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Q1) $L[\cos at] = \frac{s}{s^2+a^2}$

$\therefore \cos\theta = \frac{e^{i\theta}+e^{-i\theta}}{2}$

$\therefore \cos at = \frac{e^{i(at)}+e^{-i(at)}}{2}$

By Laplace transformation formula

$$L\{\cos at\} = \int_0^\infty e^{-st}\cos at\, dt$$

$$= \int_0^\infty e^{-st}\left(\frac{e^{i(at)}+e^{-i(at)}}{2}\right)dt$$

$$= \frac{1}{2}\int_0^\infty e^{-st}\left(e^{(ia)t}+e^{(-ia)t}\right)dt$$

$$= \frac{1}{2}\left[\int_0^\infty e^{-st}e^{(ia)t}\,dt + \int_0^\infty e^{-st}(e^{(-ia)t}\,dt\right]$$

$$= \frac{1}{2}\left[\frac{1}{s-ai}+\frac{1}{s-(-ia)}\right]$$

$$\left(\because \int_0^\infty e^{-st}e^{(ia)}\,dt = \frac{1}{s-a}\right)$$

$$= \frac{1}{2}\left[\frac{s+\cancel{a} + s-\cancel{a}}{s^2+a^2}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}s}{s^2+a^2}\right]$$

$$L[\cos at] = \frac{s}{s^2+a^2}$$

<u>Hence</u> <u>Proved</u>

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Q 2) $L\{\cos^3 t\} = ?$

Soln:- Since, $\cos 3t = 4\cos^3 t - 3\cos t$

$$\cos 3t + 3\cos t = 4\cos^3 t$$

$$\cos^3 t = \frac{\cos 3t}{4} + \frac{3\cos t}{4}$$

$$\therefore L\{\cos^3 t\} = \frac{1}{4} L\{\cos 3t\} + \frac{3}{4} L\{\cos t\}$$

$$\therefore \left[\because L\{\cos at\} = \frac{s}{s^2+a^2}\right]$$ Here a is a Coefficient

$$= \frac{1}{4}\frac{s}{s^2+a^2} + \frac{3}{4}\frac{s}{s^2+a^2}$$

$$= \frac{1}{4}\frac{s}{s^2+3^2} + \frac{3}{4}\frac{s}{s^2+1^2}$$

$$= \frac{1}{4}\left[\frac{s}{s^2+9} + \frac{3s}{s^2+1}\right]$$

$$= \frac{s}{4}\left[\frac{1}{s^2+9} + \frac{3}{s^2+1}\right]$$

$$= \frac{s}{4}\left[\frac{s^2+1 + 3(s^2+9)}{(s^2+4)(s^2+1)}\right]$$

$$= \frac{s}{4}\left[\frac{s^2+1+3s^2+27}{(s^2+9)(s^2+1)}\right]$$

$$= \frac{s}{4}\left[\frac{4s^2+28}{(s^2+9)(s^2+1)}\right]$$

$$= \frac{s}{\cancel{4}}\left[\frac{\cancel{4}(s^2+7)}{(s^2+9)(s^2+1)}\right]$$

L[cos

$$L\{\cos^3 t\} = \frac{s(s^2+7)}{(s^2+9)(s^2+1)}$$

$$\boxed{\therefore L\{\cos^3 t\} = \frac{s(s^2+7)}{(s^2+9)(s^2+1)}}$$

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Q3) Prove that $L\{e^{at}\} = \frac{1}{s-a}$

Soln:- $L\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t)\, dt$

$$L\{e^{at}\} = \int_0^\infty e^{-st} \cdot e^{at}\, dt$$

[∵ when base is same power are added]

$$= \int_0^\infty e^{-st+at} \cdot dt$$

$$= \int_0^\infty e^{-(s-a)t} \cdot dt$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty \qquad \left(\because \int e^x dx = \frac{e^x}{x}\right.$$

$$= \left[\frac{e^{-(s-a)\infty}}{-(s-a)} - \frac{e^{-(s-a)0}}{-(s-a)}\right]$$

(∵ Multiplication of anything with ∞ is ∞ and with 0 is 0)

$$= \frac{e^{\infty}}{-(s-a)} - \frac{e^{0}}{-(s-a)}$$

$$= 0 + \frac{1}{s-a}$$

[∵ e^{∞} is 0 and e^0 by is 1]

$$L\{e^{at}\} = \frac{1}{s-a}$$

Hence Proved

L[Cos

Q4) Find Laplace transformation of $f(t) = \left(\frac{\sin 4t}{t}\right)$

Soln:- By Defination of laplace transformation

$$L\{\sin at\} = \int_0^\infty e^{-st} \cdot f(t)\,dt$$

$$e^{iat} = \cos at + i\sin at$$

Taking Laplace on both sides

$$L\{e^{iat}\} = L\{\cos at + i\sin at\}$$

$$\frac{1}{s-ia} = L\{\cos at\} + i\,L\{\sin at\} \qquad \left(\therefore L\{e^{iat}\} = \frac{1}{s-ia}\right)$$

$$\frac{1}{s-ia} \times \frac{s+ia}{s+ia} = L\{\cos at\} + i\,L\{\sin at\}$$

$$\frac{s+ia}{s^2-i^2a^2} = L\{\cos at\} + i\,L\{\sin at\}$$

$$\frac{s+ia}{s^2+a^2} = L\{\cos at\} + i\,L\{\sin at\}$$

$$\frac{s}{s^2+a^2} + \frac{ia}{s^2+a^2} = L\{\cos at\} + i\,L\{\sin at\}$$

Equating real and imaginary part.

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$$L\{\cos at\} = \frac{s}{s^2+a^2}, \quad L\{\sin at\} = \frac{a}{s^2+a^2}$$

So by using above formula

$$L(\sin 4t) = \frac{4}{s^2+16}$$

$$\therefore L\left(\frac{\sin 4t}{t}\right) = \int_0^\infty \frac{4}{s^2+16}\,ds$$

$$\left[\therefore L\left[\frac{f(t)}{t}\right] = \int_0^\infty f(s)\,ds\right]$$

$$= \left[\tan^{-1}\left(\frac{s}{4}\right)\right]_0^\infty$$

$$= \frac{\pi}{2} - \tan^{-1}\left(\frac{s}{4}\right)$$

$$= \cot^{-1}\left(\frac{s}{4}\right)$$

$$\boxed{\therefore \left(\frac{\sin 4t}{t}\right) = \cot^{-1}\left(\frac{s}{4}\right)}$$

L [Cos

Q5) Find laplace transformation of $f(t) = e^{2t} \sinh 3t$

Soln:- $\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$

where $\theta = at$

$$L\{\sinh(at)\} = L\left\{\frac{e^{at} - e^{-at}}{2}\right\}$$

$$= \frac{1}{2} L\{e^{at} - e^{-at}\}$$

$$= \frac{1}{2} L\{e^{at}\} - L\{e^{-at}\} \quad \left(\because L\{e^{at}\} = \frac{1}{s-a}\right)$$

$$= \frac{1}{2} \left\{\frac{1}{s-a}\right\} - \left\{\frac{1}{s+a}\right\}$$

$$= \frac{1}{2} \left\{\frac{1}{s-a} - \frac{1}{s+a}\right\}$$

$$= \frac{1}{2} \left[\frac{\cancel{s}+a - \cancel{s}+a}{s^2 - a^2}\right]$$

$$= \frac{1}{\cancel{2}} \left[\frac{\cancel{2}a}{s^2 - a^2}\right]$$

$$\therefore L\{\sinh(at)\} = \left[\frac{a}{s^2 - a^2}\right]$$

$f(t) = e^{2t} \sinh 3t$

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So, from above equation we have,

$$L[\sinh 3t] = \frac{3}{s^2 - 9}$$

$$\therefore L[f(t)] = \frac{3}{(s-2)^2 - 9} \quad \left[\because L\{e^{at} f(t)\} = f(s-a)\right]$$

$$\therefore L[f(t)] = \frac{3}{s^2 - 4s - 5}$$

$$\boxed{\therefore L[f(t)] = e^{2t} \sinh 3t = \frac{3}{s^2 - 4s - 5}}$$

Q6) Find laplace transformation of $\sin^3 t$?

Soln:- $\sin^3 t = \left(\frac{3 \sin t - \sin 3t}{4}\right)$

$$= \frac{3}{4} \sin t - \frac{\sin 3t}{4}$$

$$\therefore L\{\sin^3 t\} = \frac{3}{4} L\{\sin t\} + -\frac{1}{4} L\{\sin 3t\}$$

$$\therefore \boxed{L\{\sin at\} = \frac{a}{s^2 + a^2}}$$

$$= \frac{3}{4}\left[\frac{a}{s^2 + a^2}\right] - \frac{1}{4}\left[\frac{a}{s^2 + a^2}\right]$$

$$= \frac{3}{4}\left[\frac{1}{s^2+1^2}\right] - \frac{1}{4}\left[\frac{3}{s^2+3^2}\right]$$

$$= \frac{3}{4}\left[\frac{1}{s^2+1}\right] - \frac{3}{4}\left[\frac{1}{s^2+9}\right]$$

$L[\text{Co}$

$$\therefore L\{\sin^3 t\} = \frac{3}{4}\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]$$

$$\boxed{\sin^3 t = \frac{3}{4}\cdot\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]}$$

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Q1) Prove that $\Gamma(n+1) = n\Gamma(n)$

Soln:- Proof we know that

$$\Gamma n = \int_0^\infty e^{-x} x^{n-1}\, dx \quad (\because \text{formula as written})$$

$$\Gamma(n+1) = \int_0^\infty e^{-x} x^{n+1-1}\, dx$$

$$= \int_0^\infty e^{-x} x^n\, dx$$

$$= \int_0^\infty \underset{U}{x^n}\, \underset{V}{e^{-x}}\, dx$$

$$\left[\because \int U.V\,dx = U\int V\,dx - \int\left[\frac{d}{dx}U\int V\,dx\right]dx\right]$$

$$= x^n \int_0^\infty e^{-x} - \int_0^\infty\left[\frac{d}{dx}x^n \int_0^\infty e^{-x}\,dx\right]dx$$

$$= x^n\left[\frac{e^{-n}}{-1}\right]_0^\infty - \int_0^\infty n x^{n-1}\,\frac{e^{-x}}{-1}\,dx$$

$$= x^n\left[\frac{e^{-\infty}}{-1} - \frac{e^{-0}}{-1}\right] + n\underbrace{\int_0^\infty e^{-x} x^{n-1}\,dx}_{\Gamma n}$$