Solution – DE

Q4)

$T(0) = 37$

$T_m = 20$

a) $\frac{dT}{dt} \propto (T - T_m)$

$\frac{dT}{dt} = k(T - T_m)$ 3

Working for the solution is not required.

b) $T = T_m + ce^{kt}$ 2

c) $T = 25$, $t = ?$

$T(0) = 37$

$T(1) = 33$

$T = 20 + ce^{kt}$

$\because T(0) = 37$

$37 = 20 + ce^{0}$

$c = 17$

$\Rightarrow T = 20 + 17e^{kt}$

$\because T(1) = 33$

$33 = 20 + 17e^{k(1)}$

$k \approx -0.26826$

$T = 20 + 17e^{-0.26826t}$

$25 = 20 + 17e^{-0.26826t}$

$\Rightarrow t \approx 4.5$ hours.

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Q3) $y'' - 4y' = 6e^{3t} - 3e^{-t}$; $y(0) = 1$; $y'(0) = -1$

$$s^2 \mathcal{L}\{y\} - s y(0) - y'(0) - 4s\mathcal{L}\{y\} + 4y(0) = \frac{6}{s-3} - \frac{3}{s+1}$$

$$\left[\mathcal{L}\{y\}(s^2 - 4s)\right] - s + 1 + 4 = \frac{6}{s-3} - \frac{3}{s+1}$$

$$\mathcal{L}\{y\}(s^2-4s) = \frac{6}{s-3} - \frac{3}{s+1} + s - 5$$

$$\mathcal{L}\{y\}(s^2-4s) = \frac{6(s+1) - 3(s-3) + (s-5)(s-3)(s+1)}{(s-3)(s+1)}$$

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$$= \frac{6s + 6 - 3s + 9 + (s-5)(s^2 - 2s - 3)}{(s-3)(s+1)}$$

$$= \frac{6s + 6 - 3s + 9 + (s^3 - 2s^2 - 3s - 5s^2 + 10s + 15)}{(s-3)(s+1)}$$

$$\mathcal{L}\{y\} = \frac{s^3 - 7s^2 + 10s + 30}{s(s-4)(s-3)(s+1)}$$

2

$$\mathcal{L}\{y\} = \frac{5/2}{s} + \frac{-3/5}{s+1} + \frac{11/10}{s-4} + \frac{-2}{s-3}$$

2

$$y = \frac{5}{2}\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - \frac{3}{5}\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + \frac{11}{10}\mathcal{L}^{-1}\left\{\frac{1}{s-4}\right\} - 2\mathcal{L}^{-1}\left\{\frac{1}{s-3}\right\}$$

$$y = \frac{5}{2} - \frac{3}{5}e^{-t} + \frac{11}{10}e^{4t} - 2e^{3t}$$

Qs 2) $y'' - 9y = \frac{9x}{e^{3x}}$

(3)

$m^2 - 9 = 0$

$m^2 = 9$

$m = \pm 3$

$y_c = c_1 e^{3x} + c_2 e^{-3x}$ $\qquad y_1 = e^{3x} \quad y_2 = e^{-3x}$

$y_p = u_1 y_1 + u_2 y_2$

(4)

$W = \begin{vmatrix} e^{3x} & e^{-3x} \\ 3e^{3x} & -3e^{-3x} \end{vmatrix}$ $\qquad W_1 = \begin{vmatrix} 0 & e^{-3x} \\ \frac{9x}{e^{3x}} & -3e^{-3x} \end{vmatrix}$ $\qquad W_2 = \begin{vmatrix} e^{3x} & 0 \\ 3e^{3x} & \frac{9x}{e^{3x}} \end{vmatrix}$

$W = -3 - 3 = -6$ $\qquad W_1 = -9xe^{-6x}$ $\qquad W_2 = 9x$

(4)

$u_1 = \int \frac{-9xe^{-6x}}{-6} dx$ $\qquad u_2 = \int \frac{9x}{-6} dx$

$u_1 = +\frac{3}{2} \int xe^{-6x} dx$ $\qquad u_2 = \frac{-3}{2} \int x dx$

$u_1 = \frac{3}{2} \left[x \int e^{-6x} dx - \int \left(\int e^{-6x} dx \right)(1) dx \right]$ $\qquad u_2 = -\frac{3}{2} \left(\frac{x^2}{2} \right)$

$u_1 = \frac{3}{2} \left[x \left(\frac{e^{-6x}}{-6} \right) - \int \frac{e^{-6x}}{-6} dx \right]$ $\qquad u_2 = -\frac{3x^2}{4}$

$u_1 = -\frac{1}{4} xe^{-6x} + \frac{1}{4} \left(\frac{e^{-6x}}{-6} \right)$

$u_1 = -\frac{1}{4} xe^{-6x} - \frac{1}{24} e^{-6x}$

(1)

$y_p = \left(-\frac{1}{4} xe^{-6x} - \frac{1}{24} e^{-6x} \right) e^{3x} + \left(-\frac{3x^2}{4} \right) e^{-3x}$

$y = c_1 e^{3x} + c_2 e^{-3x} - \frac{1}{4} xe^{-3x} - \frac{1}{24} e^{-3x} - \frac{3}{4} x^2 e^{-3x}$ → General solution f

$$\frac{1}{\sqrt{5}} \mathcal{L}^{-1}\left\{ \frac{1 \cdot \sqrt{5}}{s^2+5} \right\} + \mathcal{L}^{-1}\left\{ \frac{s}{s^2+5} \right\} + 7\mathcal{L}^{-1}\left\{ \frac{1}{s-5} \right\}$$

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$$= \frac{1}{\sqrt{5}} \sin\sqrt{5}t + \cos\sqrt{5}t + 7e^{5t}$$

Good Luck

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(d)

$$(y^2\cos x - 3x^2y - 2x)\,dx + (+2y\sin x - x^3 + \ln y)\,dy = 0$$

4

$M_y = 2y\cos x - 3x^2$ $N_x = 2y\cos x - 3x^2$

Eq is exact,

(e)

$y'' + y = x\sin x$

$m^2 + 1 = 0$

$m^2 = -1$

$m = \pm i$

4

$y_c = c_1\cos x + c_2\sin x$

~~$y_p = A\cos x + B\sin x$~~

$\because g(x) = x\sin x$

$\Rightarrow y_p = (Ax + B)\cos x + (Cx + D)\sin x$

Repetition with y_c

$\Rightarrow y_p = (Ax^2 + Bx)\cos x + (Cx^2 + Dx)\sin x$

(f)

No, because if $W = 0 \Rightarrow$ solutions are linearly dependent.

(Also, the formulae of Variation of Parameters give invalid answers because W is in denominator of formulae.)

Q1) (a)

(1) Variation of Parameters

(2) Method using Auxiliary Equation

(3) UC Method, Laplace Transform, Variation of Parameters

(4) UC Method, Variation of Parameters.

4

(b) $y = c_1 e^{-4x} + c_2 e^{-3x}$

$m_1 = -4 \qquad m_2 = -3$

$(m+4)(m+3) = 0$

$m^2 + 3m + 4m + 12 = 0$

$m^2 + 7m + 12 = 0$

$y'' + 7y' + 12y = 0$

4

(c) $y'' + 9y = 0 \quad ; \quad y_1 = 3e^x \quad ; \quad x \in (0, \infty)$

$$y_2 = 3e^x \int \frac{e^{-\int 0\, dx}}{(3e^x)^2} dx$$

$$y_2 = 3e^x \int \frac{1}{9e^{2x}} dx$$

$$y_2 = \frac{3e^x}{9} \int e^{-2x} dx$$

$$y_2 = \frac{1}{3} e^x \cdot \frac{e^{-2x}}{-2} = \frac{e^{-x}}{-6} = -\frac{1}{6e^x}$$

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