a^2\pi}{4}}$$

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(Q) Evaluate :- $\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2 \sin\theta\, dr\, d\theta$

Soln:- Let $I = \int_0^{\pi} \sin\theta \left[\frac{r^3}{3}\right]_0^{a(1-\cos\theta)} d\theta$

$$= \frac{a^3}{3}\int_0^{\pi} \sin\theta\,(1-\cos\theta)^3\, d\theta$$

Let $1-\cos\theta = t$

Derivation on both sides

$0 - (-\sin\theta) = dt$

$\sin\theta\, d\theta = dt$ $\qquad \left(\because \frac{d\cos\theta}{d\theta} = \right.$ [cut off]

when $\theta \longrightarrow 0,\ t \longrightarrow 0$

$\theta \longrightarrow \pi,\ t \longrightarrow 2$

$$\therefore I = \frac{a^3}{3}\int_0^{2} t^3\, dt$$

$$= \frac{a^3}{3}\left[\frac{t^4}{4}\right]_0^2$$

$$= \frac{a^3}{3}\left[\frac{16}{4} - 0\right]$$

$$= \frac{a^3}{3}\left[\frac{\cancel{16}^{4}}{\cancel{4}}\right]$$

$$\therefore I = \frac{4a^3}{3}$$

$$\boxed{\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2\sin\theta\, dr\, d\theta = }$$ [cut off]

3) Evaluate by changing order of Integration

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} dy \cdot dx$$

Soln:- Linear Limit of Integration

$x = 0$ to $x = a$

$y = 0$ to $y = \sqrt{a^2-x^2}$

$y^2 = a^2 - x^2$

$x^2 + y^2 = a^2$

Now by changing the order of Integration

$y = 0$ & $y = a$

$x = 0$ $x = \sqrt{a^2-x^2}$

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} dx \cdot dy = \int_{y=0}^{a} \int_{x=0}^{\sqrt{a^2-x^2}} dx \cdot dy$$

$$= \int_{y=0}^{a} [x]_0^{\sqrt{a^2-x^2}} dy$$

$$= \left[\frac{1}{2} y \sqrt{a^2-x^2} + \frac{1}{2} a^2 \sin^{-1} y/a\right]$$

$$= \frac{1}{2} a^2 \sin^{-1}(1)$$

$$= \frac{a^2}{2} \cdot \frac{\pi}{2} = \frac{a^2\pi}{4}$$

$$\boxed{\therefore \int_0^a \int_0^{\sqrt{a^2-x^2}} dy \cdot dx = \frac{a^2\pi}{4}}$$

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Q4) Find the double integration the area included be[tween] the Curves $y^2 = 4ax$ & $x^2 = 4ay$

Soln:- Given equation of Curve are $x^2 = 4ay$, $y^2 = 4$[ax]

The point of intersection are

$y^2 = 4ax$

$\therefore y^4 = 16a^2x^2$ ($\because$ Squaring on both sides)

$\therefore y^4 = 16a^2(4ay)$ (place $x^2 = 4ay$ as given)

$\therefore y^{\cancel{4}} = \cancel{16a}\ 64a^3 \cancel{y}$

$\therefore y^3 = 64a^3$

$\therefore y = 4a$

$\therefore (4a, 4a)$ is the point of intersection

we Know that, Area $= \iint dx \cdot dy$

$\therefore$ So, $y^2 = 4ax$ $\qquad$ $x^2 = 4ay$

$y = \sqrt{4ax}$ $\qquad$ $y = \frac{x^2}{4a}$

$y = 2\sqrt{ax}$

$\therefore$ Required area is $A = \int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dx \cdot dy$

$$= \int_0^{4a} dx\, [y]_{x^2/4a}^{2\sqrt{ax}}$$

$$A = \int_0^{4a} \left(2\sqrt{ax} - \frac{x^2}{4a}\right) dx$$

$$= 2\sqrt{a} \cdot \left[\frac{x^{3/2}}{3/2}\right]_0^{4a} - \frac{1}{4a}\left(\frac{x^3}{3}\right)_0^{4a}$$

$$= 2\sqrt{a} \cdot \frac{2}{3} \cdot 4^{3/2} \cdot a^{3/2} - \frac{1}{12a}\, 64a^3$$

$$= \frac{16}{3} a^2$$

$$\therefore \text{Area} = A = \frac{16}{3} a^2$$

$$\boxed{\text{Area included between the curves is } \frac{16}{3} a^2}$$