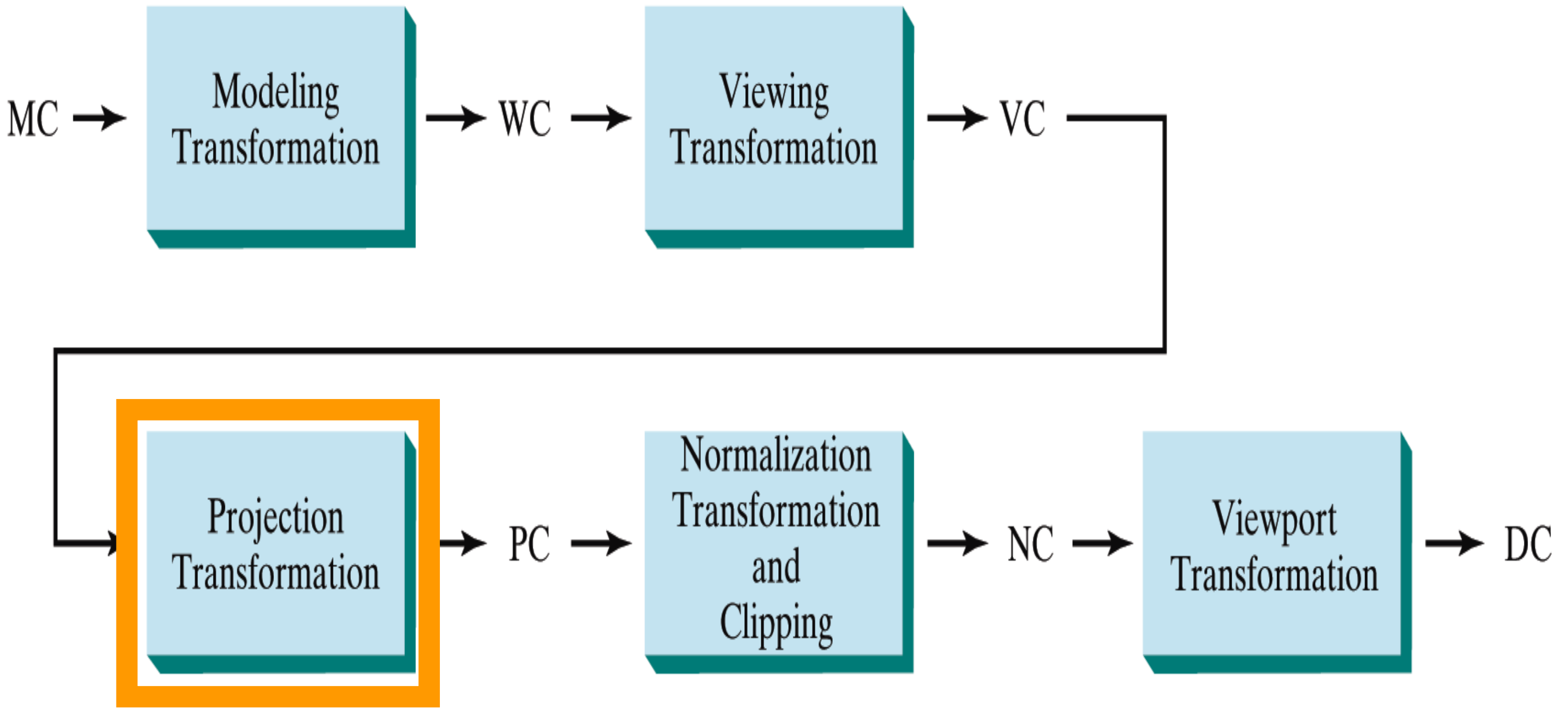
# Projection

# Viewing Pipeline

- Convert the viewing coordinate description of the scene to coordinate positions on the projection plane.

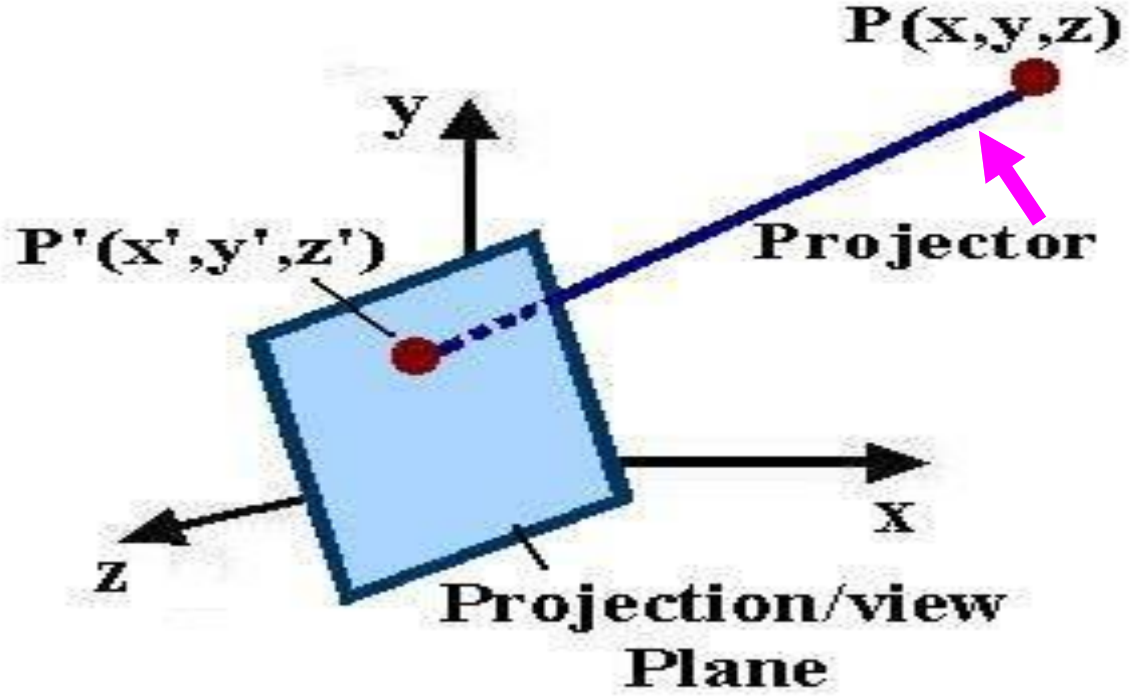
- Viewing 3D objects on a 2D display requires a mapping from 3D to 2D.



# Projection

- **Projection** can be defined as a mapping of point P(x,y,z) onto its image in the projection plane.

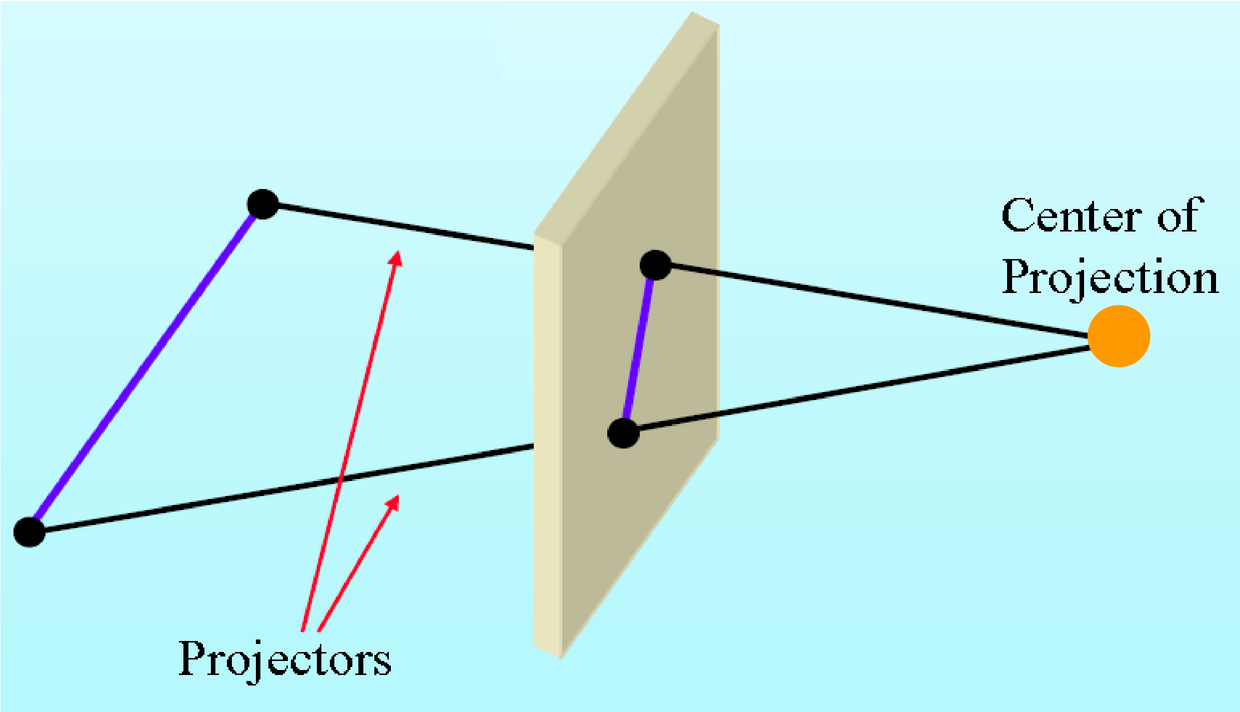
- The mapping is determined by a ***projector*** that passes through P and intersects the view plane ( ).

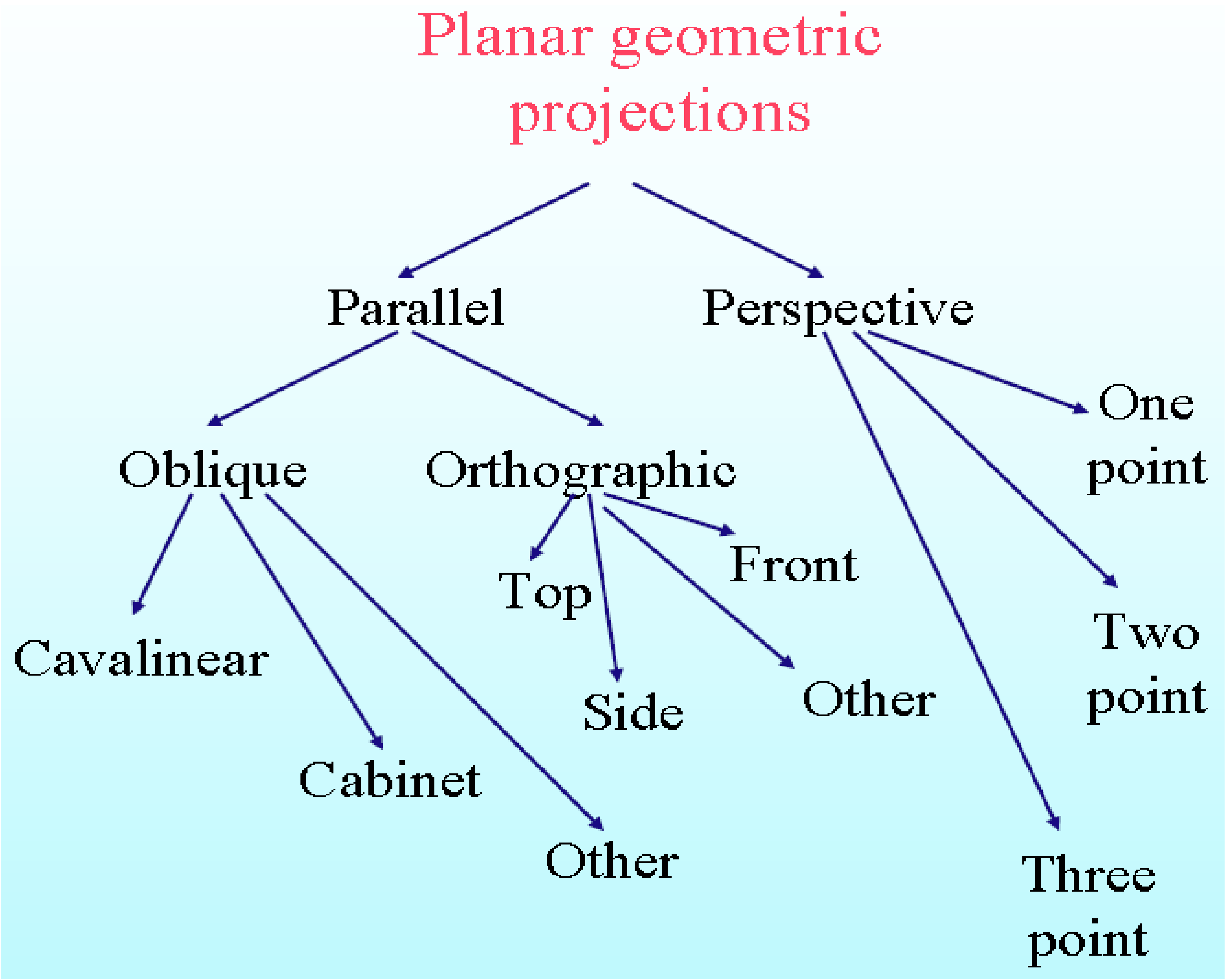


# Projection

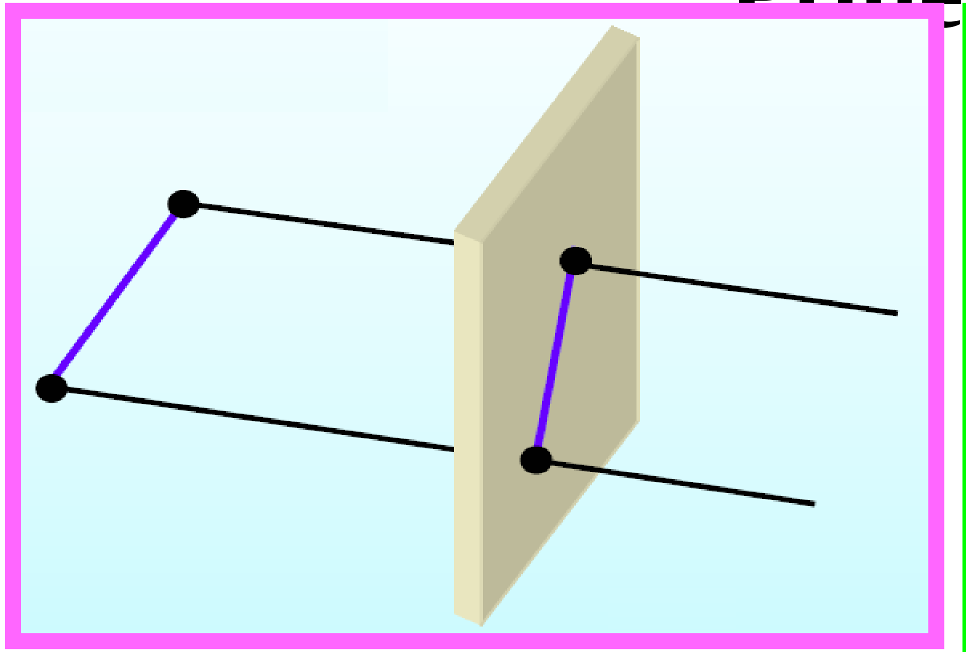
- Projectors are lines from **center (reference) of projection** through each point in the object.

- The result of projecting an object is dependent on the spatial relationship among the projectors and the view plane.

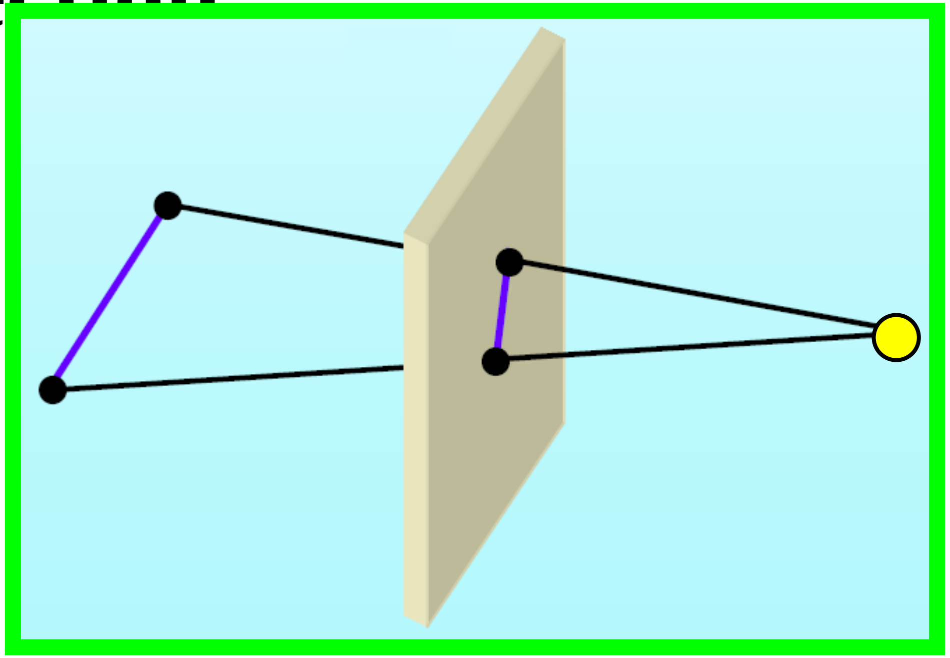



Planar geometric projections
Parallel
Perspective
Oblique
Orthographic
One point
Two point
Three point
Cavalinear
Cabinet
Other
Top
Side
Front
Other

# Projection



***Parallel Projection*** :
Coordinate position are transformed to the view plane along **parallel lines**.



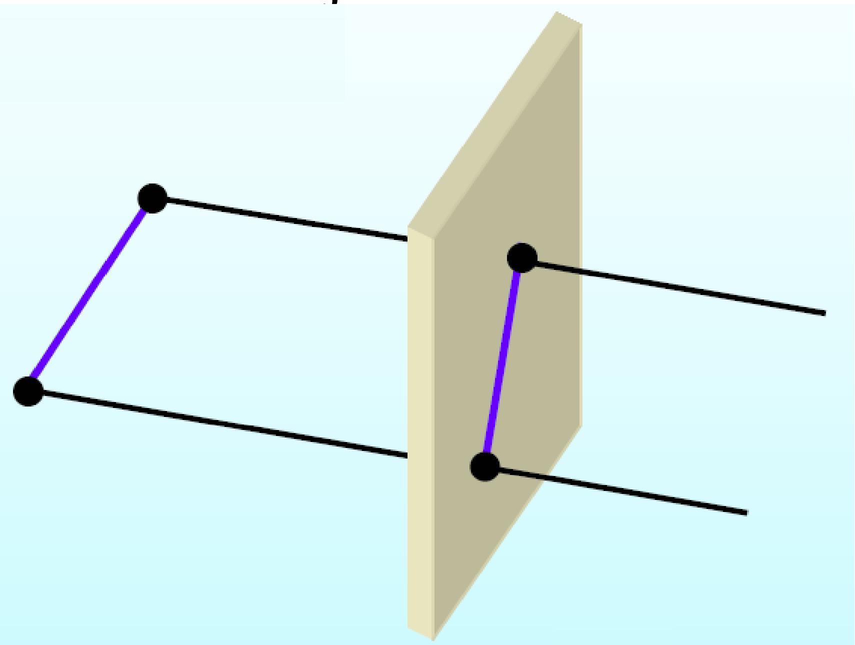
***Perspective Projection:***
Object positions are transformed to the view plane along lines that converge to the **projection reference (center) point**.

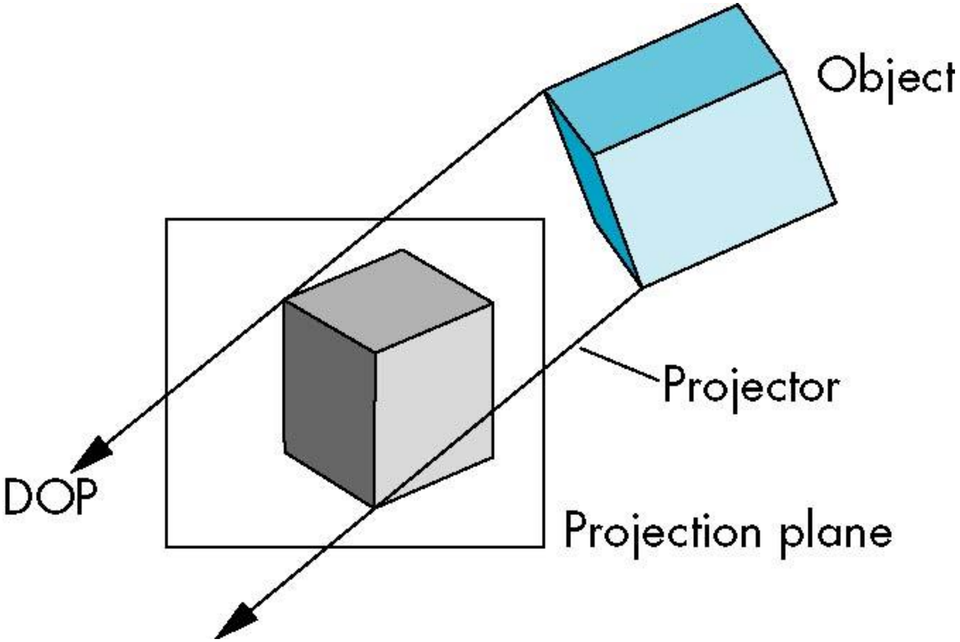
# Parallel Projection

- Coordinate position are transformed to the view plane along parallel lines.

- Center of projection at infinity results with a parallel projection.

- A parallel projection preserves relative proportion of objects, but dose not give us a realistic representation of the appearance of object.

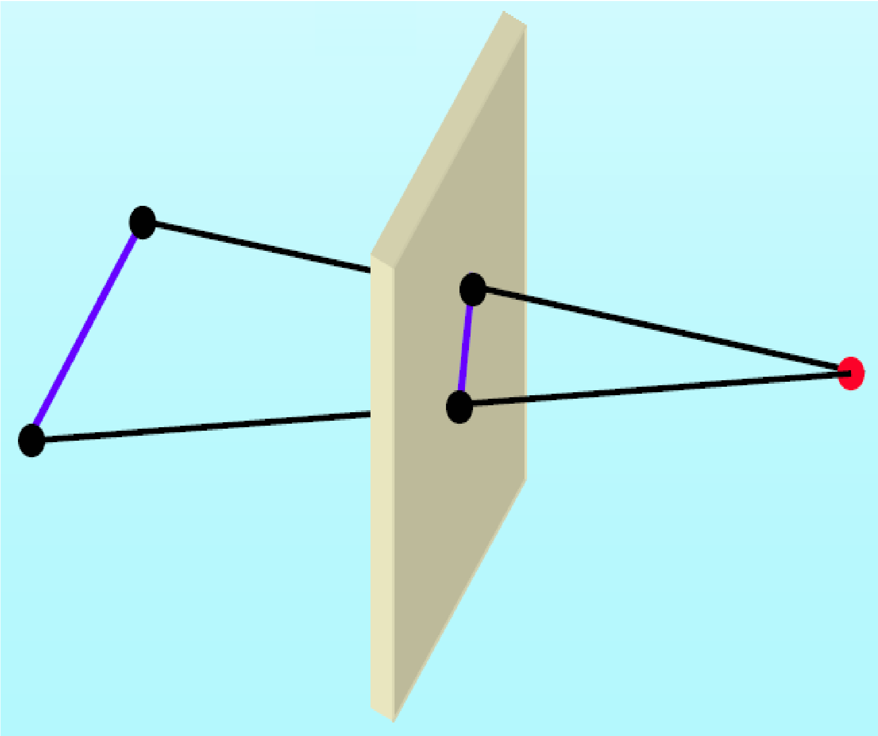


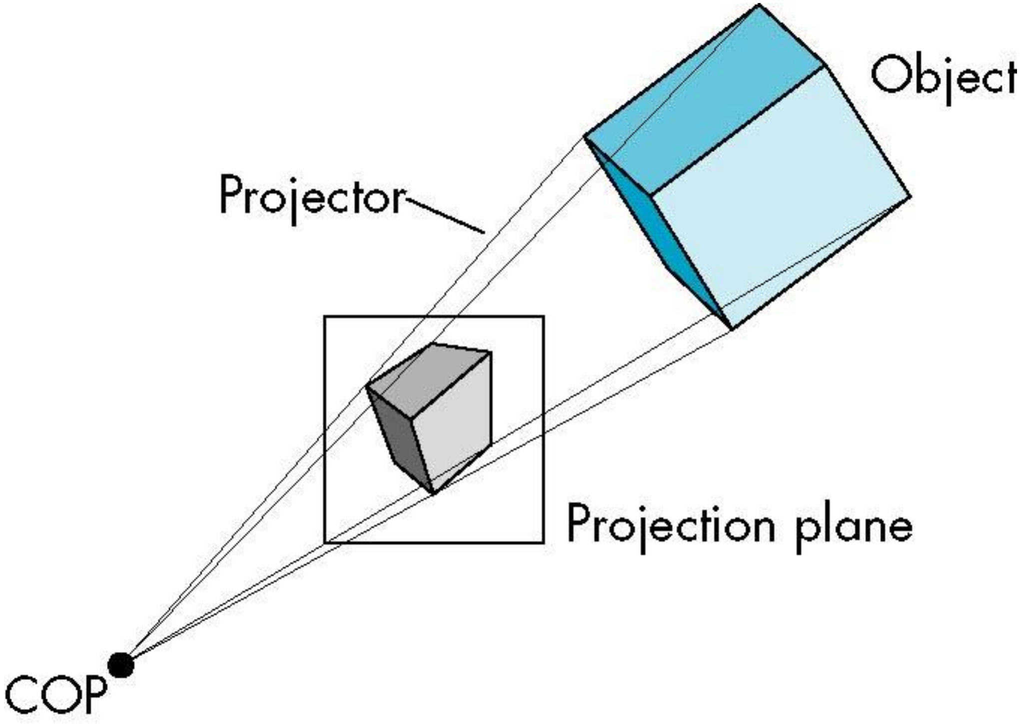


# Perspective Projection

- Object positions are transformed to the view plane along lines that converge to the **projection reference (center) point.**

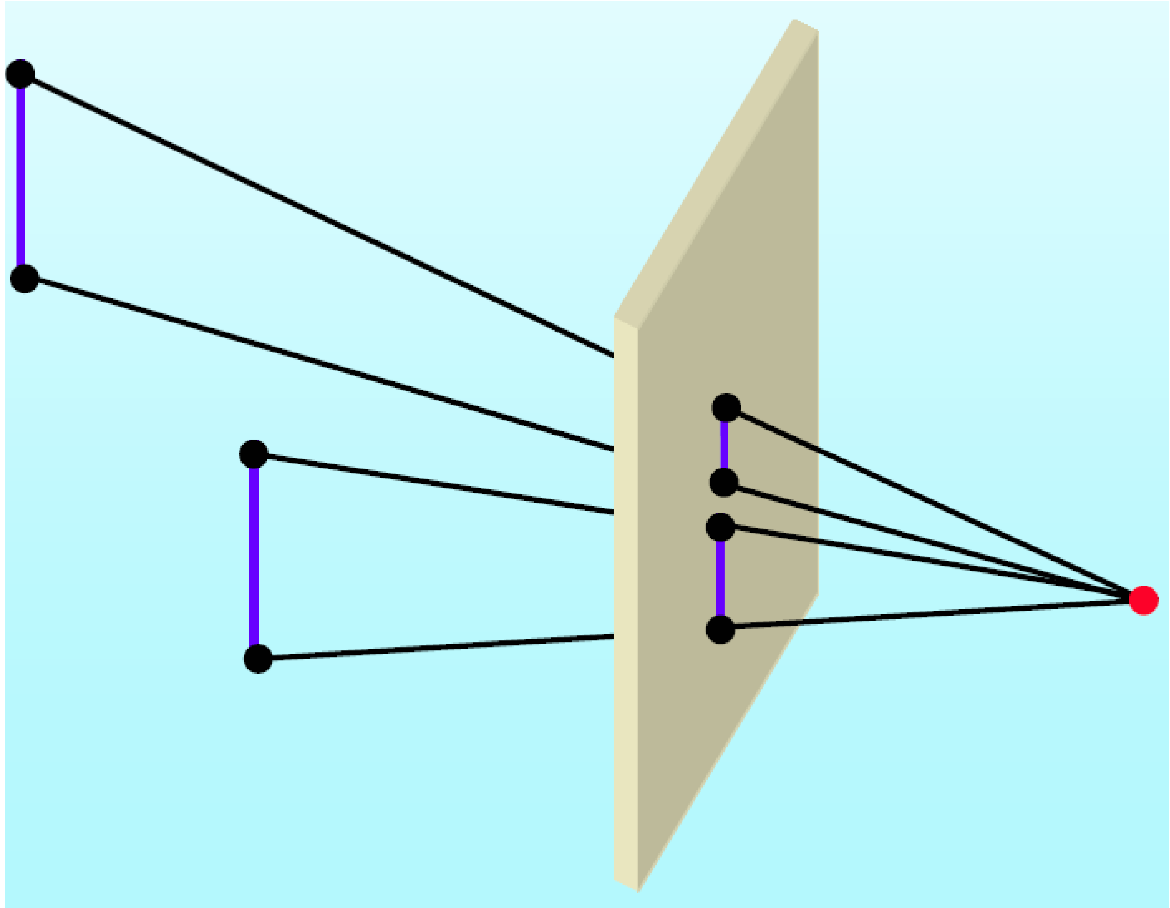
- Produces realistic views but does not preserve relative proportion of objects.



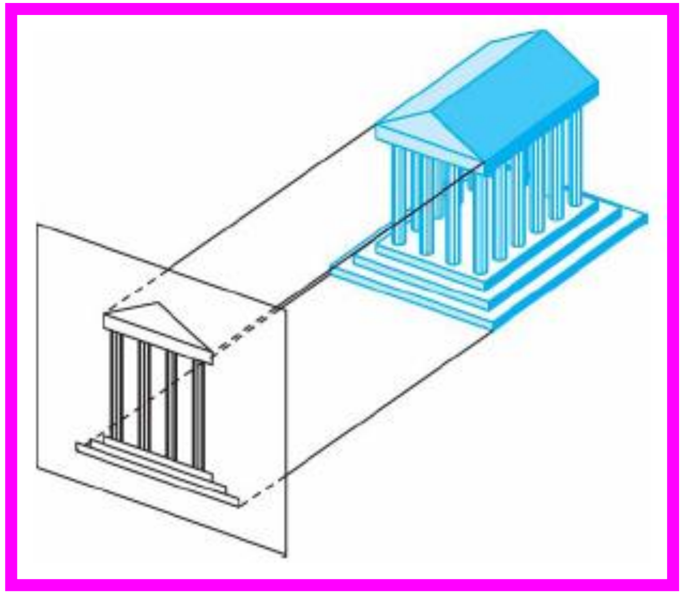


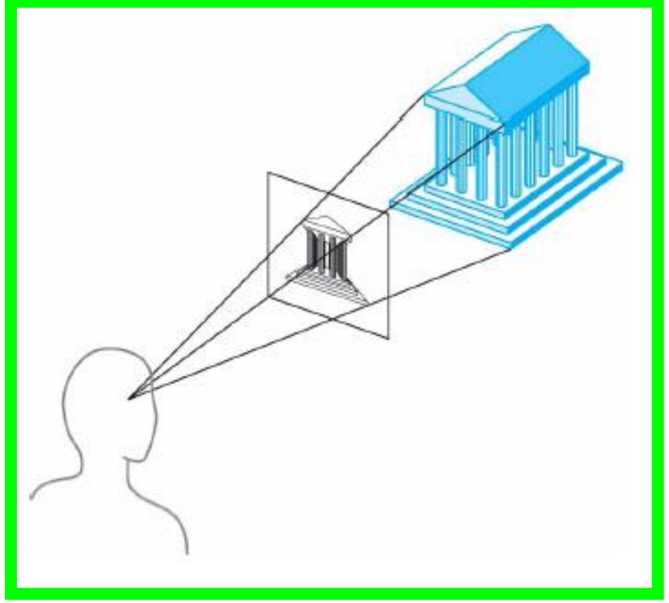
# Perspective Projection

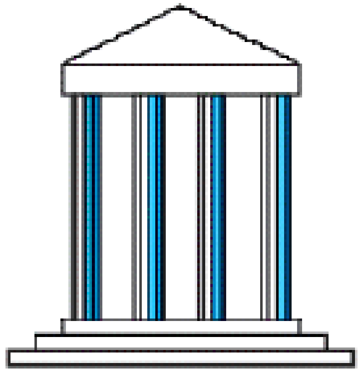
- Projections of distant objects are smaller than the projections of objects of the same size are closer to the projection plane.



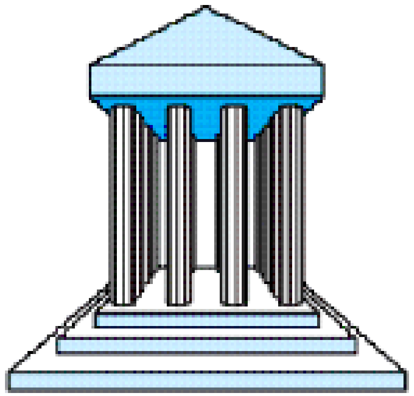
# Parallel and Perspective Projection







parallel
(orthographic)

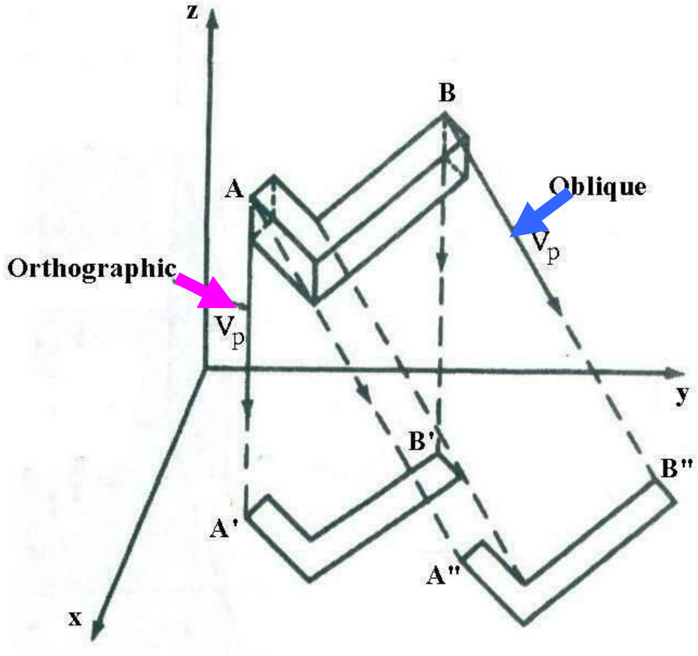


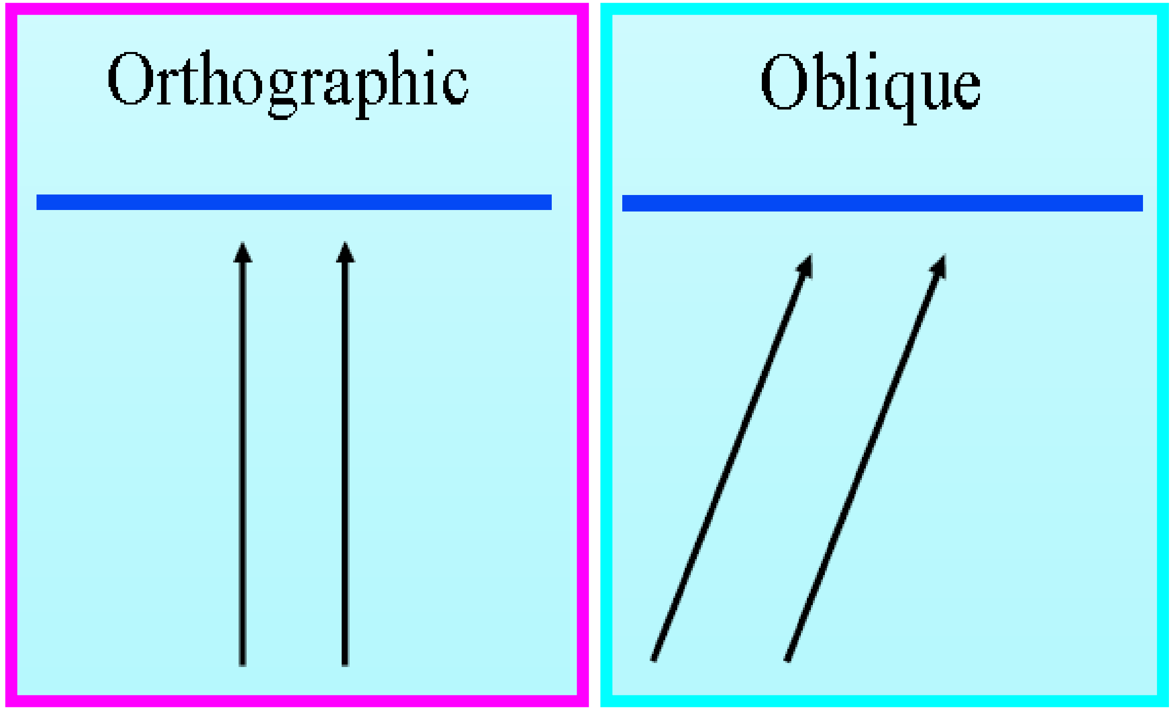
perspective

# Parallel Projection

# Parallel Projection

- **Projection vector**: Defines the direction for the projection lines (projectors).
- ***Orthographic Projection***: Projectors (projection vectors) are **perpendicular** to the projection plane.
- ***Oblique Projection***: Projectors (projection vectors) are ***not*** perpendicular to the projection plane.

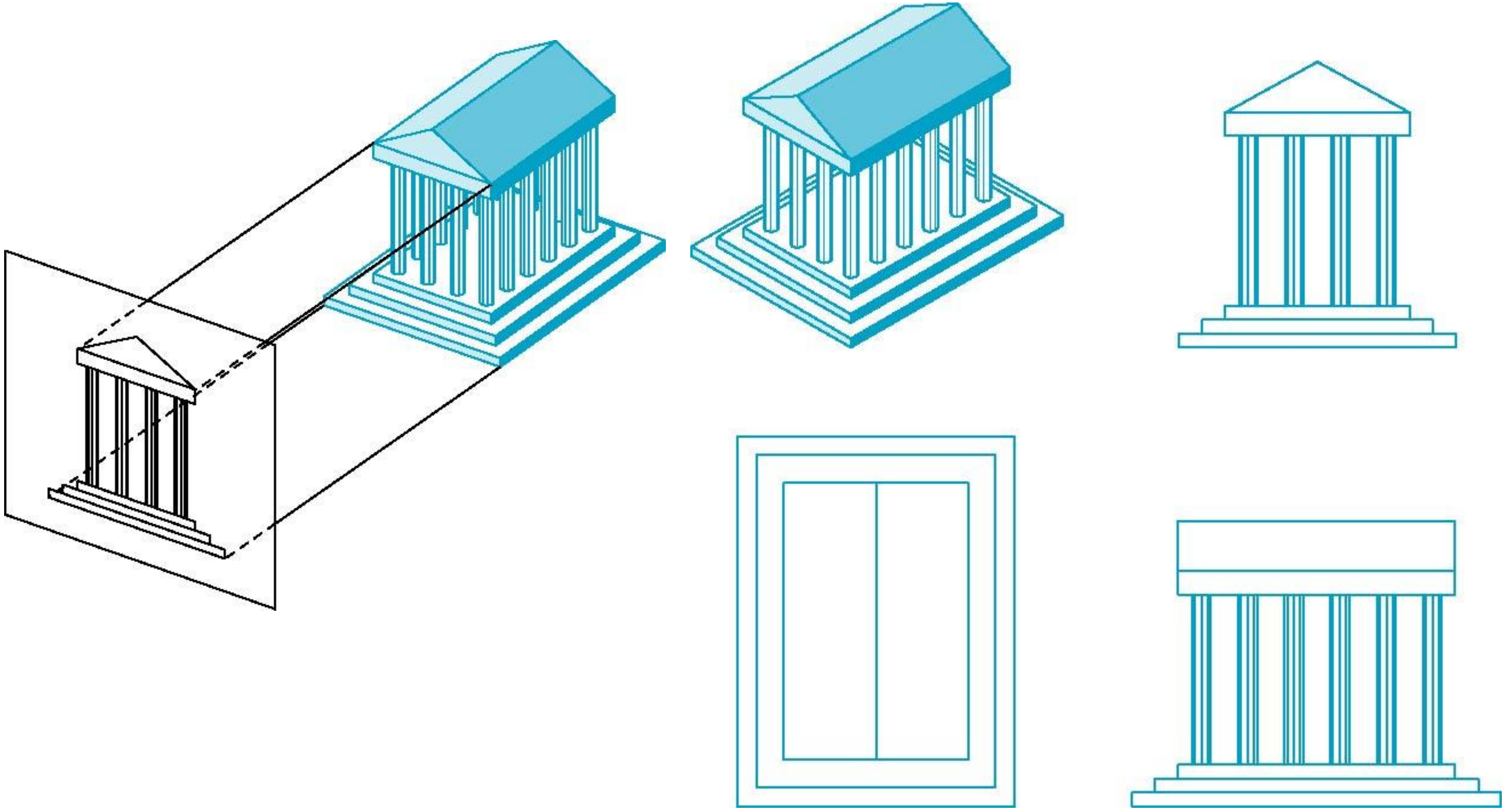


# Orthographic
# Parallel Projection

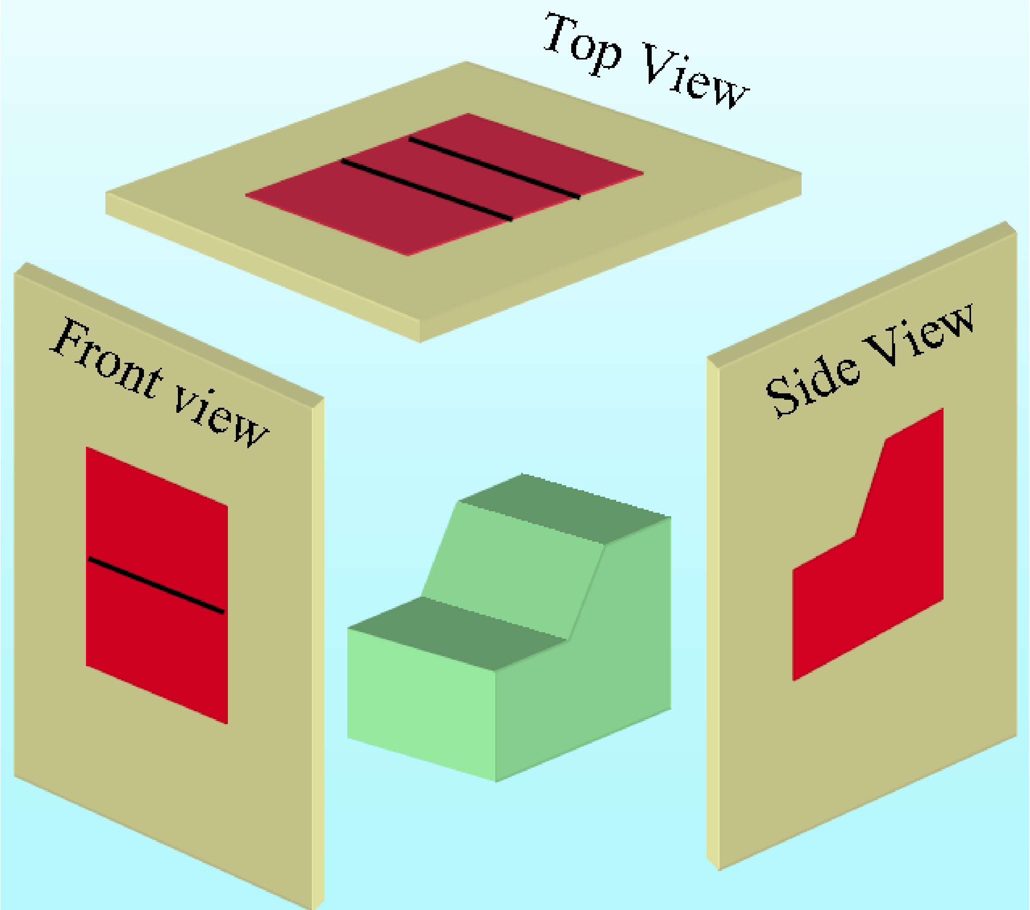
# Orthographic Parallel Projection

- Orthographic projection used to produce the **front**, **side**, and **top** views of an object.

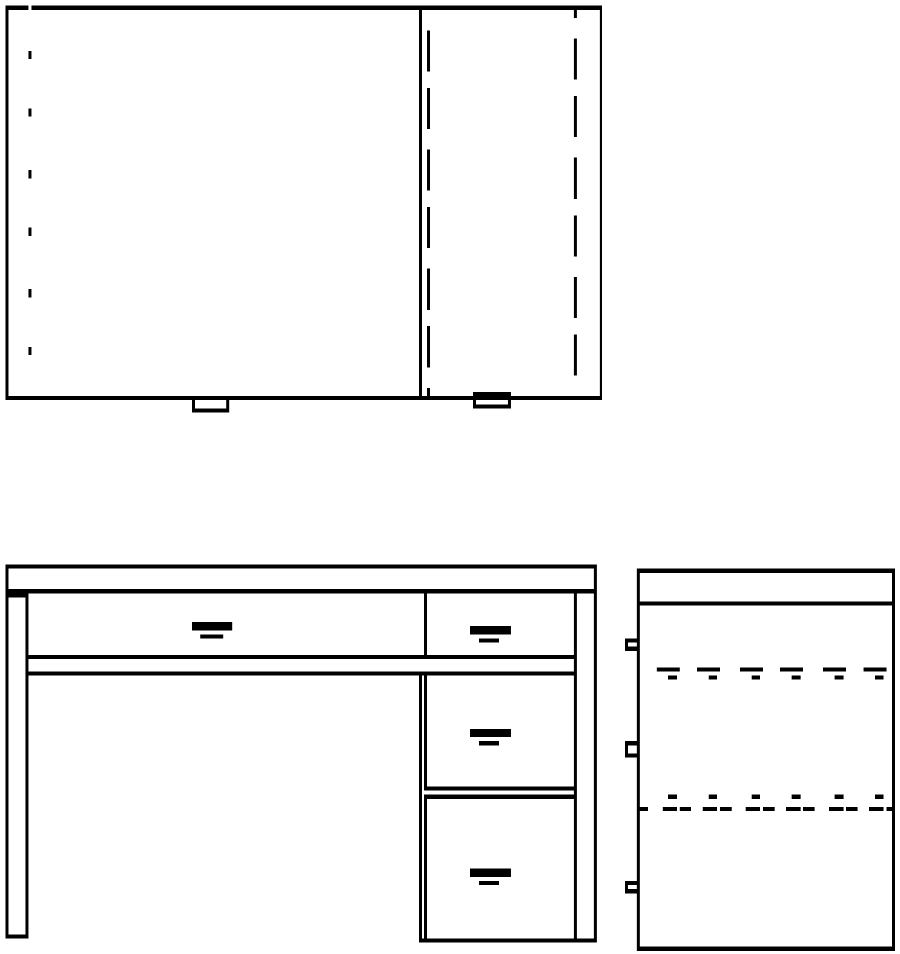


# Orthographic Parallel Projection

- ***Front***, ***side***, and ***rear*** orthographic projections of an object are called ***elevations***.
- ***Top*** orthographic projection is called a ***plan*** view.



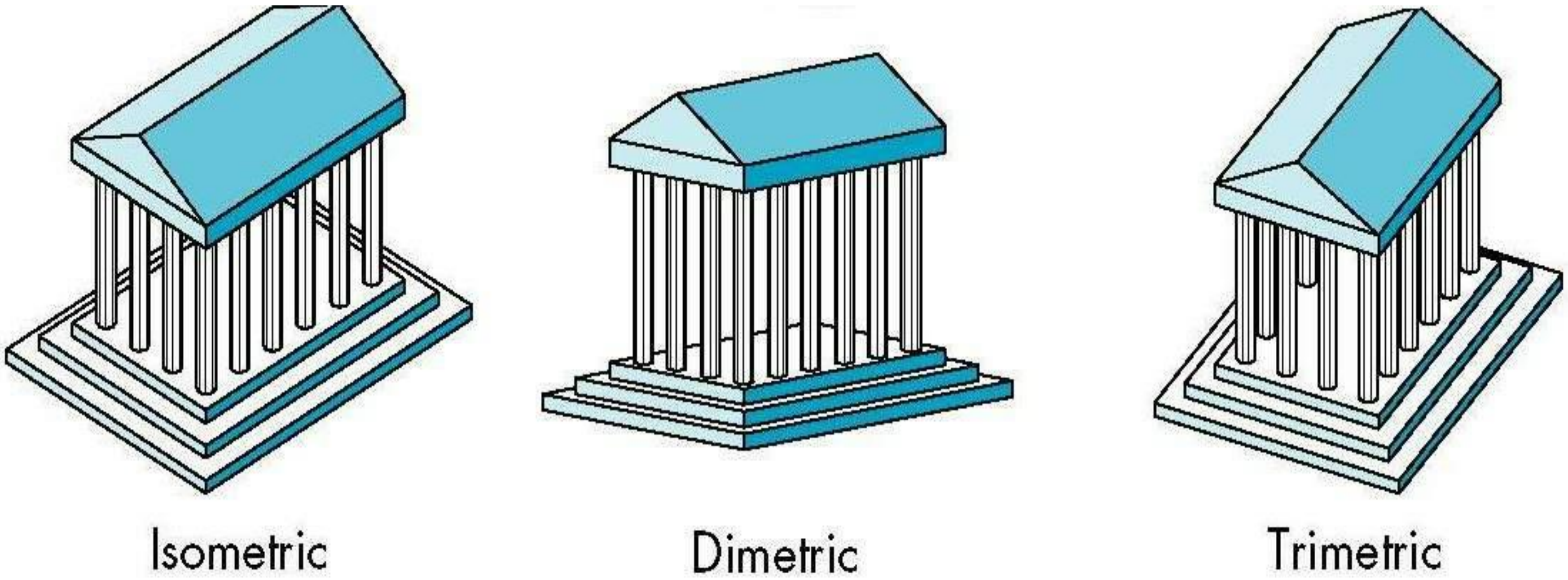
# Orthographic Parallel Projection



Multi View Orthographic

# Orthographic Parallel Projection

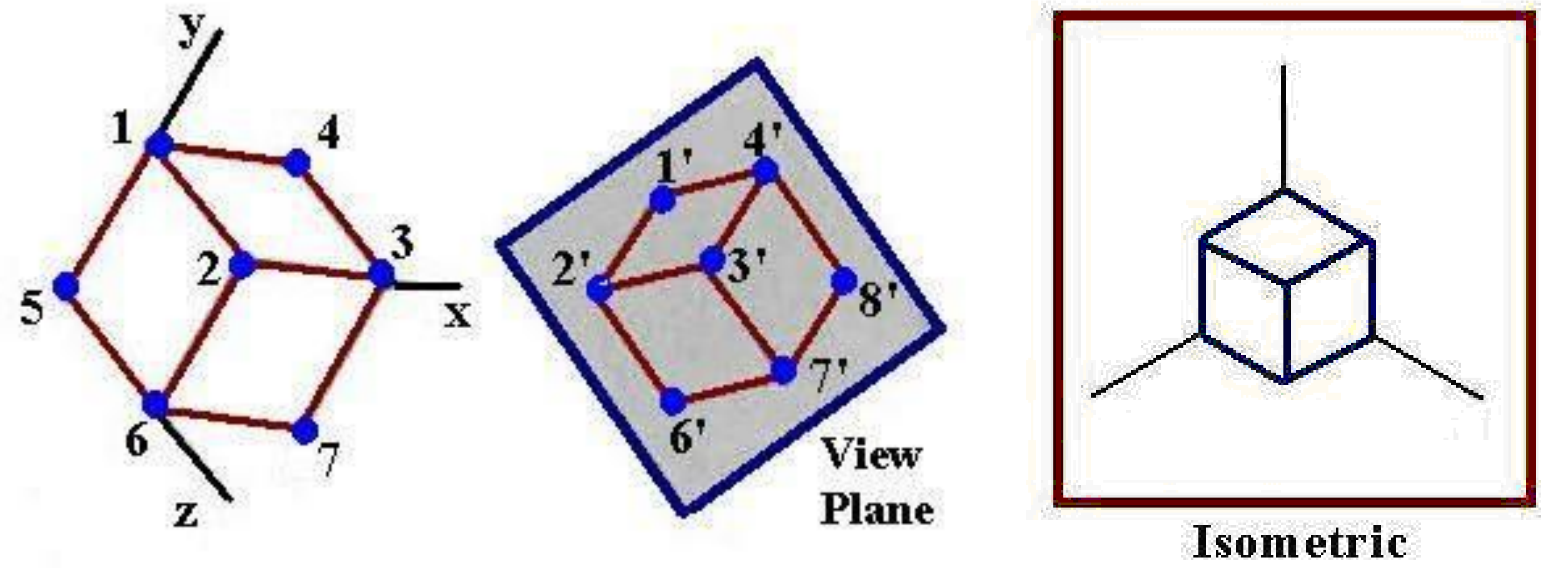
- ***Axonometric orthographic*** projections display more than one face of an object.



# Orthographic Parallel Projection

- ***Isometric Projection***: Projection plane intersects each coordinate axis in which the object is defined (principal axes) at the same distant from the origin.

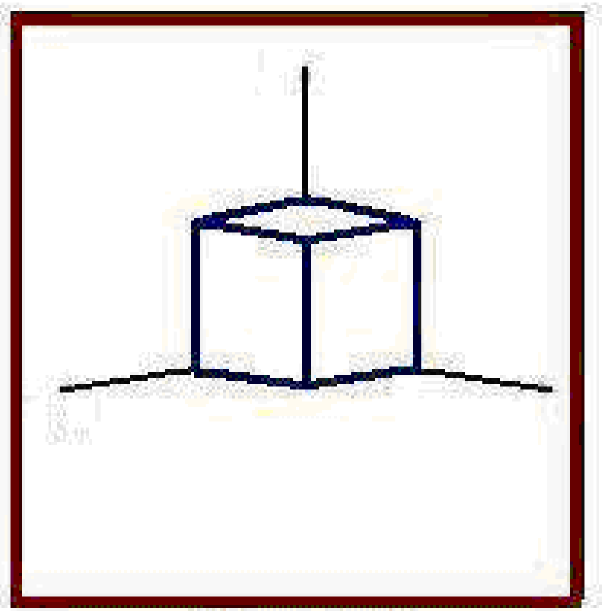
- Projection vector makes equal angles with all of the **three principal axes**.




Isometric projection is obtained by **aligning** the **projection vector** with the **cube diagonal**.

# Orthographic Parallel Projection

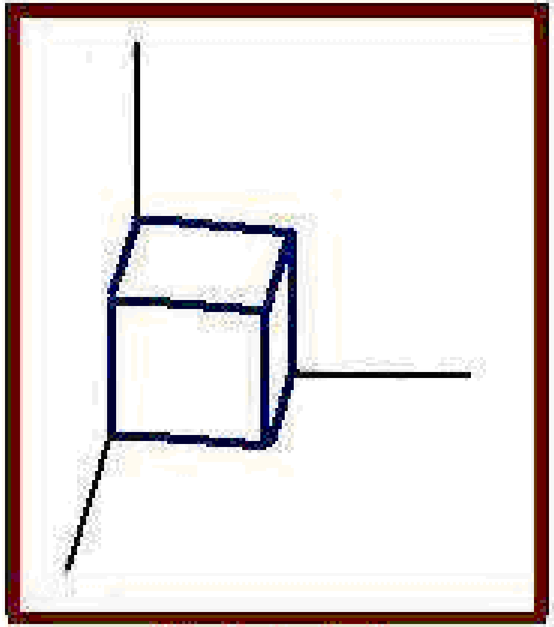
- ***Dimetric Projection***: Projection vector makes **equal angles** with exactly **two** of the principal axes.



**Dimetric**

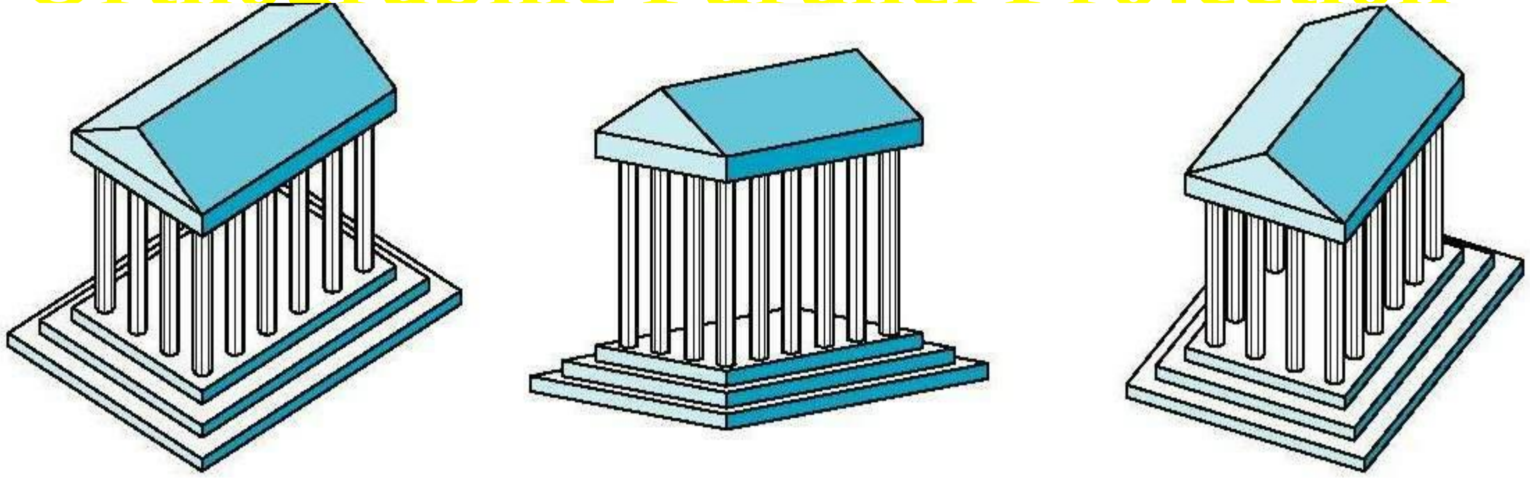
# Orthographic Parallel Projection

- ***Trimetric Projection***: Projection vector makes un**equal angles** with the three principal axes.

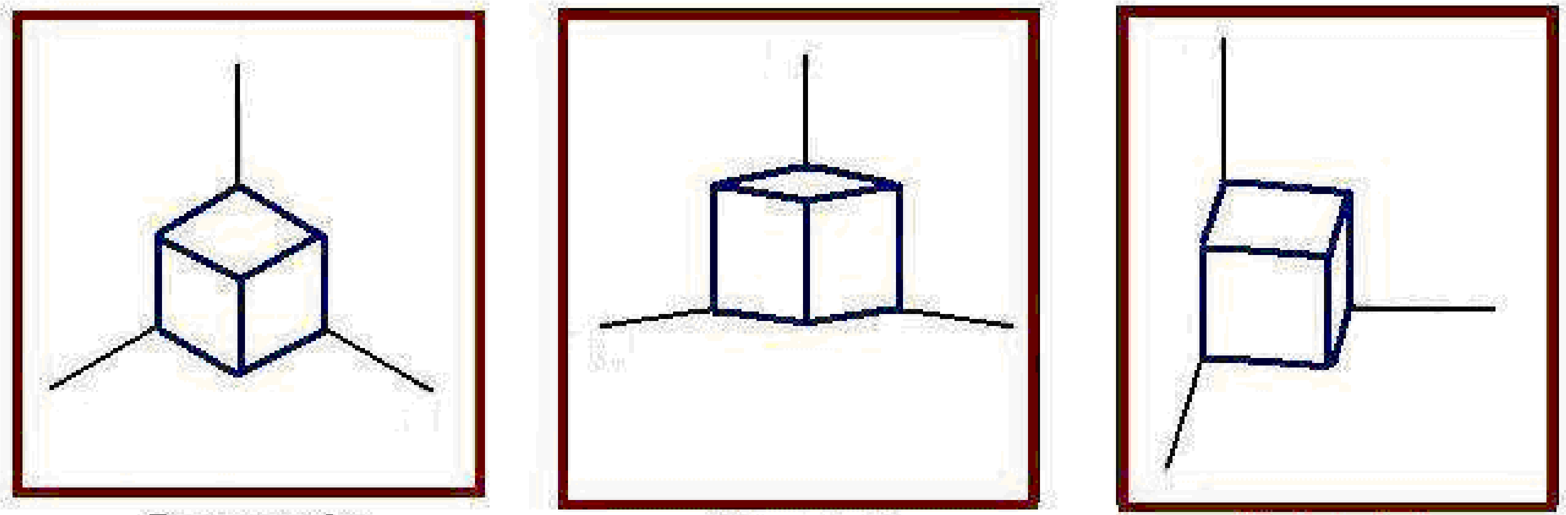


Trimetric

# Orthographic Parallel Projection



Isometric Dimetric Trimetric



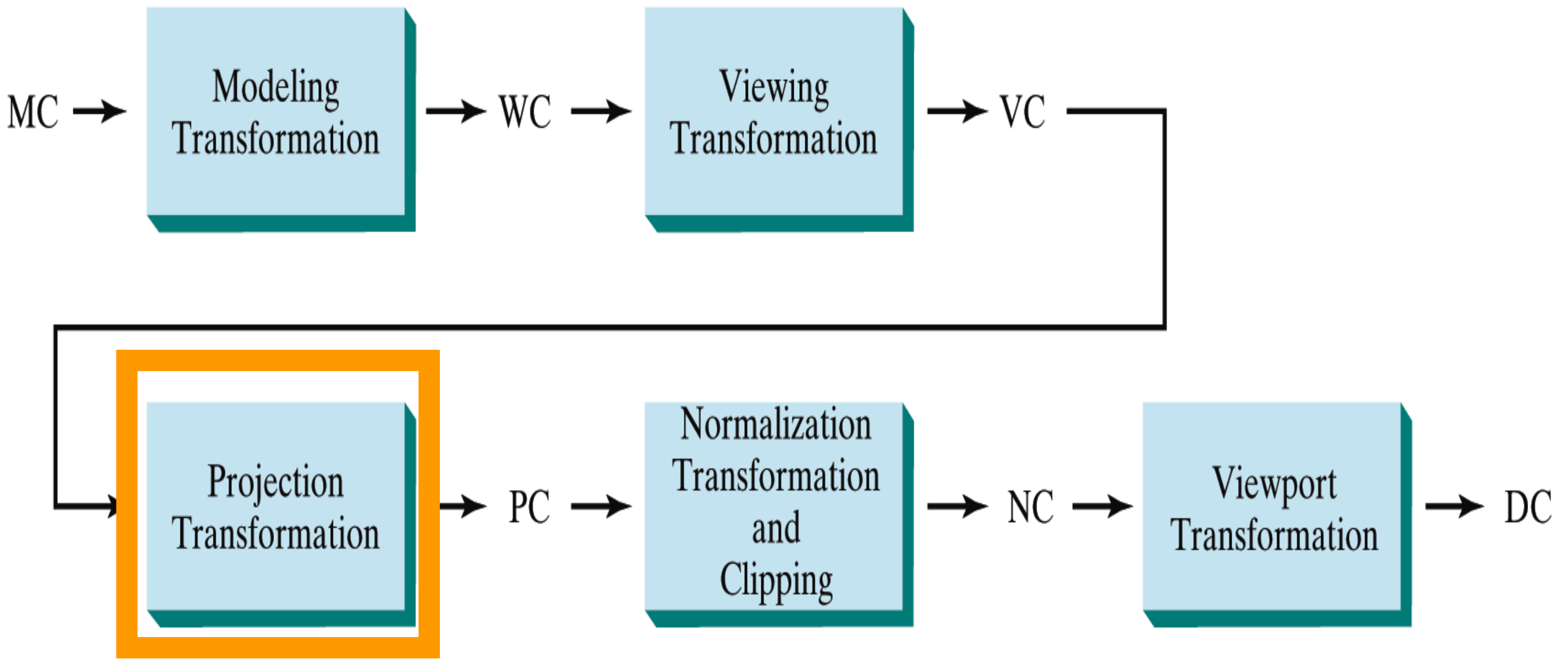
**Isometric** **Dimetric** **Trimetric**

# Orthographic Parallel Projection

# Transformation

# Orthographic Parallel Projection Transformation

- Convert the **viewing coordinate** description of the scene to coordinate positions on the **Orthographic parallel projection plane**.

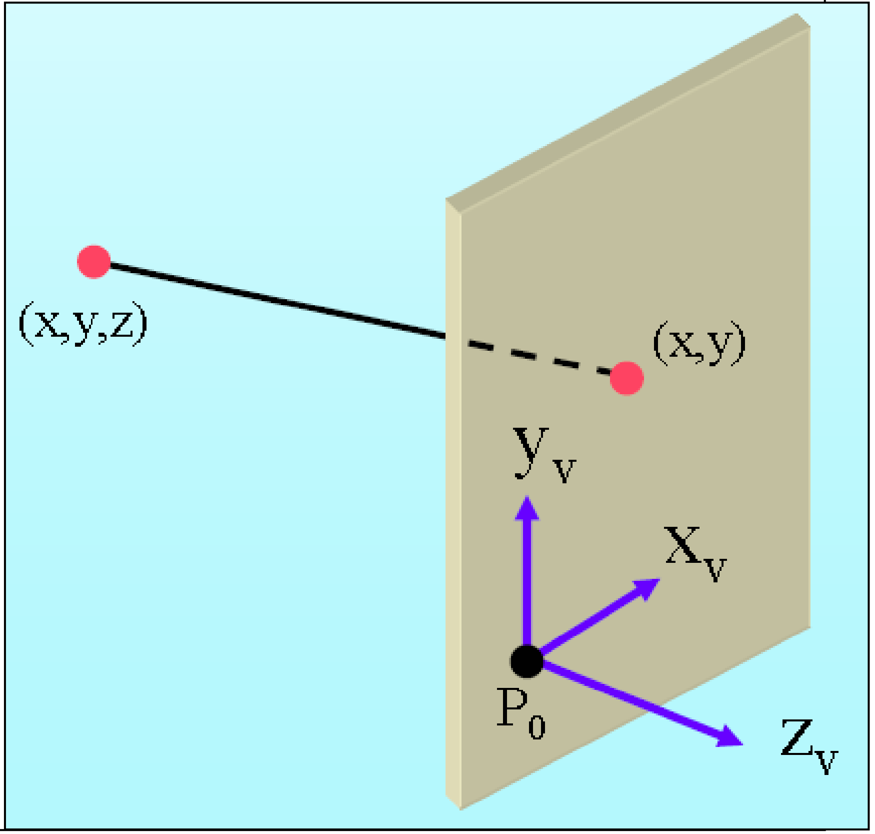


# Orthographic Parallel Projection Transformation

- Since the view plane is placed at position $z_{vp}$ along the $z_v$ axis. Then any point (x,y,z) in viewing coordinates is transformed to projection coordinates as:

$$x_p = x, \quad y_p = y$$

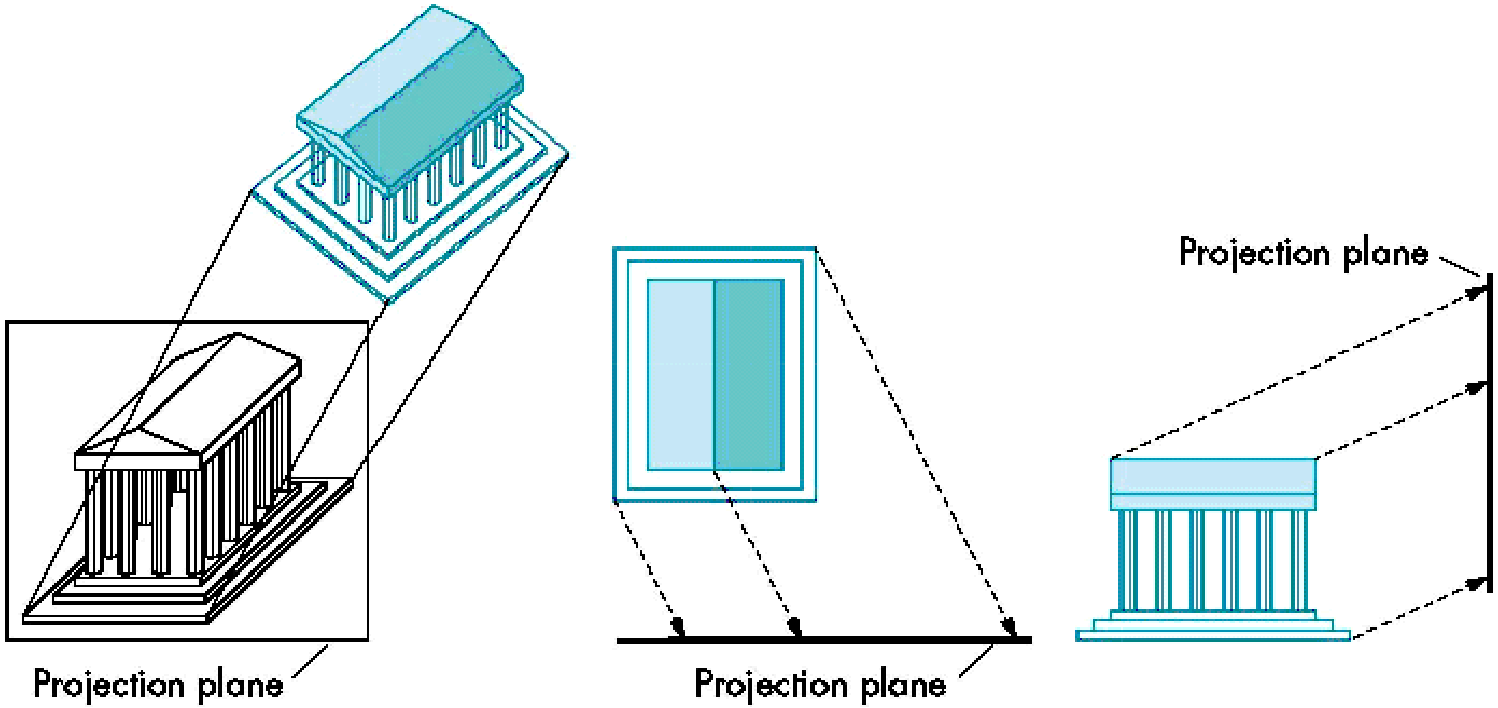
$$\mathbf{M}_{Orthographic\ \ Parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# Oblique
# Parallel Projection

# Oblique Parallel Projection

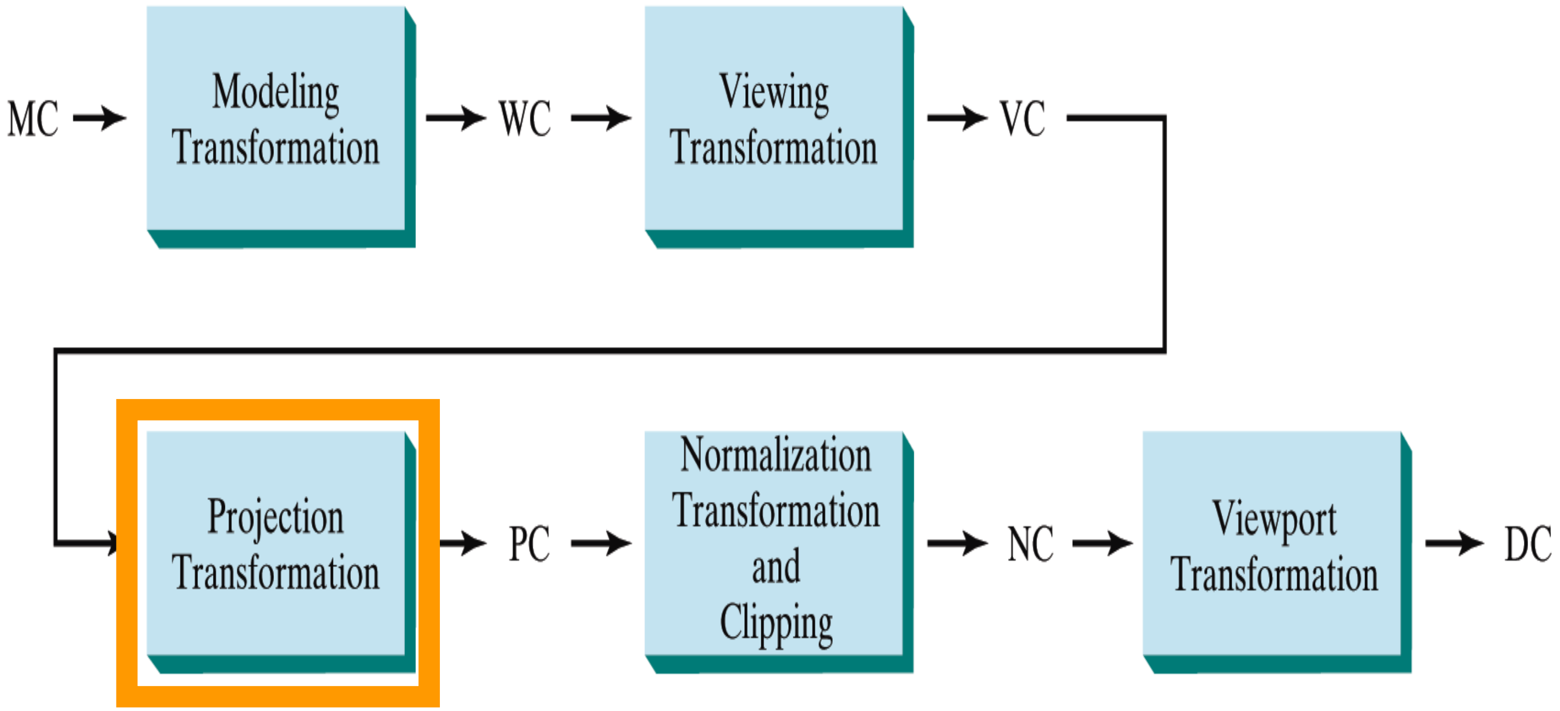
- Projection are **not** perpendicular to the viewing plane.
- Angles and lengths are preserved for faces parallel the plane of projection.
- Preserves 3D nature of an object.



# Oblique
# Parallel Projection
# Transformation

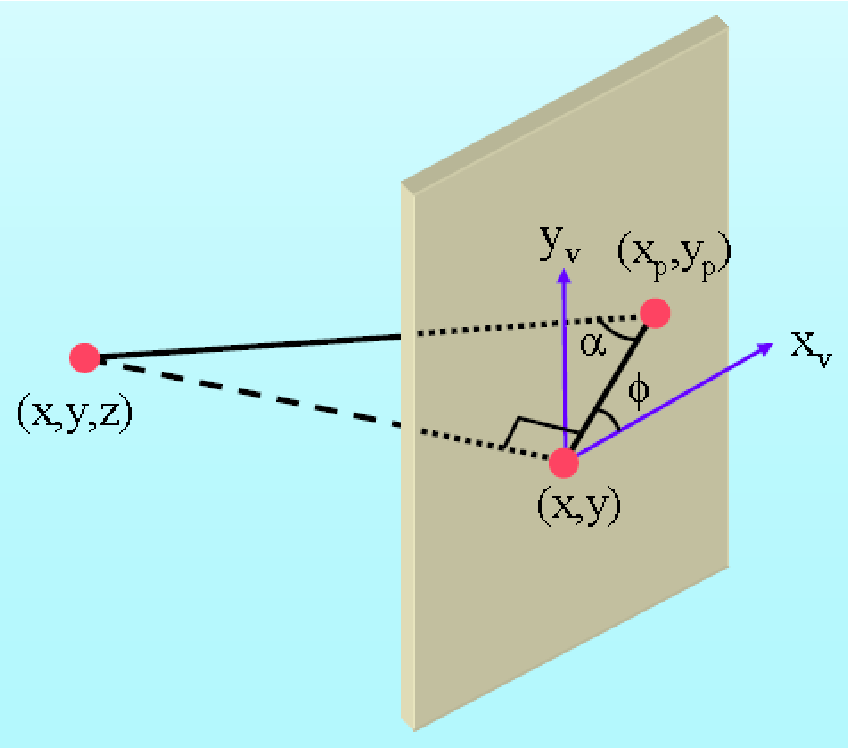
# Oblique Parallel Projection Transformation

- Convert the **viewing coordinate** description of the scene to coordinate positions on the **Oblique parallel projection plane**.



# Oblique Parallel Projection

- Point (x,y,z) is projected to position $(\mathbf{x_p,y_p})$ on the view plane. $\alpha$
- Projector (oblique) from **(x,y,z)** to $(\mathbf{x_p,y_p})$ makes an angle with the line (**L**) on the projection plane that joins $(\mathbf{x_p,y_p})$ and **(x,y).**
- Line **L** is at an angle $\varphi$ with the horizontal direction in the projection plane.



# Oblique Parallel Projection

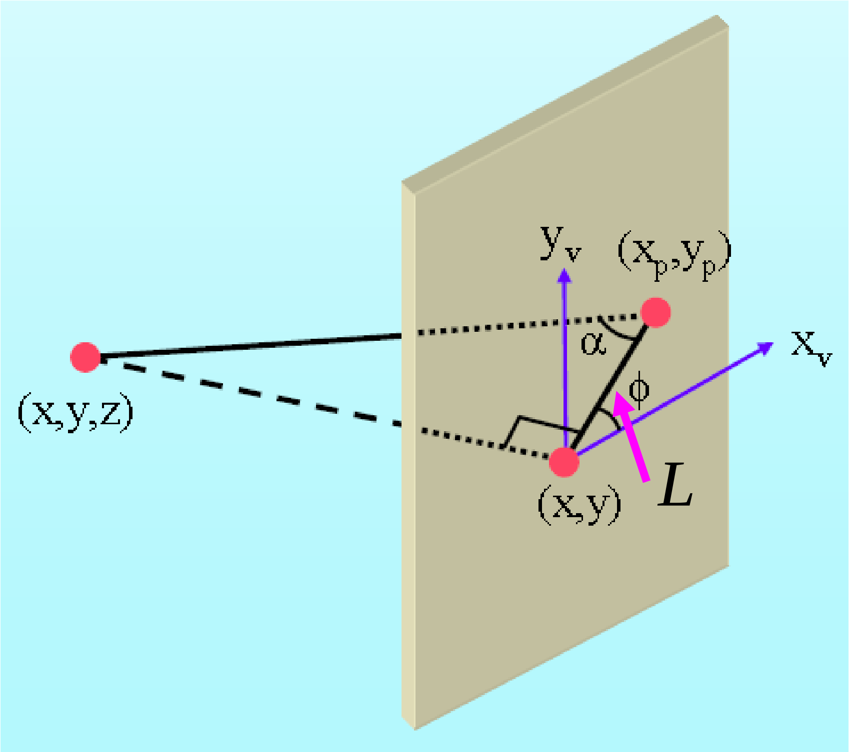
$$x_p = x + L\cos\varphi$$

$$y_p = y + L\sin\varphi$$

$$L = \frac{z}{\tan\alpha} = zL_1$$

$$x_p = x + z(L_1\cos\varphi)$$
$$y_p = y + z(L_1\sin\varphi)$$

$$\mathbf{M}_{Parallel} = \begin{bmatrix} 1 & 0 & L_1\cos\varphi & 0 \\ 0 & 1 & L_1\sin\varphi & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# Oblique Parallel Projection

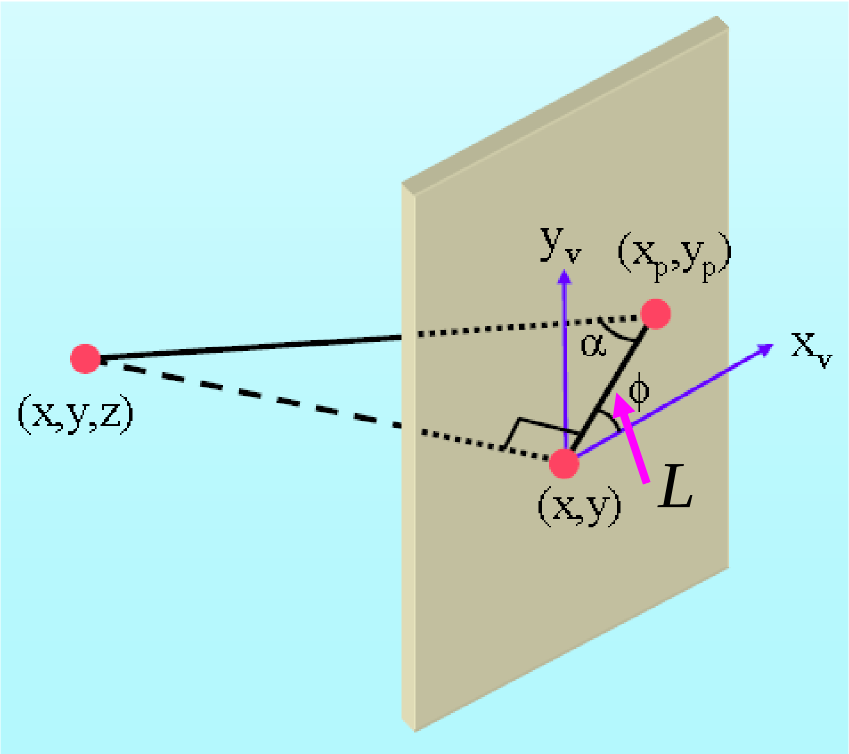
## Orthographic Projection:

$$L_1 = 0$$

$$\alpha = 90^{\circ}$$

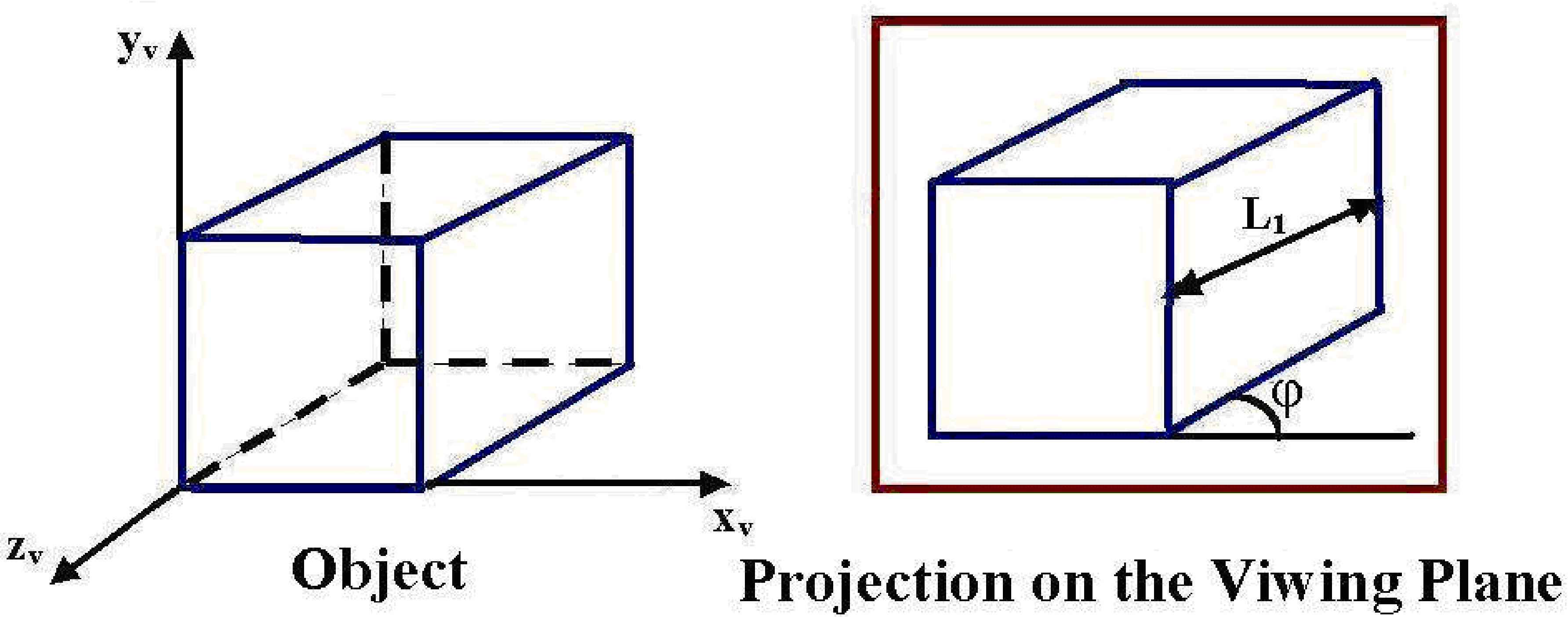
$$x_p = x, \quad y_p = y$$

$$\mathbf{M}_{Orthographic\ \ Parallel} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



# Oblique Parallel Projection

- **Angles**, **distances**, and **parallel lines** in the plane are projected accurately.



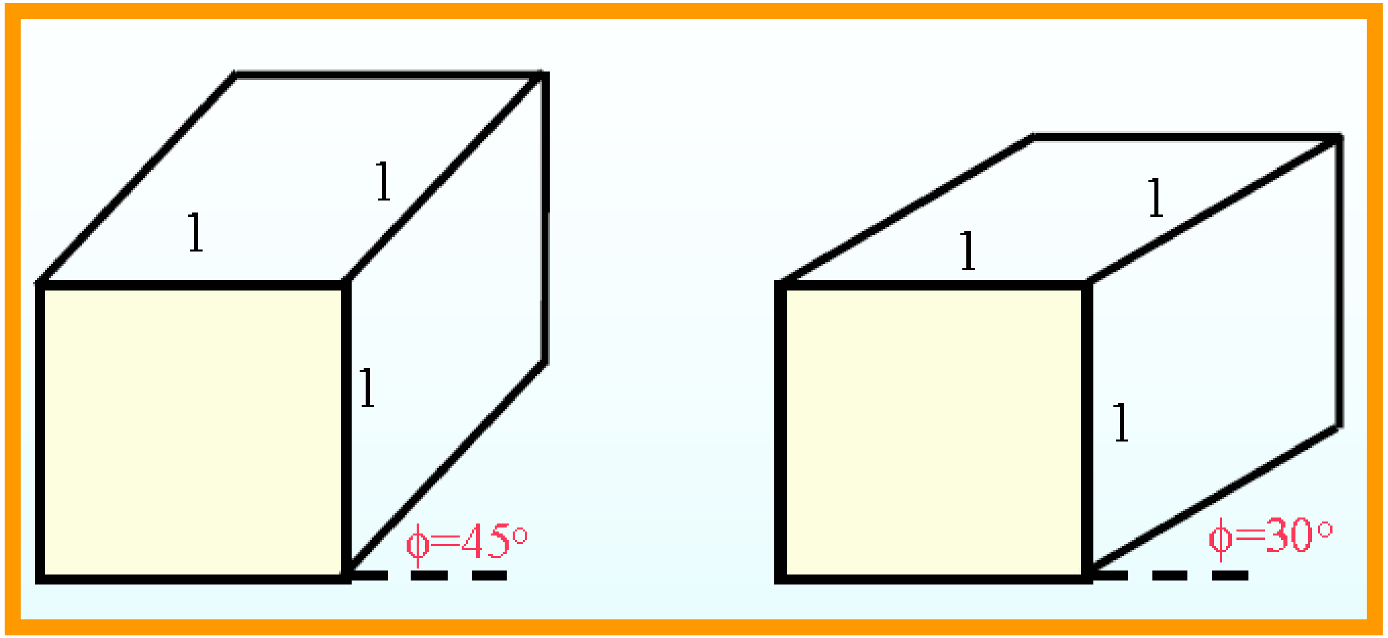
# Cavalier Projection:

# Cavalier Projection

$$\tan\alpha = 1$$

$$\alpha = 45^\circ$$

- Preserves lengths of lines perpendicular to the viewing plane.
- 3D nature can be captured but shape seems distorted.
- Can display a combination of front, and side, and top views.

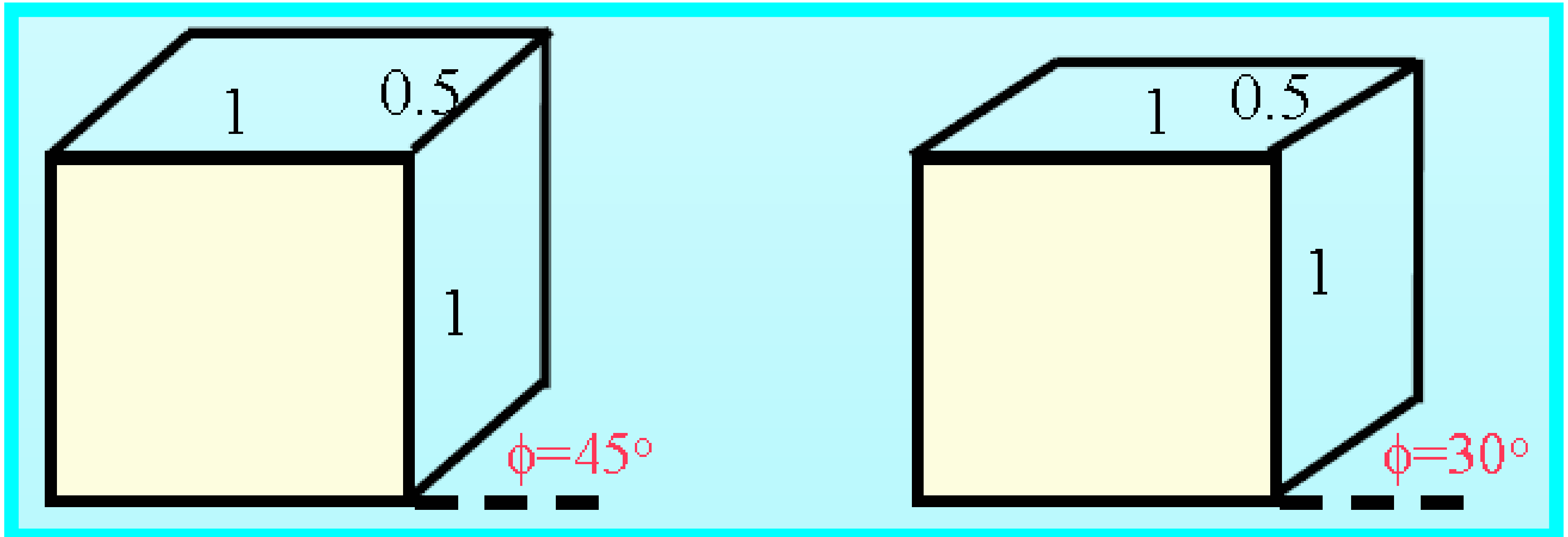


# Cabinet Projection

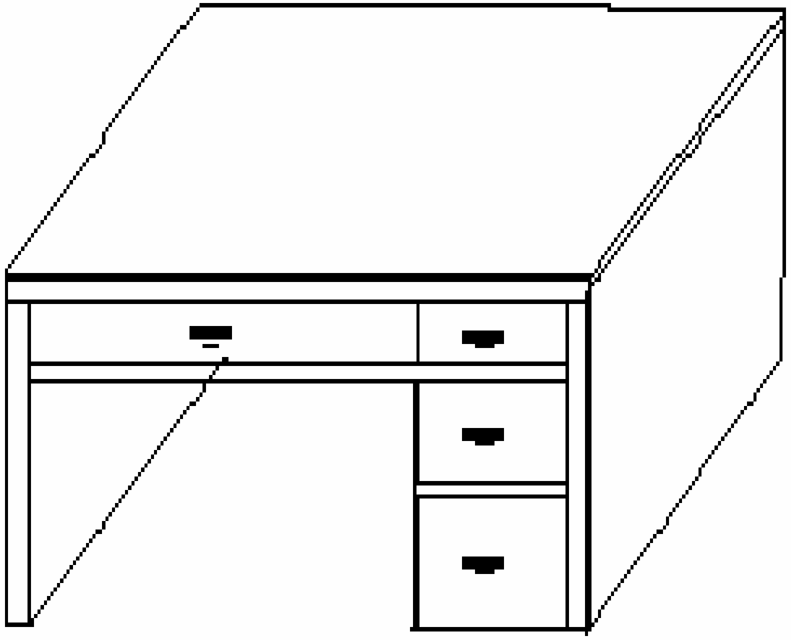
Cabinet Projection:

$$\boxed{\begin{aligned}\tan\alpha &= 2\\ \alpha &\approx 63.4^{\circ}\end{aligned}}$$

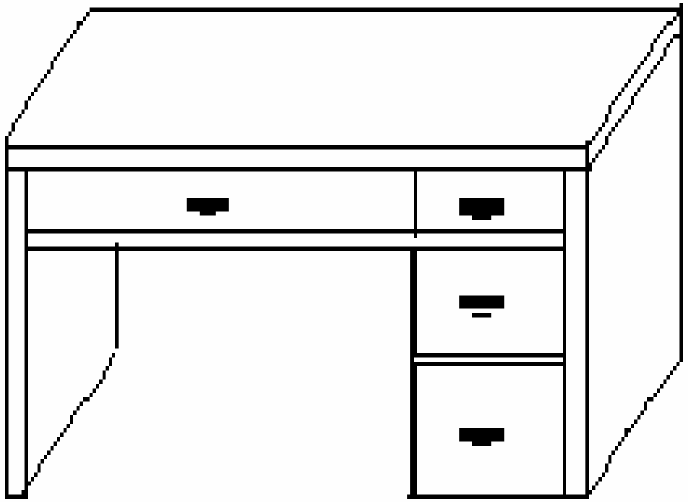
- Lines perpendicular to the viewing plane project at ½ of their **length**.
- A more realistic view than the cavalier projection.
- Can display a combination of front, and side, and top views.



# Cavalier & Cabinet Projection



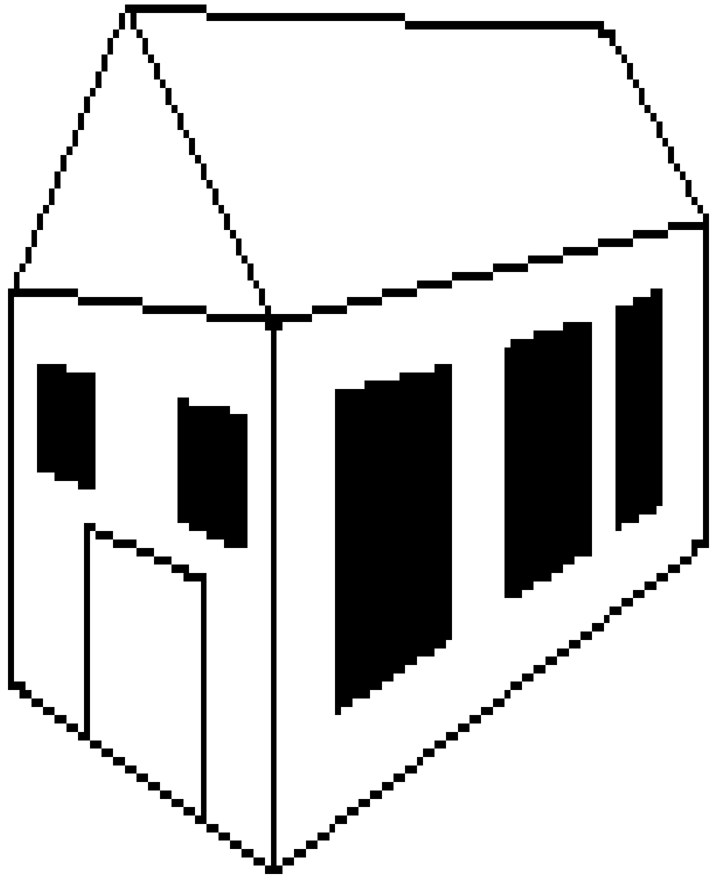
Cavalier



Cabinet

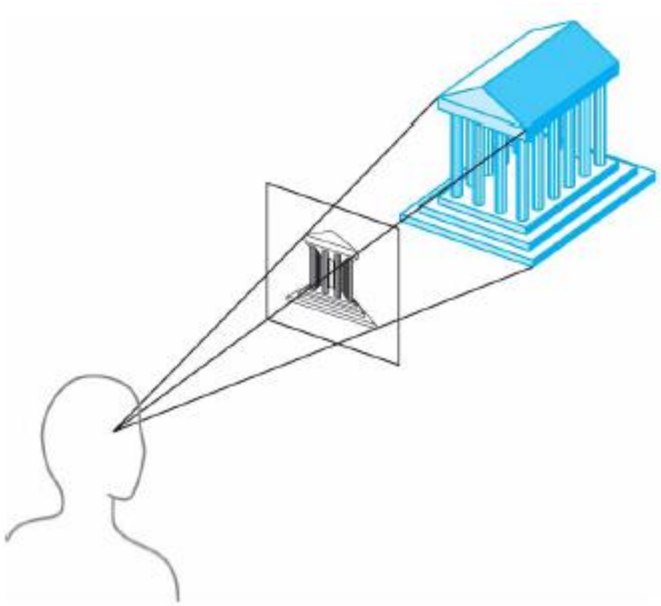
# Perspective Projection

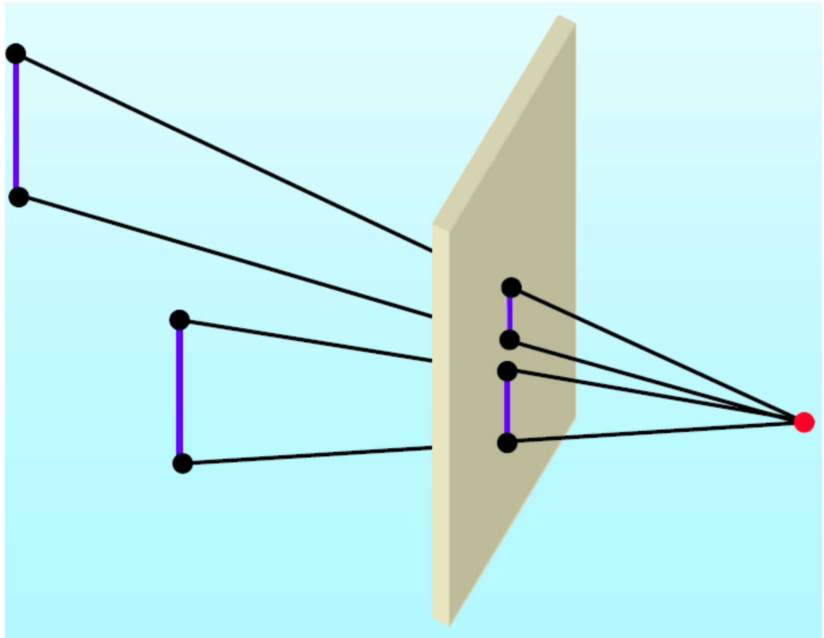
# Perspective Projection



# Perspective Projection

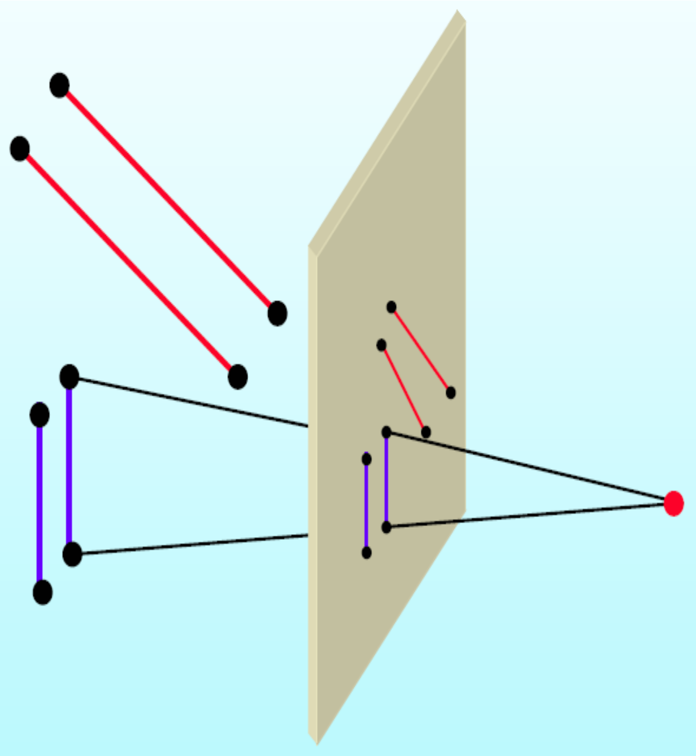
- In a perspective projection, the center of projection is at a finite distance from the viewing plane.
- Produces realistic views but does not preserve relative proportion of objects
- The size of a projection object is inversely proportional to its distance from the viewing plane.

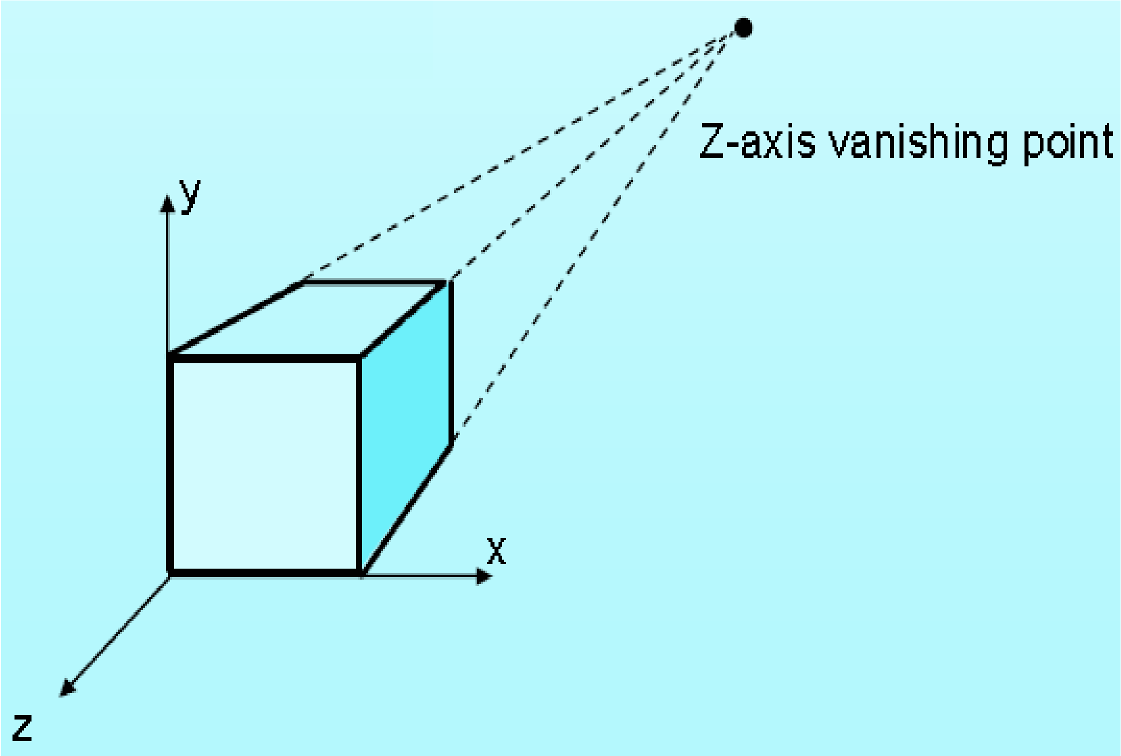




# Perspective Projection

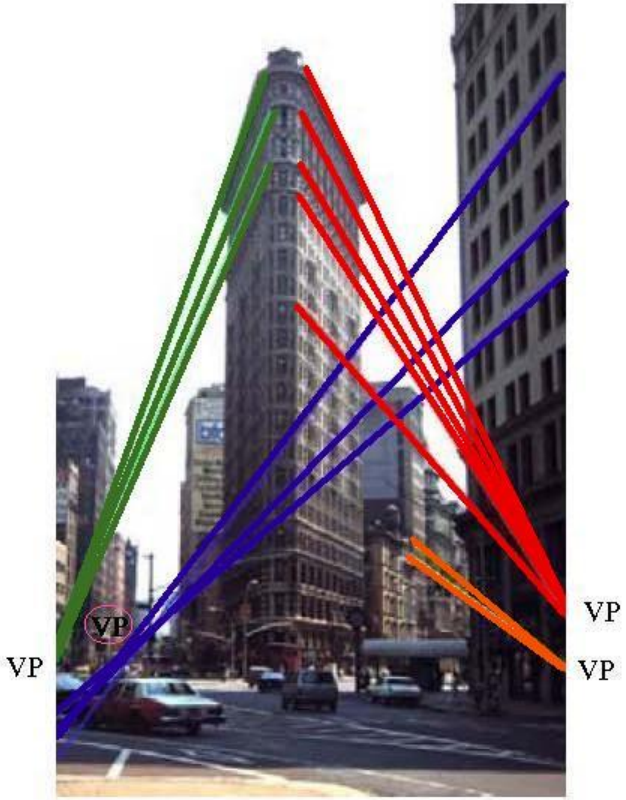
- Parallel lines that are not parallel to the viewing plane, **converge** to a ***vanishing point***.
- A vanishing point is the projection of a point at infinity.

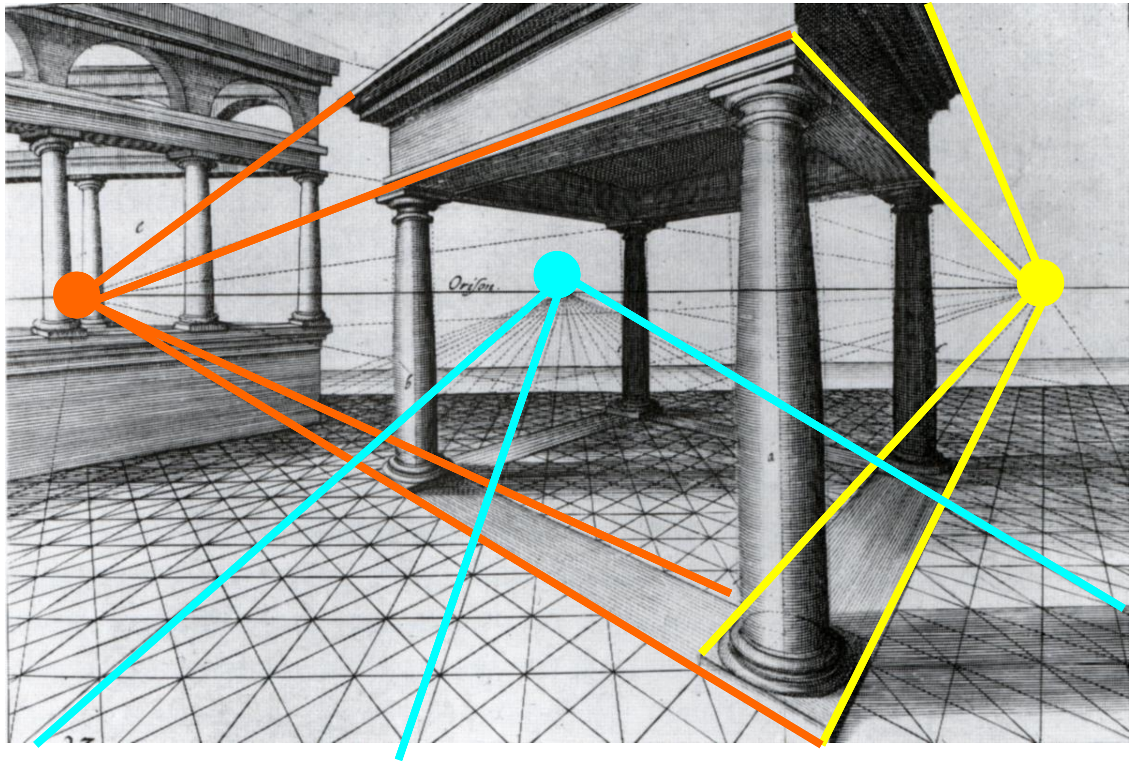




# Vanishing Points

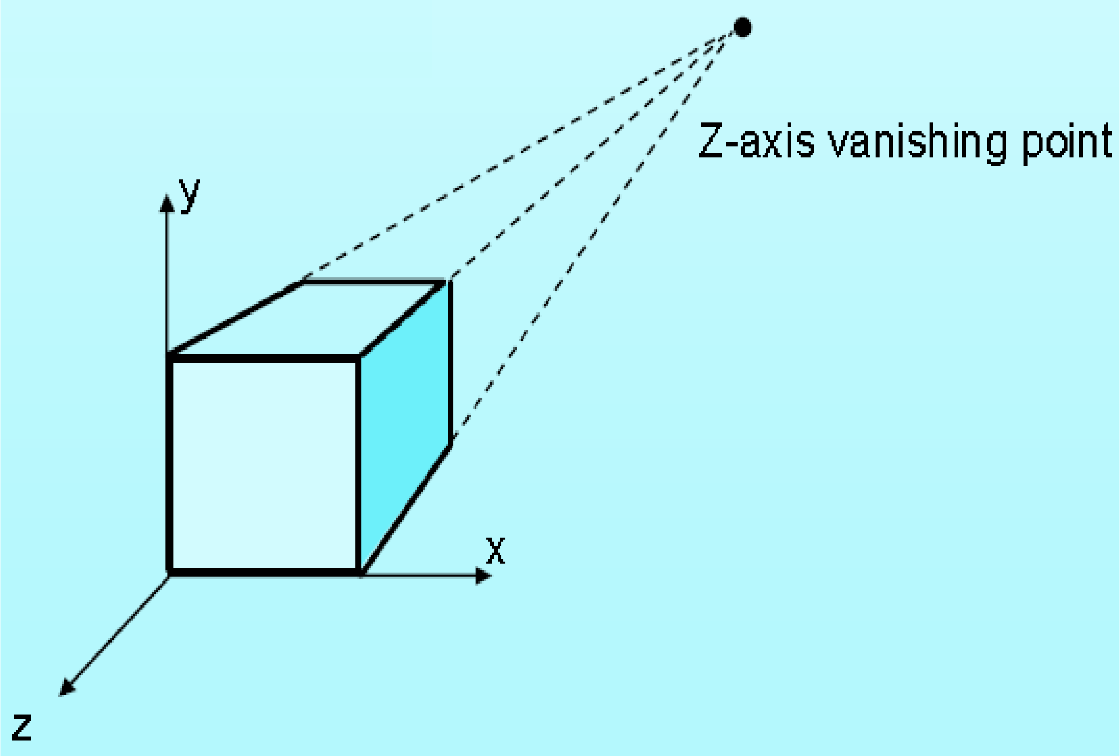
- Each set of projected parallel lines will have a separate vanishing points.
- There are infinity many **general** vanishing points.



# Perspective Projection

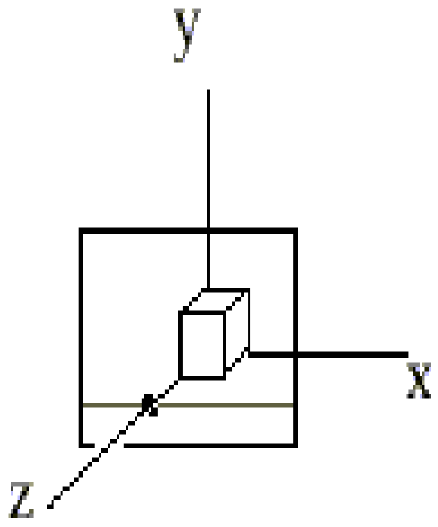
- The vanishing point for any set of lines that are parallel to one of the principal axes of an object is referred to as a **principal vanishing point**.
- We control the number of principal vanishing points (one, two, or three) with the orientation of the projection plane.

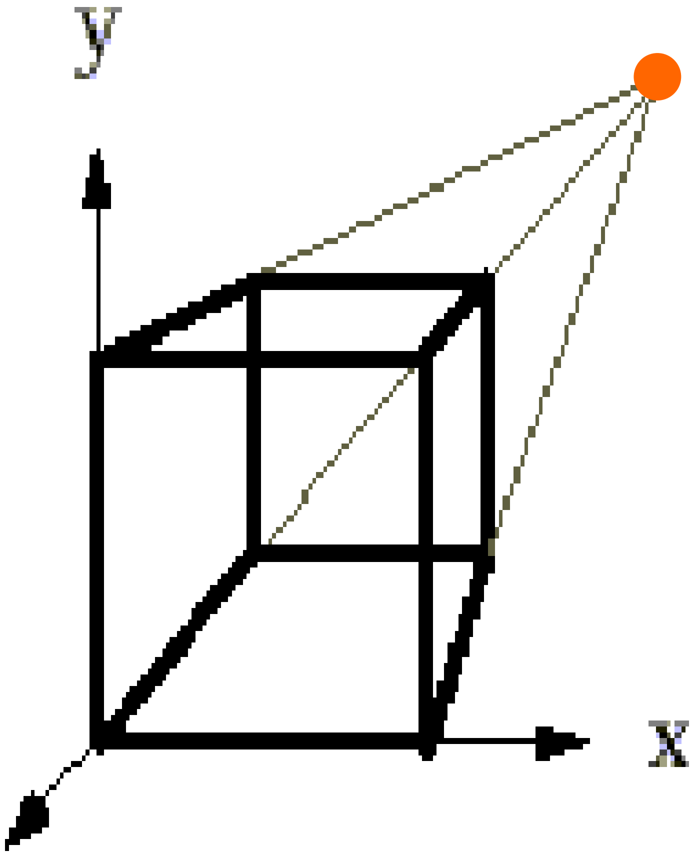


# Perspective Projection

- The number of principal vanishing points in a projection is determined by the number of principal axes **intersecting** the view plane.

# Perspective Projection

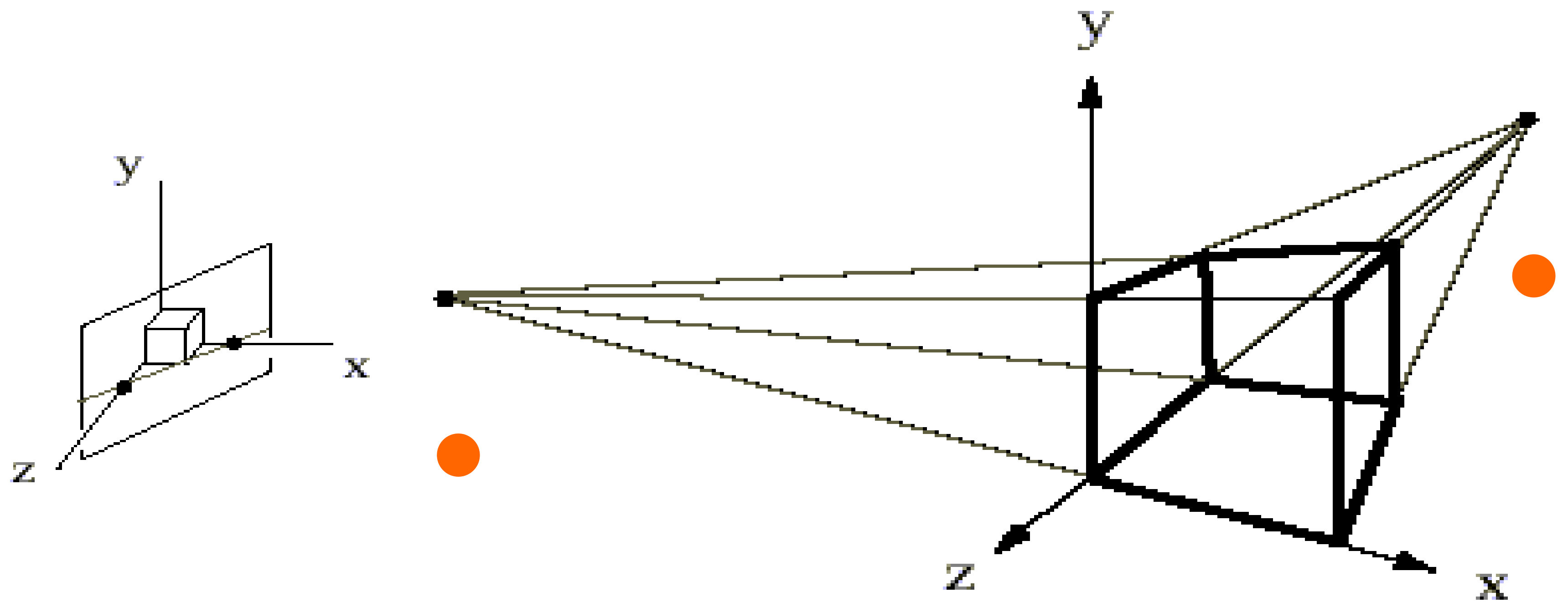



***One Point Perspective***

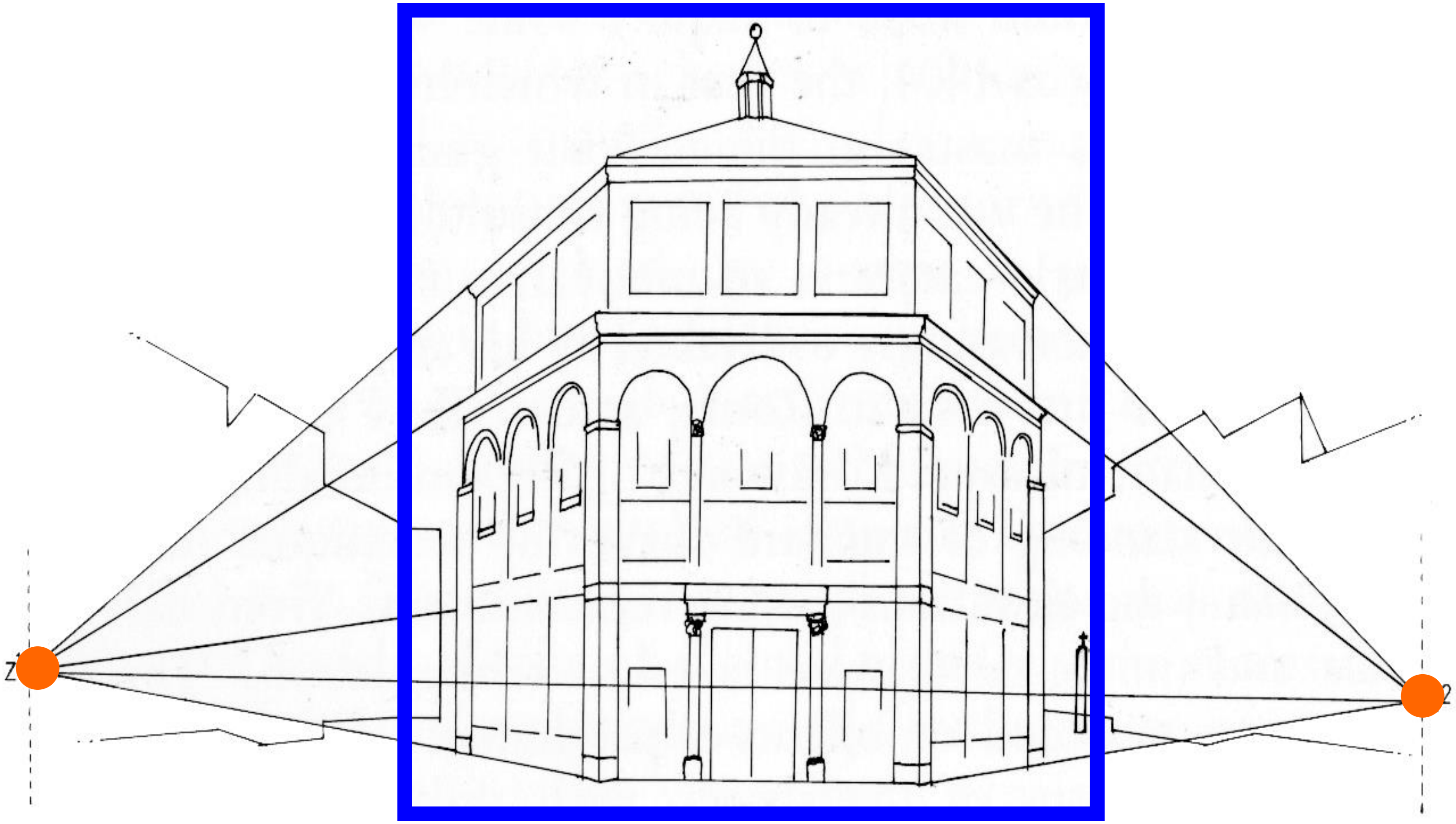
**(*z*-axis vanishing point)**

# Perspective Projection



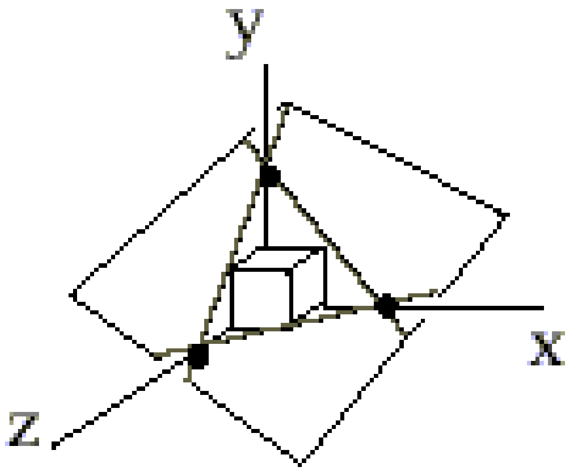

***Two Point Perspective***
**(z, and x-axis vanishing points)**

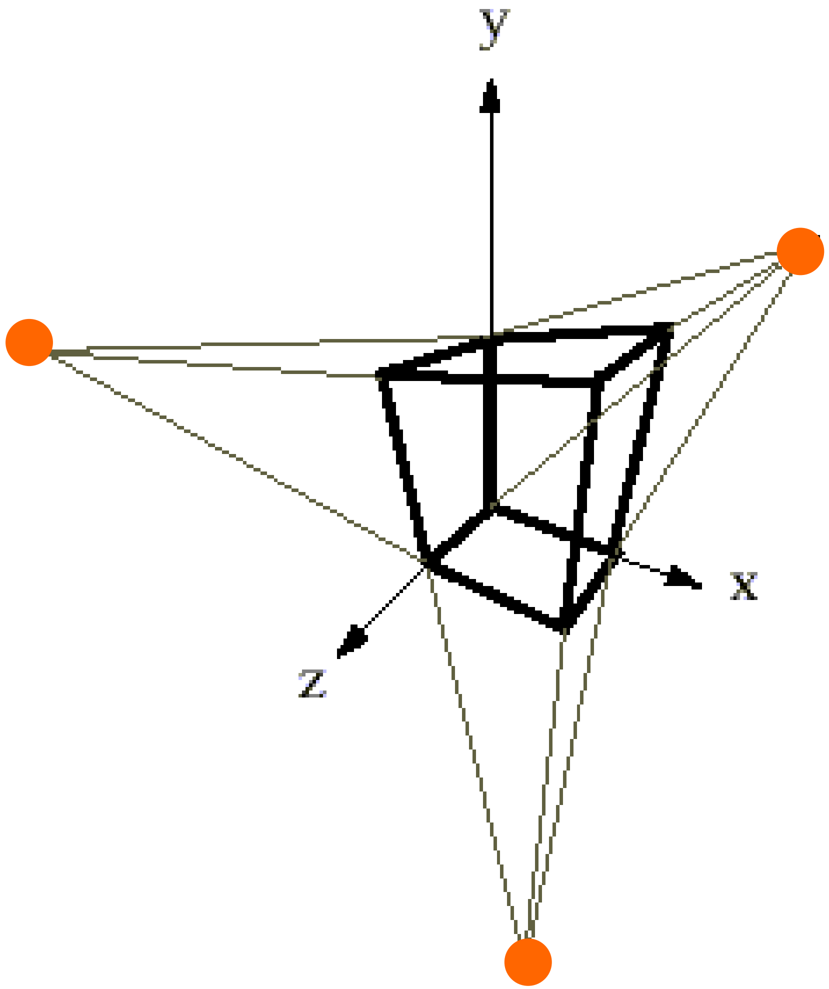
# Perspective Projection



***Two Point Perspective***

# Perspective Projection

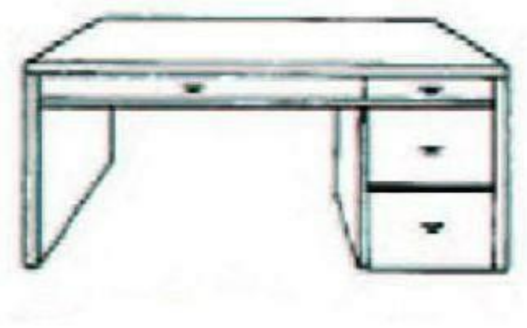



***Three Point Perspective***

**(z, x, and y-axis vanishing points)**

# Perspective Projection



One-Point Perspective Projection

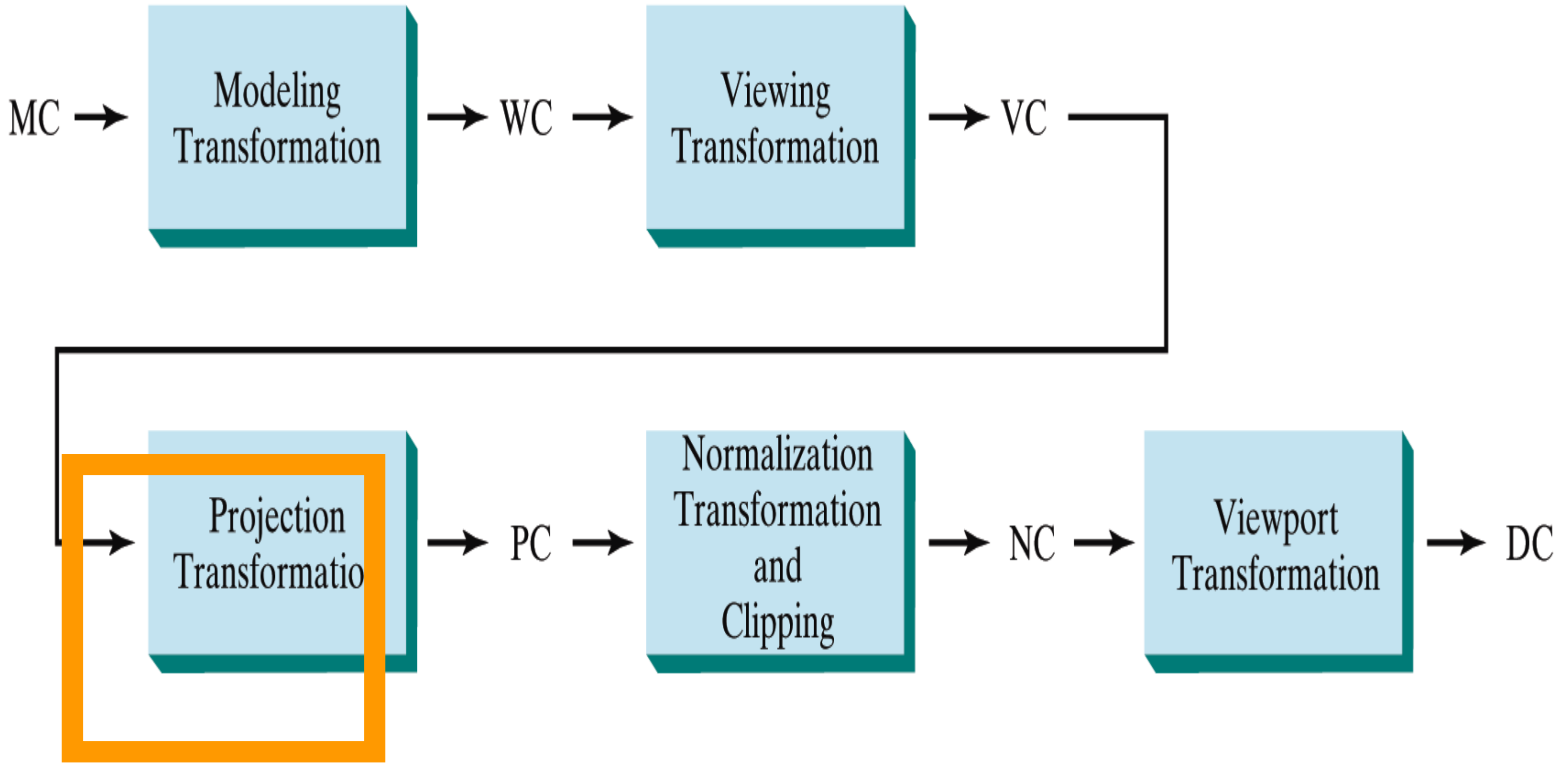
Two-Point Perspective Projection

Tree-Point Perspective Projection

# Perspective Projection Transformation

# Perspective Projection Transformation

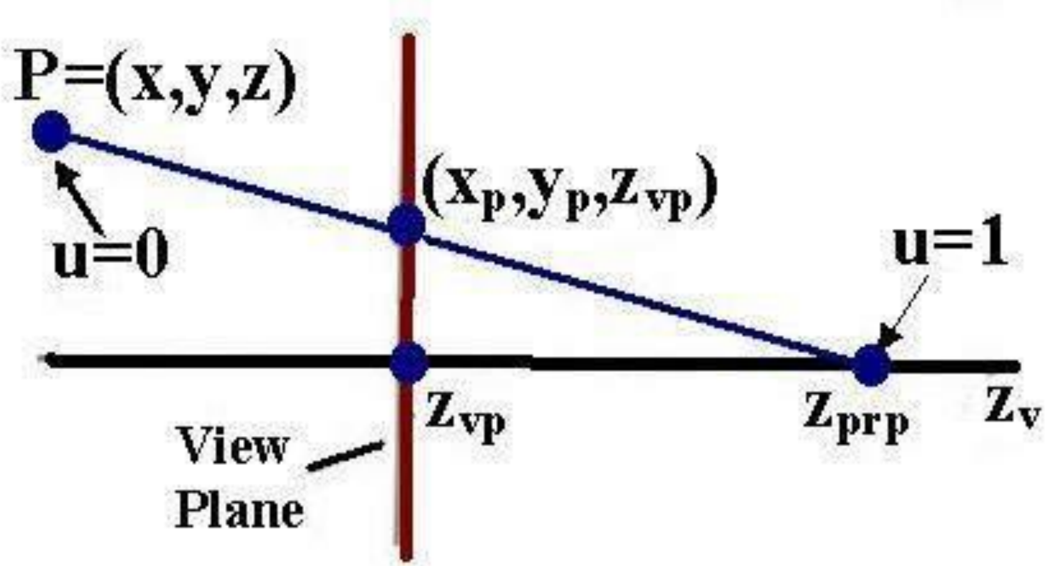
- Convert the **viewing coordinate** description of the scene to coordinate positions on the **perspective projection plane**.



# Perspective Projection Transformation

- Suppose the projection reference point at position $z_{prp}$ along the $z_v$ axis, and the view plane at $z_{vp}$.

$$x' = x - xu$$
$$y' = y - yu$$
$$z' = z - (z - z_{prp})u$$



# Perspective Projection Transformation

## On the view plane:

$$u = \frac{z_{vp} - z}{z_{prp} - z}$$

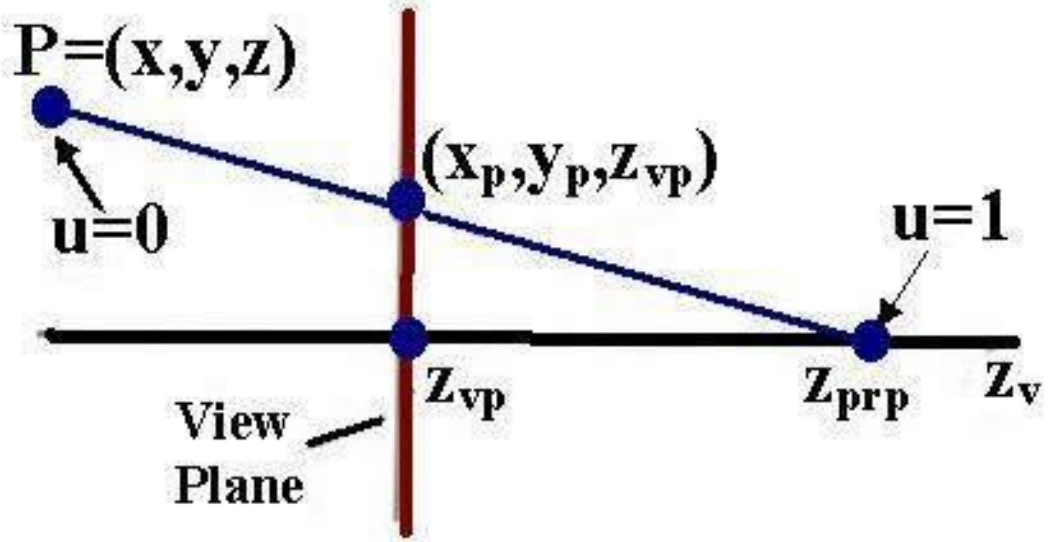
$$z' = z_{vp}$$

$$x' = x - xu$$
$$y' = y - yu$$
$$z' = z - (z - z_{prp})u$$

$$d_p = z_{prp} - z_{vp}$$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = x\left(\frac{d_p}{z - z_{prp}}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = y\left(\frac{d_p}{z - z_{prp}}\right)$$



# Perspective Projection Transformation

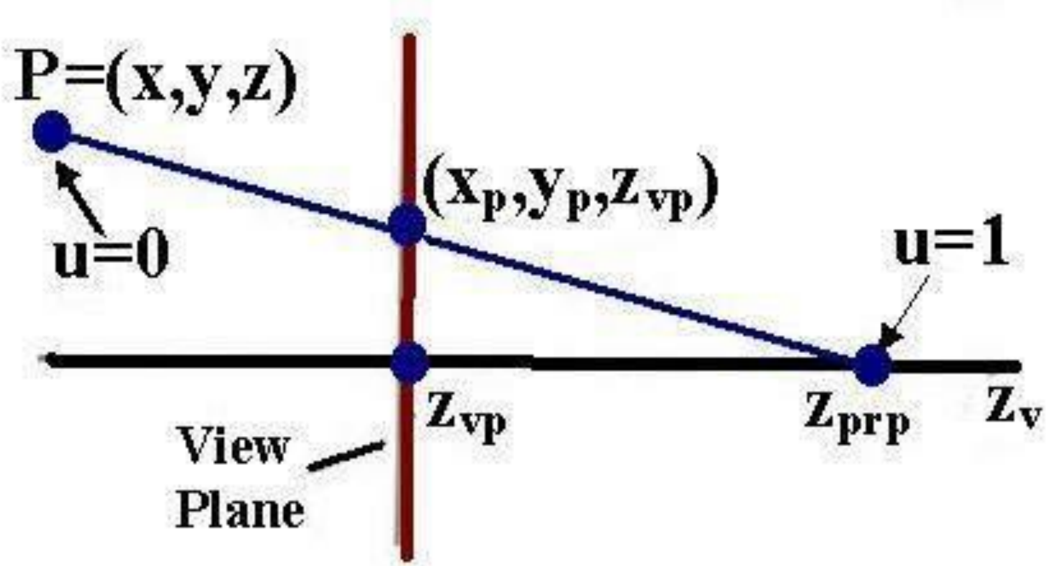
**On the view plane:** $z' = z_{vp}$

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & z_{vp}/d_p & -z_{vp}(z_{prp}/d_p) \\ 0 & 0 & 1/d_p & -z_{prp}/d_p \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = x\left(\frac{d_p}{z - z_{prp}}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = y\left(\frac{d_p}{z - z_{prp}}\right)$$

$$x_p = x_h/h, \quad y_p = y_h/h$$



# Perspective Projection Transformation

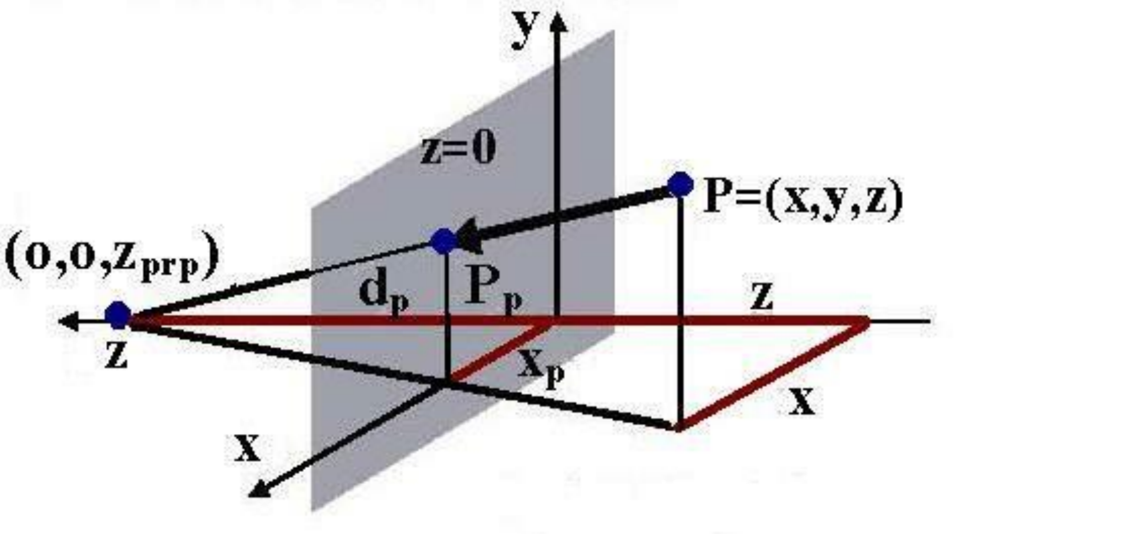
**Special Cases:** $z_{vp} = 0$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = x\left(\frac{d_p}{z - z_{prp}}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = y\left(\frac{d_p}{z - z_{prp}}\right)$$

$$x_p = x\left(\frac{z_{prp}}{z - z_{prp}}\right) = x\left(\frac{1}{z/z_{prp} - 1}\right)$$

$$y_p = y\left(\frac{z_{prp}}{z - z_{prp}}\right) = y\left(\frac{1}{z/z_{prp} - 1}\right)$$



# Perspective Projection Transformation

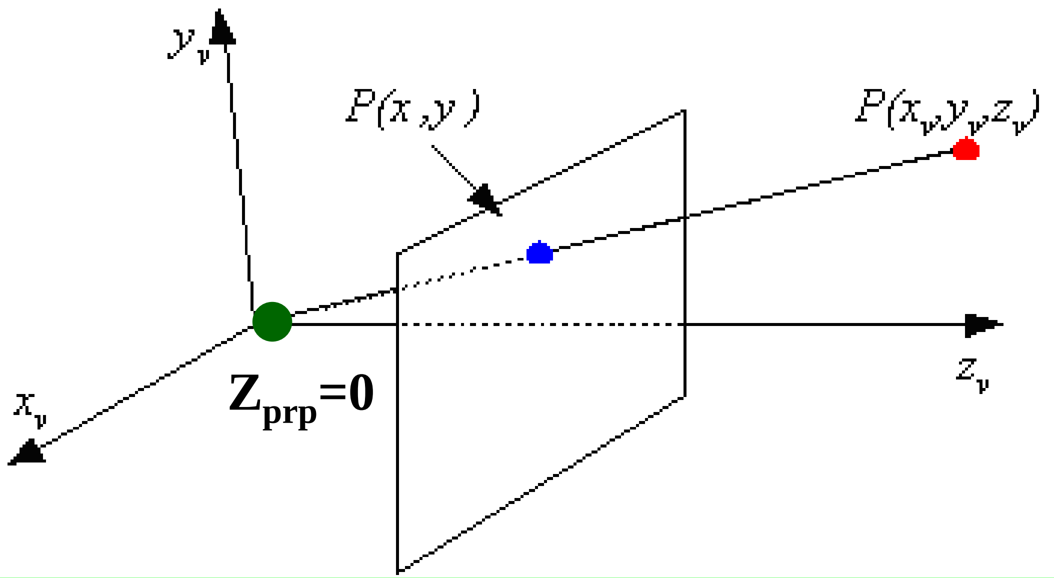
***Special Cases:*** The projection reference point is at the viewing coordinate origin: $\boxed{z_{prp} = 0}$

$$x_p = x\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = x\left(\frac{d_p}{z - z_{prp}}\right)$$

$$y_p = y\left(\frac{z_{prp} - z_{vp}}{z - z_{prp}}\right) = y\left(\frac{d_p}{z - z_{prp}}\right)$$

$$x_p = x\left(\frac{-z_{vp}}{z}\right) = x\left(\frac{-1}{z/z_{vp}}\right)$$

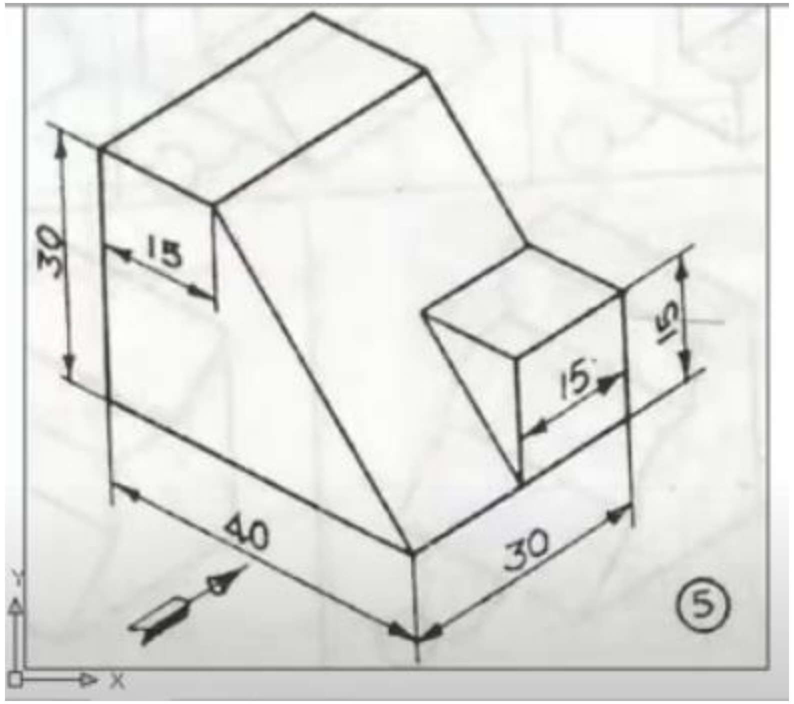
$$y_p = y\left(\frac{-z_{vp}}{z}\right) = y\left(\frac{-1}{z/z_{vp}}\right)$$



# Summary

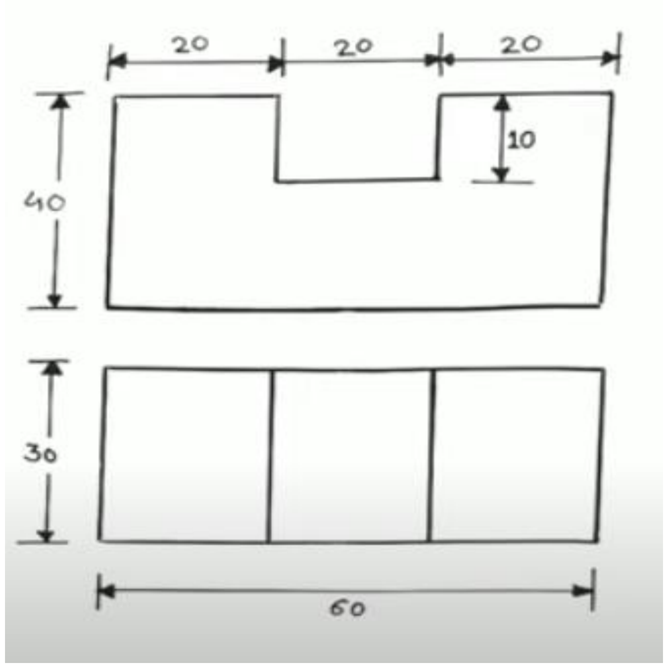
# Orthographic projections

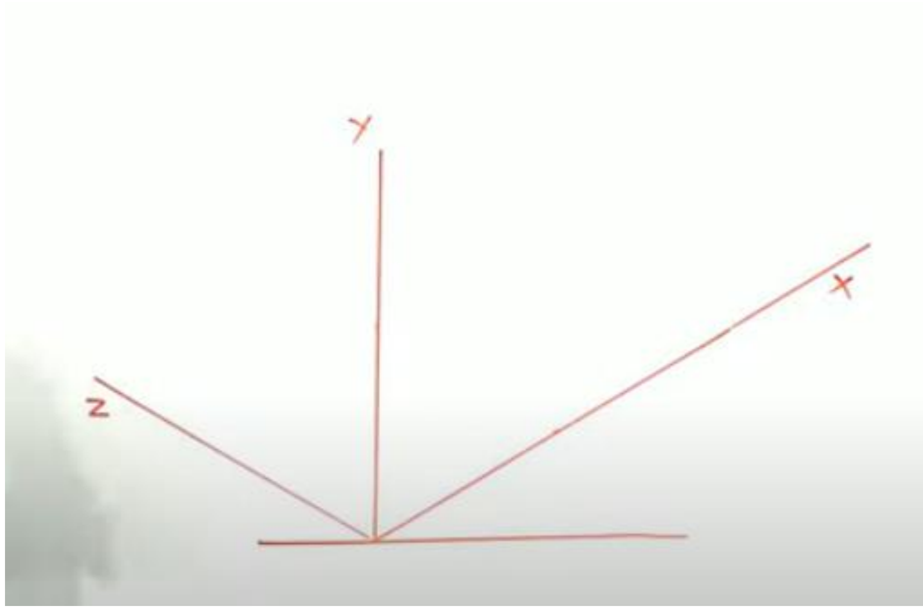
- Obtain the orthographic projection. Obtain the front and top views.



# ISOmetric projections

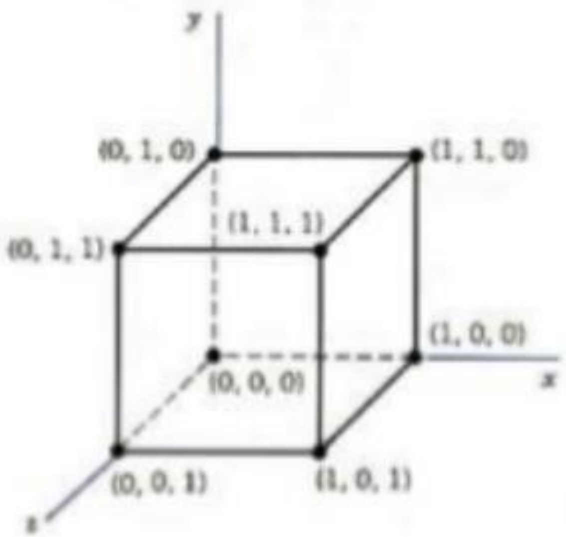
- Obtain the Isometric projections of the given diagram



# Oblique Projections

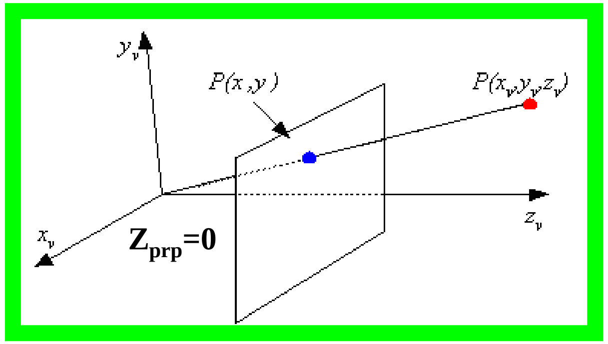
- Given the cube, find the oblique projections with alpha=45 and theta=30 and alpha=90 and theta=45.



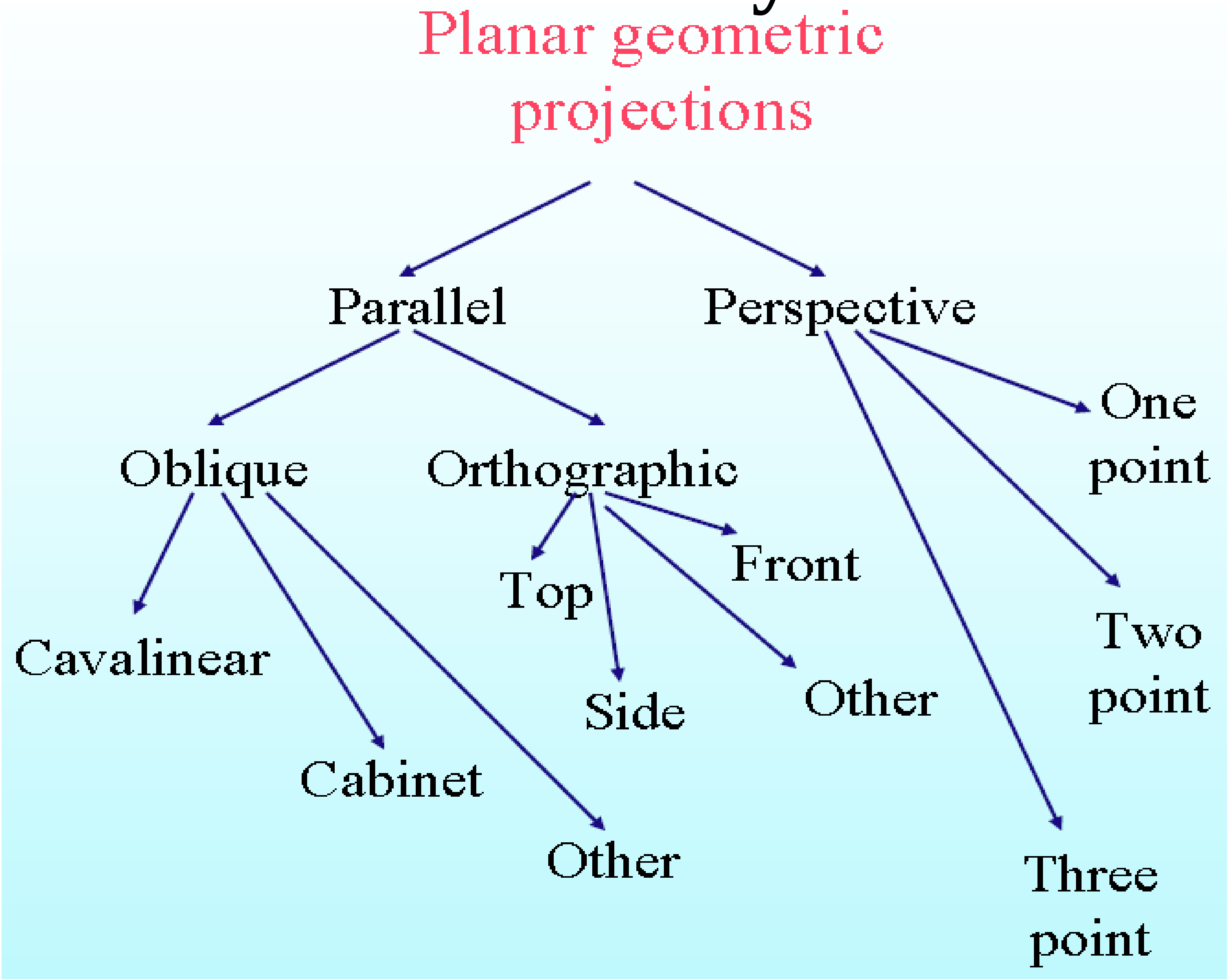
# Perspective Projection.

- A single point perspective transformation has to be performed on a triangle (0,1,0),(1,1,0) and (0,1,1) from a center zv=10 on the z-axis, followed by its projection on z=0 plane.

$$\begin{bmatrix} x_h \\ y_h \\ z_h \\ h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & z_{vp}/d_p & -z_{vp}\left(z_{prp}/d_p\right) \\ 0 & 0 & 1/d_p & -z_{prp}/d_p \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



# Summary



Planar geometric projections
Parallel
Perspective
Oblique
Orthographic
One point
Two point
Three point
Cavalinear
Cabinet
Other
Top
Side
Front
Other