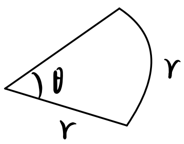


When arc length = radius, the angle θ = 1 radian (1^c) $\sim 57.3^\circ$

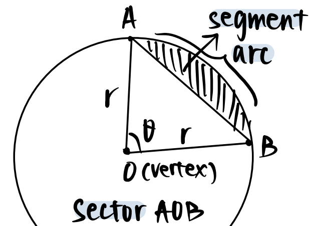
$180^\circ = \pi$ radians, degrees $\xrightarrow{\times \frac{\pi}{180} \text{ conversion factor}}$ radians

In degrees: $\begin{cases} \text{Arc length} = 2\pi r \times \frac{\theta}{360} \\ \text{Sector area} = \pi r^2 \times \frac{\theta}{360} \end{cases}$

In radians: $\begin{cases} \text{Arc length} = \theta r \\ \text{Sector area} = \frac{1}{2}\theta r^2 \end{cases}$

→ definition of a radian ex. arc length = $\frac{1^c \times r}{r\theta} = r$

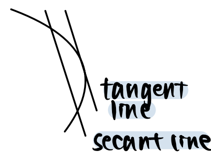
→ 2π radians in a circle, the portion is $\frac{\theta}{2\pi}$
$A = \pi r^2 \times \frac{\theta}{2\pi} = \frac{1}{2}\theta r^2$



major OR minor

Area of a triangle: $\frac{1}{2}AB\sin C$

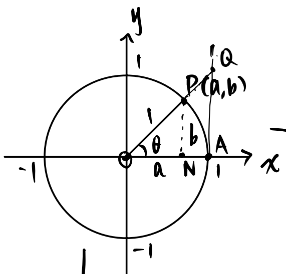
Area of a segment = $A_{sector} - A_{triangle}$



Oblique (non-right) triangles:

- Sine law: $\frac{\sin A}{a} = \frac{\sin B}{b}$
- Cosine law: $c^2 = a^2 + b^2 - 2ab\cos C$ → last resort

The unit circle ($x^2 + y^2 = 1$) is the circle with center (0,0) and radius of 1 unit



Angles are measured counterclockwise from the positive x-axis until the terminal arm is reached, may terminate in quadrants I, II, III, IV

- θ is negative for clockwise rotations

Sine and cosine in a unit circle: $\boxed{P(a,b)}$

$\sin\theta = \frac{b}{1} = b$, $\cos\theta = \frac{a}{1} = a$

can also be written as $P(\cos\theta, \sin\theta)$

$\therefore \cos^2\theta + \sin^2\theta = 1$ from pythagorean theorem

Tangent in a unit circle: expand OP → OQ, $\boxed{Q(1, \tan\theta)}$

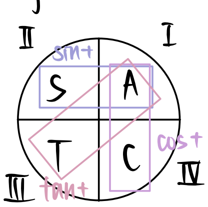
$\tan\theta = \frac{Q}{1} = Q$

position of Q relative to A is the tangent function

(on the diagram), $\triangle ONP \sim \triangle OAQ$

$\frac{AQ}{OA} = \frac{NP}{ON}$, $\frac{AQ}{1} = \frac{\sin\theta}{\cos\theta}$, $\therefore \tan\theta = \frac{\sin\theta}{\cos\theta}$ → quotient entity

Trignometric ratios:



(all students take calculus)

For $P(\sin\theta, \cos\theta)$, the point directly opposite is $(-\sin\theta, -\cos\theta)$

Co-terminal angles: from the refence angle, ex. $\frac{\pi}{3} \to \frac{2\pi}{3}$

Adding any integer multiple of 2π to θ won't change the position

- $\cos(\theta + 2k\pi) = \cos\theta$, $\sin(\theta + 2k\pi) = \sin\theta$

For tangent, $\tan(\theta + \pi) = \tan\theta$ (positive in quadrant III)

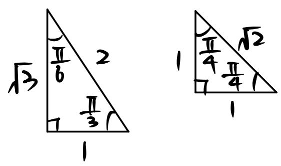
$k \in \mathbb{Z}$

$\sin(-\theta) = -\sin\theta$, $\cos(-\theta) = \cos\theta$

TRIGONOMETRIC IDENTITIES

- $\tan\theta = \frac{\sin\theta}{\cos\theta}$
- $\sin(-\theta) = -\sin\theta$
- $\cos(-\theta) = \cos\theta$
- $\sin(\pi-\theta) = \sin\theta$
- $\cos(\pi-\theta) = -\cos\theta$
- $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$
- $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$
- $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$
- $\sec\theta = \frac{1}{\cos\theta}$
- $\cot\theta = \frac{1}{\tan\theta}$
- $\cos^2\theta + \sin^2\theta = 1$
- $1+\tan^2\theta = \sec^2\theta$
- $\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

Special angles: & their integer multipliers



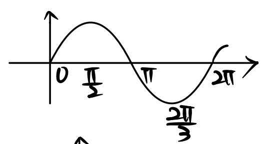
special triangles

angle	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin\theta$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos\theta$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

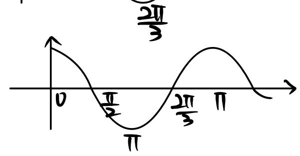
↳ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Degrees	0	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined

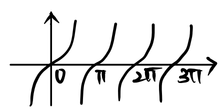
sine function:



cosine function:



tangent function:



Every integer multiplier of the 5 special angles has the same absolute value for sin, cos, tan

- Ex. $\left|\cos\frac{5\pi}{4}\right| = \cos\frac{5\pi}{4}$
  ↳ positive/negative depending on the quadrant

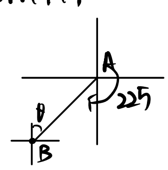
Ambiguous case of the sine law:

- 3 measures that can fix a triangle: SSS, SAS, ASA, AAS (any 2 given angles also secure the third)
  ↳ Measures that can't: AAA, SSA

there may be 0-2 possible triangles ← between 0°–180°, 2 angles have the same sine } no ambiguous case of cosine law

True bearings: measured clockwise from the y-axis, we consider true north

- A bearing is always 3 figures, ex. 72° = 072 → implied degrees, no radians in bearings ★ use cosine law
- Back bearing: ex. B is on a 225 bearing from A,
  ↳ add/subtract 180° find bearing of A for B
  ↓
  225 − 180 = 045



$\mathbb{R} \subset \mathbb{C}$ → real numbers are a subset of complex numbers

- Ex. $x^2+1=0$, $x^2=-1$, $x=\pm\sqrt{-1}=\boxed{\pm i}$
  - ↳ when defining i, $i=\sqrt{-1}$ ✗, $i^2=-1$ ✓

all complex numbers have 2 parts:
real part (Re) + imaginary part (Im)
↳ in the form $a+ib$, $a,b\in\mathbb{R}$

Let $w=a+ib$:
$Re(w)=a$, $Im(w)=b$
↳ could be wholly real or wholly imaginary

- Ex. $x^2+2x+3=0$, $x=\frac{-2\pm\sqrt{4-12}}{2}$
  $=\frac{-2\pm\sqrt{-8}}{2}=\frac{-2\pm i\sqrt{8}}{2}=-1\pm i\sqrt{2}$

Complex roots in functions: always occurs in conjugate pairs
- Quadratic: all real / all complex
- Cubic: 1 real + 2 complex / all real
- Quartic: 2 real + 2 complex / all real / all complex

Complex conjugate:
If $z=a+ib$, then $z^*=a-ib$

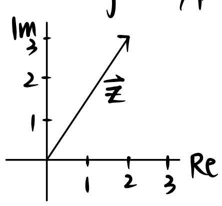
Operations with complex numbers in Cartesian form:
- Ex. Let $z=3-2i$, $w=2+5i$
  ① $z+w=5+3i$ ② $z-w=1-7i$ ③ $zw=(3-2i)(2+5i)=6+15i-4i-10i^2=16+11i$
  ④ $\frac{z}{w}=\frac{3-2i}{2+5i}=\frac{(3-2i)(2-5i)}{(2+5i)(2-5i)}=\frac{6-19i-10}{4+25}=\frac{-4-19i}{29}=-\frac{1}{29}(4+19i)$

- Ex. Show that $(z+w)^*=z^*+w^*$
  ↳ Let $z=a+ib$, $w=c+id$, $(a+ib+c+id)^*=(a+c+i(b+d))^*=a+c-i(b+d)$
  $=a-ib+c-id=z^*+w^*$
  ↳ LHS = RHS (for "show that" questions)

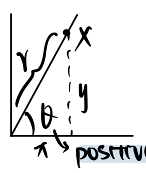
- Ex. Solve the system: $\begin{cases} z+2i=i(3-w) & ① \\ 7i(z-1)=2(z+2w) & ② \end{cases}$
  ① $z+2i=3i-iw$, $z=i-iw$
  ② $7iz-7i=2z+4w$, substitute in ① → $7i(i-iw-1)=2(i-iw)+4w$
  $-7+7w-7i=2i-2iw+4w$
  $9i-2iw-3w+7=0$, $9i+7=w(2i+3)$
  $w=\frac{9i+7}{2i+3}=\frac{(9i+7)(2i-3)}{(2i+3)(2i-3)}$
  $=\frac{-18-13i-21}{-13}$
  $=\frac{39+13i}{13}=3+i$

  $z=i-iw$
  $=i-i(3+i)$
  $=i-3i+1$
  $=1-2i$

Argand diagram/plane: $z=a+ib$ → complex numbers are vectors



Ex. $z=2+3i$



Any point P can be identified by the angle & distance from the origin

By trig. $(x,y)=(r\cos\theta, r\sin\theta)$

For complex numbers → Cartesian form: $z=a+ib=r\cos\theta+ir\sin\theta$
↳ Polar form: coordinates expressed as (r,θ),
$a+ib=|w|\cos\theta+|w|i\sin\theta=r\operatorname{cis}\theta$
→ Euler form: $re^{i\theta}=r\cos\theta+ir\sin\theta$
(Euler identity: $e^{i\pi}=-1$)

$\begin{cases} \text{Modulus: } |w|=\sqrt{a^2+b^2} \\ \text{Argument: } \arg(w)=\arctan\left(\frac{b}{a}\right) \end{cases}$

Ex. write $\sqrt{2}\,\mathrm{cis}\frac{\pi}{3}$ in Cartesian and Euler form

$\sqrt{2}\,\mathrm{cis}\frac{\pi}{3} = \sqrt{2}\cos\frac{\pi}{3} + \sqrt{2}\,i\sin\frac{\pi}{3}$ Euler form: $\sqrt{2}e^{\frac{\pi}{3}i}$

$= \sqrt{2} \times \frac{1}{2} + \sqrt{2}\,i \times \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$ Cartesian form: $\frac{\sqrt{2}}{2} + \frac{\sqrt{6}}{2}i$

Operations with complex numbers in Polar form:

- Ex. if $w = r_1\,\mathrm{cis}\,\theta_1$, $z = r_2\,\mathrm{cis}\,\theta_2$, then $wz = r_1 r_2\,\mathrm{cis}(\theta_1+\theta_2)$, $\frac{w}{z} = \frac{r_1}{r_2}\,\mathrm{cis}(\theta_1-\theta_2)$

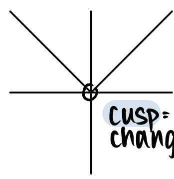
Operations with complex numbers in Euler form:

- Ex. if $w = r_1 e^{i\theta_1}$, $z = r_2 e^{i\theta_2}$, then $wz = r_1 r_2 e^{i(\theta_1+\theta_2)}$, $\frac{w}{z} = \frac{r_1}{r_2}e^{i(\theta_1-\theta_2)}$

Modulus equations & functions → basic modulus function: $f(x) = |x|$

For any modulus function $f(x)$,

$$|f(x)| = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$$



cusp = abrupt change in slope

→ Piece-wise function: the function changes depending on the domain

<u>Graphing: startplot → catalogu → abs → enter → |x|</u>

Solving modulus equations: <u>2 possibilities</u>

- Ex. $\left|\frac{x+1}{x}\right| = 2$, $\begin{cases} \frac{x+1}{x} = 2,\ 2x = x+1,\ x = 1 \\ \frac{x+1}{x} = -2,\ -2x = x+1,\ x = -\frac{1}{3} \end{cases}$
- Ex. $|2x-1| = |x+3|$, $\begin{cases} 2x-1 = x+3,\ x = 4 \\ 2x-1 = -x-3,\ x = -\frac{2}{3} \end{cases}$

Modulus inequalities:

- Ex. $|x-4| < 3$ → break into 2 parts: $-3 < x-4 < 3$

$$\left.\begin{cases} x-4 < 3,\ x < 7 \\ x-4 > -3,\ x > 1 \end{cases}\right\} 1 < x < 7$$

Partial fractions: ex. write $\frac{4x-5}{x^2-x-2}$ in partial fractions → $\boxed{\frac{A}{a} + \frac{B}{b}}$

denominators are factors

↳ Let $\frac{4x-5}{x^2-x-2} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$,

$A(x-2) + B(x+1) = 4x-5$, $Ax - 2A + Bx + B = 4x-5$

$$\left.\begin{cases} A + B = 4 \\ -2A + B = -5 \end{cases}\right\} -3A = -9,\ A = 3,\ B = 1$$

$\therefore \frac{4x-5}{x^2-x-2} = \frac{3}{x+1} + \frac{1}{x-2}$