Extra Credit Assignment

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1 SVD and Fourier Transform

Singular Value decomposition is one of the most widely used techniques in data science. Images can be considered as matrices which can then be decomposed into its singular values.

$$X = U\Sigma V^T = \sum u_i \sigma v_i^T$$

where U is the eigenvectors corresponding to XX^T and V are the eigenvectors corresponding to X^TX. Σ is the diagonal matrix where each diagonal element corresponds to the eigenvalue associated with the corresponding eigenvector.
The image chosen for the given example is an image of ETH Zurich 1 but any

Figure 1: Image used for the experiment

other picture may be provided and the code shall perform in almost similar way.

However, the eigenvalues are distributed as shown in 2.

The higher eigenvalues contains most important information about the data

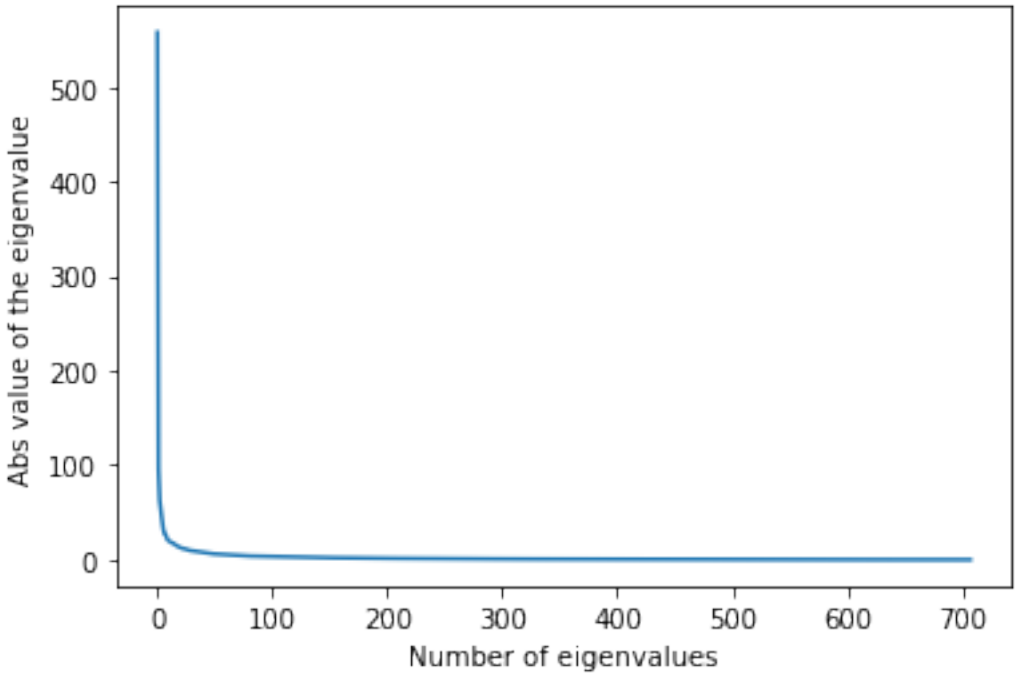

Figure 2: The absolute value of the eigenvalues.

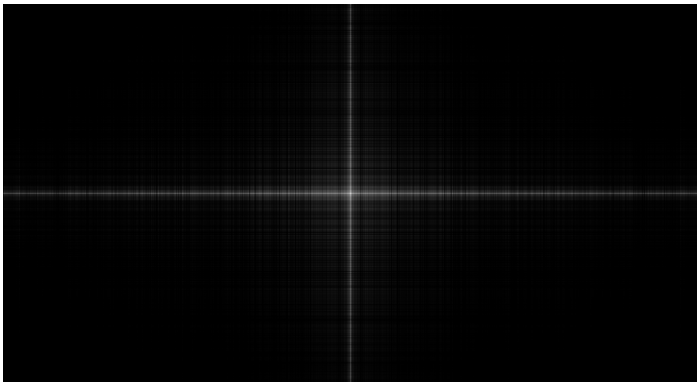

Figure 3: FT at the start

and thus we can neglect the lower eigenvalues and their corresponding components. Thus the SVD is now truncated and we can observe how the quality of

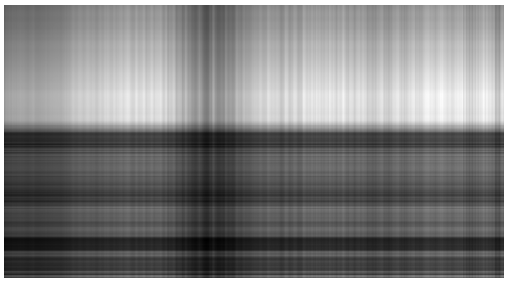

Figure 4: The reconstructed image from the first eigenvalue

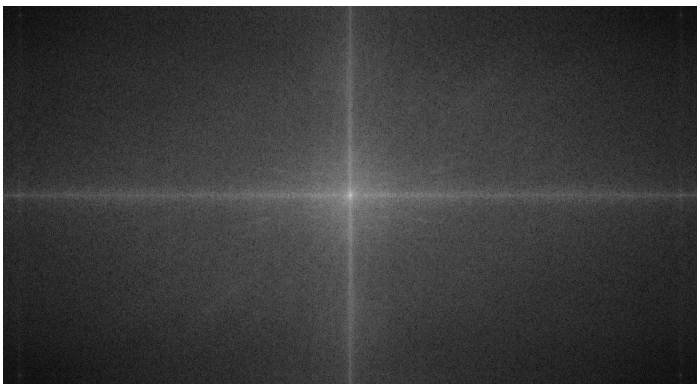

Figure 5: FT at the start

the image changes as we change the top n components. The code for doing the experiments have been provided for reproduciblity [1].

A gif was produced that shows how the reconstructed image changes with varying the leading eigenvalues. A link to the gif Image-Change.gif has been provided since the gif was too large. The gif showed that after the first few eigenvalues, the image did not change that much. However, the areas where the changes were being produced are places where there are many edges. So, the relation between the Fourier transform (FT) and the images were also taken.

At the start, when the leading eigenvalue was taken, the FT produced is 3, which is the FT corresponding to 4. At the end, the FT is the transform of the actual image and it is shown in 5. The change of the FT with respect to the change in number of leading eigenvalues have also been presented in a link FT-Change.gif.

[1] https://github.com/thearkamitra/SVD-FFT-gif