

$\text{for (int } i=1; i \leq N; i=i+1) \{ \rightarrow \text{outer } \left\lceil \frac{N-1+1}{1} \right\rceil + 1 = N+1 = N$
 $\text{for (int } j=1; j \leq i; j=j+1) \{$
 $\quad \quad \quad \left[\begin{array}{l} \text{Added} \\ \text{for} = \\ \text{equal} \end{array} \right] \left[\text{loop break} \right]$
 $\quad \quad \quad \}$
 $\}$ for every value of $i \rightarrow$ having loop for j

i	1	2	3	4	5	...	$N-2$	$N-1$	N
j	1	1	1	1	1		1	1	1
	1	2	3	4	5		$N-2$	$N-1$	N
	$\left\lceil \frac{1-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{2-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{3-1+1}{1} \right\rceil + 1$				$\left\lceil \frac{N-2-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{N-1-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{N-1+1}{1} \right\rceil + 1$
	2	3	4				$N-1$	N	$N+1$

arithmetic series

$\# \text{ pairs} \rightarrow \left\lceil \frac{N+1-2+1}{1} \right\rceil / 2 = \frac{N+2}{2}$
 $\text{pair sum} \rightarrow N+1+2 = \underline{N+3}$

$\left(\frac{N+2}{2} \right) (N+3) = \frac{1}{2} (N^2 + 5N + 6)$ *simple*
 $= \frac{1}{2} (N^2)$
 $= N^2$

$N + N^2 = O(N^2)$ answer
 \swarrow outer
 \searrow inner series

outer series has change 2

for (int i=1; i<=N; i=i+2) { $\rightarrow \left\lceil \frac{N-1+1}{2} \right\rceil + 1 = \frac{N}{2} + 1 = N$
 for (int j=1; j<=i; j=j+1) {
 }
 }
 inner series has change 1

Simplified answer

i	1	3	5	7	9	...	N-4	N-2	N
j	1	1	1	1	1		1	1	1
	1	3	5	7	9		N-4	N-2	N
	$\left\lceil \frac{1-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{3-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{5-1+1}{1} \right\rceil + 1$				$\left\lceil \frac{N-4-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{N-2-1+1}{1} \right\rceil + 1$	$\left\lceil \frac{N-1+1}{1} \right\rceil + 1$
	2	4	6				N-3	N-1	N+1

Arithmetical Series

here we have added loop terminating 1 more operation

pairs $\rightarrow \left\lceil \frac{N+1-2+1}{2} \right\rceil / 2 = \left\lceil \frac{N}{2} \right\rceil / 2 = N/4$
 pair sum $\rightarrow N+1+2 = N+3$

$$\frac{N}{4} \times (N+3) = \frac{1}{4} (N^2 + 3N) = \frac{1}{4} (N^2) = N^2$$

$$N + N^2 = O(N^2)$$

outer series has
change 2

for (int i=1; i<=N; i=i+2) { \rightarrow calculate it

$$\left\lceil \frac{N-1+1}{2} \right\rceil + 1 = \left\lceil \frac{N}{2} \right\rceil + 1 = N$$

for (int j=1; j<=i; j=j+2) {

} inner series has
change 2

}

i	1	3	5	7	9	...	N-4	N-2	N
j	1	1	1	1	1		1	1	1
	1	3	5	7	9		N-4	N-2	N
	$\left\lceil \frac{1-1+1}{2} \right\rceil + 1$	$\left\lceil \frac{3-1+1}{2} \right\rceil + 1$	$\left\lceil \frac{5-1+1}{2} \right\rceil + 1$				$\left\lceil \frac{N-4-1+1}{2} \right\rceil + 1$	$\left\lceil \frac{N-2-1+1}{2} \right\rceil + 1$	$\left\lceil \frac{N-1+1}{2} \right\rceil + 1$
	2	3	4				$\frac{N-1}{2}$	$\frac{N}{2}$	$\frac{N}{2} + 1$

arithmetic series

$$\begin{aligned} \# \text{ pairs} &\rightarrow \left\lceil \frac{\frac{N}{2} + 1 - 2 + 1}{2} \right\rceil = \left\lceil \frac{N/4}{2} \right\rceil = N/8 \\ \text{pair sum} &\rightarrow \frac{N}{2} + 1 + 2 = \frac{N}{2} + 3 \end{aligned} \quad \left[\left(\frac{N}{8} \right) \left(\frac{N}{2} + 3 \right) = \frac{N^2}{16} + \frac{3N}{8} \right]$$

$$= \frac{N^2}{16} = N^2$$

$$N + N^2 = O(N^2)$$

for (int i=1; i<=N; i=i*2) { $\lceil \lg_2(\frac{N}{1}+1) \rceil + 1 \Rightarrow \lg_2 N + 1$
 $\Rightarrow \lg_2 N$

for (int j=1; j<=i; j=j+1) {

}

}

How many terms here
 \Rightarrow complexity of loop

i	1	2	4	8	16	...	N/4	N/2	N	$\lg_2 N$
j	1	1	1	1	1		1	1	1	
	1	2	4	8	16		N/4	N/2	N	
	$\lfloor \frac{1-1+1}{1} \rfloor + 1$	$\lfloor \frac{2-1+1}{1} \rfloor + 1$	$\lfloor \frac{4-1+1}{1} \rfloor + 1$				$\lfloor \frac{N/4-1+1}{1} \rfloor + 1$		$\lfloor \frac{N-1+1}{1} \rfloor + 1$	
	2	3	5				$\frac{N}{4} + 1$	$\frac{N}{2} + 1$	N + 1	

if we separate 1 (as loop break)

1+1 2+1 4+1 $\frac{N}{4}+1$ $\frac{N}{2}+1$ N+1

(1 2 4 8 16 ... $\frac{N}{4}$ $\frac{N}{2}$ N) + (1 1 1 ... + 1)
 $\lg_2 N$ terms

largest term

$$C * N = CN = N + \lg_2 N = N$$

$$N + \lg_2 N = O(N)$$

Geometric
 Series

$$= \log(N+1) + 1$$

$55(N)$ terms

approximation

$$\Rightarrow [\text{largest term} * \# \text{ of terms}]$$

$$\Rightarrow \left(\frac{N}{2} + 1\right) * (\lg_3 N) \Rightarrow \frac{1}{2} N \lg_3 N + \lg_3 N$$

$$\Rightarrow \frac{1}{2} N f_3 N \Rightarrow N f_3 N$$

$$\Rightarrow f_3 N + N f_3 N \Rightarrow O(N f_3 N)$$

$\text{for (int } i=1; i \leq N; i=i+3) \{$
 $\quad \text{for (int } j=1; j \leq i; j=j*2) \{$
 $\quad \quad \}$
 $\quad \}$

$$\left\lceil \frac{N-1+1}{3} \right\rceil + 1 = \left\lceil \frac{N}{3} \right\rceil + 1 = N$$

terms $\left\lceil \frac{N}{3} \right\rceil$

	$(N/3) \text{ terms}$								
i	1	4	7	10	13	-----	$N-6$	$N-3$	N
j	1	1	1	1	1		1	1	1
	1	4	7	10	13		$N-6$	$N-3$	N
	$\left\lceil \log_2 \left(\frac{1}{1} + 1 \right) \right\rceil + 1$	$\left\lceil \log_2 \left(\frac{4}{1} + 1 \right) \right\rceil + 1$	$\left\lceil \log_2 \left(\frac{7}{1} + 1 \right) \right\rceil + 1$				$\left\lceil \log_2 \left(\frac{N-6}{1} + 1 \right) \right\rceil + 1$	$\left\lceil \log_2 \left(\frac{N-3}{1} + 1 \right) \right\rceil + 1$	$\left\lceil \log_2 \left(\frac{N}{1} + 1 \right) \right\rceil + 1$
	$1+1$	$3+1$	$3+1$				$\log_2(N-5)+1$	$\log_2(N-2)+1$	$\log_2(N+1)+1$
	2	4	4						

approximation $\Rightarrow \log_2(N+1) * \frac{N}{3} \Rightarrow \frac{1}{3} N * \log_2(N+1)$
 $= N \log_2(N)$

$$N/3 + N \log_2(N) = O(N \log_2(N))$$

for (int i=1; i<=N; i=i*2) { $\left\lceil \lg_2\left(\frac{N}{i}+1\right) \right\rceil + 1 = \lg_2(N+1) = \lg_2(N)$

for (int j=1; j<=i; j=j*3) {

}

}

$$* \left\lceil \lg_3\left(\frac{8}{1}+1\right) \right\rceil + 1 = 2 + 1$$

$$* \left\lceil \lg_3\left(\frac{16}{1}+1\right) \right\rceil + 1 = 3 + 1$$

$$\Rightarrow \lg_2(N)$$

i	1	2	4	8	16	...	N/4	N/2	N
j	1	1	1	1	1		1	1	1
	1	2	4	8	16		N/4	N/2	N
	$\left\lceil \lg_3\left(\frac{1}{1}+1\right) \right\rceil + 1$	$\left\lceil \lg_3\left(\frac{2}{1}+1\right) \right\rceil + 1$	$\left\lceil \lg_3\left(\frac{4}{1}+1\right) \right\rceil + 1$				$\left\lceil \lg_3\left(\frac{N/4}{1}+1\right) \right\rceil + 1$	$\left\lceil \lg_3\left(\frac{N/2}{1}+1\right) \right\rceil + 1$	$\left\lceil \lg_3\left(\frac{N}{1}+1\right) \right\rceil + 1$
	2	2	3	3	4		$\lg_3(N/4) + 1$	$\lg_3(N/2) + 1$	$\lg_3(N) + 1$

approximation

$$\Rightarrow \lg_3(N+1) + 1 * \lg_2(N)$$

$$\Rightarrow \lg_3(N) * \lg_2(N) = \lg_3 N \lg_2 N$$

$$= \lg_2(N) + \lg_3 N \lg_2 N = O(\lg_3 N \lg_2 N)$$

for (int i=1; i<=N; i=i+1) { $\left\lfloor \frac{N+1+i}{1} \right\rfloor + 1 = N+1$
 $= N$

for (int j=1; j<=i*i; j=j+1) {

}

$\Rightarrow N$ terms

i	1	2	3	4	5	...	N-2	N-1	N
j	1	1	1	1	1		1	1	1
	1	4	9	16	25		(N-2) ²	(N-1) ²	N ²

$$\left\lfloor \frac{1-1+1}{1} \right\rfloor + 1 \quad \left\lfloor \frac{4-1+1}{1} \right\rfloor + 1 \quad \left\lfloor \frac{9-1+1}{1} \right\rfloor + 1 \quad \left\lfloor \frac{16-1+1}{1} \right\rfloor + 1$$

$$1^2 + 1 \quad 2^2 + 1 \quad 3^2 + 1 \quad 4^2 + 1 \quad (N-2)^2 + 1 \quad (N-1)^2 + 1 \quad N^2 + 1$$

$$1^2 + 2^2 + 3^2 + 4^2 + \dots + (N-2)^2 + (N-1)^2 + (N)^2 + (1+1+1+\dots+1)$$

$$\frac{N(N+1)(2N+1)}{6}$$

+

N terms
 N

$$\frac{6N + N(N+1)(2N+1)}{6} \Rightarrow \frac{N(N+1)(2N+1)}{6} \Rightarrow N(N+1)(2N+1)$$

$$N + N(N+1)(2N+1) \Rightarrow O(N(N+1)(2N+1))$$

$$\Rightarrow O(N^3)$$

for (int i=N; i<=N²; i=i+2) { $\left\lceil \frac{N^2-N+1}{2} \right\rceil + 1$
 $\Rightarrow \left\lceil \frac{N^2-N}{2} \right\rceil + 1$
 $\Rightarrow N^2$ (simplified)

for (int j=i; j<=N²; j=j+2) {

}

}

i N N+2 N+4 N+6 N²-4 N²-2 N²

j N N+2 N+4 N+6 N²-4 N²-2 N²

N² N² N² N²

$\left\lceil \frac{N^2-N+1}{2} \right\rceil + 1$ $\left\lceil \frac{N^2-N-2+1}{2} \right\rceil + 1$ $\left\lceil \frac{N^2-N-4+1}{2} \right\rceil + 1$ $\left\lceil \frac{N^2-N^2+4+1}{2} \right\rceil + 1$ $\left\lceil \frac{N^2-N^2+2+1}{2} \right\rceil + 1$

$\left\lceil \frac{N^2-N+1}{2} \right\rceil + 1$ $\left\lceil \frac{N^2-N-1}{2} \right\rceil + 1$ $\left\lceil \frac{N^2-N-3}{2} \right\rceil + 1$ $\left\lceil \frac{5}{2} \right\rceil + 1$ $\left\lceil \frac{3}{2} \right\rceil + 1$ $\left\lceil \frac{1}{2} \right\rceil + 1$ $\left\lceil \frac{N^2-N^2+1}{2} \right\rceil + 1$

looks like arithmetical series.

2 ← from this end
it is adding
like Arith
series

pairs $\left\lceil \frac{N^2-N+1+1-2}{2} \right\rceil / 2 \Rightarrow \left\lceil \frac{N^2-N+1+2-4}{2} \right\rceil / 2 \Rightarrow \left\lceil \frac{N^2-N-1}{2} \right\rceil / 2 = N^2/4$

$\frac{N^2-N+1}{2} + 1 + 2 \Rightarrow \frac{N^2-N+1+6}{2} \Rightarrow \frac{N^2}{2}$

$\Rightarrow N^2/4 * N^2/2 \Rightarrow N^4/8$

$N^2 + N^4/8 \Rightarrow O(N^4)$