Find $sin\ 52^0$ using the following values.

x	$sin45^\circ$	$sin50^\circ$	$sin55^\circ$	$sin60^\circ$
$f(x)$	0.7071	0.7660	0.8192	0.8660

Solution:

x	$\sin x$	Δ	Δ^2	Δ^3
45	0.7071			
		0.0589		
50	0.7660		-0.0057	
		0.0532		-0.0007
55	0.8192		-0.0064	
		0.0468		
60	0.8660			

Here, $h = 5$ & $x = 52°$

$$p = \frac{x - x_0}{h} = \frac{52 - 45}{5} = 1.4°$$

x	$\sin x$	Δ	Δ^2	Δ^3
45	0.7071			
		0.0589		
50	0.7660		-0.0057	
		0.0532		-0.0007
55	0.8192		-0.0064	
		0.0468		
60	0.8660			

Using Newton's Forward

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \ \dots\dots\dots$$

$$y(x) = 0.7071 + 1.4(0.0589) + \frac{(1.4)(0.4)(-0.0057)}{2}$$

$$+ \frac{(1.4)(0.4)(-0.6)(-0.0007)}{6}$$

$$\approx 0.7880$$

Construct Newton's forward interpolation polynomial for the following data. **Use it to find the value of $x = 5$.**

x	4	6	8	10
y	1	3	8	16

Solution:

x	y	Δ	Δ^2	Δ^3
4	1	2	3	0
6	3	5	3	
8	8	8		
10	16			

Here, $h = 2$, $x_0 = 4$ & $x = 5$

x	y	Δ	Δ^2	Δ^3
4	1	2	3	0
6	3	5	3	
8	8	8		
10	16			

$$p = \frac{x - x_0}{h} = \frac{5-4}{2} = 0.5$$

Using Newton's Forward Interpolation formula,

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \ldots\ldots\ldots$$

$$y(x) = 1 + (0.5)(2) + \frac{(0.5)(-0.5)(3)}{2} + \frac{(0.5)(-0.5)(-1.5)(0)}{6}$$

$y(5) \approx 1.625$

Now, Here, $h = 2,\ x_0 = 4$& $x = x, p = \frac{x-x_0}{h} = \frac{x-4}{2}$

Using Newton's Forward Interpolation formula,

$$y(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots \ldots.$$

$$y(x) = 1 + \frac{(x-4)(2)}{2} + \frac{(x-4)}{2}\left(\frac{x-4}{2} - 1\right)\left(\frac{3}{2}\right)$$

$$y(x) = 1 + (x-4) + (x-4)(x-6)\left(\frac{3}{8}\right)$$

$$y(x) = \frac{8 + (8x - 32) + (x^2 - 10x + 24)(3)}{8}$$

$$y(x) = \frac{3x^2 - 22x + 48}{8}$$

The area of circle of diameter d is given by

x	80	85	90	95	100
y	5026	5674	6361	7088	7854

Use suitable interpolation to find area of circle of diameter 98. .Also Calculate the error.

Solution:

d	A	∇	∇^2	∇^3	∇^4
80	5026				
		648			
85	5674		39		
		687		1	
90	6361		40		-2
		727		-1	
95	7088		39		
		766			
100	7854				

here $h = 5,\ x = 98\ \&\ x_n = 100$

$$p = \frac{x - x_n}{h} = \frac{98 - 100}{5} = -0.4$$

d	A	∇	∇^2	∇^3	∇^4
80	5026				
		648			
85	5674		39		
		687		1	
90	6361		40		-2
		727		-1	
95	7088		39		
		766			
100	7854				

Using backward interpolation ,

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \ldots\ldots$$

$$= 7854 + (-0.4)(766) + \frac{(-0.4)(0.6)(39)}{2!} + \frac{(-0.4)(0.6)(1.6)(-1)}{3!} + \frac{(-0.4)(0.6)(1.6)(2.6)(-2)}{4!}$$

$$\boxed{y \approx 7543.0672}$$

Now exact Area= $\pi r^2 = \pi(49)^2 = 7542.9640$

error= $|exact - App| = |7542.9640 - 7543.0672|$

error= 0.1032

The population of the town is given below. Estimate the population for the year 1895 and 1930 using suitable interpolation.

year	1891	1901	1911	1921	1931
Population in thousand	46	66	81	93	101

Solution:

year	Population	Δ	Δ^2	Δ^3	Δ^4
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

here, $h = 10$, $x = 1895$ & $x_0 = 1891$

$$p = \frac{x - x_0}{h} = \frac{1895 - 1891}{10} = 0.4$$

Using Newton's Forward interpolation,

$$y(1895) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots \ldots.$$

$$= 46 + (0.4)(20) + \frac{(0.4)(-0.6)(-5)}{2!} + \frac{(0.4)(-0.6)(-1.6)(2)}{3!}$$

$$+ \frac{(0.4)(-0.6)(-1.6)(-2.6)(-3)}{4!}$$

$$= 54.8528$$

The population of the town is given below. Estimate the population for the year 1895 and 1930 using suitable interpolation.

year	1891	1901	1911	1921	1931
Population in thousand	46	66	81	93	101

Solution:

year	Population	Δ	Δ^2	Δ^3	Δ^4
1891	46				
		20			
1901	66		-5		
		15		2	
1911	81		-3		-3
		12		-1	
1921	93		-4		
		8			
1931	101				

$x = 1930, x_n = 1931 \,\&\, h = 10$

$$p = \frac{x - x_n}{h} = \frac{1930 - 1931}{10} = -0.1$$

Using Newton's Backward interpolation,

$$y(1931) = y_n + p\nabla y_n + \frac{p(p+1)\nabla^2 y_n}{2!} + \cdots$$

$$= 101 + (-0.1)(8) + \frac{(-0.1)(0.9)(-4)}{2!} + \frac{(-0.1)(0.9)(1.9)(-1)}{3!} + \frac{(-0.1)(0.9)(1.9)(2.9)(-3)}{4!}$$

$$= 100.4705$$