advanced measurement (logs, quadratics, etc)

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outline

interpretation: transforming variables

- Lin: One unit change in X leads to a β_2 unit change in Y.
- Log-Lin: One unit change in X leads to a $100 * \beta_2$ % change in Y. (guj ed4:p180 fig6.4; ed5:p163 ex6.4)
- Lin-Log: One percent change in X leads to a $\beta_2/100$ unit change in Y. (guj: ed4:p182 fig6.5; ed5:p165-6 ex6.5)
- Log-Log (aka log-linear or "linear in logs"): One percent change in X leads to a β_2 % change in Y (elasticity).

links for logs practice

- https://stats.idre.ucla.edu/other/mult-pkg/faq/general/faqhow-do-i-interpret-a-regression-model-when-some-va
- [*]http://www-stat.wharton.upenn.edu/~stine/stat621/handouts/LogsInRegression.pdf

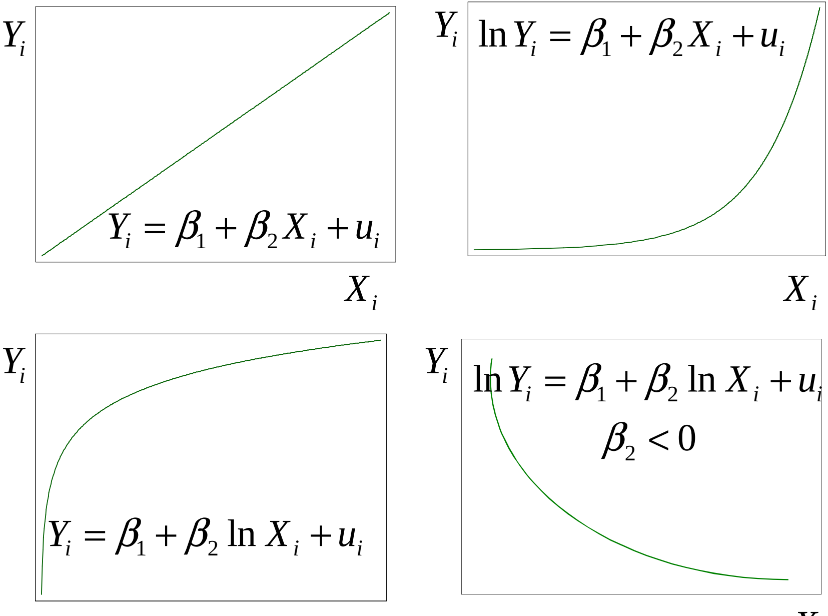
natural logarithm; for simplicity just log or ln

- log() or ln(), to reverse it exp()
- it tells us how many times 2.72 was multiplied, eg:
  - $log(7.4) = 2$, because $2.72^2 = 7.4$
  - $log(22166) = 10$, because $2.72^{10} = 22166$
- it compresses distribution!
  - it makes very big numbers relatively small
    [*]https://en.wikipedia.org/wiki/Wheat_and_chessboard_problem
  - it is often used to remove outliers
  - say wage, prices, any $ amounts are very skewed
  - and log will make them more normally distributed

log makes cool interpretations in regressions

- again it makes very big numbers small
  - actually, the bigger the number, the much more it is compressed
  - log(10)≈2; log(100)≈4; log(1000)≈7; log(10,000)≈9
- what you think will happen if you log transform your x?
- as the values of x increase, the effect on Y would be weaker
  - originally huge differences at high levels of X are now tiny
- what if we log transform Y variable? the other way round!
  - so the effect of X would be increasing

it makes a difference



lin-lin [dofile: measurement]

- eg people with more education earn higher wages...
- $Y_i = \beta_1 + \beta_2 X_i + u_i$
- This model specifies that the change is constant regardless of the level of X (because β is constant)
- $wage_i = \beta_1 + \beta_2 educ_i + u_i$
  - $\widehat{wage}_i = -0.75 + 0.75 educ_i$
  - $\widehat{wage}_{10} = \$6.75 \quad \widehat{wage}_{11} = \$7.50 \quad \Delta\widehat{wage} = \$0.75$
    the change is the same for any 1 year change in educ

relative change: log-lin

- take log of Y first, then regress lnY on X
- regression treats lnY the same as any other var
  - percent change in Y per unit change in X is
  - 100*β_2 times the unit change in X (for small changes)
- still a linear regression, but with a new DV: lnY.
  - not linear in terms of Y

eg log-lin [dofile: measurement]

- $ln(wage_i) = \beta_1 + \beta_2 educ_i + u_i$
- $\widehat{ln(wage)}_i = 1.06 + 0.08 educ_i$
- $\widehat{ln(wage)}_{10} = 1.06 + 0.08(10) = 1.86$
- this is the predicted ln(wage)
  - but what about the predicted wage?
- $\widehat{wage}_{10} = e^{1.86} = \$6.42$ exp()
- $\widehat{wage}_{11} = e^{1.94} = \$6.96$
- $\%\Delta\widehat{wage}_{10\rightarrow 11} = \frac{\$6.96 - \$6.42}{\$6.42} = 0.08 = 8\%$

the change varies in dollar terms

- but let's examine the change in wage for an additional year of graduate school, eg master's degree years.
- $\widehat{ln(wage)}_i = 1.06 + 0.08 educ_i$
- $\widehat{ln(wage)}_{17} = 1.06 + 0.08(17) = 2.42 \quad \widehat{wage}_{17} = \$11.25$
- $\widehat{ln(wage)}_{17} = 1.06 + 0.08(18) = 2.50 \quad \widehat{wage}_{18} = \$12.18$
- $\%\Delta\widehat{wage}_{17\rightarrow 18} = 0.08 = 8\%$
- the change in relative (percentage) terms is constant at 0.08 (8 percent), but the dollar change is larger.

lin-log [dofile: measurement]

- $Y_i = \beta_1 + \beta_2 lnX_i + u_i$
- we generate the natural log of education and regress dollar wage on the log of education

eg: lin-log

- wage as a function of relative change in education
- $wage_i = \beta_1 + \beta_2 ln(educ_i) + u_i$
- $\widehat{wage}_i = -10.15 + 7.54 ln(educ_i)$
- $\widehat{wage}_{10} = -10.15 + 7.54 ln(10) = 7.21$
- $\widehat{wage}_{11} = -10.15 + 7.54 ln(11) = 7.93$
- $\%\Delta\widehat{wage}_{10\rightarrow 11} = \$0.72$
- $\widehat{wage}_{17} = -10.15 + 7.54 ln(17) = 11.21$
- $\widehat{wage}_{18} = -10.15 + 7.54 ln(18) = 11.64$
- $\%\Delta\widehat{wage}_{17\rightarrow 18} = \$0.43$
- for a 1% (0.01) change in X, the change in Y is $\beta_2/100$, in this case 0.0754

relative change in education

educ	%change	educ	%change
1		11	10%
2	100%	12	9%
3	50%	13	8%
4	33%	14	8%
5	25%	15	7%
6	20%	16	7%
7	17%	17	6%
8	14%	18	6%
9	13%	19	6%
10	11%	20	5%

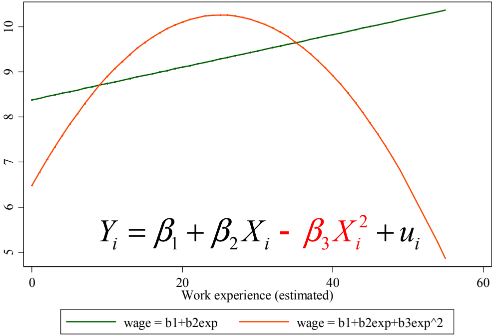
The relative change in education per year is declining because the base is getting larger. So the lin-log model will predict a smaller impact on wage each year (see graph few slides back)

and now quadratic regression

- ◇ not bivariate regression anymore, but trivariate
- ◇ the third var is just a sq of 2nd var
- ◇ and it does what logs do–fits a curve as opposed to a line
- ◇ and i think it is more intuitive than logs or reciprocals !
- ◇ the idea is that quadratic coef is smaller than linear, and opposite sign
- ◇ but as X gets bigger, its square get huge, and so quadratic coef with opposite sign overpowers first term and curve flips to positive or negative

quadratic model

If a *non-linear relationship* between X and Y is suspected, a *polynomial function of* X can be used to model it.

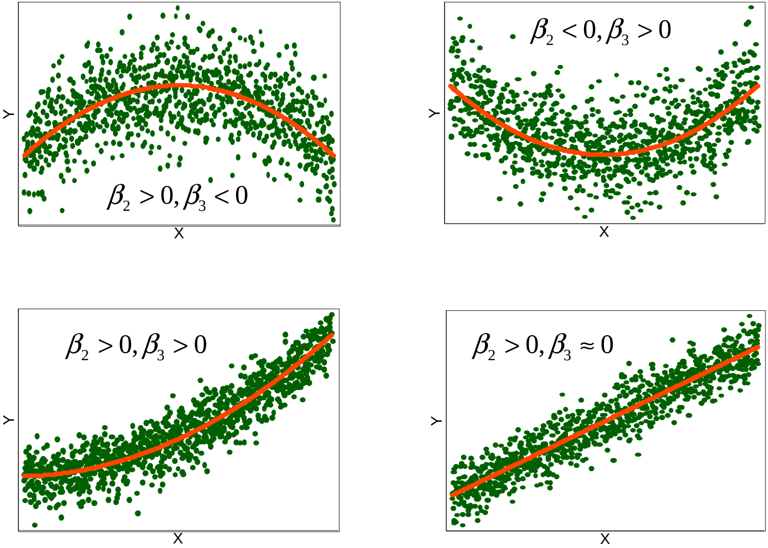


when it flips:

$$X_i^* = -\frac{\beta_2}{2\beta_3}$$

This curve reaches a maximum wage at the point were the marginal effect of experience is zero.

quadratic model



quadratic: interpretation

- the slope changes with X, it is not constant
- the best way to show the quadratic relationship is to graph it
- there is always a tipping point, but it may be outside the range of the data; in fact, the estimated line may be approximately linear for the observed data range even if the quadratic term is significant !
- the t test on squared term has a null hypothesis of linearity
  - if it is not significant, only linear term is left
- dofile: quadratic,bonus