CS 228 Minor Autumn 2022 Quiz 1

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1. Is the formula $p \to (q \to \neg p)$ satisfiable? If yes, give a satisfying assignment. If not, say why.

2. Is the formula $(p \vee \neg(p \wedge q))$ valid? Why?

3. Using the proof rules seen in class, can you give a proof for the sequent $p \vee q, r \vdash (p \wedge r) \vee (q \wedge r)$

4. What is the size of the formula $\neg(p \to \neg(q \vee (\neg r \wedge \neg(s \to p))))$? Draw the parse tree and explain.

5. Suppose we add to the proof rules of propositional logic, two new proof rules
   (a) $\frac{\varphi \vee \psi}{\varphi}$ called $\vee\mathtt{elim}_1$
   (b) $\frac{\varphi \vee \psi}{\psi}$ called $\vee\mathtt{elim}_2$
   resulting in a new proof engine. Then the new proof engine is

   (a) Sound and complete
   (b) Sound but not complete
   (c) Complete but not sound
   (d) Neither sound nor complete

   Explain you answer.

6. Consider the formula $\varphi = (p \to (q \to p))$. Which of the following apply? Check all those which apply, and explain your answer.
   (a) $p, \neg q \vdash \varphi$
   (b) $\neg p, q \vdash \varphi$
   (c) $\neg p, \neg q \nvdash \varphi$
   (d) $p, q \vdash \varphi$
   (e) $\models \varphi$

7. Suppose we remove LEM from the proof rules of propositional logic. Then the resultant proof system is
   (a) Unsound and not complete
   (b) Sound but not complete since we removed a proof rule
   (c) Sound and complete, since we do not need LEM as a proof rule anyway!
   (d) Unsound but complete

   Explain your answer.

8. The rule $\perp$ elimination indicates that
   (a) We do not need $\perp$ in any proof, hence can be eliminated

(b) If we obtain $\perp \to \psi$ as part of a proof, we can eliminate $\perp$ and conclude ψ
(c) If we obtain $\perp$ in a proof, then we can conclude any formula ψ after that, since $\perp \to \psi$ holds good always
(d) If we obtain $\perp$ in a proof, then we can conclude any formula ψ after that, since anything can be concluded in an inconsistent system

Check all those which apply, and explain your answer.

9. Which among these is a Horn formula? Check all those which apply.
(a) $(\neg p \lor (q \to s))$
(b) $\neg(p \to q)$
(c) $\neg p \land \neg r \land (r \lor s)$
(d) $p \lor q \land s$

10. Consider the formula $\varphi = \neg p \lor (r \to (s \land p))$. What is the smallest n for which $Res^n(\varphi) = Res^{n+1}(\varphi)$?
(a) 2
(b) 3
(c) 1
(d) None of the above

Explain your answer.

11. Compute $Res^*(\varphi)$ for φ in the above question. Check all those which apply.
(a) $\emptyset \in Res^*(\varphi)$, but not in $Res^2(\varphi)$, hence φ is satisfiable
(b) $\emptyset \notin Res^*(\varphi)$, hence φ is satisfiable.
(c) To check for validity of φ it is enough to check if $\emptyset \notin Res^*(\neg\varphi)$
(d) To check for validity of φ it is enough to check if $\emptyset \in Res^*(\neg\varphi)$

12. Consider the formula $\varphi = (x \lor y) \land (\neg x \lor \neg y)$. In DNF, what is the size of φ? Explain why.

13. Given φ in DNF, which of the following is equivalent to the validity of φ?
(a) At least one clause of φ is satisfiable
(b) At least one clause of φ is valid
(c) All clauses of φ are satisfiable
(d) All clauses of φ are valid
(e) None of the above

Explain why.