# dummies and interactions

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### **outline**

#### intuition

- dummies and interactions are fun!
- this is one of the most interesting things in regression
- you can test some interesting hypotheses
- · and you can contribute to the literature

### what is it?

- dummies identify nominal or ordinal characteristics, such as gender, race, region, religion, or education (measured as highest degree attained)
- dummies are binary indicators of a specific attribute
- · you either have the attribute or you do not
- · 1 if the condition is true and 0 otherwise
- · say male dummy=1 if a guy; 0 if a girl
- o never name a dummy like 'gender', which is impossible to figure out what it means!

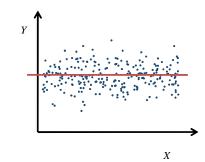
### what is it?

- can use dummies to create separate intercepts and/or slopes for subgroups within one regression
- dummies must always be interpreted relative to
  - · "base case," "omitted category", "reference group"

# regression on constant only

$$\Rightarrow \beta_2 = 0$$

$$\diamond \ \hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = \bar{Y}$$



- remember sums of squares discussions?our best bet before regression
- · our best prediction of Y is mean of Y

# now add a dummy A = A + B = A

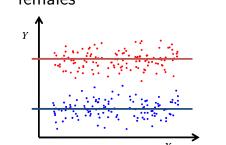
$$\diamond Y_i = \beta_1 + \beta_2 female_i + u_i$$

 $\Rightarrow \text{ if } \textit{female}_i = 1 \quad \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2(1) = \hat{\beta}_1 + \hat{\beta}_2$   $\cdot E[Y|\textit{female} = 1] = \hat{\beta}_1 + \hat{\beta}_2$ 

$$\Rightarrow \text{ if } female_i = 0 \quad \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2(0) = \hat{\beta}_1$$

•  $E[Y|female = 0] = \beta_1$ • hence  $\beta_2$  is the difference between  $\overline{Y}$ 

 $\diamond$  hence,  $\beta_2$  is the difference between Y for males and females

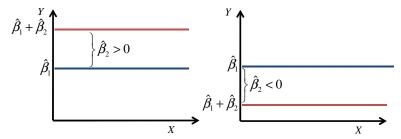


#### schematic

$$\diamond Y_i = \beta_1 + \beta_2 female_i + u_i$$

$$\diamond \ \hat{\beta}_2 = \bar{Y}_{\textit{female}} - \bar{Y}_{\textit{male}}$$

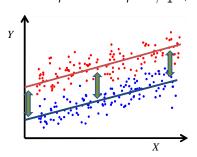
this is like a t-test!



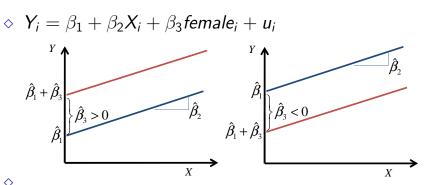
### and add a continuous var

$$\diamond Y_i = \beta_1 + \beta_2 X_i + \beta_3 \text{female}_i + u_i$$

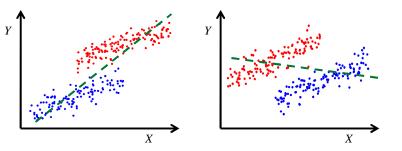
 $\text{$\diamond$ if } \textit{female}_i = 1 \quad \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{\beta}_3 (1) = (\hat{\beta}_1 + \hat{\beta}_3) + \hat{\beta}_2 X_i \\ \text{$\diamond$ if } \textit{female}_i = 0 \quad \hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{\beta}_3 (0) = (\hat{\beta}_1) + \hat{\beta}_2 X_i \\ \end{aligned}$ 



### schematic



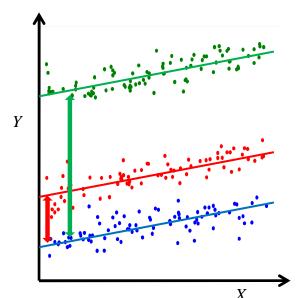
# bias from omitting a dummy...



- difficult to know if there should be a dummy or not
- can experiment; and again use theory! it will tell you!

# ordinal vars: blackboard: asst assoc full prof

omit one category (ie pick base case)

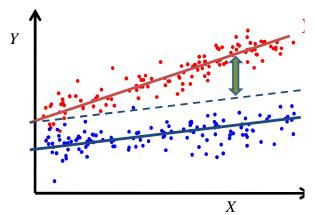


### choosing the base case

- be meaningful, eg pick typical or default situation
- · eg in my paper on wrk hrs I picked 40,
- · and dummies are then relative to the typical case
- think about what hypotheses you are most interested in
- remember that a different base case can change which coefficients are significant!
- make your choice(s) clear in your tables and text

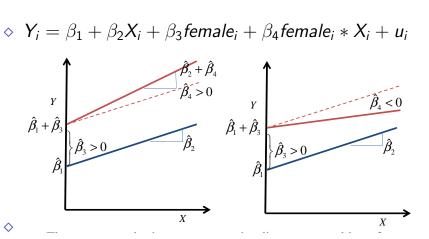
### continuous/dummy interactions

 $\diamond Y_i = \beta_1 + \beta_2 X_i + \beta_3 \text{female}_i + \beta_4 \text{female}_i * X_i + u_i$ 





### schematic



### interaction of dummies

- if there is an interaction effect between two variables, the effect of one variable depends on the level of the other
- eg the effect of marriage on wage depends on gender.
- interactions go both ways:
- · the effect of gender depends on marital status, too

### interaction of dummies

$Y_i = \beta_1 + \beta_2$ female $+ \beta_3$ married $+ \beta_4$ female $*$ married $+ u_i$										
ı	Male Female		Gender							
			Difference							
Unmarried	$\hat{oldsymbol{eta}}_{\!\scriptscriptstyle 1}$	$\hat{oldsymbol{eta}}_1 + \hat{oldsymbol{eta}}_2$	$\hat{oldsymbol{eta}}_2$							
Married	$\hat{oldsymbol{eta}}_1 + \hat{oldsymbol{eta}}_3$	$\hat{\beta}_1 + \hat{\beta}_2 + \hat{\beta}_3 + \hat{\beta}_4$	$\hat{oldsymbol{eta}}_2+\hat{oldsymbol{eta}}_4$							
Effect of Marriage	$\hat{oldsymbol{eta}}_3$	$\hat{eta}_3 + \hat{eta}_4$	$\hat{eta}_{\!\scriptscriptstyle 4}$							



### example [let's calc tab from reg]

. table married female, c(mean wage) row col f(%7.2f)

  Married	male	Gender female	Total
no yes	8.35 10.88	8.26 7.68	8.31 9.40
Total	9.99	7.88	9.02

- . gen femxmar = female\*married
- . reg wage female married femxmar

wage		Std. Err.		P> t	=	. Interval]
$\hat{eta}_{\scriptscriptstyle 2}$ female		.7350367	-0.13	0.897	-1.539132	1.348754
$\hat{eta}_{\scriptscriptstyle 3}$ married	2.521222	.6120814	4.12	0.000	1.318819	3.723626
$\hat{eta}_{_4}$ femxmar	-3.09704	.9072785	-3.41	0.001	-4.879344	-1.314737
$\hat{eta}_{_1}$ _cons	8.354677	.4936728	16.92	0.000	7.384882	9.324473

### dummy practice

- in addition to the dofile, see the links on the website for the code
- let's especially focus on the dummy variables
- we'll do it in the class if we have time...

### interactions of continuous variables

$$Y_{i} = \beta_{1} + \beta_{2}X_{2i} + \beta_{3}X_{3i} + \beta_{4}(X_{2i}X_{3i}) + u_{i}$$

$$\frac{\Delta Y_{i}}{\Delta X_{2i}} = \beta_{2} + \beta_{4}X_{3i}$$

$$\frac{\Delta Y_{i}}{\Delta X_{3i}} = \beta_{3} + \beta_{4}X_{2i}$$

#### interactions links

- again, interactions are a great way to contribute
- See sections 3.7 and 3.8 https://stats.idre.ucla.edu/stata/webbooks/reg/ chapter3/regression-with-statachapter-3-regression-with-categorical-predictors/
- http://www.stata.com/support/faqs/stat/anoregcoef.html
- http://nd.edu/~rwilliam/stats2/151.pdf
- http://www.stata.com/manuals13/rmarginsplot.pdf: scroll down to examples
- it is terrific command!