University of Central Punjab
Faculty of Information Technology
Linear Algebra
Final Term Exam

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Course Code: CSSS2753, SESS2743	Marks: 50
Duration : 150 minutes	Section:
Date : 15th July, 2023	Program: BSCS, BSSE

Name: [redacted] Registration Numb[redacted]

Instructions

1. Write your name and registration number on the answer sheet and sign it before submission.
2. Attempt all questions.
3. All your calculations should be on your answer sheet.
4. You have first 5 minutes to read the paper carefully and then start solving the paper.

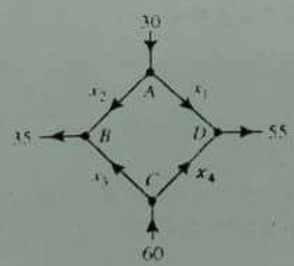
Question 1: [10 marks]

The accompanying sketch represents the flow of traffic in a city in the United States. The traffic olume through certain blocks during an hour has been measured. Suppose that the vehicles eaving the area per minute were exactly the same as those entering it.

a) Set up a system of linear equations for the network shown in figure.

b) Solve it by using Gauss elimination method.

c) If the traffic flow on the road connecting C and D is 20 cars per hour, then what would be the number of cars on the road connecting A and B.



Question 2: [6+4 marks]

a) Consider the triangle ΔABC given by the vertices $A(2,2)$, $B(4,2)$ and $C(2,5)$. Find the vertices of the triangle after shearing it along the x-axis by factor 2 and then translating the sheared triangle with vector $\vec{a} = (2,3)$. Also, sketch the image of the original and transformed triangles.

b) Describe the matrix operator corresponding to $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and show its effect on the equation of the circle

$$x^2 + y^2 = 4.$$

Question 3: [10 marks]

Find all the eigenvalues of the given matrix. Also, find eigenvector for ***negative*** eigenvalue of the matrix

$$A = \begin{bmatrix} 2 & -3 & 0 \\ 2 & -5 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Question 4: [6+4 marks]

i) Let $V = R^3 = \{(x,y,z), x,y,z \in R\}$ be a set of ordered triples of real number with the following operations of addition and scalar multiplication:

$$\overline{u_1} + \overline{u_2} = (x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1 + x_2 - 2,\ y_1 + y_2, z_1 + z_2 + 3)$$
$$k(x_1, y_1, z_1) = (1, ky_1, kz_1).$$

a. Compute $\overline{u_1} + \overline{u_2}$ if $\overline{u_1} = (1,-1,\ 3)$, $\overline{u_2} = (-4,2,-7)$.

b. Compute $k\overline{u_1}$ if $\overline{u_1} = (2,3,-5)$ and $k = -2$.

c. Find the zero vector of the set.

d. Find the additive inverse of the vector $(3,2,5)$.

e. Prove or disprove the property: $k(\overline{u_1} + \overline{u_2}) = k\overline{u_1} + k\overline{u_2}$.

f. Prove or disprove the property: $1\overline{u} = \overline{u}$

ii) Let $V = R^2 = \{(x,y),\ x \in R, y \in R\}$ be a vector space under standard addition and scalar multiplication. Prove or disprove the following subsets are ***subspaces***.

a. $W_1 = \{(x,y): 0 \le x \le 1, 0 \le y \le 1\}$

b. $W_2 = \{(x,0): x \in R\}$

Question 5: [3+3+2+2marks]

Let $S = \{(1,0,\ 3), (2,1,4), (1,0,0)\}$

a) Express $(2,3,1)$ as a linear combination of the given set of vectors in S.

b) Does the vectors in set S are linearly independent?

c) Does S Spans $\mathbb{R}^3$?

d) Does S form basis of $\mathbb{R}^3$?