Design & Analysis of Algorithm
Final Examination (Summer 2022)

Reg. No: _____ Time: 150 mins

Q1: (10 Marks)

Write a proper function that will complete a task in an **array A of size N** (where values are in non-decreasing order). Function has to find an index in the array where value at the index and index are the same. If **found** return that index, otherwise return -1.

Array index start from 0

The time complexity of the algorithm shall be **O(lgn)**.

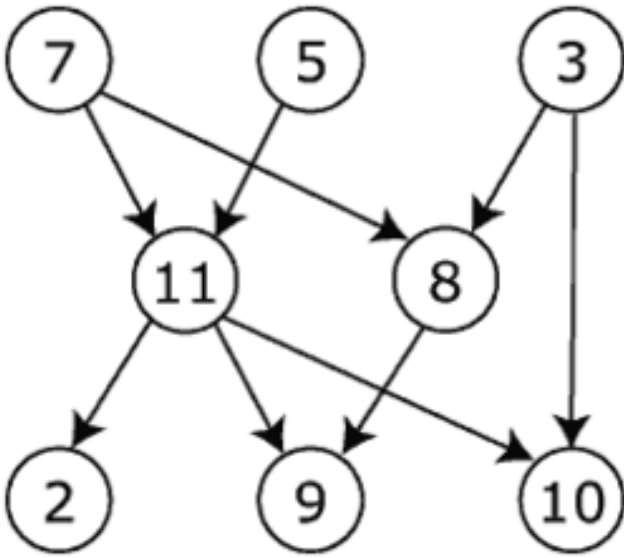
Example:

Input: 12, 16, 21, 27, 33, 33, 54, 58, 70, 88 **Output**: -1	**Input:** -12, -2, 0, 2, 2, 3, 4, 7, 30, 38 **Output**: 7

Q2: (5+5+5 Marks)

```
DFS(G)
1  for each vertex u ∈ G.V
2      u.color = WHITE
3      u.π = NIL
4  time = 0
5  for each vertex u ∈ G.V
6      if u.color == WHITE
7          DFS-VISIT(G, u)

DFS-VISIT(G, u)
1  time = time + 1
2  u.d = time
3  u.color = GRAY
4  for each v ∈ G.Adj[u]
5      if v.color == WHITE
6          v.π = u
7          DFS-VISIT(G, v)
8  u.color = BLACK
9  time = time + 1
10 u.f = time
```

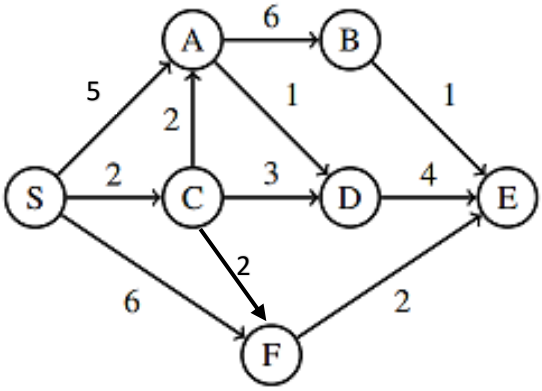


Apply DFS algorithm on the given graph and answer the following questions. The source node is '**7**'. (**Note**: The adjacent nodes of a vertex are to be traversed in increasing order.)

a) Write the type of each edge of the above graph.
b) Write updated details for all the vertices when DFS-VISIT (G, u) called for the 5th time (Just Called) write the values of their (**'Parent'**, '**color'**, '**d-time', 'f-time')**.
c) Write down details for all the vertices when **Line 10** of DFS-VISIT (G, u) completed its work for 3rd time - write the values of their (**'Parent'**, '**color'**, '**d-time', 'f-time')**.

Q3: (9+7 Marks)

```
DIJKSTRA (G, w, s)
1.  INITIALIZE-SINGLE-SOURCE(G, s)
2.  X = Ø
3.  H = G.V
4.  While H ≠ Ø
5.      u = EXTRACT-MIN(H)
6.      X = X U {u}
7.      for each vertex v ∈ G. Adj[u]
8.          if v. d > u. d + w(u, v)
9.              v. d = u. d + w(u, v)
10.             v. π = u
```



Apply Dijkstra's Shortest Path Algorithm on the given graph and answer the following questions. The source node is '**A**'. (**Note**: **The adjacent nodes of a vertex are to be traversed in alphabetical order**.) – After Initialization is completed, keep track of every update in the graph

a) Write down the first 8 updates done on the above graph, list every update from first to the 8th [**vertex_name (Parent, distance)**], comma separated.
b) What are the '**distance'** and '**predecessor'** values of nodes that are not in the heap, **H**, when **Line-10** is executed for the 8th time. [List all nodes with [**vertex_name (Parent, distance)**]

Q4: (5+5 Marks)

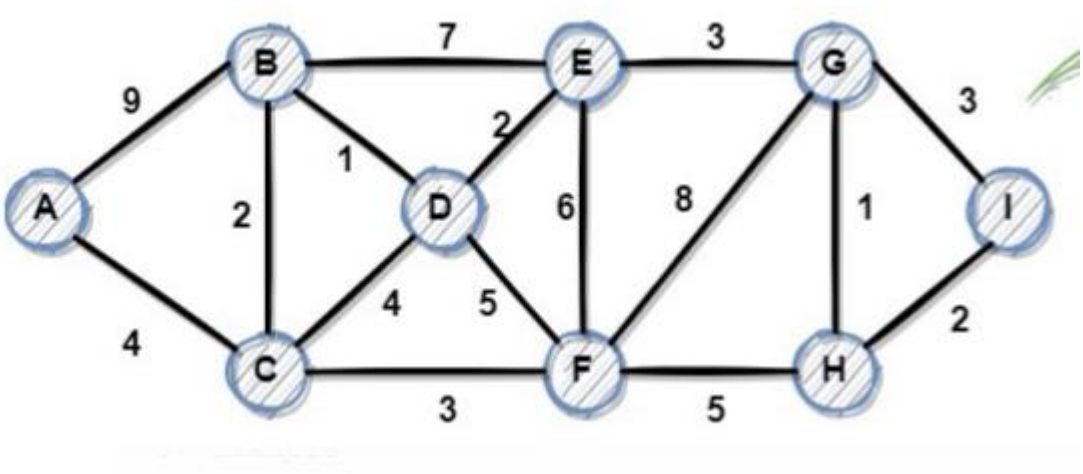
```
INSERTION-SORT(A)
1  for j = 2 to A.length
2      key = A[j]
3      // Insert A[j] into the sorted sequence A[1 .. j − 1].
4      i = j − 1
5      while i > 0 and A[i] > key
6          A[i + 1] = A[i]
7          i = i − 1
8      A[i + 1] = key
```

Solve the following for A = {11, 15, 24, 2, 16, 1, 3}. (**Note**: **Indexing** starts from **1**.)

a) Write the value of '**key'** and contents of '**A**' when **Line-8** is reached for the 3rd time (but not executed).
b) Write the values of '**key', 'A[i]'** and contents of '**A**' when **Line-5** is executed for the 10th time. **[Carefully understand the execution]**

Q5: (2+4+4+5 Marks)

```
MST-KRUSKAL(G, w)
1  A = ∅
2  for each vertex v ∈ G.V
3      MAKE-SET(v)
4  sort the edges of G.E into nondecreasing order by weight w
5  for each edge (u, v) ∈ G.E, taken in nondecreasing order by weight
6      if FIND-SET(u) ≠ FIND-SET(v)
7          A = A ∪ {(u, v)}
8          UNION(u, v)
9  return A
```



Apply KRUSKAL's MST Algorithm on the given graph and answer the following questions.
Note: (write every edge in alphabetical order, AB, BD, DE, etc)
When more than one edge has the same weight, selection should be done in Alphabetical order.

a) What is the weight of the MST?
b) Draw the partial MST that is formed immediately before, the rejection of the second edge
c) Write all the rejected edges in order of rejection (first to Last).
d) Draw the Partial MST when Line-8 is executed for the 6th time