DE- Solution

Q1)

a) Marks:3

$$x^2y'' - 7xy' + 15y = 0$$

$$y = x^m$$
$$y' = mx^{m-1}$$
$$y'' = m(m-1)x^{m-2}$$

$$x^2 m(m-1)x^{m-2} - 7x\,m x^{m-1} + 15x^m = 0$$

$$x^m\left[m^2 - m - 7m + 15\right] = 0$$

$$x^m\left[m^2 - 8m + 15\right] = 0$$

$\because x^m \neq 0$

$$\Rightarrow \boxed{m_1 = 5}, \boxed{m_2 = 3}$$

$$m = \frac{-(-8) \pm \sqrt{64 - 4(1)(15)}}{2(1)}$$

$$m = \frac{8 \pm 2}{2}$$

$$m_1 = 5,\ m_2 = 3$$

b) $y' = y$ Marks:2

$$y = ce^x$$

$y(0) = 3$ → from figure

$$3 = ce^0$$

$$c = 3$$

$$\boxed{y = 3e^x}$$

$$y = ce^x$$

$y(1) = -2$ → from figure

$$-2 = ce^1$$

$$c = -\frac{2}{e} \approx 0.73575$$

$$\boxed{y = -\frac{2}{e}e^x = -2e^{x-1}}$$

c) Marks:2

$$(e^w \sin w)\,dy - (5y^2 \sin y)\,dw = 0$$

Order = 1

Degree = 1

It is non-linear in both of the variables: w & y.

d) $(2xy^2 + ye^x)dx + (2x^2y - ke^x - 1)dy = 0$

Marks = 3

$M = 2xy^2 + ye^x$; $N = 2x^2y - ke^x - 1$

$\frac{\partial M}{\partial y} = 4xy + e^x$; $\frac{\partial N}{\partial x} = 4xy - ke^x$

For the eq. to be exact,

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$$\Rightarrow \cancel{4xy} + e^x = \cancel{4xy} - ke^x$$

$$\cancel{e^x} = -k\cancel{e^x}$$

$$\boxed{k = -1}$$

Q2)

a) Separable ODE:

$-\frac{1}{2}\int x\,d\cdot$

$-\frac{1}{2}\ \frac{x^2}{2}$

Marks: 5

$$y \ln|x| \frac{dx}{dy} = \frac{(y+1)^2}{x}$$

$$\int x \ln|x|\, dx = \int \frac{(y+1)^2}{y} dy$$

$$\ln|x| \int x\,dx - \int\left(\int x\,dx\right)\left(\frac{1}{x}\right)dx = \int\left(\frac{y^2}{y} + \frac{1}{y} + \frac{2y}{y}\right) dy$$

$$\frac{x^2}{2}\ln|x| - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx = \int\left(y + \frac{1}{y} + 2\right) dy$$

$$\boxed{\frac{x^2}{2}\ln|x| - \frac{x^2}{4} = \frac{y^2}{2} + \ln|y| + 2y + C}$$

Q2)

b) Linear IVP:

Marks: 5

$$(1+x)y' - y = \frac{x+1}{x} \quad ; \quad y(2) = 2$$

$$y' - \frac{1}{1+x} y = \frac{1}{x}$$

$$P(x) = -\frac{1}{1+x}$$

$$I.F = e^{-\int \frac{1}{1+x} dx} = e^{-\ln|1+x|} = \frac{1}{1+x}$$

$$\frac{1}{1+x} y = \int \left(\frac{1}{1+x}\right) \cdot \frac{1}{x} dx$$

$$\frac{1}{1+x} y = \int \frac{1}{x(1+x)} dx$$

Taking $\frac{1}{x(1+x)} = \frac{A}{x} + \frac{B}{1+x}$

$$1 = A(1+x) + Bx$$

$$1 = A + Ax + Bx$$

$A = 1$, $A + B = 0$
$B = -A$
$B = -1$

$$\frac{1}{x(1+x)} = \frac{1}{x} - \frac{1}{1+x}$$

$$\Rightarrow \frac{1}{1+x} y = \int \frac{1}{x} - \frac{1}{1+x} dx$$

$$\boxed{\begin{gathered} \frac{y}{1+x} = \ln|x| - \ln|1+x| + C \\ y(2) = 2 \\ \Rightarrow C = 1.07213 \end{gathered}}$$

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Q3)

Marks:3

a) $\frac{dA}{dt} \propto A$

$\frac{dA}{dt} = kA \; ; \; A(0) = A_0$

$\Rightarrow \quad A = A_0 e^{kt}$

Marks:7

b) $A_0 = 150$

$A(13) = \frac{150}{2} = 75$

$\Rightarrow \quad A = 150 \, e^{kt}$

$75 = 150 \, e^{13k}$

$k \approx -0.053319$

$A = 150 e^{-0.053319 t}$

$t = 8 \, , \, A = ?$

$A = 150 e^{-0.053319(8)}$

$\boxed{A = 97.9 \text{ mg}}$

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