Student Name: ________________	**Registration No:______________**
Course Title: Differential Equations	**Course Code: CSSS2763**
Course Instructor: Ms. Fatima Asif	**Exam Type: Quiz # 2**

(1)

Question#1

You can find the temperature inside your refrigerator without putting a thermometer inside. Take a can of soda from the refrigerator, let it warm for half an hour, then record its temperature. Let it warm for another half an hour, and record its temperature again. Suppose that the readings are T(1/2) = 45°F and T(1) = 55°F. Assuming that the room temperature is 70°F, what is the temperature inside the refrigerator?

Solution: Taking the time one half hour after the soda was removed from the refrigerator to be the "zero time" (and stating the given information in an appropriate way), we have the boundary value problem

$$\frac{dT}{dt} = k(70 - T)$$
$$T(0) = 45$$
$$T(1/2) = 55$$

and we know that the solution of this boundary value problem is

$$T = 70 - 25\left(\frac{3}{5}\right)^{2t}.$$

To check this formula for reasonableness, we observe that the formula gives us

$$T(0) = 70 - 25\left(\frac{3}{5}\right)^{2(0)} = 45$$

and

$$T\left(\frac{1}{2}\right) = 70 - 25\left(\frac{3}{5}\right)^{2\left(\frac{1}{2}\right)} = 55.$$

The temperature of the refrigerator is the temperature of the can of soda at time $t = -1/2$, so we see that the temperature of the refrigerator is

$$\begin{aligned} T\left(-\frac{1}{2}\right) &= 70 - 25\left(\frac{3}{5}\right)^{2\left(-\frac{1}{2}\right)} \\ &= 70 - 25\left(\frac{5}{3}\right) \\ &\approx 28.3^\circ F. \end{aligned}$$

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(2)

<u>Question#1</u>

Carbon 14 is a common form of carbon which decays over time. The amount of Carbon 14 contained in a preserved plant is modeled by the equation

$$A(t) = 10e^{kt}$$

Time in this equation is measured in years from the moment when the plant dies (t=0) and the amount of Carbon 14 remaining in the preserved plant is measured in micrograms (a microgram is one millionth of a gram). So when $t = 0$ the plant contains 10 micrograms of Carbon-14.

1. The half-life of Carbon 14, that is the amount of time it takes for half of the Carbon 14 to decay, is approximately 5730 years. Use this information to find the constant c.
2. If there is currently one microgram of Carbon-14 remaining in the preserved plant, approximately when did the plant die?

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a. Since the half life of Carbon 14 is 5730 years, this means that after 5730 years there will only be 5 micrograms of Carbon 14 left in the preserved plant:

$$f(5730) = 10e^{-5730c} = 5.$$

To solve for c, notice that c is in the exponent and so we need to take a logarithm to isolate c. Since the base of the exponent is e, the natural logarithm is appropriate. Dividing by 10 first (to isolate the exponential expression) and taking the logarithm of both sides gives

$$-5730c = \ln e^{-5730c} = \ln \frac{1}{2} = -\ln 2.$$

Solving for c gives

$$c = \frac{\ln 2}{5730}$$

which is approximately equal to 0.000121.

b. To find when $f(t) = 1$ we use the value of c from part a and are left with the equation

$$10e^{-\frac{\ln 2}{5730}t} = 1.$$

Dividing by 10 and taking the natural logarithm on both sides, as above, gives

$$-\frac{\ln 2}{5730}t = \ln e^{\left(-\frac{\ln 2}{5730}t\right)} = \ln \frac{1}{10}$$

or

$$t = -5730\frac{\ln \frac{1}{10}}{\ln 2}.$$

Thus, evaluating on a calculator, gives a value of approximately 19,035 years since the plant has died. Note that if the approximate value 0.000121 is used in place of $\frac{\ln 2}{5730}$ then an approximate value of 19,030 years is found instead.

In either case, it is more appropriate to report the time since the plant has died as approximately 19,000 years since these measurements are never completely precise.