

Ques 1
 ① $\text{for } (\text{int } i=N; i<=3N^2; i=i+3) \{ \rightarrow \lceil f_3\left(\frac{3N^2}{N} + 1\right) \rceil + 1 \Rightarrow \lceil f_3(3N+1) \rceil + 1 \Rightarrow f_3(3N)$
 ~~$f_3(N)$~~

$\text{for } (\text{int } j=1; j<=i; j=j+3) \{$

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 $i \quad N \quad | \quad 3N \quad | \quad 9N \quad | \quad 27N \quad | \quad 81N \quad \dots \quad | \quad \frac{N^2}{3} \quad | \quad N^2 \quad | \quad 3N^2$
 $j \quad 1 \quad | \quad 1$
 $N \quad , \quad 3N \quad , \quad 9N \quad , \quad 27N \quad , \quad 81N \quad , \quad N^2/3 \quad , \quad N^2 \quad , \quad 3N^2$

$\left[\frac{N-1}{3} + 1 \right] + 1 \quad \left[\frac{3N-1}{3} + 1 \right] + 1 \quad \left[\frac{9N-1}{3} + 1 \right] + 1$
 $\frac{N+1}{3} \quad N+1 \quad 3N+1 \quad 9N+1 \quad 27N+1 \quad \frac{N^2+1}{3} \quad \frac{N^2+1}{3} \quad N^2+1$
 $\left\{ \frac{N}{3}, N, 3N, 9N, \dots, \frac{N^2}{3}, N^2, N^2 \right\} + [1+1+\dots+1] \text{ separating one}$
 geometric Series.

$$C * N^2 = CN^2$$

$$CN^2 + f_3 N = CN^2$$

$$1 * \underline{\underline{f_3(N)}}$$

tecml

$$CN^2 + f_3 N = \underline{\underline{O(N^2)}} \text{ Answe.}$$

⑦ $\text{for } (\text{int } i=3; i \leq 3N; i=i+3) \{ \quad \lceil \frac{3N-3+1}{3} \rceil + 1 = \lceil \frac{3N-2}{3} \rceil + 1 \Rightarrow 3N/3 \Rightarrow N$

$\text{for } (\text{int } j=i; j \leq 3N; j=j+3) \{$

}

i	3	6	9	12	15	18	... - - - - -	$3N-6$	$3N-3$	$3N$
j	3	6	9	12	15			$3N-6$	$3N-3$	$3N$
	$3N$	$3N$	$3N$	$3N$	$3N$			$3N$	$3N$	$3N$

$$\left\lceil \mathfrak{f}_3\left(\frac{3N+1}{3}\right) \right\rceil + 1 \quad \left\lceil \mathfrak{f}_3\left(\frac{3N+1}{6}\right) \right\rceil + 1 \quad \left\lceil \mathfrak{f}_3\left(\frac{3N+1}{9}\right) \right\rceil + 1$$

can't separate $\mathfrak{f}_3(N+1) + 1$ $\left\lceil \mathfrak{f}_3\left(\frac{N+1}{2}\right) \right\rceil + 1 \quad \left\lceil \mathfrak{f}_3\left(\frac{N+1}{3}\right) \right\rceil + 1$
 to $\mathfrak{f}_3(N+1) + 1$ $\left\lceil \mathfrak{f}_3\left(\frac{N}{2}\right) \right\rceil + 1 \quad \left\lceil \mathfrak{f}_3\left(\frac{N}{3}\right) \right\rceil + 1$
 smaller change but not same? // approximation

$$\frac{N * \mathfrak{f}_3(N)}{\# \text{terms}} = N \mathfrak{f}_3 N$$

$$\left\lceil \mathfrak{f}_3\left(\frac{3N+1}{3N-6}\right) \right\rceil + 1 \quad \left\lceil \mathfrak{f}_3\left(\frac{3N+1}{3N-3}\right) \right\rceil + 1 \quad \left\lceil \mathfrak{f}_3\left(\frac{3N+1}{3N}\right) \right\rceil + 1$$

$$|| \quad \mathfrak{f}_3(2) + 1$$

$$N \mathfrak{f}_3 N + N = O(N \mathfrak{f}_3 N)$$

③ for (int i=4; i<=2N; i=i*4) { $\rightarrow \left[f_4\left(\frac{2N}{4}+1\right) \right] + 1 \Rightarrow \left[f_4\left(\frac{N}{2}+1\right) \right] + 1 \Rightarrow f_4\left(\frac{N}{2}\right)$
 $\Rightarrow f_4(N)$

for (int j=i; j<=2N; j=j*2) {

i	4	16	64	256	...
j	4	16	64	256	...
	↓	↓	↓	↓	...
	2N	2N	2N	2N	...

$$\left[f_2\left(\frac{2N}{4}+1\right) \right] + 1 \quad \left[f_2\left(\frac{2N}{16}+1\right) \right] + 1 \quad \left[f_2\left(\frac{2N}{64}+1\right) \right] + 1$$

$$\left[f_2\left(\frac{N}{2}+1\right) \right] + 1 \quad \left[f_2\left(\frac{N}{8}+1\right) \right] + 1 \quad \left[f_2\left(\frac{N}{32}+1\right) \right] + 1$$

$$\left[f_2\left(\frac{N}{64}+1\right) \right] + 1 \quad \left[f_2\left(\frac{N}{128}+1\right) \right] + 1$$

Series like an arithmetic Series

* Series like an arithmetic Series
 $f_2(N) + ?$ complex work

$\frac{N}{8}$	$\frac{N}{2}$	$2N$
$\frac{N}{16}$	$\frac{N}{4}$	$\frac{1}{2}N$
$\frac{N}{32}$	$\frac{N}{8}$	$\frac{1}{4}N$

$$\left[f_2\left(\frac{2N}{N/8}+1\right) \right] + 1 \quad \left[f_2\left(\frac{2N}{N/16}+1\right) \right] + 1$$

$$\left[f_2(16+1) \right] + 1 \quad \left[f_2\left(\frac{2N}{N/32}+1\right) \right] + 1$$

$$\left[f_2(4+1) \right] + 1$$

$$\left[f_2(2+1) \right] + 1$$

$$f_2(65)+1 \quad f_2(17)+1 \quad f_2(5)+1 \quad f_2(2)+1$$

$$\frac{9+1}{10} \quad \frac{7+1}{8} \quad \frac{5+1}{6} \quad \frac{3+1}{4} \quad \frac{1+1}{2}$$

* can be solved as approximation.

$$\Rightarrow f_2(n) * f_4^4 = f_2^n * f_4^n$$

$$f_2^n f_4^n + f_4^n = \underline{\underline{O(f_2^n f_4^n)}}$$