

# probability

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# outline

why bother? and intuition

computing probability

conditional probability and independence

## what is it?

- the numerical measure of the likelihood that the event will occur (the proportion of times the outcome would occur)
- it ranges from 0 to 1
- 0 means impossible
  - never 0, almost nothing is impossible eg  $\frac{1 \text{ (crashes)}}{10m \text{ (flights)}}$
- 1 means certain
  - also almost never 1; almost nothing is certain
- probability is really the basis of all statistics

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## making right decisions

- people and orgs make mistakes bc miscalculate probab
- eg gambling and lotteries
- smoking (hundreds or thousands of % increased risks!)
- flying v driving, etc
- 9/11 killed additional thousands  
because people chose to drive (Wheelan, 2013, p.72-3)
- think about probabilities when making a decision
- the easiest (but already informative and helpful):
  - $\frac{\text{occurrences}}{\text{total}}$ ; eg :  $\frac{\text{cancers}}{\text{smokers}}$ ,  $\frac{\text{crashes}}{\text{miles}}$ ,  $\frac{\text{crashes}}{\text{hours travelled}}$
  - so depends how you measure ! car v flight: hours v miles !

## why is probability relevant to MPA student?

- probability may be confusing !
- but probability improves thinking/decision making
  - it is everywhere !
- usually don't realize it, but we calculate prob all the time!
  - and it helps our lives enormously if can do it better!
- how likely that this will be on exam—should i study it?
- how likely i will get caught if i am speeding?
- how likely a student will drop out?
- how likely that graduation rate is above some value?

## important for organizations!

- eg identify teachers who cheat or doctors who overcharge
- (just identify outliers, and unlikely events (nontypical))
- (or overall changes/trends (for everyone) that are suspect (collusion?))
- public policy makers determining dangerous items  
eg swimming pools kill more kids than guns
  - (just see how many there are and how many people die from them!—simple des stat)
- for more interesting examples see Levit's Freakonomics  
<http://www.freakonomics.com/>

## evolutionary and counterintuitive!

- evolution made us to survive in an environment that is long time gone!
- so is our cognitive function and probability calculation off!
- and so we need statistics to help us think better!
- eg: overestimate prob of memorable/flashy events
- terrorist attack and airplane crash are similar to mistaking stick for snake etc—better be extra careful and see even if it's not there
- we underestimate, on the other hand
- much more deadly effects of sugar and fat (which were always rare and desirable)



## i used to have a policy about undocumented

### emergencies

- and i do not have it anymore! thank god for probability!
- there were 7 students and 4 of them had their grandmas die
- the reported probability was too high to be plausible!
- so what's the prob of grandma dying during this semester?
- say avg grandma is expected to last at least (if not more) about 10 years, or 40 three-month periods in a year (about semster long)
- so about  $1/40$ , so for 10 students class: every 4 semsters one grandama dead

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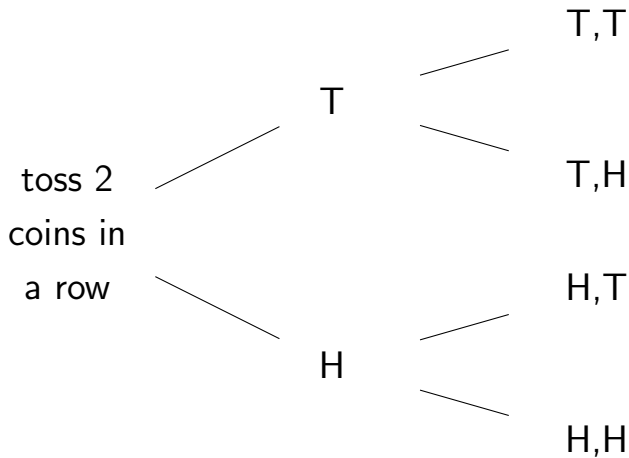
computing probability

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## concepts

- event  $(A)$ ,  $(B)$ , etc: a set of outcomes, eg  $A = \{T, T\}$
- sample space  $(S)$ : the finest grain, mutually exclusive, collectively exhaustive listing of all possible outcomes
- mutually exclusive (disjoint):
  - 2 outcomes can't occur at the same time, eg  $T$  and  $H$
- collectively exhaustive:
  - 1 outcome must occur: eg  $T$  or  $H$
- eg toss a coin twice
- $S = \{H, H\}, \{H, T\}, \{T, H\}, \{T, T\}$
- $A = \{T, T\}$

# tree



## table

- |    |    |
|----|----|
| HH | HT |
| TH | TT |

- 1st row H in first flip
- 2nd row T in first flip
- 1st column H in second flip
- 2nd column T in second flip

## exercises...

- what is the probability of getting 2 T in 2 flips ?
- we just showed with tree and table that there are 4 possible events, and only one outcome with 2 T, so  $P = \frac{1}{4}$
- how about at least 1 T ?
- $P = \frac{3}{4}$
- how about exactly 1 T ?
- $P = \frac{2}{4}$

## some useful properties and useful language

- intersection (and)  $\cap$  (both have to happen)
  - multiply prob ( $= < 1$ ), so less likely to have both (frownie)
- union (or)  $\cup$  (at least one has to happen)
  - add prob, so more likely to have both (smiley)
- eg  $A = H \ B = H$ 
  - $A \cap B = \{H, H\}$
  - $A \cup B = \{H, H\}, \{H, T\}, \{T, H\}$
- if mutually exclusive:  $P(A \cup B) = P(A) + P(B)$
- if mutually exclusive and collectively exhaustive:  
$$P(A) + P(B) = 1$$
  - eg  $A = \text{tail} \ B = \text{head}; A = \text{male} \ B = \text{female}$

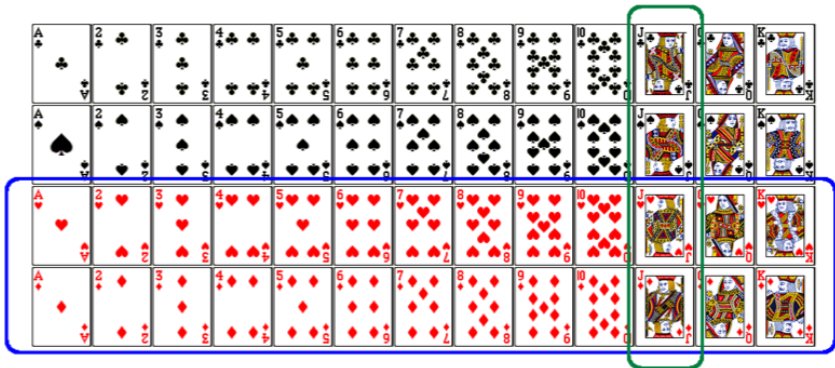
## cards examples

- $P(\text{heart}) = \frac{1}{4}$
- mutually exclusive
$$P(A \cup B) = P(A) + P(B)$$
- $P(\text{ace or king}) = P(\text{ace}) + P(\text{king}) = \frac{1}{13} + \frac{1}{13} = 2/13$
- not mutually exclusive
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
- $P(\text{ace or black}) = P(\text{ace}) + P(\text{black}) - P(\text{ace and black}) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = 7/13$
- $P(\text{heart}|\text{red}) = 1/2$  because sample space is reduced to 26 red cards (will get back to it at the end!)



# Union of non-disjoint events

What is the probability of drawing a jack or a red card from a well shuffled full deck?



$$P(\text{jack or red}) = P(\text{jack}) + P(\text{red}) - P(\text{jack and red})$$

$$= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{52}$$

What is the probability that a randomly sampled student thinks marijuana should be legalized or they agree with their parents' political views?

<i>Legalize MJ</i>	<i>Share Parents' Politics</i>		<i>Total</i>
	<i>No</i>	<i>Yes</i>	
No	11	40	51
Yes	36	78	114
Total	47	118	165

**(a)  $(40 + 36 - 78) / 165$**

**(b)  $(114 + 118 - 78) / 165$**

**(c)  $78 / 165$**

**(d)  $78 / 188$**

**(e)  $11 / 47$**

# Product rule for independent events

$$P(A \text{ and } B) = P(A) \times P(B)$$

$$\text{Or more generally, } P(A_1 \text{ and } \dots \text{ and } A_k) = P(A_1) \times \dots \times P(A_k)$$

**You toss a coin twice, what is the probability of getting two tails in a row?**

$$\begin{aligned} &P(\text{T on the first toss}) \times P(\text{T on the second toss}) \\ &= (1 / 2) \times (1 / 2) = 1 / 4 \end{aligned}$$



**Roughly 20% of undergraduates at a university are vegetarian or vegan. What is the probability that, among a random sample of 3 undergraduates, at least one is vegetarian or vegan?**

**(a)  $1 - 0.2 \times 3$**

**(b)  $1 - 0.2^3$**

**(c)  $0.8^3$**

**(d)  $1 - 0.8 \times 3$**

**(e)  $1 - 0.8^3$**

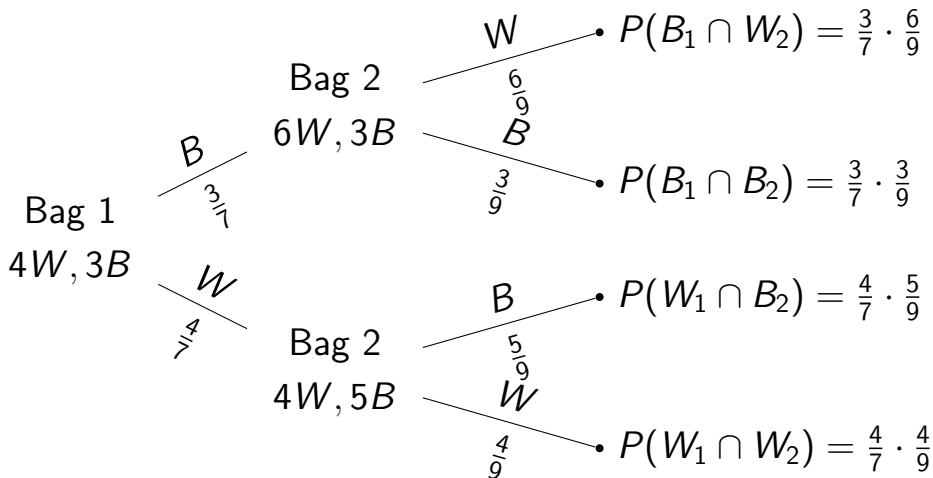
**$P(\text{at least 1 from veg})$**

**$= 1 - P(\text{none veg})$**

**$= 1 - 0.8^3$**

**$= 1 - 0.512 = 0.488$**

## tree example

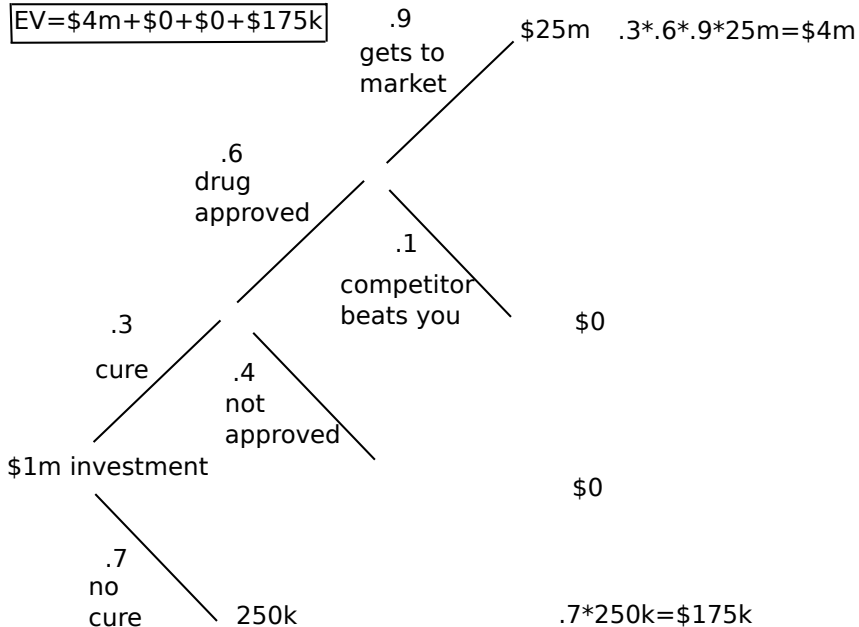


[http://www.onemathematicalcat.org/Math/Algebra\\_II\\_obj/prob\\_tree\\_diagrams.htm](http://www.onemathematicalcat.org/Math/Algebra_II_obj/prob_tree_diagrams.htm)

## Expected Value (Wheelan, 2013, p83)

- just multiply value (\$ amount) by associated probability(ies) AND add them up
- and this is how much you are expected to get on average

$$EV = \$4m + \$0 + \$0 + \$175k$$





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## conditional probability

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$
- you have  $P(A \cap B)$  in numerator because both A and  $(\cap)$  B need to happen to be conditional on B, if A happens but not B, then it cannot be conditional on B

## probability of intersection and independence

- probability of intersection:

- $P(A \cap B) = P(A) * P(B|A) = P(B) * P(A|B)$

- A and B are independent if

- $P(A|B) = P(A)$  or

$$P(B|A) = P(B)$$

- (if one equality is true, the other one is true as well)

- then if A and B are independent:

- $P(A \cap B) = P(A) * P(B|A) = P(A) * P(B)$

- <http://davidmlane.com/hyperstat/A127969.html>

<http://davidmlane.com/hyperstat/A129515.html>

<https://www.mathsisfun.com/data/probability-events-independent.html>

<https://www.mathsisfun.com/data/probability-events-conditional.html>

## table practice (all numbers in the body are "n")

Category	Type of Policy (%)			Total %
	Fire	Auto	Other	
Fraudulent	6	1	3	10
Nonfraudulent	14	29	47	90
Total	20	30	50	100

- $P(\text{fire}) = \frac{20}{100} = .2$
- $P(F|\text{fire})$  is  $6/20$  or  $.06/.2$
- $P(\text{fire}|F)$  is  $.06/.1$

## strategy

- yes, probability can be confusing...but
- probability is not a rocket science
- if you think about it you can figure it out
- formulas may be more confusing than revealing
- you can just use tables or possibly best to use trees...
- i don't care what method you use

LEVITT, S. D. AND S. J. DUBNER (2010): Freakonomics, vol. 61, Sperling & Kupfer.

WHEELAN, C. (2013): Naked statistics: stripping the dread from the data, WW Norton & Company.