D.E

First Order Linear O.D.E

① Separation of variable

$$\frac{dy}{dx} = \frac{f(x)}{g(y)}$$

$$g(y)\,dy = f(x)\,dx$$

$$\int g(y)\,dy = \int f(x)\,dx$$

② Linear O.D.E

$$\frac{dy}{dx} + p(x)y = Q(x)$$

$$I.F = e^{\int P(x)\,dx}$$

By putting I.F in the above eq.

L.H.S Always $= \frac{d}{dx}\left(I.F \times \text{Depedent variable}\right)$

$$\int \frac{d}{dx}(I.F \times D.V) = \int -$$

③ Exact First O.D.E

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

If $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

then

$$\int M \cdot dx + \int N(\text{Free of } x \text{ Terms})\,dy = c$$

④ Non-Exact

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If $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

then ① $M_y - N_x = ?$

② If $\frac{M_y - N_x}{N} = f(x)$ only

then $I.F = e^{\int\left(\frac{M_y - N_x}{N}\right)dx}$

③ If $\frac{M_y - N_x}{M} = f(y)$ only

then $I.F = e^{-\int\left(\frac{M_y - N_x}{M}\right)dy}$

Multiply I.F with the eq.

then $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ (If not $=$ then check ur Q. again)

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Derivative of Trigonometric

$d/dx(\sin x) = \cos x$

" $\cos x = -\sin x$

" $\tan x = \sec^2 x$

" $\cot x = -\csc^2 x$

" $\sec x = \sec x \tan x$

" $\csc x = -\csc x \cot x$

$\sec x = \frac{1}{\cos x}$, $\sin 2x = 2\sin x \cos x$, $\int u.v = u\int v - \int\int v \frac{du}{dv}$

$\int \ln x\,dx = x\ln(x) - x + c$ | $\frac{d}{dx}\left(\frac{a^x}{\log a}\right) = a^x$, $\int a^x\,dx = \frac{a^x}{\log a} + c$

Ki wekh rea SHOREA..

(5) Growth & Decay Model

$$\frac{dP}{dt} \propto P$$

$$\frac{dP}{dt} = kP$$

$$\int \frac{1}{P}\, dP = \int k\, dt$$

$$\ln P = Kt + c$$

$$P = e^{Kt+c}$$

$$P = e^{kt} \cdot e^{c}$$

$$P = e^{kt} \cdot c$$

The sol. of above Model is $\boxed{P = c \cdot e^{kt}}$

If $K = +ve \rightarrow$ Growth

If $k = -ve \rightarrow$ Decay

$C\text{-}14 \rightarrow t = 5730$

(6) Newton's Law of Cooling / Warming

$$\frac{dT}{dt} \propto (T - T_m)$$

$$\frac{dT}{dt} = K(T - T_m)$$

The sol. of above model is $\boxed{T = T_m + ce^{kt}}$

$\because$ T = Temp of body

T_m = Temp of environment

t = time

Derivatives of inverse Trigonometric

$$\frac{d}{dx} \sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$$

$$\text{"} \quad \tan^{-1}x = \frac{1}{1+x^2}$$

$$\text{"} \quad \cot^{-1}x = \frac{-1}{1+x^2}$$

$$\text{"} \quad \sec^{-1}x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\text{"} \quad \csc^{-1}x = \frac{-1}{|x|\sqrt{x^2-1}}$$

① **General n^{th} order D·E**

$$a_n \frac{d^n y}{dx^n} + a_{n-1} \frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_2 \frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx}$$

② **Wronskian**

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \qquad W = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

$\neq 0$ Linear Independent | $= 0$ Non-Linear

$y = y_c + y_p$

y_c → Complementary Solution

y_p → Particular solution

③ **Reduction of Order**

$$y_2 = y_1(x) \int \frac{e^{-\int P(x)\,dx}}{(y_1(x))^2}\,dx$$

$$y'' + P(x)y' + Q(x)y = 0$$

④ **Homo. D.E with const. co-efficient**

Put

$y = e^{mx}$, $y' = me^{mx}$, $y'' = m^2 e^{mx}$

① Real & Distinct : $y_c = c_1 e^{mx} + c_2 e^{mx}$

② Real & Repeated : $y_c = c_1 e^{mx} + c_2 x e^{mx}$

③ Complex : $y_c = e^{\alpha x}[c_1 \cos\beta x + c_2 \sin\beta x]$

⑤ **Undetermined Co-efficient**

$y = y_c + y_p$

y_c is same as ④

For y_p Check Non Homo. Part

Constant : $y_p = A$ ①

e^x : $y_p = Ae^x$ ②

$\sin x / \cos x$: $y_p = A\cos x + B\sin x$ ③

Polynomial

Linear : $y_p = Ax + B$ ④

Quadratic : $y_p = Ax^2 + Bx + C$ ⑤

Cubic : $y_p = Ax^3 + Bx^2 + Cx + D$ ⑥

$(9x-2)e^{5x}$: $y_p = (Ax+B)e^{5x}$ ⑦

$x^2 e^{5x}$: $y_p = (Ax^2 + Bx + C)e^{5x}$ ⑧

⑨ → $e^{3x}\sin 4x$: $y_p = Ae^{3x}\cos 4x + Be^{3x}\sin 4x$

⑩ → $6x^2 \sin 4x$: $y_p = (Ax^2 + Bx + C)\cos 4x + (Ex^2 + Fx + G)\sin 4x$

⑥ Variation of Parameter

$y'' + P(x)y' + Q(x)y = f(x)$

$\cosh x = \frac{e^x + e^{-x}}{2}$

$\sinh x = \frac{e^x - e^{-x}}{2}$

$y = y_c + y_p$

y_c is same as ④

$y_p = u_1 y_1 + u_2 y_2$

$u_1 = \int \frac{W_1}{W}, \quad u_2 = \int \frac{W_2}{W}$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \neq 0$

$W_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix}$

$W_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix}$

⑦ Laplace Transformation

$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$

$L[1] = 1/s, \quad L[e^{kt}] = \frac{1}{s-k}$

$L[t^n] = \frac{n!}{s^{n+1}}$

$L[\sin kt] = \frac{k}{s^2+k^2}$

$L[\sinh kt] = \frac{k}{s^2-k^2}$

$L[\cos kt] = \frac{s}{s^2+k^2}$

$L[\cosh kt] = \frac{s}{s^2-k^2}$

$L[f'(t)] = s f(s) - f(0)$

$L[f''(t)] = s^2 f(s) - s f(0) - f'(0)$