3.6

(a)

$$Z \in (-\infty, +\infty)$$

Thus, Set

$$W = |Z - f(B)| \in [0, +\infty)$$

$$\sum_{Z} P(Z = z | B_1, B_2, \dots, B_n) = \left(\frac{1-\alpha}{1+\alpha}\right) \sum_{z=-\infty}^{+\infty} \alpha^{|Z-f(B)|}$$

$$= \left(\frac{1-\alpha}{1+\alpha}\right)\left(\sum_{z=-\infty}^{-1} \alpha^{|Z-f(B)|} + \alpha^0 + \sum_{z=1}^{+\infty} \alpha^{|Z-f(B)|}\right)$$

$$= \left(\frac{1-\alpha}{1+\alpha}\right)\left(2\sum_{w=1}^{+\infty} \alpha^w + 1\right) = \left(\frac{1-\alpha}{1+\alpha}\right)\left(\frac{2\alpha}{1-\alpha} + 1\right)$$

$$= \left(\frac{1-\alpha}{1+\alpha}\right)\left(\frac{1+\alpha}{1-\alpha}\right) = 1$$

(b)

Sample N times:

$$B_1, B_2, \dots B_k \dots, B_{10} \sim P(B_k = 1) = \frac{1}{2}$$

Estimate from ratio:

$$P(B_i = 1|Z = 128) \approx \frac{\sum_{j=1}^{N} I(1, (\mathrm{B_i})_\mathrm{j}) P(Z = 128|B_1, B_2, \dots, B_{10})}{\sum_{j=1}^{N} P(Z = 128|B_1, B_2, \dots, B_{10})}$$

$$= \frac{\sum_{j=1}^{N} I(1, (\mathrm{B_i})_\mathrm{j}) \left(\frac{1-\alpha}{1+\alpha}\right) \alpha^{|128-f(B)|}}{\sum_{j=1}^{N} \left(\frac{1-\alpha}{1+\alpha}\right) \alpha^{|128-f(B)|}}$$

$$= \frac{\sum_{j=1}^{N} I(1, (\mathrm{B_i})_\mathrm{j}) \left(\frac{1}{10}\right)^{|128-f(B)|}}{\sum_{j=1}^{N} \left(\frac{1}{10}\right)^{|128-f(B)|}}$$

Where

$$f(B) = \sum_{i=1}^{10} 2^i B_i$$

$$i \in \{2,5,8,10\}$$

(c)