probablility

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outline

why bother? and intuition

theory

computing probability

conditional probability and independence

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what is it?

- the numerical measure of the likelihood that the event will occur (the proportion of times the outcome would occur)
- it ranges from 0 to 1
- 0 means impossible
  - never 0, almost nothing is impossible eg $\frac{1\ (crashes)}{10m\ (flights)}$
- 1 means certain
  - also almost never 1; almost nothing is certain
- probability is really the basis of all statistics

making right decisions

- people and organizations make many mistakes because they miscalculate probability
  - eg gambling and lotteries
- smoking (hundreds or thousands of % increased risks!)
- flying v driving, etc
  - 9/11 killed additional thousands because people chose to drive (Wheelan, 2013, p.72-3)
- think about probabilities when making a decision
- the easiest (but already informative and helpful):
  - $\frac{occurences}{total}$; eg : $\frac{cancers}{smokers}$; $\frac{crashes}{miles}$; $\frac{crashes}{hours\ travelled}$
  - so depends how you measure ! car v flight: hours v miles !

why is probabilty relevant to MPA student?

- now we'll do some probability, which may be confusing !
- but probability improves thinking/decision making
  - ○ it is everywhere !
- we usually do not realize it, but we calculate probability all the time!
  - ○ and it helps if we can do it better !
- how likely it is that this will be on exam–should i study it?
- how likely it is i will get caught if i am speeding?
- how likely it is a student will drop out?
- how likely it is that graduation rate is above some value?

important for organizations!

- education agency looking at test scores to determine which teachers cheat
- checking doctors who overcharge
- (just identify outliers, and unlikely events (nontypical))
- (or overall changes/trends (for everyone) that are suspect (collusion?))
- public policy makers determining dangerous items eg swimming pools kill more kids than guns
  - (just see how many there are and how many people die from them!–simple des stat)
- for more interesting examples see Levit's Freakonomics `http://www.freakonomics.com/`

evolutionary and counterintuitive!

- evolution made us to survive in an environment that is long time gone!
- and so is our cognitive fucntion and probability calcualtion off!
- and so we need statistics to help us think better!
- eg: we overestimate probability of events that are memorable, flashy like
  - terrorist attack and airplane crash like we mistake stick for snake etc
- we understimate:
  - bad effect of sugar and fat, which were always rare

i used to have a policy...

- ... about undocumented emergencies
- and i do not have it anymore
- because there were about 7 people in the class
- and 4 of them had their grandmas die
- the reported probability was too high !
- what's the probability of your grandma dying during this semester?
  - say average grandma is expected to last at least (if not more) about 10 years, or 40 three-month periods in a year (about semster long)
  - so about $1/40$, so for 10 students class, every 4 semsters one grandama dead

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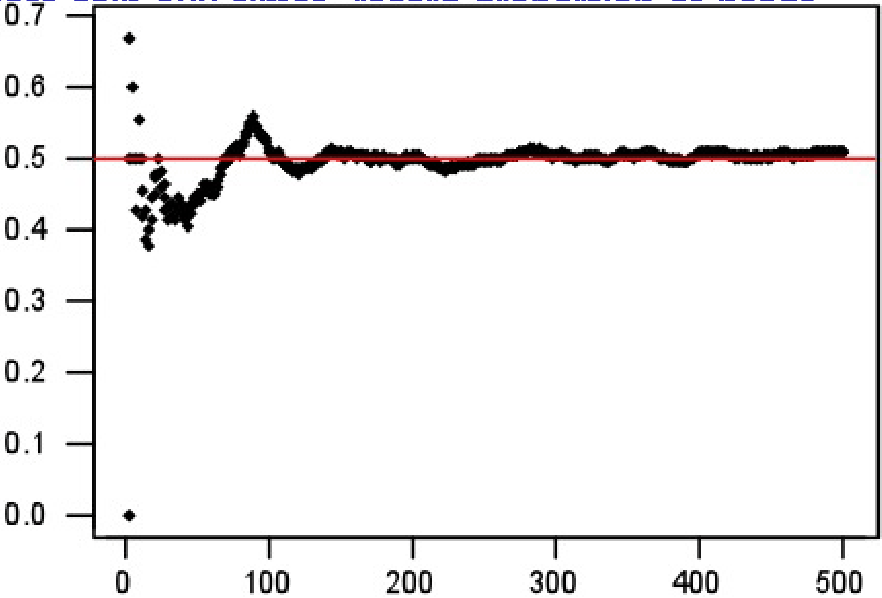
concepts

- random experiment: procedure whose outcome cannot be predicted in advance
- outcome: result of an experiment
- event (A), (B), etc: a set of outcomes
- sample space (S): the finest grain, mutually exclusive, collectively exhaustive listing of all possible outcomes
- mutually exclusive (disjoint):
  - 2 outcomes cannot occur at the same time, eg both male and female
- collectively exhaustive:
  - 1 outcome in sample space must occur: eg male or female

theory and reality

- before experiment: theory says that the probability of getting H is 0.5
- if you keep on tossing the coin, over the many tosses on average half of the time you will be getting H

toss coin 500 times, record proportion of heads

- 

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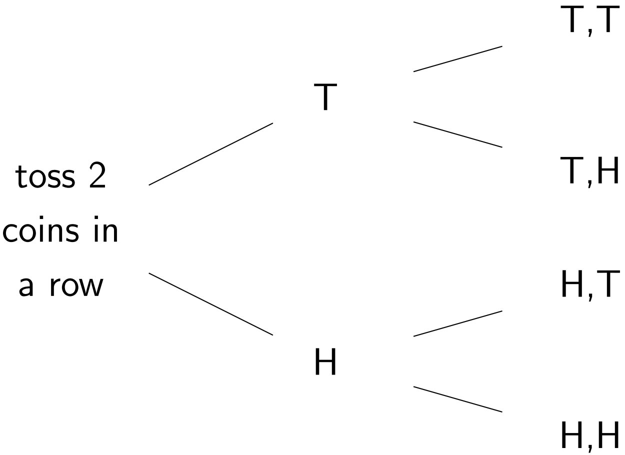
eg toss a coin twice

- $S = \{H, H\}, \{H, T\}, \{T, H\}, \{T, T\}$
- $A = \{T, T\}$
- complement of event A: set of all outcomes not in A
   - $A^c = \bar{A} = A' = \{H, H\}, \{H, T\}, \{T, H\}$

tree



table

HH	HT
TH	TT
- 1st row H in first flip
- 2nd row T in first flip
- 1st column H in second flip
- 2nd column T in second flip

exercises...

- what is the probability of getting 2 T in 2 flips ?
- we just showed with tree and table that there are 4 possible events, and only one outcome with two T, so $P = \frac{1}{4}$
- how about at least 1 T ?
- $P = \frac{3}{4}$
- how about exactly 1 T ?
- $P = \frac{2}{4}$

definitions

- if two events are mutually exclusive, the probability that either one event occurs is
  $P(\textit{either } A \textit{ or } B) = P(A) + P(B)$
- in addition if they are collectively exhaustive
  $P(A) + P(B) = 1$
- eg $A = \textit{male}$ $B = \textit{female}$
- eg $A = \textit{tail}$ $B = \textit{head}$

operators

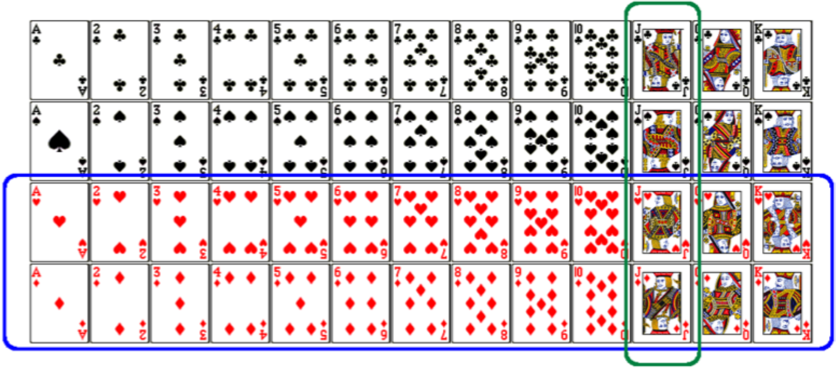
- intersection (and) $\cap$ (both have to happen)
  - multiply prob ($=< 1$), so less likely to have both (frownie)
- union (or) $\cup$ (at least one has to happen)
  - add prob, so more likely to have both (smiley)
- eg $A = H$ $B = H$
  - $A \cap B = \{H, H\}$
  - $A \cup B = \{H, H\}, \{H, T\}, \{T, H\}$

cards examples

- $P(heart) = \frac{1}{4}$
- mutually exclusive
  $P(A \cup B) = P(A) + P(B)$
  - $P(ace\ or\ king) = P(ace) + P(king) = \frac{1}{13} + \frac{1}{13} = 2/13$
- not mutually exclusive
  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - $P(ace\ or\ black) = P(ace) + P(black) -$
    $P(ace\ and\ black) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = 7/13$
- $P(heart|red) = 1/2$ because sample space is reduced to 26 red cards (will get back to it at the end!)

Union of non-disjoint events

What is the probability of drawing a jack or a red card from a w shuffled full deck?



$$P(\textit{jack or red}) = P(\textit{jack}) + P(\textit{red}) - P(\textit{jack and red})$$

$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52}$$

- **What is the probability that a randomly sampled student thinks marijuana should be legalized <u>or</u> they agree with their parents' political views?**

	Share Parents' Politics		
Legalize MJ	No	Yes	Total
No	11	40	51
Yes	36	78	114
Total	47	118	165

(a) (40 + 36 - 78) / 165

(b) (114 + 118 - 78) / 165

(c) 78 / 165

(d) 78 / 188

(e) 11 / 47

- # Product rule for independent events

P(A and B) = P(A) x P(B)

Or more generally, P(A1 and … and Ak) = P(A1) x … x P(Ak)

You toss a coin twice, what is the probability of getting two tails in a row?

P(T on the first toss) x P(T on the second toss)

= (1 / 2) x (1 / 2) = 1 / 4

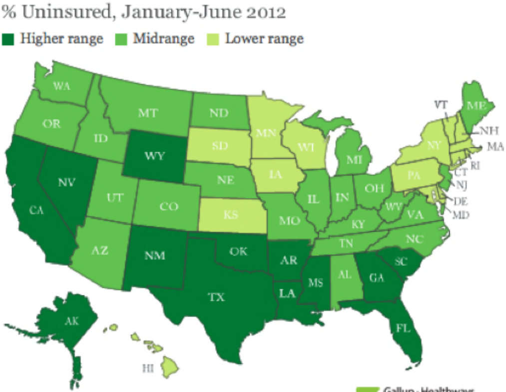
A recent Gallup poll suggests that 25.5% of Texans do not have health insurance as of June 2012. Assuming that the uninsured rate stayed constant, what is the probability that two randomly selected Texans are both uninsured?

(a) 25.5^2

(b) 0.255^2

(c) 0.255×2

(d) $(1 - 0.255)^2$



Roughly 20% of undergraduates at a university are vegetarian or vegan. What is the probability that, among a random sample of 3 undergraduates, at least one is vegetarian or vegan?

(a) $1 - 0.2 \times 3$

(b) $1 - 0.2^3$

(c) 0.8^3

(d) $1 - 0.8 \times 3$

(e) $1 - 0.8^3$

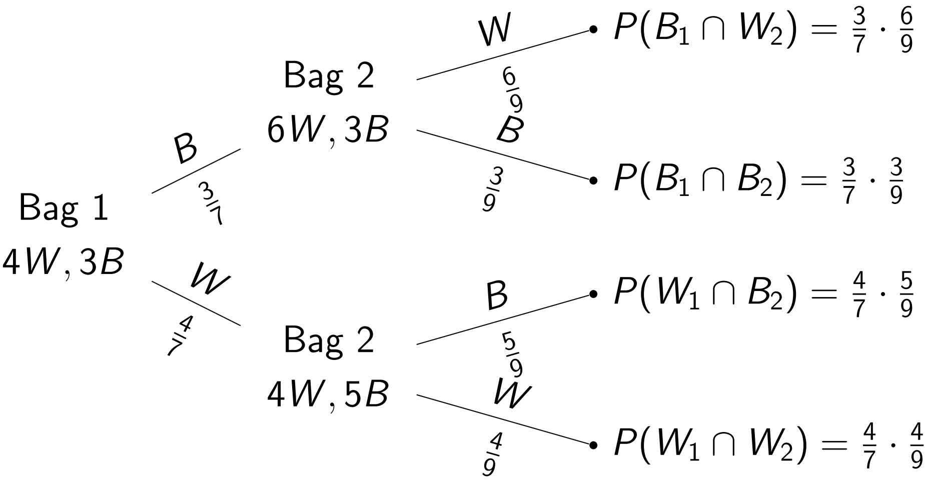
P(at least 1 from veg)

$= 1 -$ P(none veg)

$= 1 - 0.8^3$

$= 1 - 0.512 = 0.488$

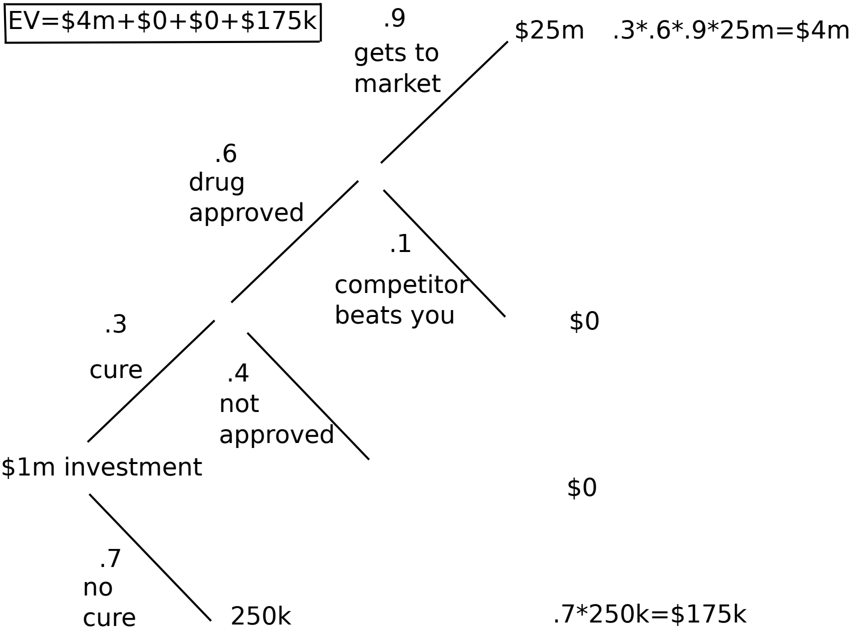
tree example



http://www.onemathematicalcat.org/Math/Algebra_II_obj/prob_tree_diagrams.htm

Expected Value (Wheelan, 2013, p83)

- just multiply value ($ amount) by associated probability(ies) AND add them up
- and this is how much you are expected to get on average


EV=$4m+$0+$0+$175k
.9
gets to
market
$25m
.3*.6*.9*25m=$4m
.6
drug
approved
.1
competitor
beats you
$0
.3
cure
.4
not
approved
$1m investment
$0
.7
no
cure
250k
.7*250k=$175k

outline

conditional probability

- $\boxed{P(A|B) = \frac{P(A \cap B)}{P(B)}}$
- you have $P(A \cap B)$ in numerator because both A and ($\cap$) B need to happen to be conditional on B, if A happens but not B, then it cannot be conditional on B

probability of intersection and independence

- probability of intersection:
  - $P(A \cap B) = P(A) * P(B|A) = P(B) * P(A|B)$
- A and B are independent if
- $P(A|B) = P(A)$ or
  $P(B|A) = P(B)$
  - (if one equality is true, the other one is true as well)
- then if A and B are independent:
  - $P(A \cap B) = P(A) * P(B|A) = P(A) * P(B)$
  - http://davidmlane.com/hyperstat/A127969.html

    http://davidmlane.com/hyperstat/A129515.html

    https://www.mathsisfun.com/data/probability-events-independent.html

    https://www.mathsisfun.com/data/probability-events-conditional.html

table practice (all numbers in the body are "$\cap$")

Category	Type of Policy (%) Fire	Auto	Other	Total %
Fraudulent	6	1	3	10
Nonfraudulent	14	29	47	90
Total	20	30	50	100

- $P(fire) = \frac{20}{100} = .2$
  - $P(F|fire)$ is $6/20$ or $.06/.2$
  - $P(fire|F)$ is $.06/.1$

strategy

- yes, probability can be confusing...but
- probability is not a rocket science
- if you think about it you can figure it out
- formulas may be more confusing than revealing
- you can just use tables or possibly best to use trees...
- i don't care what method you use

Levitt, S. D. and S. J. Dubner (2010): Freakonomics, vol. 61, Sperling & Kupfer.

Wheelan, C. (2013): Naked statistics: stripping the dread from the data, WW Norton & Company.