

$\text{for (int } i=1; i \leq N; i = i+1) \{$ $\rightarrow \text{outer } \left\lceil \frac{N-1+1}{1} \right\rceil + 1 = N+1 = N$
 $\quad \quad \quad \text{for (int } j=1; j \leq i; j = j+1) \{$ $\left[\begin{array}{l} \text{Added} \\ \text{for } = \\ \text{equal} \end{array} \right] \left[\begin{array}{l} \text{loop break} \\ \downarrow \end{array} \right]$
 $\quad \quad \quad \}$

$\}$ for every value of $i \rightarrow$ having loop for j

i	1	2	3	4	5	$N-2$	$N-1$	N
j	1	1	1	1	1		1	1	1
	1	2	3	4	5		$N-2$	$N-1$	N

$\left\lceil \frac{1-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{2-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{3-1+1}{1} \right\rceil + 1$				$\left\lceil \frac{N-2-1+1}{1} \right\rceil + 1$	\cdots	$\left\lceil \frac{N-1+1}{1} \right\rceil + 1$
2	3	4				$N-1$	N	$N+1$

arithmetical series

$$\begin{aligned}
 \# \text{ pairs} &\rightarrow \left\lceil \frac{N+1-2+1}{1} \right\rceil / 2 = \frac{N+2}{2} \\
 \text{pair sum} &\rightarrow N+1+2 = \underline{\underline{N+3}} \quad \left(\frac{N+2}{2} \right) (N+3) = \frac{1}{2} (N^2 + 5N + 6) \text{ Simplify} \\
 &= \frac{1}{2} (N^2) \\
 &= N^2
 \end{aligned}$$

outer $\leftarrow N + N^2 = O(N^2)$ answer
 \downarrow
 inner series

Outer series has change 2

for (int i=1; i<=N; i=i+2) { } → $\left[\frac{N-1+1}{2} \right] + 1 = \frac{N}{2} + 1 = N$

for (int j=1; j<=i; j=j+1) { } ↓
 (No of terms) simplified answer

inner series has change 1

}

i	1	3	5	7	9	N-4	N-2	N
j	1	1	1	1	1		1	1	1
	1	3	5	7	9		N-4	N-2	N

$\left[\frac{1-1+1}{2} \right] + 1 \quad \left[\frac{3-1+1}{2} \right] + 1 \quad \left[\frac{5-1+1}{2} \right] + 1 \quad \left[\frac{N-4-1+1}{2} \right] + 1 \quad \left[\frac{N-1-1+1}{2} \right] + 1$

2 4 6
 ↴ ↴
 ↴ ↴
 arithmetical Series here we have added loop terminating 1 more operation

$$\# \text{ pairs} \rightarrow \left[\frac{N+1-2+1}{2} \right] / 2 = \left[\frac{N}{2} \right] / 2 = N/4$$

$$\text{pair sum} \rightarrow N+1+2 = N+3$$

$$\left. \begin{aligned} &\frac{N \times (N+3)}{4} \\ &= \frac{1}{4} (N^2 + 3N) \\ &= \frac{1}{4} (N^2) = N^2 \end{aligned} \right\} \text{(iteration)}$$

$$N + N^2 = O(N^2)$$

~~outer series has
change 2~~

~~for (int i=1; i<=N; i=i+2) { calculate it~~

$$\lceil \frac{N-1+1}{2} \rceil + 1 = \lceil \frac{N}{2} \rceil + 1 = N$$

~~for (int j=1; j<=i; j=j+2) {~~

~~}~~ ~~inner Series has
change 2~~

~~}~~

i 1 3 5 7 9 ... N-4 N-2 N

j 1 1 1 1 1 1 1 1

1 3 5 7 9 N-4 N-2 N

$$\lceil \frac{1-1+1}{2} \rceil + 1 \quad \lceil \frac{3-1+1}{2} \rceil + 1 \quad \lceil \frac{5-1+1}{2} \rceil + 1 \quad \lceil \frac{N-4-1+1}{2} \rceil + 1 \quad \lceil \frac{N-1+1}{2} \rceil + 1$$

$$2 \quad 3 \quad 4 \quad \frac{N-1}{2} \quad \frac{N}{2} \quad \frac{N+1}{2}$$

arithmetical Series

$$\begin{aligned} \# \text{ pairs} &\rightarrow \left[\frac{N+1-2+1}{2} \right] / 2 = \lceil \frac{N}{4} \rceil / 2 = \frac{N}{8} \\ \text{pair sum} &\rightarrow \frac{N}{2} + 1 + 2 = \frac{N}{2} + 3 \end{aligned} \quad \left[\left(\frac{N}{8} \right) \left(\frac{N}{2} + 3 \right) = \frac{N^2}{16} + \frac{3N}{8} \right. \\ &\quad \left. = \frac{N^2}{16} = N^2 \right]$$

$$N + N^2 = O(N^2)$$

for (int $i=1$; $i \leq N$; $i=i*2$) { $\left\lceil \lg_2 \left(\frac{N+1}{1} \right) \right\rceil + 1 \Rightarrow \lg_2^{N+1}$
 $\Rightarrow \underline{\underline{\lg_2^N}}$

for (int $j=1$; $j \leq i$; $j=j+1$) {

}

}

how many terms here
 \Rightarrow complexity of outer loop

i	1	2	4	8	16	---	$N/4$	$N/2$	N	\lg_2^N
j	1	1	1	1	1		1	1	1	
	1	2	4	8	16		$N/4$	$N/2$	N	
	$\left\lceil \frac{1-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{2-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{4-1+1}{1} \right\rceil + 1$				$\left\lceil \frac{N-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{N-1+1}{1} \right\rceil + 1$		
2	3	5					$\frac{N}{4} + 1$	$\frac{N}{2} + 1$	$N + 1$	

if we separate 1 (as loop break)

$$1+1 \quad 2+1 \quad 4+1 \quad \frac{N}{4}+1 \quad \frac{N}{2}+1 \quad N+1$$

$$(1 \quad 2 \quad 4 \quad 8 \quad 16 \quad \dots \quad \frac{N}{4} \quad \frac{N}{2} \quad N) + (1 \quad 1 \quad 1 \quad \dots \quad + 1) \quad \lg_2^N \text{ terms}$$

Largest term

$$\ll C * N = CN = N + \lg_2^N = N$$

$$N + \lg_2^N = O(N)$$

Geometric Series

2

3

$\mathfrak{g}^{s(N)}$ terms

$$i \ 1 \ | 3 \ | 9 \ | 27 \ 81 \cdots N/9 \ N/3 \ N$$

$$\begin{array}{r|rrr|rr|rr} j & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 3 & 9 & 27 & 81 & N/9 & N/3 \\ \hline \end{array}$$

$$\left\lceil \frac{1-1+1}{2} \right\rceil + 1 = \left\lceil \frac{3-1+1}{2} \right\rceil + 1 = \left\lceil \frac{9-1+1}{2} \right\rceil + 1 = \left\lceil \frac{N-1+1}{2} \right\rceil + 1 = \left\lceil \frac{N+1}{2} \right\rceil + 1$$

in start it didn't look like geometric, at end
it may look like geometric

approximation

\Rightarrow [largest term * # of terms]

$$\Rightarrow \left(\frac{N}{2}+1\right) * (\lg_3 N) \Rightarrow \frac{1}{2} N \lg_3 N + \lg_3 N$$

$$\Rightarrow \frac{1}{2} N \lg_3 N \Rightarrow N \lg_3 N$$

$$\Rightarrow f_3N + Nf_3N \Rightarrow O(Nf_3N)$$

for (int i=1; i<=N; i=i+3) { $\left\lceil \frac{N-1+1}{3} \right\rceil + 1$
~~= $\left\lceil \frac{N}{3} \right\rceil + 1 = N$~~

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for (int j=1; j<=10; j=j+2) {
```

2

2

$(N/3)$ terms

i | 1 4 7 10 13 ----- $N-6$ $N-3$ N

j | 1 1 1 1 1 1 1 1

$$1, \quad 4, \quad 7, \quad 10, \quad 13, \quad N-6, \quad N-3, \quad N$$

$$\left\lceil \lg_2 \left(\frac{N}{2} + 1 \right) \right\rceil + 1 \quad \left\lceil \lg_2 \left(\frac{N}{2} + 1 \right) \right\rceil + 1 \quad \left\lceil \lg_2 \left(\frac{N}{2} + 1 \right) \right\rceil + 1 \quad \left\lceil \lg_2 \left(\frac{N-6}{2} + 1 \right) \right\rceil + 1 \quad \left\lceil \lg_2 \left(\frac{N}{2} + 1 \right) \right\rceil + 1$$

\downarrow
 $1+1$
 2

$3+1$
 4

$3+1$
 4

$\lg_2(N-5)+1$
 $\lg_2(N+1)+1$

$\lg_2(N-2)+1$

$$\text{approximation} \Rightarrow f_2(N+1) * \frac{N}{3} \Rightarrow \frac{1}{3} N * f_2(N+1)$$

$$= N f_2(N)$$

$$N_3 + Nf_2(N) = O(Nf_2(N))$$

for (int $i=1; i \leq N; i=i+2$) { $\left[\lg_2\left(\frac{N}{i}+1\right) \right]^{e1}$ } $= \lg_2\left(\frac{N+1}{2}\right)^{e1}$
 $= \lg_2^2(N)$

for (int $j=1; j \leq i \Rightarrow j=j+3$) {

}

}

$$* \left[\lg_3\left(\frac{8}{j}+1\right) \right]^{e1} = 3^{e1}$$

$$* \left[\lg_3\left(\frac{16}{j}+1\right) \right]^{e1} =$$

$$\Rightarrow \lg_2(N)$$

i	1	2	4	8	16	...	$N/4$	$N/2$	N
-----	---	---	---	---	----	-----	-------	-------	-----

j	1	1	1	1	1	...	1	1	1
-----	---	---	---	---	---	-----	---	---	---

1	2	4	8	16	$N/4$	$N/2$	N
\downarrow	\downarrow	\downarrow	\downarrow	\downarrow			

$$\left[\lg_3\left(\frac{2}{1}+1\right) \right]^{e1} \left[\lg_3\left(\frac{2}{2}+1\right) \right]^{e1} \left[\lg_3\left(\frac{4}{4}+1\right) \right]^{e1}$$

$$\frac{1+1}{2} \quad \frac{1+1}{2} \quad \frac{2+1}{3}$$

$$\left[\lg_3\left(\frac{N/4}{1}+1\right) \right]^{e1} \left[\lg_3\left(\frac{N/2}{1}+1\right) \right]^{e1} \left[\lg_3\left(\frac{N}{1}+1\right) \right]^{e1}$$

$$\lg_3\left(\frac{N}{4}+1\right) \lg_3\left(\frac{N}{2}+1\right) \lg_3(N+1)$$

$$2 \quad 2 \quad 3 \quad 3 \quad 4 \quad \dots \quad \lg_3\left(\frac{N}{4}+1\right) + \lg_3\left(\frac{N}{2}+1\right) + \lg_3(N+1)$$

approximation

$$\Rightarrow \lg_3(N+1) + \lg_2(N)$$

$$\Rightarrow \lg_3(N) * \lg_2(N) = \lg_3 N \lg_2 N$$

$$= \lg_2(N) + \lg_3 N \lg_2 N = O(\lg_3 N \lg_2 N)$$

for (int $i=1$; $i \leq N$; $i = i + 1$) { $\left[\frac{N+1+i}{1} \right]^{e1} = N^{+1}$

for (int $j=1$; $j \leq i * i$; $j = j + 1$) {

}

$\Rightarrow N$ terms

i	1	2	3	4	5	---	$N-2$	$N-1$	N
-----	---	---	---	---	---	-----	-------	-------	-----

j	1	1	1	1	1		1	1	1
-----	---	---	---	---	---	--	---	---	---

1	4	9	16	25	$(N-2)^2$	$(N-1)^2$	N^2
---	---	---	----	----	-----------	-----------	-------

$$\left\lceil \frac{1-i+1}{1} \right\rceil + 1 \quad \left\lceil \frac{4-i+1}{1} \right\rceil + 1 \quad \left\lceil \frac{9-i+1}{1} \right\rceil + 1 \quad \left\lceil \frac{16-i+1}{1} \right\rceil + 1$$

$$1^2 + 1 \quad 2^2 + 1 \quad 3^2 + 1 \quad 4^2 + 1 \quad (N-2)^2 + 1 \quad (N-1)^2 + 1 \quad N^2 + 1$$

$$\underbrace{1^2 + 2^2 + 3^2 + 4^2 + \dots + (N-2)^2 + (N-1)^2 + (N)^2}_{N(N+1)(2N+1)} + (1+1+1+\dots+1)$$

$$\frac{N(N+1)(2N+1)}{6} + N \text{ terms}$$

$$\frac{6N + N(N+1)(2N+1)}{6} \Rightarrow \frac{N(N+1)(2N+1)}{6} \Rightarrow N(N+1)(2N+1)$$

$$N + N(N+1)(2N+1) \Rightarrow O(N(N+1)(2N+1))$$

$$\Rightarrow O(N^3)$$

for (int $i=N$; $i \leq N^2$; $i=i+2$) {
 $\quad \quad \quad \left\{ \begin{array}{l} \left(\frac{N^2-N+1}{2} \right) + 1 \\ \Rightarrow \left(\frac{N^2-N}{2} \right) + 1 \\ \Rightarrow N^2 \end{array} \right. \text{(simplified)} \right.$

for (int $j=i$; $j \leq N^2$; $j=j+2$) {

}

}

i	N	$N+2$	$N+4$	$N+6$	N^2-4	N^2-2	N^2
-----	-----	-------	-------	-------	-------	-------	---------	---------	-------

j	N	$N+2$	$N+4$	$N+6$			N^2-4	N^2-2	N^2
-----	-----	-------	-------	-------	--	--	---------	---------	-------

N^2	N^2	N^2	N^2				N^2	N^2	N^2
-------	-------	-------	-------	--	--	--	-------	-------	-------

\downarrow	\downarrow	\downarrow	\downarrow				\downarrow	\downarrow	\downarrow
$\left[\frac{N^2-N+1}{2} \right] + 1$	$\left[\frac{N^2-N-1}{2} \right] + 1$	$\left[\frac{N^2-N-3}{2} \right] + 1$	$\left[\frac{N^2-N-5}{2} \right] + 1$				$\left[\frac{N^2-N+4+1}{2} \right] + 1$	$\left[\frac{N^2-N+2+1}{2} \right] + 1$	

$\left[\frac{N^2-N+1}{2} \right] + 1$	$\left[\frac{N^2-N-1}{2} \right] + 1$	$\left[\frac{N^2-N-3}{2} \right] + 1$	$\left[\frac{N^2-N-5}{2} \right] + 1$	$\left[\frac{5}{2} \right] + 1$	$\left[\frac{3}{2} \right] + 1$	$\left[\frac{1}{2} \right] + 1$	$\left[\frac{N^2-N+1}{2} \right] + 1$
----------------------------------------	----------------------------------------	----------------------------------------	----------------------------------------	----------------------------------	----------------------------------	----------------------------------	----------------------------------------

Look like arithmetical series.

from this end
 it is adding
 like Arith
 series

pairs $\left[\frac{N^2-N+1+1-2}{2} \right] / 2 \Rightarrow \left[\frac{N^2-N+1+2-4}{2} \right] / 2 \Rightarrow \left[\frac{N^2-N-1}{2} \right] / 2 = N^2 / 4$
 # pair sum

$$\frac{N^2-N+1}{2} + 1 + 2 \Rightarrow \frac{N^2-N+1+6}{2} \Rightarrow \frac{N^2}{2}$$

$$\Rightarrow N^2 / 4 + N^2 / 2 \Rightarrow N^4 / 8$$

$$N^2 + N^4 / 8 \Rightarrow O(N^4)$$