List the names and salaries of the employees along with the total number of hours spent on projects if they have spent less than 20 hours total.

$A \leftarrow {}_{essn}\mathfrak{F}_{sum(hrs)}(\text{Works\_On})$

$B \leftarrow \sigma_{sum(hrs) \leq 20}(A)$

$C \leftarrow B \bowtie_{essn = ssn} \text{Employee}$

$D \leftarrow \pi_{fname, lname, salary, sum(hrs)}(C)$

```
Select E.fname, E.lname, E.salary, sum(W.hrs)
    from Employee E left Join WorksOn W
      on E.ssn = W.ESSN
   group by E.SSN
  having sum(W.hrs) <= 20;
```

Functional Dependency (FD)

Domain	Range
a	1
b	2
c	3
	4

$a \to 1$, $b \to 3$, $c \to 3$

y= square(x)

```
square (int x)
{
    return x * x;
}
```

If for every unique value of X
I get the same value for Y
then X functionally defines Y
or Y is functionall dependent on X

x	y
3	9
4	16
3	9
5	25
8	64
1	1
4	16
-3	9
-1	1
0	0

$x \to y$

attributes

records →

records →

A	B	C	D	E
a1	b1	c1	d1	e1
a2	b1	C2	d2	e1
a3	b2	C1	d1	e1
a4	b2	C2	d2	e1
a5	b3	C3	d1	e1

Table R

$C \to D$ ✓

$D \to C$ ✗

$D \to E$ ✓

$A \to B$ ✓

$B \to A$ ✗

$CD \to B$ ✗

$CD \to E$ ✓

$AC \to D$ ✓

$AD \to C$ ✓

Armstrong Axioms

1. If $B \subseteq A$ then
$A \rightarrow B$
Reflexivity
$A \rightarrow A$

name, age, address $\rightarrow$ age

2. If $A \rightarrow B$ then
$AX \rightarrow BX$
Augmentation

name $\rightarrow$ sex
name, age $\rightarrow$ sex, age

3. If $A \rightarrow B$ & $B \rightarrow C$ then
$A \rightarrow C$
Transitivity

roll # $\rightarrow$ address
address $\rightarrow$ phone
roll # $\rightarrow$ phone

4. If $A \rightarrow B$ & $A \rightarrow C$
then $A \rightarrow BC$
Union

roll# $\rightarrow$ name
roll # $\rightarrow$ address
roll# $\rightarrow$ name, address

5. If $A \rightarrow B$ & $X \rightarrow Y$ then
$AX \rightarrow BY$
Composition

roll# $\rightarrow$ s_name
cid $\rightarrow$ teacher.name
roll #, cid $\rightarrow$ s_name, teacher's name

6. If $AX \rightarrow BY$ then
$AX \rightarrow B$ & $AX \rightarrow Y$
Decomposition

reverse of union

7. If $A \rightarrow B$ & $BC \rightarrow D$ then
$AC \rightarrow D$
Pseudo Transitivity

$R(A, B, C, D, E, F)$

$A \rightarrow B$ $B \rightarrow C$ $C \rightarrow D$ $E \rightarrow F$ $F \rightarrow E$

$A \rightarrow A$ reflexivity

$A \rightarrow B$ given

$A \rightarrow C$ $A \rightarrow B$, $B \rightarrow C$ transitivity

$A \rightarrow D$ transitivity

$A \rightarrow E$ not possible

$AF \rightarrow E$ $F \rightarrow E$

$A \rightarrow ABCD$ Union

Closure

$R(A, B, C, D, E)$

$A \to BC \qquad C \to B \qquad D \to E \qquad E \to D$

(1)

$A^+ \to ABC$

$B^+ \to B$

$C^+ \to CB$

$D^+ \to DE$

$E^+ \to ED$

(2)

$AB^+ \to ABC$

$AC^+ \to ACB$

$\boxed{AD^+ \to ABCDE}$

$\boxed{AE^+ \to ABCDE}$

$BC^+ \to BC$

$BD^+ \to BDE$

$BE^+ \to BED$

CD

CE

DE

(3)

ABC

ABD

ABE

ACD

ACE

$\boxed{ADE^+ \to ADEBC}$

BCD

BCE

BDE

CDE

(4)

ABCD

ABCE

ACDE

ABDE

BCDE

(5) ABCDE

31

Super key : Any set of attributes that can be used to determine the values of all the attributes of the relation

# of possible FDs. $= 2^n - 1$

5 attributes $= 2^5 - 1 = 31$

6 " $= 2^6 - 1 = 63$

HW

R(A, B, C, D, E)

$A \rightarrow BC \quad C \rightarrow B \quad D \rightarrow E \quad E \rightarrow D$

R(A B C D E)

$A \rightarrow B \quad B \rightarrow C \quad D \rightarrow C$

Find closure of all possible combinations of attributes using the given functional dependencies

Also identify the super keys.

Hand written Submission at start of class on 19th Jan 22

R(A, B, C, D, E)

Default Super key

SK = { AD, AE, ABD, ABE, ACD, ACE, ADE, ABCD, ABCE, ABDE, ACDE, ABCDE }

2 2 3 3 3 3 3 4 4 4 4 5

Superkey : Any set of attributes that can uniquely identify the whole record

Default Superkey : Set of all attributes of the relation

Size of Superkey : # of attributes in the superkey

Candidate key : Superkeys of the smallest size (CK)

Primary key : One of the candidate keys is chosen as PK

R(A, B, C, D, E) $AB \to C$ $C \to E$ $D \to A$

find the CKs in the system

$A^+ \to A$	$AB^+ \to ABCE$	$\boxed{BD^+ \to BDACE}$
$B^+ \to B$	$AC^+ \to ACE$	$BE^+ \to BE$
$C^+ \to CE$	$AD^+ \to AD$	$CD^+ \to CDAE$
$D^+ \to DA$	$AE^+ \to AE$	$CE^+ \to CE$
$E^+ \to E$	$BC^+ \to BCE$	$DE^+ \to DEA$

CK = { BD }

PK = { BD }

L	B	R
B	A	E
D	C	

$R(W, X, Y, Z)$ $Z \to WXY$ $X \to W$

$\bar{D}$	D
Z	W X Y

$Z^+ \to ZWXY$ s.k

$W^+ \to W$

$X^+ \to XW$

$Y^+ \to Y$

$Z^+ \to WXYZ$

L	B	R
Z	X	W Y

$Ck = \{Z\}$

$R(A\ B\ C\ D\ E\ F)$ $A \to C$ $C \to B$ $B \to D$ $D \to E$ $E \to A$

$\bar{D}$	D
A F	B C D E

$A^+ \to ACBDE$ ✓

$B^+ \to BDE$

$C^+ \to CBDE$

$D^+ \to DE$

$E^+ \to E$

L	B	R
A F	B C D	E

$AF^+ \to AFCBDE$

$Ck = \{A\}$

$Ck = \{AF\}$

$R(A\ B\ C\ D\ E)$ $A \to C$ $E \to D$ $B \to C$

$\bar{D}$	D
A B E	C D

$ABE^+ \to ABECD$ ✓

L	B	R
A B E		C D

$Ck = \{ABE\}$

$R(V\ W\ X\ Y\ Z)$

$Z \to WXY$ $X \to W$ $Y \to Z$

$\bar{D}$	D
V	W X Y Z

$V^+ \to V$ ←

$VW^+ \to VW$ ←

$VX^+ \to VXW$

$VY^+ \to VYZWX$ ✓

$VZ^+ \to VZWXY$ ✓

L	B	R
V	X Y Z	W

$Ck = \{VY, VZ\}$

R(A B C D E)   A → C   C → B   B → D   D → E   E → A

$\bar{D}$	D
	A
	B
	C
	D
	E

L	B	R
	A	
	B	
	C	
	D	
	E	

A^+ → ACBDE ✓

B^+ → BDEAC ✓

C^+ → CBDEA ✓

D^+ → DEACB ✓

E^+ → EACBD ✓

CK = {A, B, C, D, E}

R(ABCDEFGHIJ)

AB → EF   A → C   B → DGHI   H → I   AE → BDFGHI

L	B	R
A	B	C
J	E	D
	H	F
		G
		I

AJ^+ → AJC

AJB → AJBCEFDGHI ✓

AJE → AJEBDFGHIC ✓

AJH → AJHCI

CK = {AJB, AJE}

XBE

XBH

XEH

XBEH

R(A B C D)

AB → C   AB → D   C → A   D → B

L	B	R
	A	
	B	
	C	
	D	

A^+ → A AB^+ → ABCD ✓

B^+ → B   AC → AC

C^+ → AC   AD → ADBC ✓

D^+ → BD   BC → BCAD ✓

BD → BD

CD → CDAB ✓

CK = {AB, AD, BC, CD}

FD - HW # 2

Hand written , Due on Friday 21st Jan 2022

till 3:00 PM

Office : F-303 3rd Floor A - Building

(8) R(A B C D E)

B → C , D → A

L	B	R
B		A
D		C
E		

$BDE^+ \to$ B D E C A

Ck = {BDE}

(12) R(A B C D E F G H)

AB → C   AC → B   AD → E   B → D   BC → A   E → G

L	B	R
F	A	G
H	B	
	C	
	D	
	E	

$FH^+ \to$ FH

$FHA^+ \to$ FHA

$FHB^+ \to$ FHBD

$FHC^+ \to$ FHC

$FHD^+ \to$ FHD

FHE → FHEG

$FHAB^+ \to$ ABCDEFGH ✓

$FHAC^+ \to$ ABCDEFGH ✓

$FHAD^+ \to$ A DEFGH

$FHAE^+ \to$ A EFGH

$FHBC^+ \to$ ABCDEFGH ✓

$FHBD^+ \to$ BD F H

$FHBE^+ \to$ BDEFGH

$FHCD^+ \to$ CD F H

$FHCE^+ \to$ C EFGH

FHDE → DE FGH

Ck = {ABFH, ACFH, BCFH}

Normalization

0 - Normal Form : System must have FDs

Convert the data to a relation so that you can find the Functional Dependencies in the data

1st Normal Form

Find the Primary Key for your relation.

System must be in 0-NF

AND

System must have a primary key

Prime Attributes

Attributes which are part/present in any candidate key

2nd Normal Form

System must be in 1-NF

AND

All non prime attributes must be dependent on the whole PK

OR

No partial dependency between non prime attributes and the PK is allowed.

3rd Normal Form

System must be in 2nd Normal Form

AND

All non prime attributes must be directly dependent on the primary key

Item	Color	Price	Tax	Qty
T-Shirt	Red, Blue	12.00	0.60	5, 3
Polo	Red, Yellow	15.00	0.60	8, 2
T-Shirt	Red, Black	12.00	0.60	5, 9
Sweatshirt	Blue, Black	25.00	1.25	12, 2

lets Convert to relation

Item	Color	Price	Tax	Qty
Tshirt	Red	12.00	0.60	5
Tshirt	Blue	12.00	0.60	3
Polo	Red	15.00	0.60	8
Polo	Yellow	15.00	0.60	2
Tshirt	Black	12.00	0.60	9
S.Shirt	Blue	25.00	1.25	12
S.shirt	Black	25.00	1.25	2

item → price

price → tax

item, color → qty

lets find the candidate key

L	B	R
item	price	tax
color		qty

$\text{item, color}^{+} \rightarrow \text{item, color, price, tax, qty}$

CK : { (item, color) }

so Primary key = (item, color)

R1(item, color, price, tax, qty)

R1 is in 1.NF

Prime Attributes : { item , color }

Non Prime Attributes : { price, tax, qty }

Prime Attributes : { item , color }

Non Prime Attributes : { ~~price~~, ~~tax~~, qty }

item$^+$ → item , price , tax

color$^+$ → color

R2 (<u>item</u>, price, tax)

R3 (<u>item</u>, <u>color</u> , qty)

R2 & R3 are in 2nd NF

item$^+$ → item , price , tax

R3 (<u>item</u> , <u>color</u> , qty)

R4 (<u>item</u> , price)

R5 (<u>price</u> , tax)

R3, R4 & R5 are in 3 NF.

<u>R3</u>

item	color	qty
T.shirt	Red	5
T.shirt	Blue	3
T.shirt	Black	9
Polo	Red	8
Polo	Yellow	2
S.shirt	Blue	12
S.shirt	Black	2

<u>R4</u>

item	price
T.shirt	12.00
Polo	15.00
S.shirt	25.00

<u>R5</u>

price	tax
12.00	0.60
15.00	0.60
25.00	1.25

Boyce Codd Normal Form (BCNF)

Advance form of 3 NF

All FDs of the form $X \rightarrow Y$ then either (i) it is a trivial FD $Y \subseteq X$

(ii) X is a candidate / superkey for R

Patient id	P. name	App No	Time	Doctor
1	Sara	0	9:00	Ahsan
2	Saleem	0	9:00	Hassan
3	Usman	1	10:00	Ahsan
4	Ali	0	13:00	Hassan
5	Farooq	1	14:00	Ahsan

Pat ID → Pat Name

Pat ID, AppNo → Time, Doctor

Time → AppNo

L	B	R
PNo	AppNo	PName
	Time	Doctor

1 NF

$PNo^+ \rightarrow$ PNo, PName

$PNo, AppNo^+ \rightarrow$ PNo, PName, AppNo, Time, Doctor ✓

$PNo, Time^+ \rightarrow$ PNo, PName, AppNo, Time, Doctor ✓

CK = {(PNo, AppNo), (PNo, Time)}

1NF R1(PNo, PName, AppNo, Time, Doctor) → 1NF

2NF

$PNo^+ \rightarrow$ PNo, PName

$AppNo^+ \rightarrow$ AppNo

R2(PNo, PName)

R3 (PNo, AppNo, Time, Doctor)

} 2NF

R2 (PNo , PName)

R3 (PNo , ApNo , Time , Doctor)

R2 & R3 are in 3NF

		non trivial	superkey of R
(i)	PNo → PName	✓	✓
(ii)	PaNo , AppNo → Time , Doctor	✓	✓
(iii)	Time → AppNo	✓	✗

R4 (Time , AppNo)

R5 (PNo , Time , Doctor)

$\boxed{\text{R2, R4 \& R5 are in BCNF}}$

R(C# (A), CName (B), CPhone (C), CAddress (D), E# (E), EName (F), EPhone (G), EAddress (H), ESalary (I), Pid (J), PName (K),
PDesc (L), Pprice (M), O# (N), Odate (O), Qty (P))

C# (A) → CName (B), CPhone (C), CAddress (D)
E# (E) → EName (F), EPhone (G), EAddress (H), ESalary (I)
Pid (J) → Pname (K), PDesc (L), Pprice (M)
O# (N) → Odate (O), C# (A), E# (E)
Pid (J), O# (N) → Qty (P)

$A \rightarrow BCD$
$E \rightarrow FGHI$
$J \rightarrow KLM$
$N \rightarrow OAE$
$JN \rightarrow P$

1NF

$(JN)^+ \rightarrow$ J N K L M O A E B C D F G H I P

L	B	R	
J	A	B	
N	E	C	
		D	
		F	K
		G	L
		H	M
		I	O
			P

R1(A B C D E F G H I J K L M N O P)

NPA = {~~A~~ ~~B~~ ~~C~~ ~~D~~ ~~E~~ ~~F~~ ~~G~~ ~~H~~ ~~I~~ ~~K~~ ~~L~~ ~~M~~ ~~O~~ P}

2NF

$J^+ \rightarrow$ J K L M
$N \rightarrow$ N O A E B C D F G H I

R2(J K L M)

R3(A B C D E F G H I N O)

R4(J N P)

3rd NF

R2(J K L M) ✓

R3(A B C D E F G H I N O)

R4(J N P)

R2(J K L M)
R5(A B C D)
R6(E F G H I)
R7(N O A E)
R4(J N P)

3 NF

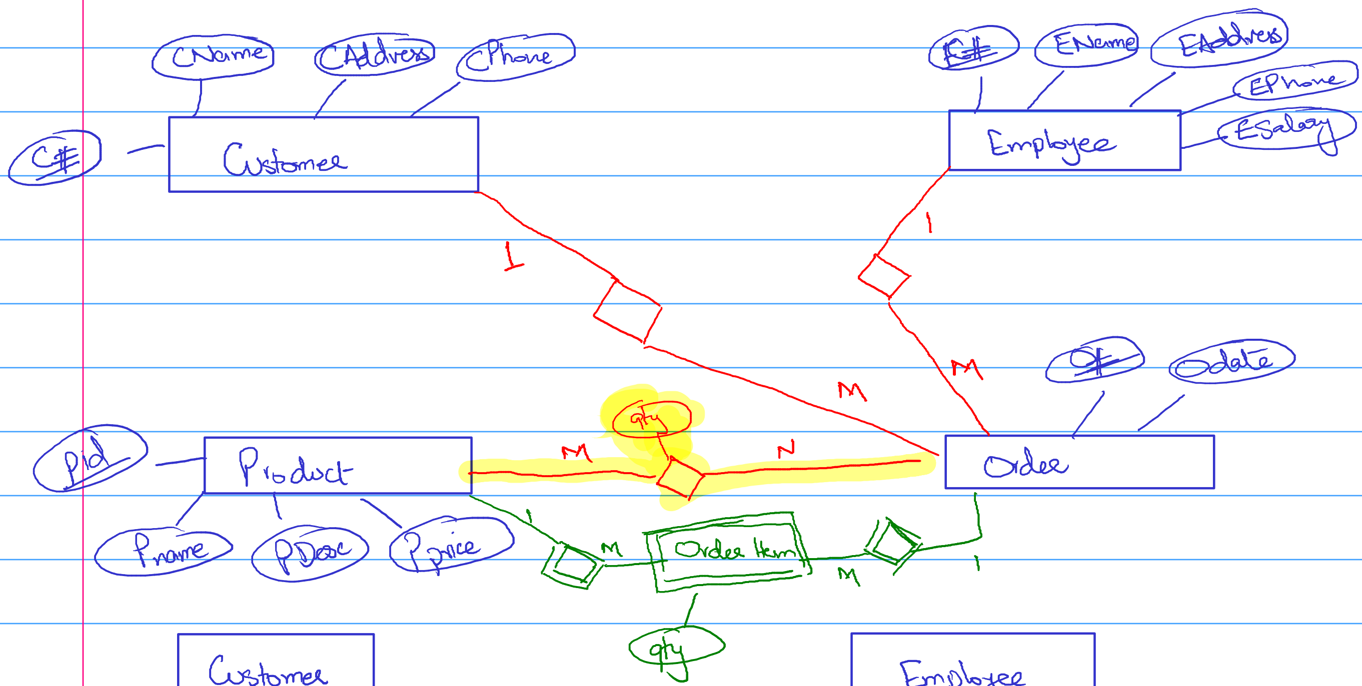
R2 (J K L M) R2 (Pid, PName, PDesc, Pprice)

R5 (A B C D) R5 (C#, CName, CPhone, CAddress)

R6 (E F G H I) R6 (E#, EName, EPhone, EAddres, ESalary)

R7 (N O A E) R7 (O#, ODate, C#, E#)

R4 (J N P) R4 (Pid, O#, Qty)



Customer
- <u>C#</u>
- CName
- CAddress
- CPhone

Employee
- <u>E#</u>
- EName
- EAddress
- EPhone
- ESalary

Product
- <u>Pid</u>
- PName
- PDesc
- Pprice

Order
- <u>O#</u>
- Odate
- C#
- E#

Order Item
- <u>O#</u>
- <u>Pid</u>
- Qty

ERD

Customer (C#, CName, CAddress, CPhone)

Employee (E#, EName, EAddress, EPhone, ESalary)

Product (Pid, PName, PDesc, Pprice)

Order (O#, Odate, C#, E#)

OrderItem (O#, Pid, Qty)

Normalization

R2 (Pid, PName, PDesc, Pprice)

R5 (C#, CName, CPhone, CAddress)

R6 (E#, EName, EPhone, EAddress, ESalary)

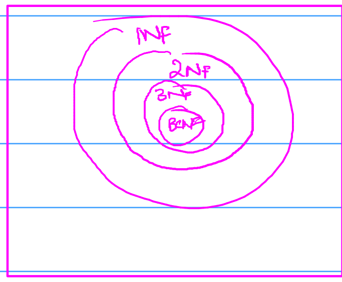
R7 (O#, ODate, C#, E#)

R4 (Pid, O#, Qty)

$A \rightarrow BC$

$A \rightarrow B$

$A \rightarrow C$



$\alpha \rightarrow \beta$

$A \rightarrow B$

$A \rightarrow CD$

$AE \rightarrow FG$

$H \rightarrow F$

$\alpha \rightarrow \{SK, CK, P, PC, NP, NPC\}$

$\beta \rightarrow \{P, NP\}$

	1NF	2NF	3NF	BCNF
SK → P	✓	✓	✓	✓
SK → NP	✓	✓	✓	✓
CK → P	✓	✓	✓	✓
CK → NP	✓	✓	✓	✓
P → P	✓	✓	✓	
P → NP	✓			
PC → P	✓	✓	✓	
PC → NP	✓			
~~NP → P~~	~~✓~~			
NP → NP	✓	✓		
NPC → P	✓	✓	✓	
NPC → NP	✓			
	System should have FD	no partial dependency α is CK/SK α & β is prime or α & β is non prime	no transitive dependency α is CK/SK or β is prime	α must be CK/SK

L	B	R
B	A, C, D	E

R(A B C D E)

AB → CD

D → A

BC → DE

CK = {AB, BC, BD}

PA = {A, B, C, D}

	BCNF	3NF
AB → C	✓	✓
AB → D	✓	✓
D → A	✗	✓
BC → D		✓
BC → E		✓

3NF

R(A B C D E F G H)

AB → C

A → DE

B → F

F → GH

CK = {AB}

PA = {A, B}

NPA = {C, D, E, F, G, H}

	BCNF	3NF	2NF	1NF
AB → C	✓	✓	✓	✓
A → D	✗	✗	✗	✓
A → E				✓
B → F				✓
F → G				✓
F → H				✓

1NF

R (A B C D E F G H I J K L M N)

$A \rightarrow B$ CK = {ACE}

$C \rightarrow D$ R1 ($\underline{A}$ B $\underline{C}$ D $\underline{E}$ F G H I J K L M N)

$E \rightarrow F$ 1NF

$F \rightarrow G$ $A^+ \rightarrow A\,B\,H\,I$ R2 ($\underline{A}$ B H I)

$A \rightarrow H$ $C^+ \rightarrow C\,D$ R3 ($\underline{C}$ D)

$H \rightarrow I$ $E^+ \rightarrow E\,F\,G$ R4 ($\underline{E}$ F G)

$AC \rightarrow J$ $AC^+ \rightarrow A\,C\,\cancel{B}\,\cancel{H}\,\cancel{I}\,\cancel{D}\,J\,K$ R5 ($\underline{A}\,\underline{C}$ J K)

$J \rightarrow K$ $AE^+ \rightarrow A\,E\,\cancel{B}\,\cancel{H}\,\cancel{I}\,\cancel{F}\,\cancel{G}$ R6 ($\underline{C}\,\underline{E}$ L)

$CE \rightarrow L$ $CE^+ \rightarrow C\,E\,\cancel{D}\,\cancel{F}\,\cancel{G}\,L$ R7 ($\underline{A}\,\underline{C}\,\underline{E}$ M N)

$ACE \rightarrow M$

$M \rightarrow N$ R2, R3, R4, R5, R6, R7 are in 2NF

R2 (A B H I) R8 ($\underline{A}$ B H), R9 ($\underline{H}$ I)

R3 ($\underline{C}$ D) ✓

R4 ($\underline{E}$ F G) R10 ($\underline{E}$ F) R11 ($\underline{F}$ G)

R5 ($\underline{A}\,\underline{C}$ J K) R12 ($\underline{A}\,\underline{C}$ J) R13 ($\underline{J}$ K)

R6 ($\underline{C}\,\underline{E}$ L) ✓

R7 ($\underline{A}\,\underline{C}\,\underline{E}$ M N) R14 ($\underline{A}\,\underline{C}\,\underline{E}$ M) R15 ($\underline{M}$ N)

R3, R6, R8, R9, R10, R11, R12, R13, R14, R15 are in 3NF

Equivalence of FD Sets

R (A B C)

$F_1 = \{ A \to BC \quad B \to C \quad AB \to A \}$

$F_2 = \{ A \to B \quad B \to C \}$

Checking F_1

$A^+ \to ABC$ ✓

$B^+ \to BC$ ✓

$AB \to ABC$ ✓

$F_1 \subseteq F_2 \longrightarrow (1)$

Check F_2 wrt F_1

$A \to ABC$ ✓

$B \to C$ ✓

$F_2 \subseteq F_1 \longrightarrow (2)$

from (1) & (2) F_1 is equivalent to F_2

$R(A\ B\ C\ D\ E\ F\ G)$

$F_1 = \{A \to AB,\ B \to C,\ A \to B,\ AB \to C,\ D \to A,\ ADE \to FG\ \ A \to F\}$

$F_2 = \{A \to B,\ B \to C,\ D \to A,\ DE \to G\}$

Checking F_2 w.r.t F_1

$\underline{\underline{A \to B}}$	$A^+ \to ABCF$	✓
$\underline{\underline{B \to C}}$	$B^+ \to BC$	✓
$\underline{\underline{D \to A}}$	$D^+ \to DA$	✓
$\underline{\underline{DE \to G}}$	$DE^+ \to DEABCFG$	✓

$$F_2 \subseteq F_1 \longrightarrow (1)$$

lets check F_1 wrt F_2

$A \to AB$	$A^+ \to ABC$	✓
$B \to C$	$B^+ \to BC$	✓
$A \to B$	$A^+ \to ABC$	✓
$AB \to C$	$AB^+ \to ABC$	✓
$D \to A$	$D^+ \to DA$	✓
$ADE \to FG$	$ADE^+ \to ABCG$	X
$A \to F$	$A^+ \to ABC$	X

$$F_1 \not\subseteq F_2 \longrightarrow (2)$$

from (1) & (2) F_1 is not equivalent to F_2

R(A B C)

$F = \{A \to BC \quad B \to C \quad AB \to A\}$

Is this the simplest possible FD set for this data

Step 1 : Only single attribute on RHS of every FD

$F = \{A \to B, \; A \to C, \; B \to C, \; AB \to A\}$

Step 2 : Try to reduce every complex L.H.S

AB $\qquad A^+ \to ABC$

AB can be reduced to A only

$F = \{A \to B, \; A \to C, \; B \to C, \; A \to A\}$ → trivial

$F = \{A \to B, \; A \to C, \; B \to C\}$

Step 3: For every FD check if alternate path is available

$A \to B \qquad A^+ - \{A \to B\} \to AC \qquad$ required

$A \to C \qquad A^+ - \{A \to C\} \to ABC \qquad$ so redundant

$F = \{A \to B, \; B \to C\}$

$B \to C \qquad B^+ - \{B \to C\} \to B \qquad$ required

$F = \{A \to B, \; B \to C\}$

$F = \{ A \to AB \quad B \to C \quad A \to B \quad AB \to C \quad D \to A \quad ADE \to FG \quad A \to F \}$

Step 1

trivial

Duplicate

$F = \{ A \to A,\ A \to B,\ B \to C,\ A \to B,\ AB \to C,\ D \to A,\ ADE \to F,$
$ADE \to G,\ A \to F \}$

$F = \{ A \to B,\ B \to C,\ AB \to C,\ D \to A,\ ADE \to F,\ ADE \to G,\ A \to F \}$

Step 2

AB $\quad A^+ \to ABCF \quad$ so B is not required

$F = \{ A \to B \quad B \to C \quad A \to C \quad D \to A \quad ADE \to F \quad ADE \to G \quad A \to F \}$

ADE $\quad A^+ \to ABCF$

$D \to DABCF \quad$ A can be replaced by D

E

$F = \{ A \to B \quad B \to C \quad A \to C \quad D \to A \quad DE \to F \quad DE \to G \quad A \to F \}$

Step 3

$A \to B \quad A^+ - \{A \to B\} \to ACF \quad$ required

$B \to C \quad B^+ - \{B \to C\} \to B \quad$ required

$A \to C \quad A^+ - \{A \to C\} \to ABCF \quad$ redundant

$F = \{ A \to B,\ B \to C,\ D \to A,\ DE \to F,\ DE \to G,\ A \to F \}$

$D \to A \quad D^+ - \{D \to A\} \to D \quad$ required

$DE \to F \quad DE^+ - \{DE \to F\} \to DEABCF \quad$ redundant

$F = \{ A \to B,\ B \to C,\ D \to A,\ DE \to G,\ A \to F \}$

$DE^+ - \{DE \to G\}$

$A^+ - \{A \to F\}$