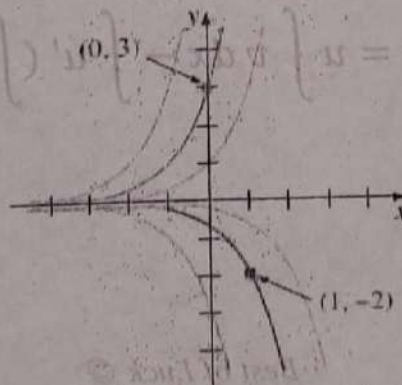## INSTRUCTIONS
1. Write your name and registration number on the Question Paper and Answer Sheet.
2. Write with blue/black permanent ink pen.
3. All your rough work and calculations should also be available on the answer sheet.
4. Make sure your calculator is in radian mode. Exchange of calculators is not allowed.
   - **No cheat sheet, notes, handbooks or any kind of sharing allowed.**

Q1. Marks: [3+2+2+3]

a. Find values of m so that the function $y = x^m$ is a solution of the given differential equation.

$$x^2 y'' - 7xy' + 15y = 0$$

b. $y = ce^x$ is a one-parameter family of solutions of a simple first order differential equation $y' = y$. Two solution curves passing through the marked points are shown in the figure below. Find the particular solution in each case.

$$y = ce^x \quad \frac{xb(xb\,\sigma)}{}) \quad {}^{1}(0,3)$$

$$y' = xce^{x-1}$$

$$xce^{x-1} = ce^x \checkmark$$

(1, -2)

c. State the order and degree of the following differential equation. Identify the variable in which it is linear.

$$[e^w \sin(w)]dy - [5y^2 \sin(y)]dw = 0$$

d. Find the value of k so that the following differential equation is exact.

$$(2xy^2 + ye^x)dx + (2x^2 y - ke^x - 1)dy = 0$$

Q2. Marks: [5+5]

a. Solve the following separable differential equation.

$$y \ln|x| \frac{dx}{dy} = \frac{(y + 1)^2}{x}$$

b. Solve the following linear initial value problem.

$$(1 + x)y' - y = \frac{x + 1}{x} \quad ; \quad y(2) = 2$$

Q3. Marks: [3+7]

The amount of medication in Rory's bloodstream decreases at a rate that is proportional at any time to the amount of the medication in the bloodstream at that time.
Rory takes 150 milligrams of medication initially. The amount of medication is halved every 13 hours.

a. Write the differential model and its solution for the given scenario.
b. How many milligrams of the medication are in Rory's bloodstream after 8 hours?

$$A_o = 150$$

$$t = 13 \text{ hours}$$

$$A = 150 \, e^{k(13)}$$

$$0.693 = 13 \, k$$

Note: Formula for Integration by Parts:

$$\int uv \, dx = u \int v \, dx - \int u' \left(\int v \, dx \right) dx$$