Course: Differential Equations

Course Code: CSSS2736

Credit Hours: 3

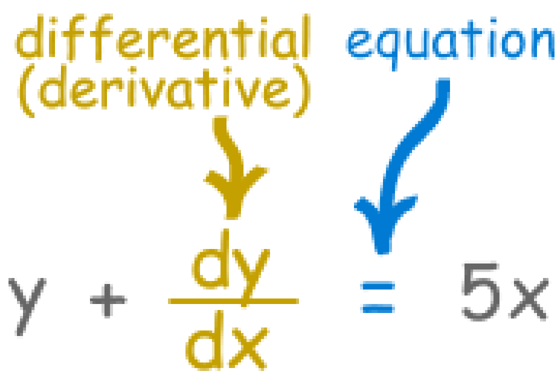
Instructor: Dr. Sumaira Sharif

Email: sumaira.sharif@ucp.edu.pk

Office: B-Building F313 (7th Cabin)

Office Hours: Displayed before office

What is a Differential Equation?



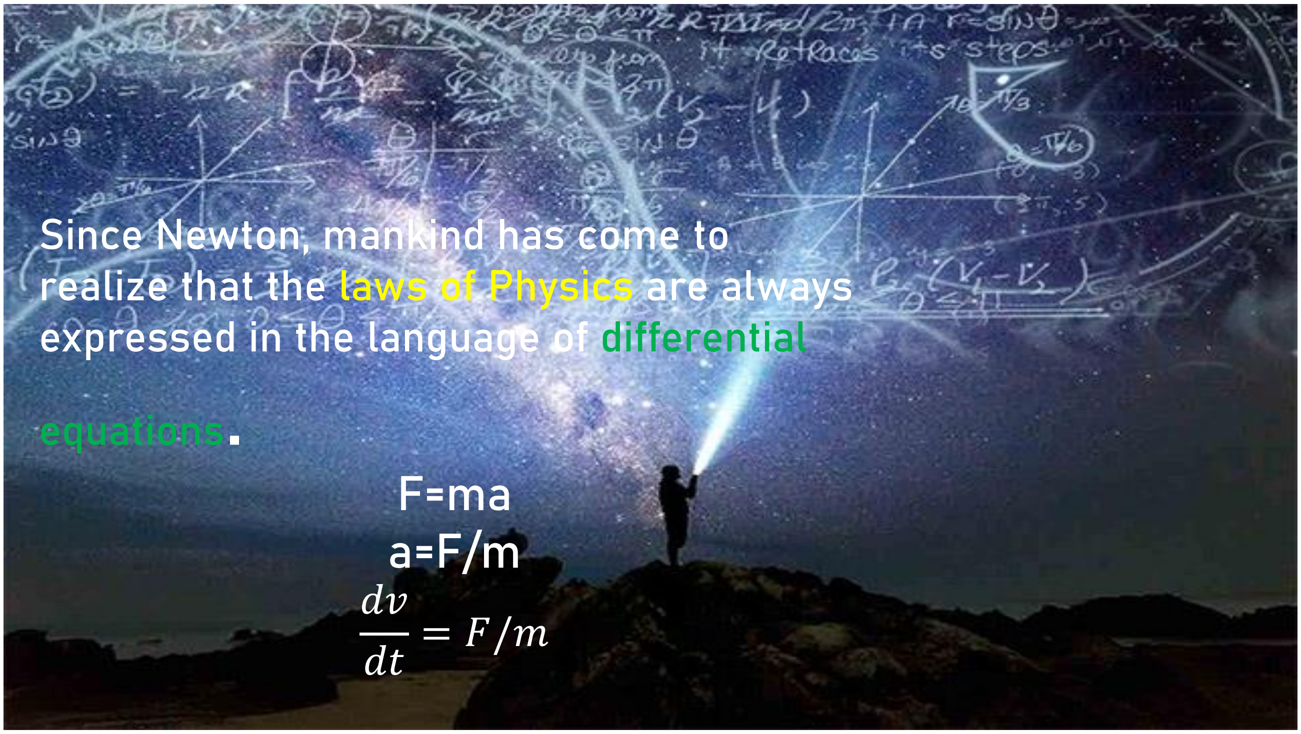
Definition

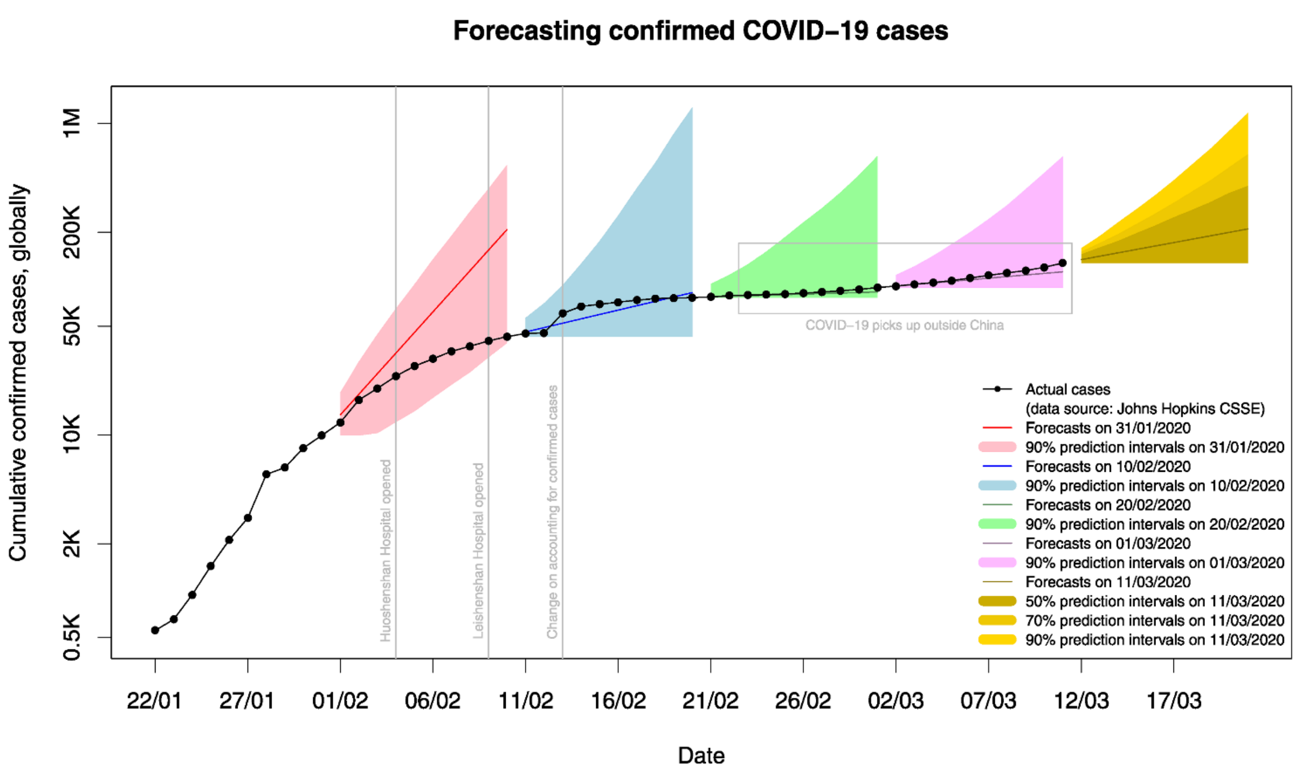
An equation containing the derivatives of one or more dependent variables, with respect to one or more independent variables, is said to be a differential equation (DE).

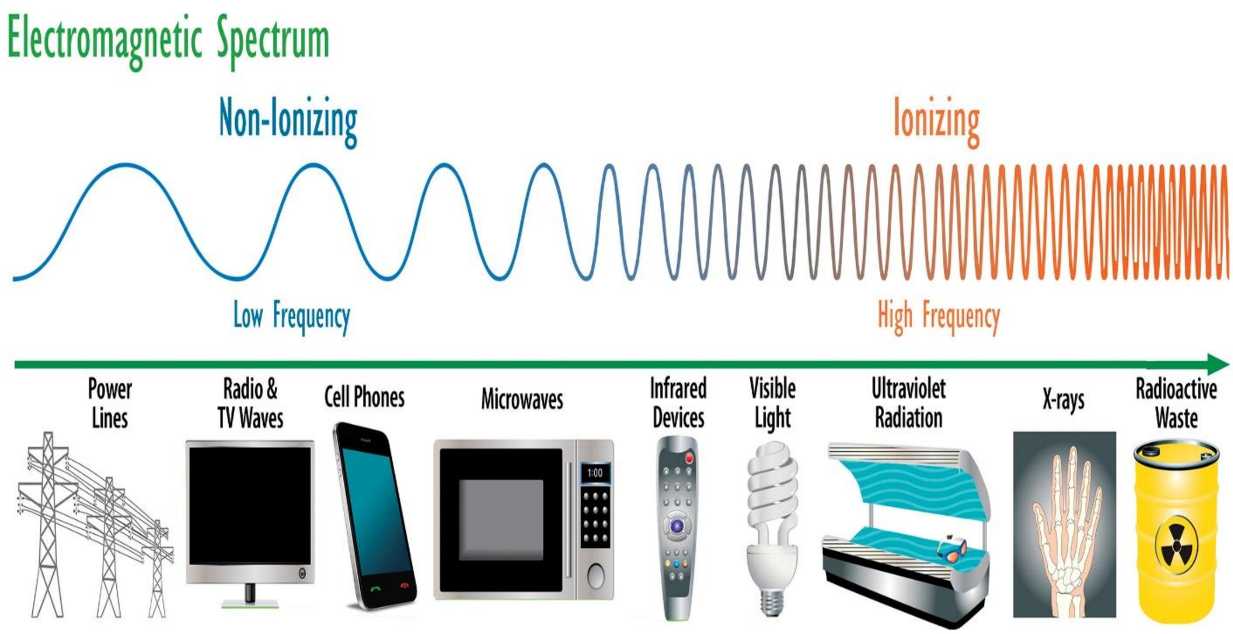
Contents of the course

- Differential Equations as Mathematical Models
- Classification and geometric representation of DEs
- First Order Differential Equations (Separable, Linear and Exact DEs)
- Modelling with First Order DEs
- Higher Order Homogeneous/non homogeneous DEs
- Series Solutions of Linear DEs
- Solution of DEs by Laplace Transform

Understanding and Real Life Applications of Differential Equations

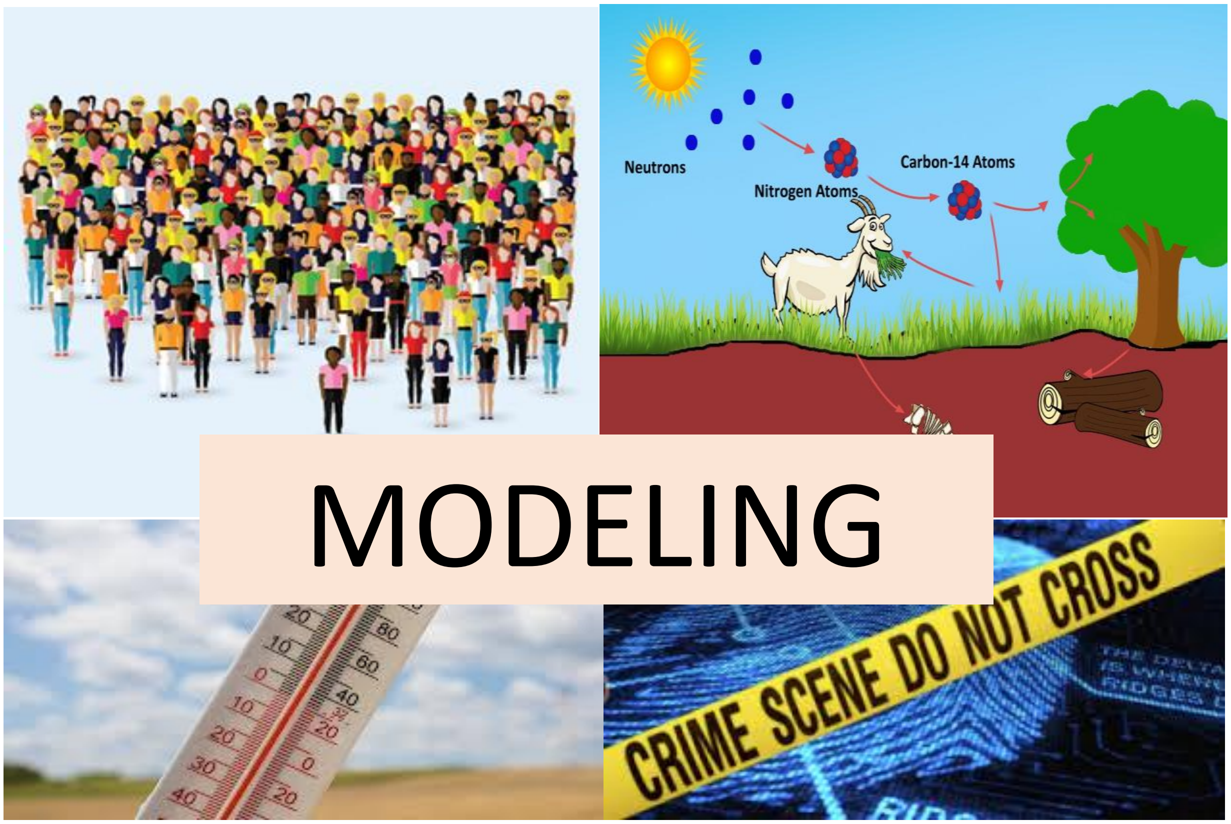






All these scenarios can be represented mathematically in the form of Differential equations. But we will start our mathematical modelling with very simple scenarios, like population model, radioactive decay model, Newton's law of cooling/warming model.

MODELING



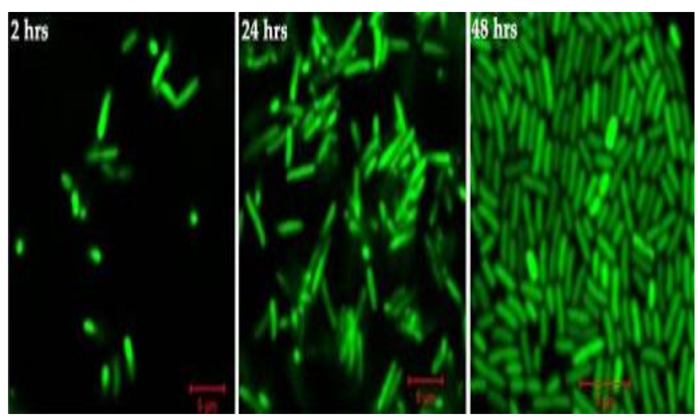
Population

We are studying a population of bacteria undergoing binary fission. In particular, the population doubles every three hours.

1. How many bacteria are present after 51 hours if a culture is inoculated with 1 bacterium?

2. With how many bacteria should a culture be inoculated if there are to be 81,920 bacteria present on hour 42?

3. How long would it take for an initial population of 6 to reach a size of 12,288 bacteria?



How to Model it?

Let P represents the population of bacteria at any time t. Since,

$$\frac{dP}{dt} \propto P$$

$$\frac{dP}{dt} = kP \quad ; \qquad P(0) = P_0$$

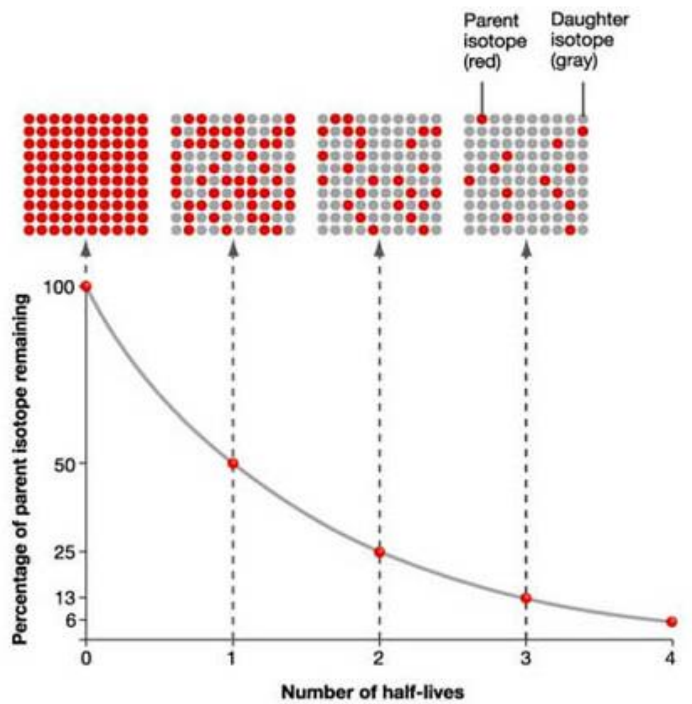
After solving this mathematical model, we can answer all three questions raised by the researchers/biologists about this bacterial population.

Also, we can have model of human population of a town or country to estimate population after few years. Since this model is a differential equation, so we have to learn the method to solve this particular types of differential equation. That's why we will learn types of DEs first and some methods to solve them accordingly.

Radioactive Decay

What is radioactive decay?

For example radium, Ra-226, over time the highly radioactive radium, Ra-226, transmutes into the radioactive gas radon, Rn-222. To model the phenomenon of **radioactive decay,** it is assumed that the rate dA/dt at which the nuclei of a substance decay is proportional to the amount (more precisely, the number of nuclei) $A(t)$ of the substance remaining at time t.



$$\frac{dA}{dt} \propto A$$

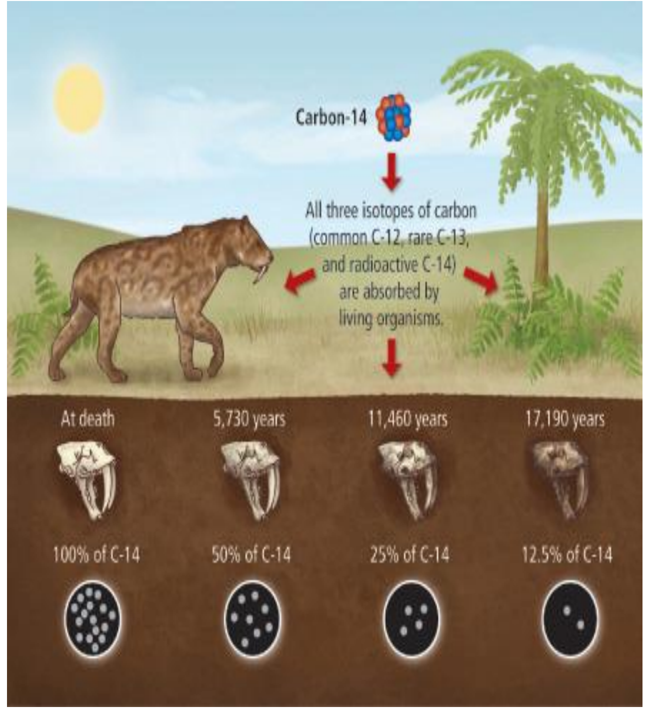
$$\frac{dA}{dt} = kA, \qquad A(0) = A_0$$

We can see that decay model and population models are represented by almost same initial value problem. For growth, $k < 0$, and for decay, $k > 0$.

* The model for decay also occurs in biological applications such as determining the **half-life** of a drug in a body—the time that it takes for 50% of a drug to be eliminated from a body by excretion or metabolism.

Let's have a particular case of radioactive decay, the carbon dating. It is so important that the person who used it first was awarded the Nobel prize for chemistry in 1960.

Carbon Dating



You are presented with a document which purports to contain the recollections of a Mycenaean soldier during the Trojan War. The city of Troy was finally destroyed in about 1250 BC, or about 3250 years ago.

Given the amount of carbon-14 contained in a measured sample cut from the document, there would have been about 1.3×10^{-12} grams of carbon-14 in the sample when the parchment was new, assuming the proposed age is correct. According to your equipment, there remains 1.0×10^{-12} grams.

Is there a possibility that this is a genuine document? Or is this instead a recent forgery? Justify your conclusions.

Model

Let A be the amount of C-14 remaining at any time t. Since rate of decay is proportional to the amount of substance present at time t, so

$$\frac{dA}{dt} \propto A$$

$$\boldsymbol{\frac{dA}{dt} = kA, \quad A(0) = A_0}$$

Here in this model we have an additional calculated information that Carbon-14 has a half-life of 5730 years. That already known information will reduce our calculations, making problem solution very direct. To find that document is fake or original, we will first learn to solve such form of models/DEs.