4.4

$$L = \sum_t logP(x_t|x_{t-1}, x_{t-2}, x_{t-3})$$

Where

$$P(x_t|x_{t-1}, x_{t-2}, x_{t-3})$$
$$= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_t - a_1x_{t-1} - a_2x_{t-2} - a_3x_{t-3})^2\right)$$

Take it into L, we can get

$$L = \sum_t \log\left(\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x_t - a_1x_{t-1} - a_2x_{t-2} - a_3x_{t-3})^2\right)\right)$$
$$L = \sum_t \left(\log\left(\frac{1}{\sqrt{2\pi}}\right) - \frac{1}{2}(x_t - a_1x_{t-1} - a_2x_{t-2} - a_3x_{t-3})^2\right)$$
$$L = -\frac{1}{2}\sum_t (\log(2\pi) + (x_t - a_1x_{t-1} - a_2x_{t-2} - a_3x_{t-3})^2)$$

To maximize L, let

$$\frac{\partial L}{\partial a_i} = 0 \qquad i = 1,2,3$$

So we can get

$$\sum_t (x_t - a_1x_{t-1} - a_2x_{t-2} - a_3x_{t-3})x_{t-i} = 0 \qquad i = 1,2,3$$
$$\sum_t (a_1x_{t-1} + a_2x_{t-2} + a_3x_{t-3})x_{t-i} = \sum_t x_tx_{t-i} \qquad i = 1,2,3$$

Or denote it in matrix as

$$A\boldsymbol{a} = \boldsymbol{X}$$

Where

$$A = A_{ij} = \begin{pmatrix} \sum_t x_{t-1}x_{t-1} & \sum_t x_{t-1}x_{t-2} & \sum_t x_{t-1}x_{t-3} \\ \sum_t x_{t-2}x_{t-1} & \sum_t x_{t-2}x_{t-2} & \sum_t x_{t-2}x_{t-3} \\ \sum_t x_{t-3}x_{t-1} & \sum_t x_{t-3}x_{t-2} & \sum_t x_{t-3}x_{t-3} \end{pmatrix}$$

$$= 10^9 * \begin{pmatrix} 3.63557 & 3.63752 & 3.64025 \\ 3.63752 & 3.64300 & 3.64553 \\ 3.64025 & 3.64553 & 3.65160 \end{pmatrix}$$

$$X = X_{ij} = \begin{pmatrix} \sum_t x_t x_{t-1} \\ \sum_t x_t x_{t-2} \\ \sum_t x_t x_{t-3} \end{pmatrix} = 10^9 * \begin{pmatrix} 3.62910 \\ 3.63121 \\ 3.63403 \end{pmatrix}$$

The numerical result is reached by programming

we can get the inverse of A:

$$A^{-1} = 10^{-7} \begin{pmatrix} 2.85178 & -2.68694 & -0.160442 \\ -2.68694 & 5.36530 & -2.67780 \\ -0.160442 & -2.67780 & 2.83603 \end{pmatrix}$$

Then

$$\boldsymbol{a} = A^{-1}\boldsymbol{X} = 10^{-7} \begin{pmatrix} 2.85178 & -2.68694 & -0.160442 \\ -2.68694 & 5.36530 & -2.67780 \\ -0.160442 & -2.67780 & 2.83603 \end{pmatrix}$$

$$* 10^9 * \begin{pmatrix} 3.62910 \\ 3.63121 \\ 3.63403 \end{pmatrix} = \begin{pmatrix} 0.95067 \\ 0.01560 \\ 0.03190 \end{pmatrix}$$

(b)

For year 2001, similar to year 2000, we can get

$$A = A_{ij} = \begin{pmatrix} \sum_t x_{t-1}x_{t-1} & \sum_t x_{t-1}x_{t-2} & \sum_t x_{t-1}x_{t-3} \\ \sum_t x_{t-2}x_{t-1} & \sum_t x_{t-2}x_{t-2} & \sum_t x_{t-2}x_{t-3} \\ \sum_t x_{t-3}x_{t-1} & \sum_t x_{t-3}x_{t-2} & \sum_t x_{t-3}x_{t-3} \end{pmatrix}$$

$$= 10^9 * \begin{pmatrix} 1.03568 & 1.03582 & 1.03658 \\ 1.03582 & 1.03682 & 1.03749 \\ 1.03658 & 1.03749 & 1.03907 \end{pmatrix}$$

$$X = X_{ij} = \begin{pmatrix} \sum_t x_t x_{t-1} \\ \sum_t x_t x_{t-2} \\ \sum_t x_t x_{t-3} \end{pmatrix} = 10^9 * \begin{pmatrix} 1.03391 \\ 1.03407 \\ 1.03491 \end{pmatrix}$$

Then

$$A^{-1} = 10^{-6} * \begin{pmatrix} 1.185193 & -1.066724 & -1.172534 \\ -1.066724 & 2.072425 & -1.005108 \\ -1.172534 & -1.005108 & 1.121518 \end{pmatrix}$$

$\boldsymbol{a} = A^{-1}\boldsymbol{X}$

$= 10^{-6}$

$$* \begin{pmatrix} 1.185193 & -1.066724 & -1.172534 \\ -1.066724 & 2.072425 & -1.005107 \\ -1.172534 & -1.005108 & 1.121518 \end{pmatrix} \begin{pmatrix} 1033911229.265101 \\ 1034068446.344900 \\ 1034912098.350900 \end{pmatrix}$$

$$= \begin{pmatrix} 0.97189 \\ -0.06685 \\ 0.09318 \end{pmatrix}$$

To compute root mean squared error, we can use forumula

$$RMSD = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_t' - x_t)^2}$$

Where x_t' is the estimated of maximum likelihood x_t, according to the properties of Gaussian function we can get:

$$x_t' = a_1 x_{t-1} + a_2 x_{t-2} + a_3 x_{t-3}$$

Take it into RMSD, we can get

$$RMSD = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(x_t - a_1 x_{t-1} - a_2 x_{t-2} - a_3 x_{t-3})^2}$$

Because $x_0, x_1, x_2 is\ not\ estimated$ we just start from x_3

Take it into RMSD, we can get

$$RMSD_{2000} = 147719.48786534887$$

$$RMSD_{2001} = 14891.964415063494$$

A lower RMSE doesn't necessarily meant it performed better since the price in 2001 dropped about half, so the same percentage error will only result in 1/4 RMSE for 2001.

(c)

```
CSE250A0403 ×
"C:\Program Files\Java\jdk1.8.0_281\bin\java.exe" ...


===================4.4(a)===================
===============A==============
_____matrix_____
3.6355695068406982E9        3.6375226812277007E9        3.640246084574798E9
3.6375226812277007E9        3.6430018754187984E9        3.6455300713603005E9
3.640246084574798E9     3.6455300713603005E9        3.651597333563498E9
===============X==============
_____vector_____
3.6290974384690013E9        3.6312082884379973E9        3.6340303181715007E9
================reversed A=====================
_____matrix_____
2.851779411214503E-7        -2.6869371929156277E-7        -1.6044170416322285E-8
-2.6869371929156277E-7       5.36529914031267E-7      -2.6777998417208107E-7
-1.6044170416322285E-8       -2.6777998417208107E-7       2.836032066918198E-7
================solution of a=====================
_____vector_____
0.9506733661381332      0.015601330716549455        0.03189568503967166
================RMSD=====================
147719.48786534887
```

```
===================4.4(b)===================
===============A==============
_____matrix_____
1.0356759655175996E9        1.0358192035126007E9        1.0365825456692998E9
1.0358192035126007E9        1.0368159621070997E9        1.0374908186850007E9
1.0365825456692998E9        1.0374908186850007E9        1.0390670818302997E9
===============X==============
_____vector_____
1.0339112292651007E9        1.0340684463448999E9        1.0349120983509004E9
================reversed A====================
_____matrix_____
1.1851934399765474E-6       -1.0667243377201148E-6      -1.1725337930478298E-7
-1.0667243377201148E-6      2.072425147960771E-6        -1.0051076125520681E-6
-1.1725337930478298E-7      -1.0051076125520681E-6      1.1215182798098521E-6
================solution of a====================
_____vector_____
0.9718870435069675      -0.06684675603514734        0.09318344326243277
================RMSD====================
14891.964415063494

Process finished with exit code 0
```