CS70 Extra Problems 0

Andrew Chan

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1 Warm-Up: Liars and Truth-Tellers

You arrive on an island with 2 types of people: liars and truth-tellers. Everyone knows everything about each other, liars always lie, and truth-tellers always tell the truth.

Problem 1.1. You encounter Alice, Bob, and Charlie.
Alice says something but you can't hear her.
Bob says "Alice said she is a liar."
Charlie says "Don't listen to Bob, he is a liar."
What types of people are Bob and Charlie?

Problem 1.2. You have heard stories of gold on the island, and are allowed to ask a single yes-or-no question to a person from the island. What do you ask?

2 Bite-Sized Questions

Problem 2.1. Prove or disprove: $\sqrt{xy}$ is always irrational for distinct primes x, y.

Problem 2.2. Prove or disprove: The sum of 2 irrational numbers is always irrational.

Problem 2.3. In a group of 400 people, at least k of them share a birthday with another person. What is the minimum value of k?

Problem 2.4. Given 5 people anywhere on earth, prove that there exists a closed hemisphere containing at least 4 of them. (*Hint:* A great circle is defined by _ points)

3 Long Division... Forever

Problem 3. In decimal long division, the quotient $\frac{p}{q}$ is given by a recursive algorithm:

$$\text{div}(p, q) = \lfloor \frac{p}{q} \rfloor + 0.1 * \text{div}(p - \lfloor \frac{p}{q} \rfloor,\ 0.1 * q)$$

Using the definition of long division, show that the decimal representation of any rational number will have digits that eventually repeat forever.