

# advanced measurement (logs, quadratics)

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## outline

## interpretation: transforming variables

- ◇ Lin: One unit change in  $X$  leads to a  $\beta_2$  unit change in  $Y$ .
- ◇ Log-Lin: One unit change in  $X$  leads to a  $100 * \beta_2$  % change in  $Y$ . (guj ed4:p180 fig6.4; ed5:p163 ex6.4)
- ◇ Lin-Log: One percent change in  $X$  leads to a  $\beta_2/100$  unit change in  $Y$ . (guj: ed4:p182 fig6.5; ed5:p165-6 ex6.5)
- ◇ Log-Log (aka log-linear or “linear in logs”): One percent change in  $X$  leads to a  $\beta_2$  % change in  $Y$  (elasticity).

# logs practice

- ◇ <https://stats.idre.ucla.edu/other/mult-pkg/faq/general/faqhow-do-i-interpret-a-regression-model-when-some-variables-are-not-significant>
- ◇ <http://www-stat.wharton.upenn.edu/~stine/stat621/handouts/LogsInRegression.pdf>

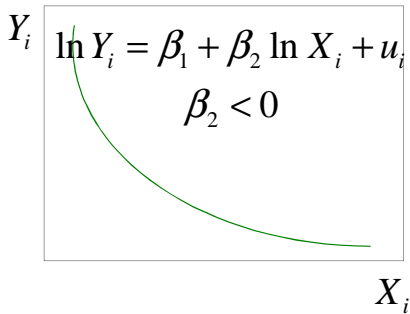
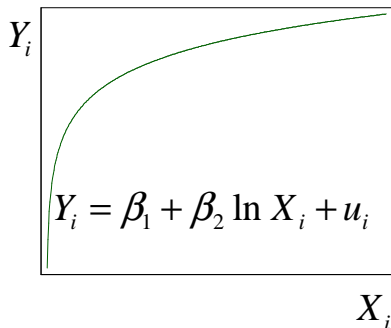
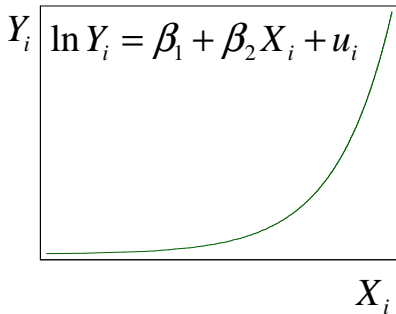
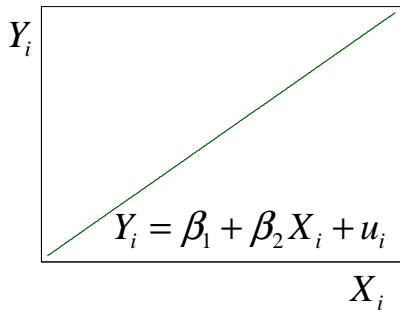
## natural logarithm; for simplicity just log or ln

- ◇ `log()` or `ln()`, to reverse it `exp()`
- ◇ it tells us how many times 2.72 was multiplied, eg:
  - $\log(7.4) = 2$ , because  $2.72^2 = 7.4$
  - $\log(22166) = 10$ , because  $2.72^{10} = 22166$
- ◇ it compresses distribution!
- ◇ it makes very big numbers relatively small  
[https://en.wikipedia.org/wiki/Wheat\\_and\\_chessboard\\_problem](https://en.wikipedia.org/wiki/Wheat_and_chessboard_problem)
- ◇ it is often used to remove outliers
- ◇ say wage, prices, any \$ amounts are very skewed
- ◇ and log will make them more normally distributed

## log makes cool interpretations in regressions

- ◇ again it makes very big numbers small
- actually, the bigger the number, the much more it is compressed
- $\log(10) \approx 2$ ;  $\log(100) \approx 4$ ;  $\log(1000) \approx 7$ ;  $\log(10,000) \approx 9$
- ◇ what you think will happen if you log transform your  $x$ ?
- ◇ as the values of  $x$  increase, the effect on  $Y$  would be weaker
- originally huge differences at high levels of  $X$  are now tiny
- ◇ what if we log transform  $Y$  variable? the other way round!
- so the effect of  $X$  would be increasing

it makes a difference



## lin-lin [dofile: measurement]

- ◇ eg more education earn higher wages
- ◇  $Y_i = \beta_1 + \beta_2 X_i + u_i$
- ◇ the change is constant regardless of the level of  $X$   
(because  $\beta$  is constant)
- ◇  $wage_i = \beta_1 + \beta_2 educ_i + u_i$
- $\widehat{wage}_i = -0.75 + 0.75 educ_i$
- $\widehat{wage}_{10} = \$6.75 \quad \widehat{wage}_{11} = \$7.50 \quad \Delta \widehat{wage} = \$0.75$   
the change is the same for any 1 year change in educ



## relative change: log-lin

- ◇ take log of Y first, then regress  $\ln Y$  on X
- ◇ regression treats  $\ln Y$  the same as any other var
  - percent change in Y per unit change in X is
  - $100 * \beta_2$  times the unit change in X (for small changes)
- ◇ still a linear regression, but with a new DV:  $\ln Y$ .
  - not linear in terms of Y

## eg log-lin [dofile: measurement]

- ◇  $\ln(wage_i) = \beta_1 + \beta_2 educ_i + u_i$
- ◇  $\widehat{\ln(wage)}_i = 1.06 + 0.08 educ_i$
- ◇  $\widehat{\ln(wage)}_{10} = 1.06 + 0.08(10) = 1.86$
- ◇ this is the predicted  $\ln(wage)$
- but what about the predicted wage?
- ◇  $\widehat{wage}_{10} = e^{1.86} = \$6.42$  exp()
- ◇  $\widehat{wage}_{11} = e^{1.94} = \$6.96$
- ◇  $\% \Delta \widehat{wage}_{10 \rightarrow 11} = \frac{\$6.96 - \$6.42}{\$6.42} = 0.08 = 8\%$

## the change varies in dollar terms

- ◇ but let's examine the change in wage for an additional year of graduate school, eg master's degree years.
- ◇  $\widehat{\ln(wage)}_i = 1.06 + 0.08educ_i$
- ◇  $\widehat{\ln(wage)}_{17} = 1.06 + 0.08(17) = 2.42$      $\widehat{wage}_{17} = \$11.25$
- ◇  $\widehat{\ln(wage)}_{18} = 1.06 + 0.08(18) = 2.50$      $\widehat{wage}_{18} = \$12.18$
- ◇  $\% \Delta \widehat{wage}_{17 \rightarrow 18} = 0.08 = 8\%$
- ◇ the change in relative (percentage) terms is constant at 0.08 (8 percent), but the dollar change is larger.

## lin-log [dofile: measurement]

- ◇  $Y_i = \beta_1 + \beta_2 \ln X_i + u_i$
- ◇ we generate the natural log of education and regress dollar wage on the log of education

## eg: lin-log

- ◇ wage as a function of relative change in education
- ◇  $wage_i = \beta_1 + \beta_2 \ln(educ_i) + u_i$
- ◇  $\widehat{wage}_i = -10.15 + 7.54 \ln(educ_i)$
- ◇  $\widehat{wage}_{10} = -10.15 + 7.54 \ln(10) = 7.21$
- ◇  $\widehat{wage}_{11} = -10.15 + 7.54 \ln(11) = 7.93$
- ◇  $\% \Delta \widehat{wage}_{10 \rightarrow 11} = \$0.72$
- ◇  $\widehat{wage}_{17} = -10.15 + 7.54 \ln(17) = 11.21$
- ◇  $\widehat{wage}_{18} = -10.15 + 7.54 \ln(18) = 11.64$
- ◇  $\% \Delta \widehat{wage}_{17 \rightarrow 18} = \$0.43$
- ◇ for a 1% (0.01) change in X, the change in Y is  $\beta_2/100$ ,  
in this case 0.0754

## relative change in education

educ	%change	educ	%change
1		11	10%
2	100%	12	9%
3	50%	13	8%
4	33%	14	8%
5	25%	15	7%
6	20%	16	7%
7	17%	17	6%
8	14%	18	6%
9	13%	19	6%
10	11%	20	5%

relative change in

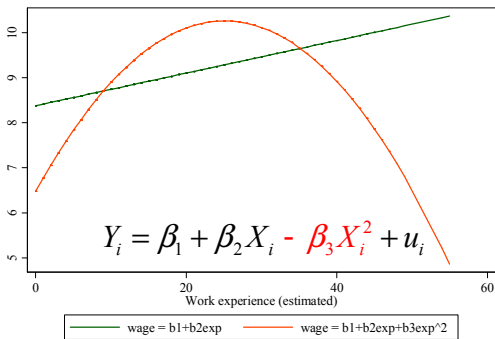
education per year is declining because the base is getting larger. Lin-log will predict a smaller impact on wage each year (see graph few slides back)

## quadratic regression

- ◇ does what logs do—fits a curve as opposed to a line
- ◇ i think more intuitive than logs
- ◇ the idea is that quadratic coef is smaller than linear, and opposite sign
- ◇ but as  $X$  gets bigger, its square get huge, and so quadratic coef with opposite sign overpowers first term and curve flips to positive or negative

## quadratic model

If a *non-linear relationship* between  $X$  and  $Y$  is suspected, a *polynomial function of  $X$*  can be used to model it.



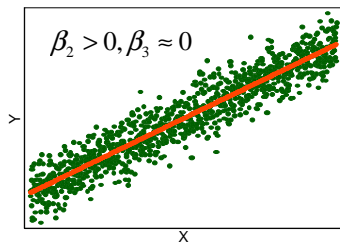
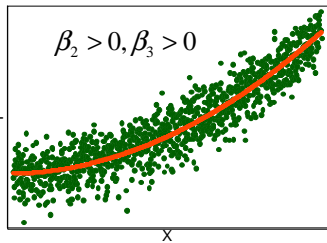
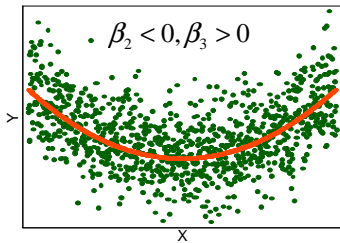
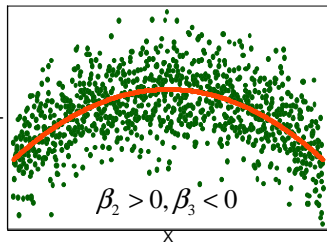
when it flips:

$$X_i^* = -\frac{\beta_2}{2\beta_3}$$

This curve reaches a maximum wage at the point where the marginal effect of experience is zero.



## quadratic model



## quadratic: interpretation

- ◇ the slope changes with  $X$ , it is not constant
- ◇ best graph it
- ◇ there is always a tipping point, but it may be outside the range of the data; in fact, the estimated line may be approximately linear for the observed data range even if the quadratic term is significant !
- ◇ t test on squared term has a null hypothesis of linearity
  - if it is not significant, only linear term is left
- ◇ dofile: quadratic,bonus