Exponents:

$\underbrace{\square}_{\text{base}}^{\overbrace{\square}^{\text{exponent}}}$ } a power

For example, 4^5 is { 4 to the power of 5 / 4 to the 5^{th} power

Laws of exponents:

1. Multiplication of powers: $a^m \cdot a^n = a^{m+n}$
2. Division of powers: $a^m / a^n = a^{m-n}$

} bases must be the same!

3. Power of a power: $(a^m)^n = a^{mn}$
4. Power of a product: $(ab)^m = a^m b^m$
5. Power of a quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
6. The zero exponent: $a^0 = 1$, $a \neq 0, \infty$
7. Negative exponents: $a^{-m} = \left(\frac{1}{a}\right)^m$, $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$ → find the reciprocal value
8. Rational exponents: $a^{\frac{m}{n}} = \sqrt[n]{a^m}$, $a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$

HAESE SL 5A, HL 2A

Algebraic expansion and factorisation:

- Ex. expand and simplify $x^{-\frac{1}{2}}(x^{\frac{3}{2}} + 2x^{\frac{1}{2}} - 3x^{-\frac{1}{2}})$

$$= x^{-\frac{1}{2}+\frac{3}{2}} + 2x^{-\frac{1}{2}+\frac{1}{2}} - 3x^{-\frac{1}{2}-\frac{1}{2}}$$
$$= x^1 + 2x^0 - 3x^{-1}$$
$$= x + 2 \cdot 1 - 3 \cdot \frac{1}{x}$$
$$= x + 2 - \frac{3}{x}$$

HAESE SL 5B, HL 2B

Exponential equations: the variable you are trying to find is the exponent

- Equate indices: if $a^m = a^n$, then $m = n$

↳ only with same base!

HAESE SL 5C, HL 2C

Exponential functions:

- Base function: $y=a^x$, where $a>0$, $a\neq 1$, $x\in\mathbb{R}$
  - As $x\to\infty$, $y\to\infty$
  - As $x\to-\infty$, $y\to 0^+$, HA: $y=0$

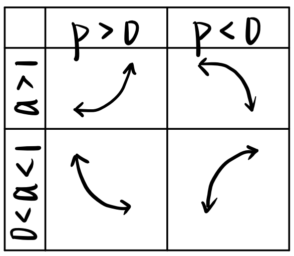
General exponential function form: $y=p\cdot a^{x-h}+k$, where $a>0$, $a\neq 1$, $p\neq 0$

if a^{-x}, it is reflected over the y axis

- a: how steep the graph increases/decreases
- h: horizontal translation
- k: vertical translation, HA: $y=k$

(translation by a vector of $\binom{h}{k}$)

Values of p, a and the graph:



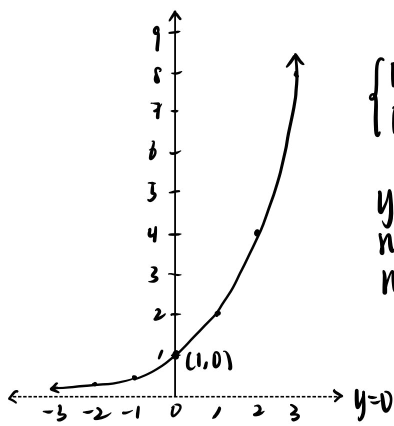
Sketching exponential functions:

- HA
- y-int
- 2 other points
- labelled points and axis!

Graphing functions:

1. Make a table of values
   - Ex. $f(x)=2^x$

x	-3	-2	-1	0	1	2	3
$f(x)$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8



D: $\{x \mid x\in\mathbb{R}\}$
R: $\{y \mid y>0, y\in\mathbb{R}\}$

y-int: (1,0)
no x-int, asymptotic to negative x axis (HA: $y=0$)

HAESE SL 5D, HL 2D

Growth and decay:

for $y=p\cdot a^{x-h}+k$, where $a,p>0$, $a\neq 1$, growth if $a>1$, decay if $a<1$

- Percentage increase: $\left(\frac{\text{new}-\text{old}}{\text{old}}\right)\cdot 100\%$

Growth: ex. 100 mice, population increases by 1.2 mice each week, find the population after n weeks:

1.2% each week

$f(n)=100\cdot(1+1.2\%)^n$

1 = 100%, percentage of initial population

Decay: ex. 20g weight, weight decreases by 5% each year, find the weight after n years

$f(n)=20\cdot(1-5\%)^n$

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Growth in money - compound interest

- Ex. \$5000 to start, interest $4\%/a$, compounded monthly for 5 years
  ↳ $\underbrace{5000}_{\text{initial amount}} \cdot \left(1+\underbrace{\frac{4\%}{12}}_{\text{percentage growth per month}}\right)^{12\cdot 5}$ ↳ 12 months per year × 5 months
  ↳ per annum/year
  $= \$6104.98$ at the end of 5 years
- OR, at the end of a period = original amount $\cdot \left(1+\underbrace{\boxed{\frac{\text{interest/year}}{\text{\# of payments/year}}}}_{\text{this is a percentage}}\right)^{\text{\# of payments/year} \cdot \text{\# of years}}$

Ex. the number of people who have heard a rumor increases by 2% every minute. Calculate the time for the number of people to triple

- Here, the initial number of people isn't given, but it is still solvable
↳ The initial number is 100% of the initial number, or 1
  ↳ Over a time t, the number of people triples: $1\times3=3$, it is now 300% of the original
  $\therefore (\underline{1}+7\%)^t = \underline{3}$, thus, you can solve for t

HAESE SL 5E.1, 5E.2; HL 2E.1, 2E.2

Logarithm: inverse of the exponential function

- If $a^x = b$, $x = \log_a b$

base: a; exponent: x; answer: b; exponent: x; base: a → base is always at the base; answer: b

- If a log is written on one side, the other side must be an exponent
- You cannot take logs of negative numbers!
  - ↳ Ex. $\log_a b$ → a & b must be positive

$\begin{cases} \text{Common logarithm: } \log \to \log_{10} \\ \text{Natural logarithm: } \ln \to \log_e,\ e \approx 2.71818 \end{cases}$

Logarithms on calculator:

$\begin{cases} \log(x) = \log_{10}(x) \\ \ln(x) = \log_e(x) \end{cases}$

For other log bases:

$\begin{cases} \text{"math" menu} \to \text{go down to "A" to logBASE} \to \log_{\square}\square \\ \text{"alpha" + "window"} \to \text{"5" logBASE} \end{cases}$

HAESE SL 6A, 6B, 6D; HL 3A, 3B, 3D

Log laws:

1. $\log_a(xy) = \log_a x + \log_a y$
2. $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
3. $\log_a x^y = y \log_a x$

log laws essentially have the same principles as exponent laws

- Same laws still apply for log or ln

$\begin{cases} \log_a a^x = x \\ a^{\log_a x} = x \end{cases}$

HAESE SL 6C, HL 3C

Logarithmic equations: taking or removing the log on each side of the equation

- Doesn't work with $\log(\square)$ if $\square$ includes addition or subtraction being performed
  - ↳ Ex. $\log(3+x) \neq \log 3 + \log x$ or $\log(3x)$,
  - and $\underline{\log(3+x) = \log(x^2)}$
    - ↳ cannot directly take out logs
  - but $\underline{\log(3x) = \log(x^2)}$
    - ↳ this equals to $3x = x^2$

HAESE SL 6E, HL 3E

Change of base rule: $\log_b a = \frac{\log_c a}{\log_c b}$

- $\log_{a^b} c = \frac{1}{b}\log_a c$

HAESE SL 6F, HL 3F

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Log functions:

Exponential function ex. $y=2^x$ ↳ in log form, it is $\log_2 y = x$ } the same function

Log functions are the inverses of exponential functions

- Proof: let $f(x)=\log_a x$, $g(x)=a^x$
  ↳ $f(g(x))=\log_a a^x = x$, $g(f(x)) = a^{\log_a x} = x$
  $\therefore f(g(x)) = g(f(x)) = x$.
  $\therefore f(x)$ and $g(x)$ are inverse functions

$\therefore$ if $f(x)=a^x$, $f^{-1}(x)=\log_a x$

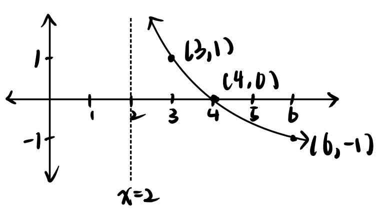
HA: $y=0$ VA: $x=0$

inverted functions flip the x and y values

Graphing log functions:

- Ex. $f(x) = 1-\log_2(x-2)$
  $= -\log_2(x-2)+1$ ↳ base function $f(x)=\log_2 x$, reflected over the x axis and translated by a vector of $\binom{2}{1}$

1. Domain: $\{x \mid x>2,\ x\in\mathbb{R}\}$, VA: $x=2$ → no y-ints
2. Range: $\{y \mid y\in\mathbb{R}\}$
3. x-int: $-\log_2(x-2)+1=0$, $-\log_2(x-2)=-1$, $\log_2(x-2)=1$, $\underset{\log_2 2 = 1}{\underline{x-2=2}}$, $x=4$, $(4,0)$
4. Other points: $f(3)=1$, $f(6)=-1$



HAESE SL 6H, HL 3H
+ end of unit review sets