

Exercise 43

Ex: Use Newton-Raphson method to find the root of the equation  $f(x) = 3x + \sin x - e^x = 0$  which is close to 0.

$$f(x) = 3x + \sin x - e^x; \quad f'(x) = 3 + \cos x - e^x.$$

we begin with  $x_0 = 0$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0 - \frac{(-1)}{3} = 0.33333$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.33333 - \frac{(-0.068418)}{2.54934} = 0.36017$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 0.36017 - \frac{(-6.279 \times 10^{-4})}{2.50226} = 0.3604217.$$

After three iterations, the root is correct to three digits.

Note that the error after three iterations  $(x_3 - x_2)$  is 0.0002517 which is about one-third of the square of the previous error.  
i.e. The previous error  $x_2 - x_1 = 0.02684$ .

$$\text{One-third of square of error} = \frac{(0.02684)^2}{3} = \underline{0.0002401}.$$