Geometric Manipulation

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Geometric Manipulation

Module 2

Module No. 2	Geometric Manipulation	8 hours
• Homogeneous coordinates • Affine transformations (translation, rotation, scaling, shear) • Concatenation • Matrix stacks and • Use of model view matrix in OpenGL for these operations		

What do you understand in this video?

https://www.youtube.com/watch?v=lR0kyOdT86E&feature=youtu.be

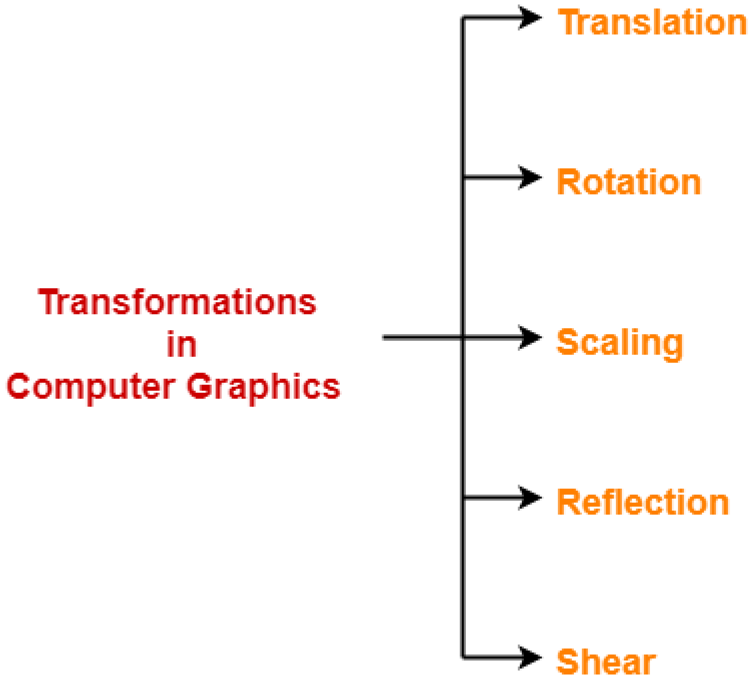
- Introduction to geometric transformations (T,R,S)
- Sets & Staging
- compositing
- Camera movement
- Computer animation

Here are some definitions:

- **Point** – set of values that denotes a location in a space. Example $(1, 1)$, in homogeneous coordinates $(1, 1, 1)$.
- **Vector** – set of values that denotes a direction in a space. Example $(1, 1)$, in homogeneous coordinates $(1, 1, 0)$.
- **Orthogonal vectors** – perpendicular vectors, angle between them is 90°.
- **Line** – set of points that satisfy a linear equation of two variables. Example $x + y = 2$.
- **Line segment** – part of a line that is bounded by two distinct points.
- **Plane** – set of points that satisfy a linear equation of three variables. Example $x + y + z = 2$.
- **Dot product –** also called scalar product and inner product. $v \cdot u = |v| \cdot |u| \cdot cos(\angle uv)$.

Geometric Transformation

- **Transformation** is a process of modifying and re-positioning the existing graphics.
- **Geometric Transformation**: The object itself is transformed relative to the coordinate system or background.
- 2D and 3D transformations



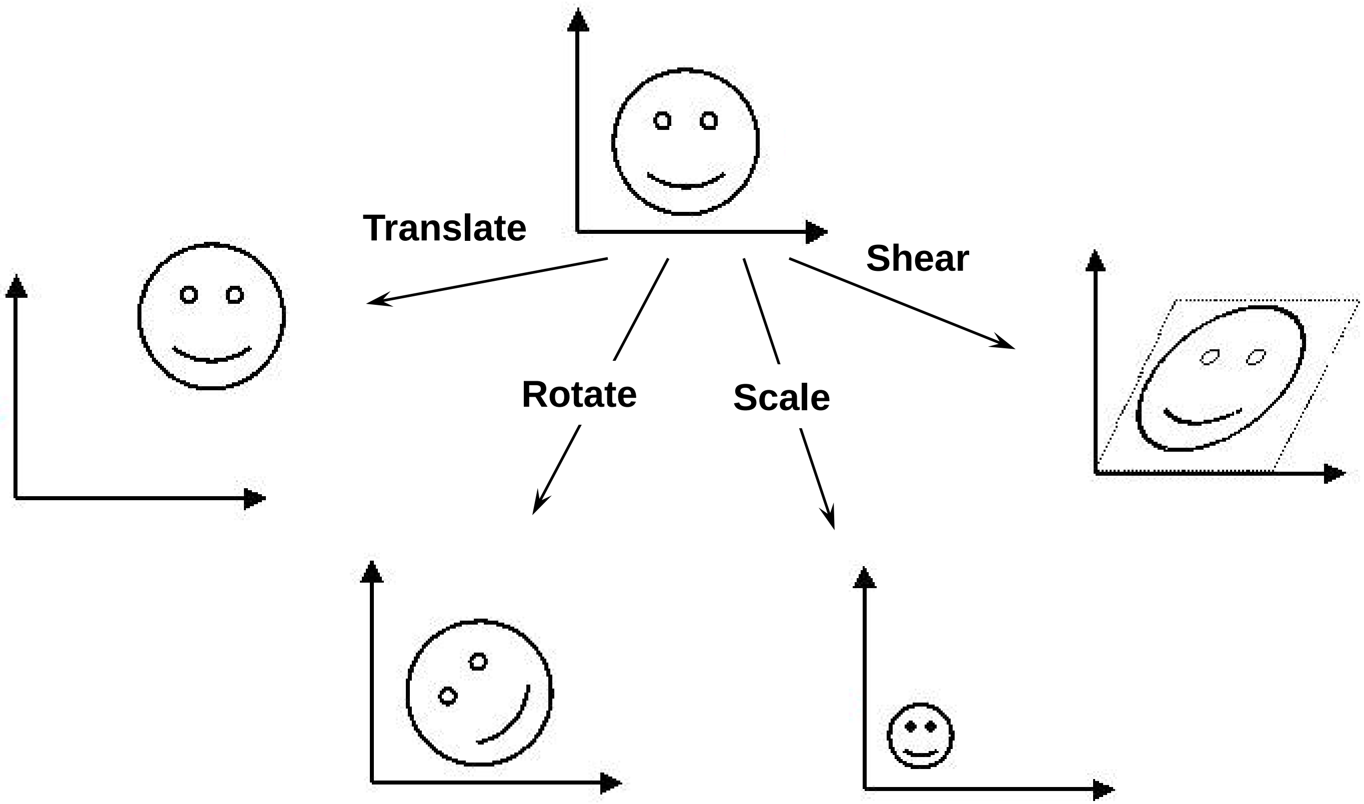
Transformation

- Transformation plays a major role in graphics
- All the transformation can be represented as matrix

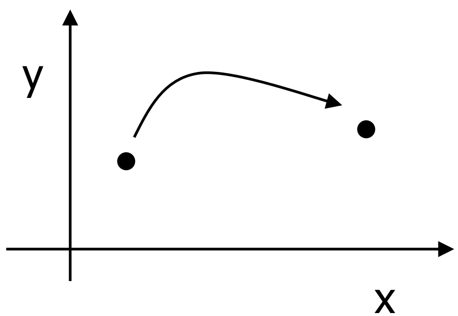
2D transformation

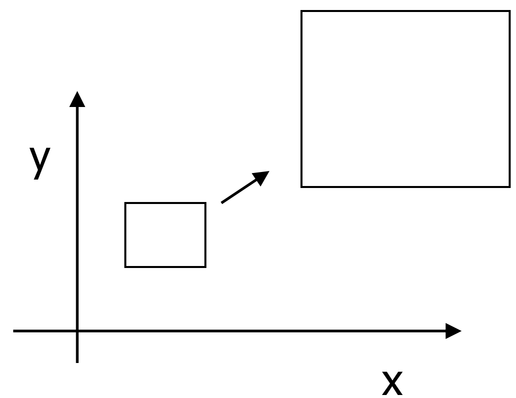
- Considering a 2D object , transformation is to change the object's
  - Positioning(Translation)
  - Orientation(rotation)
  - Size(Scaling)
  - Shapes(Shearing)
  - Mirroring(Reflection)

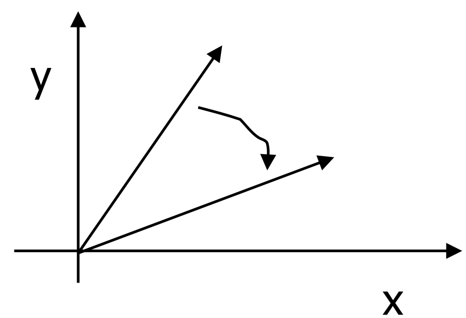
2D Geometrical Transformations



2D Transformations







OpenGl transformations:

glTranslatef (tx, ty, tz);

glRotatef (theta, vx, vy, vz)

glScalef (sx,sy,sz)

……

P5.js

Translate()

Rotate()

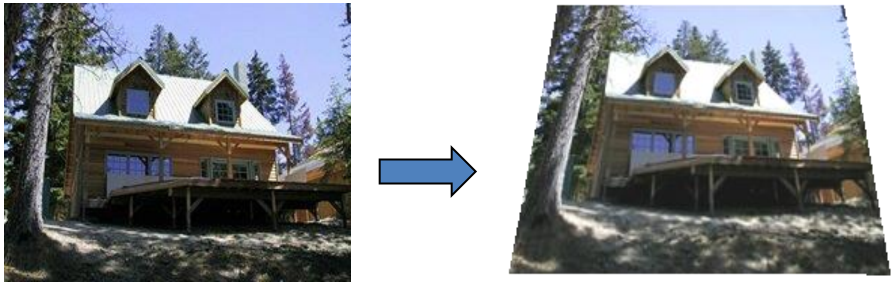
Scale()

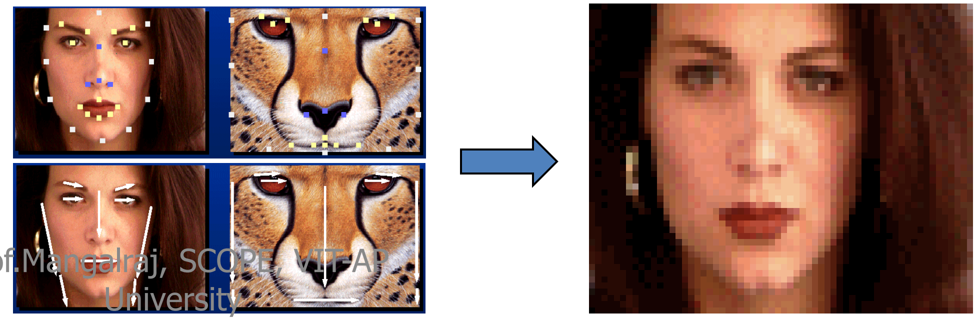
…

Applications of 2D Transformations

- 2D geometric transformations

- Animation
- Image warping
- Face morphing
- Cricket





Translation

- A translation moves an object to a different position on the screen. You can translate a point in 2D by adding translation coordinate (tx, ty) to the original coordinate X,Y to get the new coordinate X′,Y′.

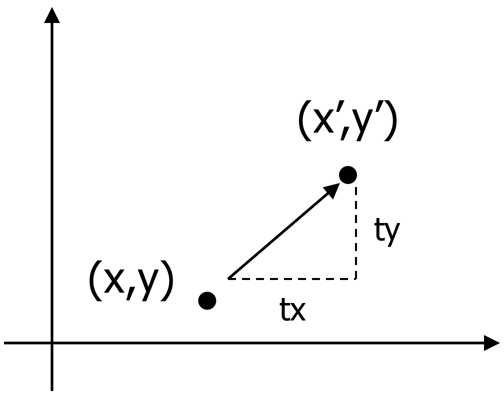
- We translate objects in the (x, y) plane to new positions by adding translation vectors to the coordinates of their vertices.

1. Translation

- Re-position a point along a straight line
- Given a point (x,y), and the translation distance (tx,ty)

The new point: (x', y')

$$x' = x + tx$$
$$y' = y + ty$$

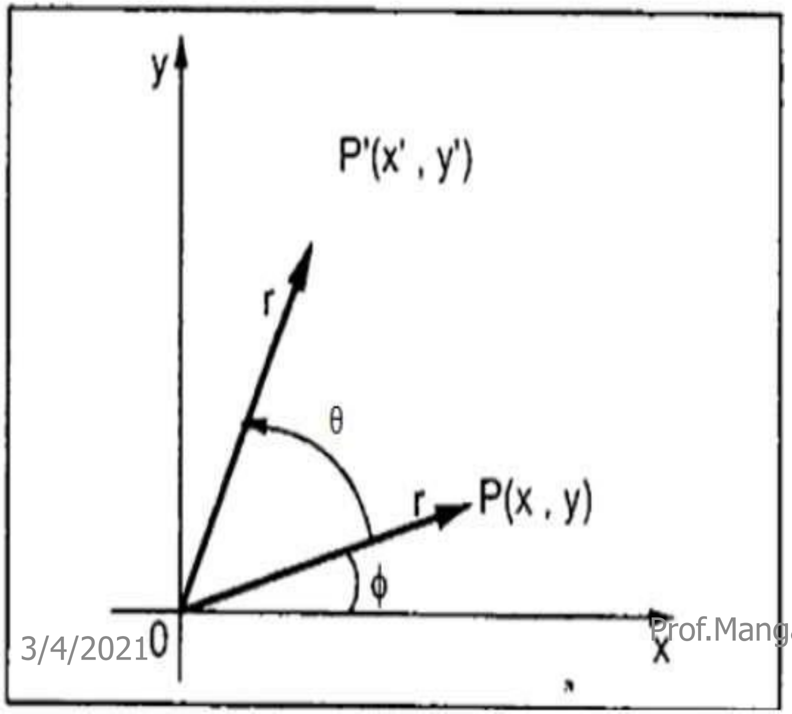


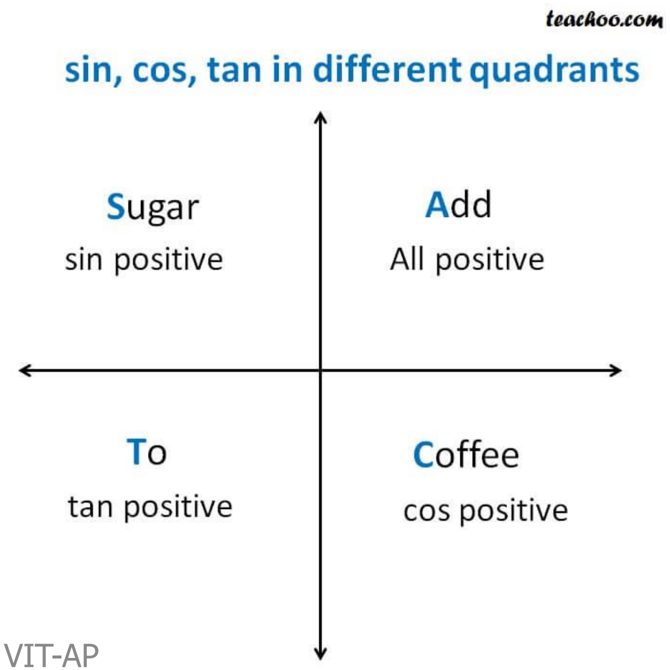
OR $P' = P + T$ where $P' = \begin{vmatrix} x' \\ y' \end{vmatrix}$ $p = \begin{vmatrix} x \\ y \end{vmatrix}$ $T = \begin{vmatrix} tx \\ ty \end{vmatrix}$

2. Rotation

- In rotation, we rotate the object at particular angle θ theta from its origin. From the following figure, we can see that the point P X,Y is located at angle φ from the horizontal X coordinate with distance r from the origin.

- Let us suppose you want to rotate it at the angle θ. After rotating it to a new location, you will get a new point P' X′,Y′.





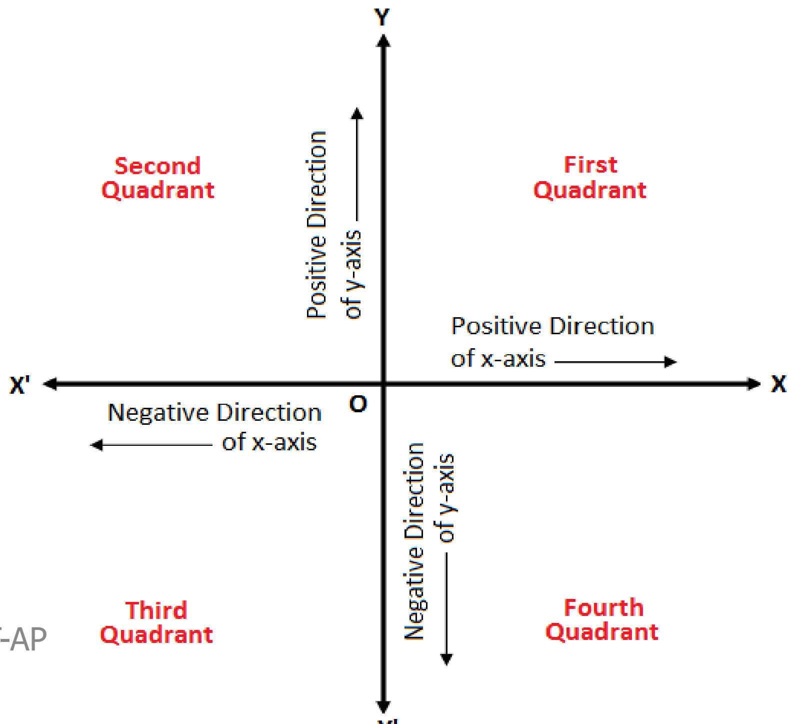
- Using standard trigonometric the original coordinate of point P(X,Y) can be represented as (Clockwise - -ve && anti cloak +ve)
- $X = r\cos\phi$......(1)
- $Y = r\sin\phi$......(2)

Same way we can represent the point P’ X', Y' as −

- $x' = r\cos(\phi+\theta) = r\cos\phi\cos\theta - r\sin\phi\sin\theta$.......(3)
- $y' = r\sin(\phi+\theta) = r\cos\phi\sin\theta + r\sin\phi\cos\theta$.......(4)

Substituting equation 1 & 2 in 3 & 4 respectively, w will get

- $x' = x\cos\theta - y\sin\theta$
- $y' = x\sin\theta + y\cos\theta$



Representing the above equation in matrix form,

$$[X'Y'] = [XY]\begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} OR$$

P' = P . R

Where R is the rotation matrix

$$R = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix}$$

3.Scaling

- To change the size of an object, scaling transformation is used. In the scaling process, you either expand or compress the dimensions of the object.
- Scaling can be achieved by multiplying the original coordinates of the object with the scaling factor to get the desired result.

- Let us assume that the original coordinates are X,Y, the scaling factors are (S_X, S_Y) and the produced coordinates are X′,Y′. This can be mathematically represented as shown below −

$$X' = X \cdot S_X \qquad \text{and} \qquad Y' = Y \cdot S_Y$$

The scaling factor S_X, S_Y scales the object in X and Y direction respectively. The above equations can also be represented in matrix form as below –

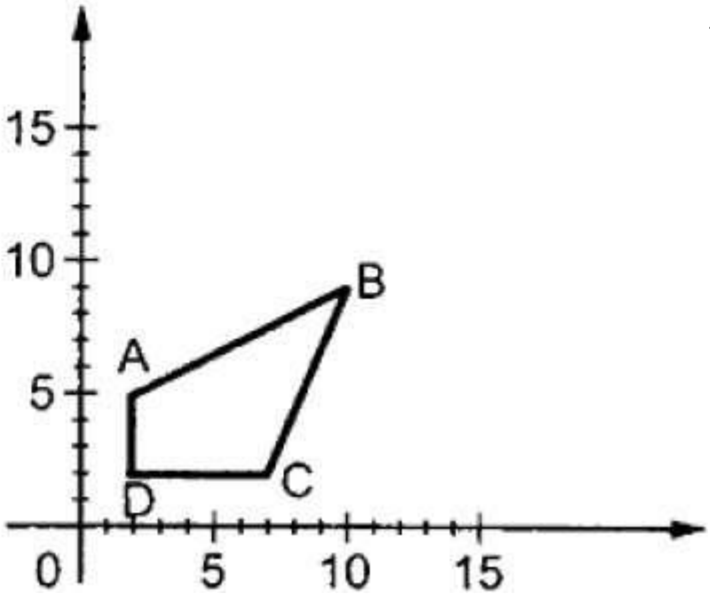
$$\begin{pmatrix} X' \\ Y' \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix} \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

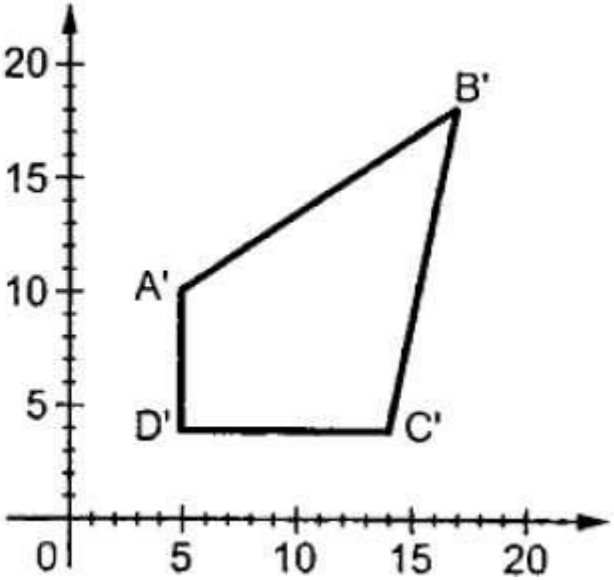
OR

$$\mathbf{P' = P \,.\, S}$$

Where S is the scaling matrix. The scaling process is shown in the following figure.

The scaling process is shown in the following figure.





If we provide values less than 1 to the scaling factor S, then we can reduce the size of the object. If we provide values greater than 1, then we can increase the size of the object.

Review…

- Translate: P' = P+T
- Scale: P' = S.P
- Rotate: P' = R.P

- Spot the odd one out…
  - Multiplying versus adding matrix…
  - Ideally, all transformations would be the same..
    - easier to code
- Solution: Homogeneous Coordinates

Put it all together

- Translation:

$$\begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} x \\ y \end{vmatrix} + \begin{vmatrix} t_x \\ t_y \end{vmatrix}$$

- Rotation:

$$\begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix} \cdot \begin{vmatrix} t_x \\ t_y \end{vmatrix}$$

- Scaling:

$$\begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} s_x & 0 \\ 0 & s_y \end{vmatrix} \cdot \begin{vmatrix} t_x \\ t_y \end{vmatrix}$$

Points to reminder

- Though we are converting 2d projection coordinates into 3d coordinates, the formulae of Rotation, Translation, Scaling should not be changed.

3x3 2D Translation Matrix

$$\begin{vmatrix} x' \\ y' \end{vmatrix} = \begin{vmatrix} x \\ y \end{vmatrix} + \begin{vmatrix} tx \\ ty \end{vmatrix}$$

Use 3 x 1 vector

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

- Note that now it becomes a matrix-vector multiplication
- We are converting to homogenous coordinate system.

Homogenous Coordinates

- If you want to perform sequence of transformation then you have to use 3 X 3 matrix. For.ex. we need to follow a sequential process −
    - Translate the coordinates,
    - Rotate the translated coordinates, and then
    - Scale the rotated coordinates to complete the composite transformation.
- So convert 2X2 tranf.matrix to 3X3 tranf.matric by adding one extra dummy coordinate w.
- In this way, we can represent the point by 3 numbers instead of 2 numbers, which is called Homogenous Coordinate system.
- The homogeneous coordinates representation of (X, Y) is (X, Y, 1).

All the transformation should be treated in a consistent way

- Translation → addition
- Rotation and Scaling → multiplication

All the transformation should be treated in a consistent way

- This can only happen, if all the points are expressed in homogeneous coordinates.
- Finally, in homogeneous coordinates, all the three transformation can be treated as multiplication.

Homogeneous coordinates

- Represents coordinates in 2 dimensions with a vector

$$\begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \longrightarrow \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ 1\,w \end{bmatrix}$$

- Basically we are adding a 3rd coordinate to every 2d point
- Now (x,y) become (x,y,w)

Or, 3x3 Matrix Representations

- Translation:

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

- Rotation:

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

- Scaling:

$$\begin{vmatrix} x' \\ y' \\ 1 \end{vmatrix} = \begin{vmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

Why use 3x3 matrices?

Why Use 3x3 Matrices?

- So that we can perform all transformations using matrix/vector multiplications

- This allows us to *pre-multiply* all the matrices together

- The point (x,y) needs to be represented as

(x,y,1) -> this is called Homogeneous coordinates!

Reference Link

- https://www.youtube.com/watch?v=7VUGSKuCEu4

Take practice test in T,R,S

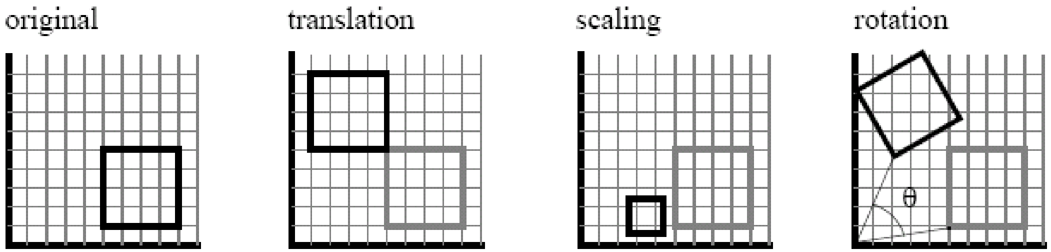
- https://www.khanacademy.org/math/geometry-home/transformations

Day -9

Matrix representations of Translation, rotation and scaling

Translation P’ = T+P

$$= \begin{bmatrix} t_x \\ t_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix}$$



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} d_x \\ d_y \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Rotation P’ = R(θ).P

$$= \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

Scaling P’ = S.P

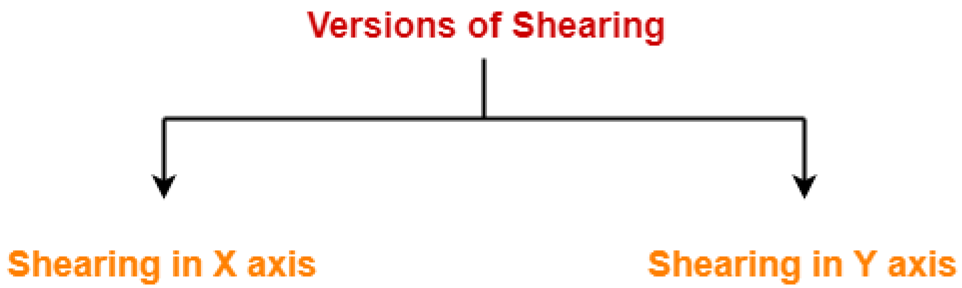
$$= \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

4.Shearing

- 2D Shearing is an ideal technique to change the shape of an existing object in a two dimensional plane.
- There are two shear transformations X-Shear and Y-Shear.
- One shifts X coordinates values and other shifts Y coordinate values. However; in both the cases only one coordinate changes its coordinates and other preserves its values. Shearing is also termed as Skewing.

Shearing

- . In a two dimensional plane, the object size can be changed along X direction as well as Y direction.



- Consider a point object O has to be sheared in a 2D plane.
- Let-
- Initial coordinates of the object O = (X_{old}, Y_{old})
- Shearing parameter towards X direction = Sh_x
- Shearing parameter towards Y direction = Sh_y
- New coordinates of the object O after shearing = (X_{new}, Y_{new})

Shearing in X Axis-

- Shearing in X axis is achieved by using the following shearing equations-
- $X_{new} = X_{old} + Sh_x \times Y_{old}$
- $Y_{new} = Y_{old}$
- In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & Sh_x \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In X axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In X axis)

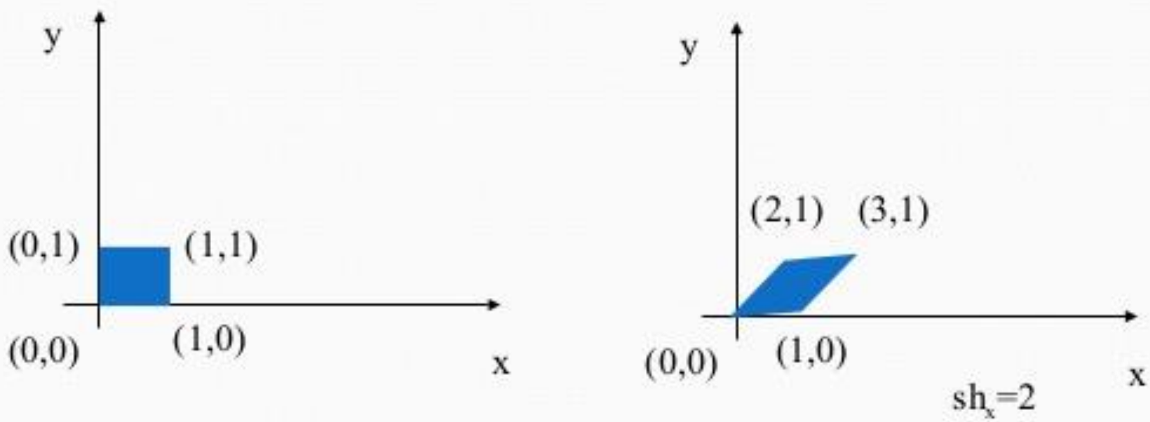
(Homogeneous Coordinates Representation)

X shear

- Preserve Y coordinates but change the X coordinates values

$$x' = x + sh_x \cdot y$$
$$y' = y$$

$$\begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Shearing in Y Axis-

- Shearing in Y axis is achieved by using the following shearing equations-
- $X_{new} = X_{old}$
- $Y_{new} = Y_{old} + Sh_y \times X_{old}$
- In Matrix form, the above shearing equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Sh_y & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Shearing Matrix

(In Y axis)

For homogeneous coordinates, the above shearing matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Shearing Matrix

(In Y axis)

(Homogeneous Coordinates Representation)

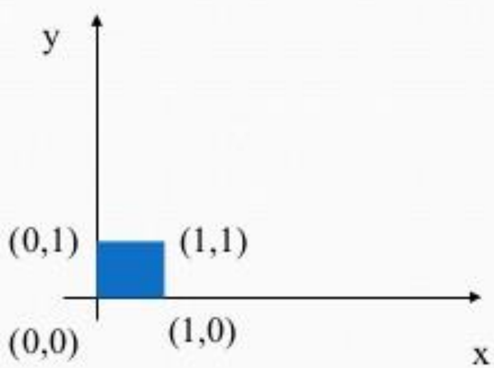
Y shear

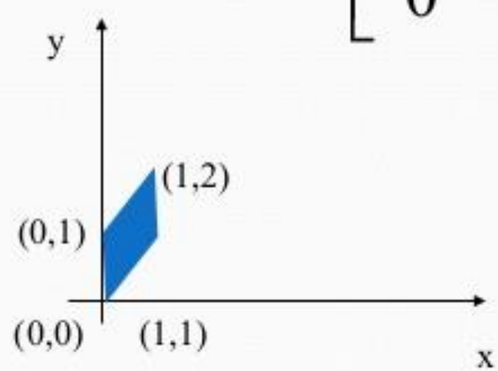
- Preserve X coordinates but change the Y coordinates values

$$x' = x$$

$$y' = y + Sh_y \cdot x$$

$$\begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

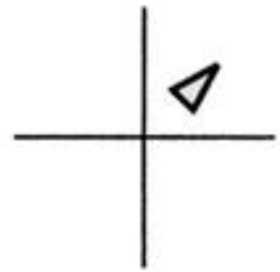
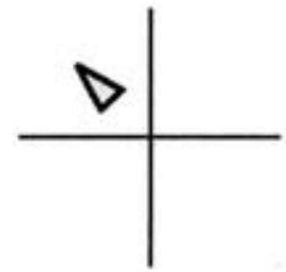
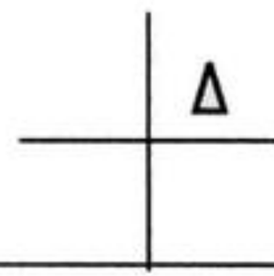
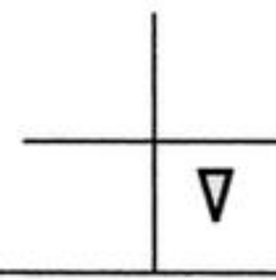
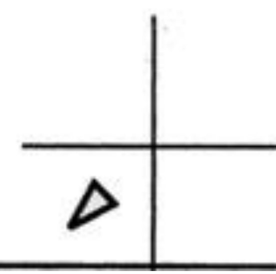




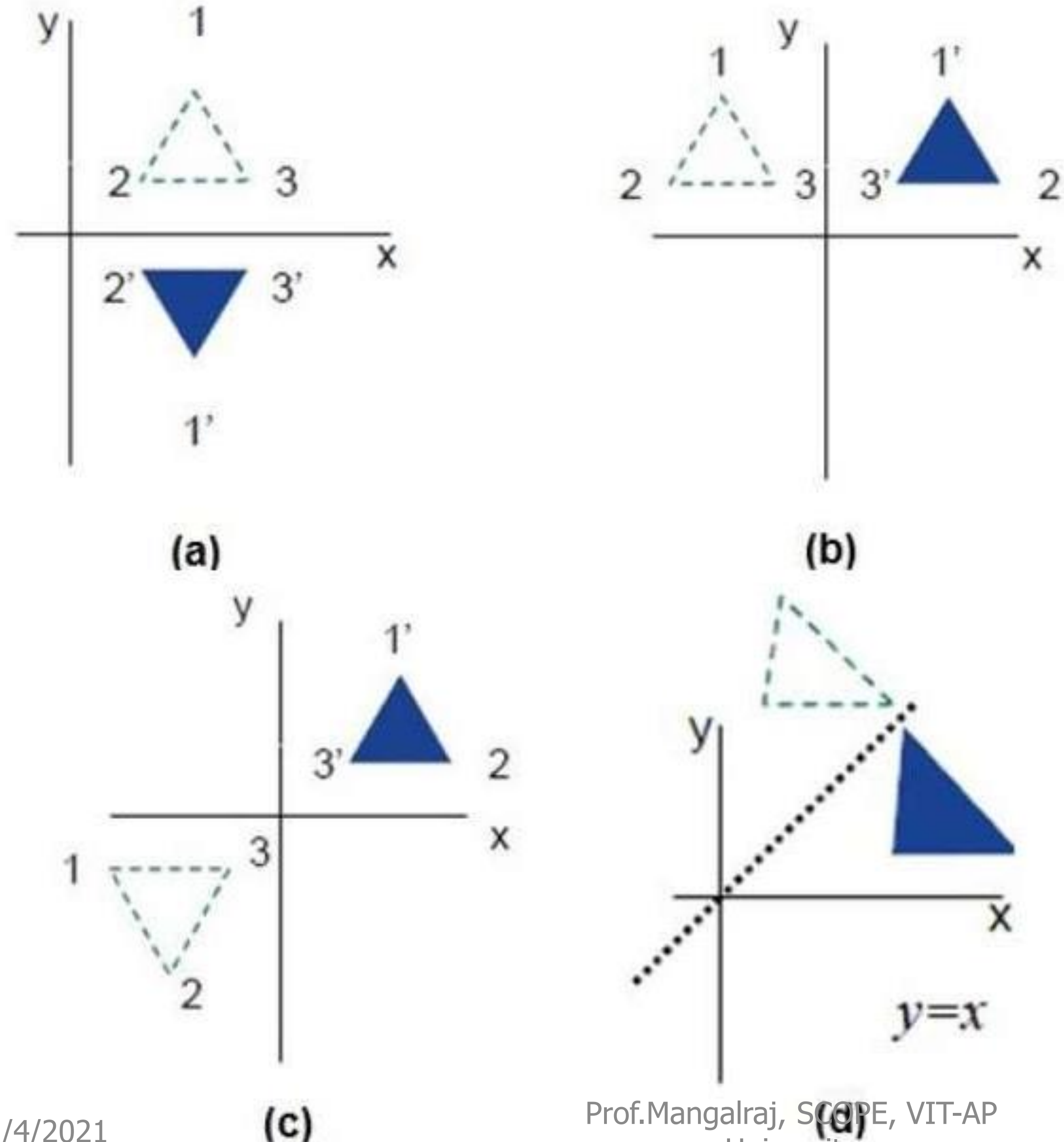
5.Reflections

- Reflection is the mirror image of original object. In other words, we can say that it is a rotation operation with 180°. In reflection transformation, the size of the object does not change.

- The following figures show reflections with respect to X and Y axes, and about the origin respectively.
- **We can represent Reflection by using four ways-**
- **1. Reflection along X-axis:** In this kind of Reflection, the value of X is positive, and the value of Y is negative.
- **2**. **Reflection along Y-axis:** In this kind of Reflection, the value of X is negative, and the value of Y is positive.

Reflection	Transformation matrix	Original image	Reflected image
Reflection about Y axis	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
Reflection about x axis	$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		
Reflection about origin	$\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$		

Guess the reflections ?



a. Ref. about X axis

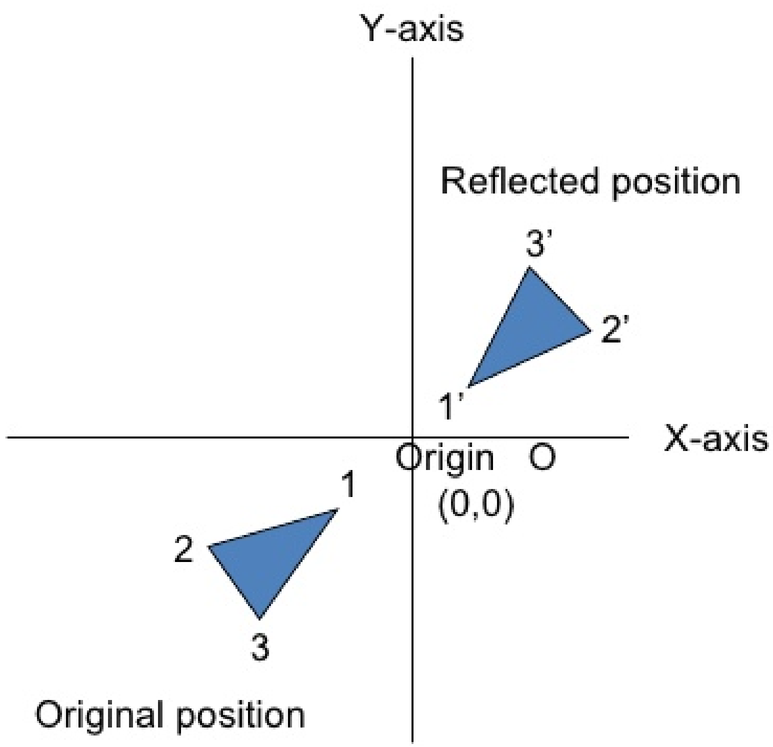
b. Ref.about Y axis

c. Refl. About origin

d. Ref. about xy line

Reflection

Reflection of an object relative to an axis perpendicular to the xy plane and passing through the coordinate origin



$$\begin{vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

The above reflection matrix is the rotation matrix with angle=180 degree.

This can be generalized to any reflection point in the xy plane. This reflection is the same as a 180 degree rotation in the xy plane using the reflection point as the pivot point.

Reflection

- Let-
- Initial coordinates of the object O = (X_{old}, Y_{old})
- New coordinates of the reflected object O after reflection = (X_{new}, Y_{new})
- **Reflection On X-Axis:**
- This reflection is achieved by using the following reflection equations-
- $X_{new} = X_{old}$
- $Y_{new} = -Y_{old}$

In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along X Axis)

(Homogeneous Coordinates Representation)

Reflection On Y-Axis:

- This reflection is achieved by using the following reflection equations-
- Xnew = -Xold
- Ynew = Yold
- In Matrix form, the above reflection equations may be represented as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

For homogeneous coordinates, the above reflection matrix may be represented as a 3 x 3 matrix as-

$$\begin{bmatrix} X_{new} \\ Y_{new} \\ 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \\ 1 \end{bmatrix}$$

Reflection Matrix

(Reflection Along Y Axis)

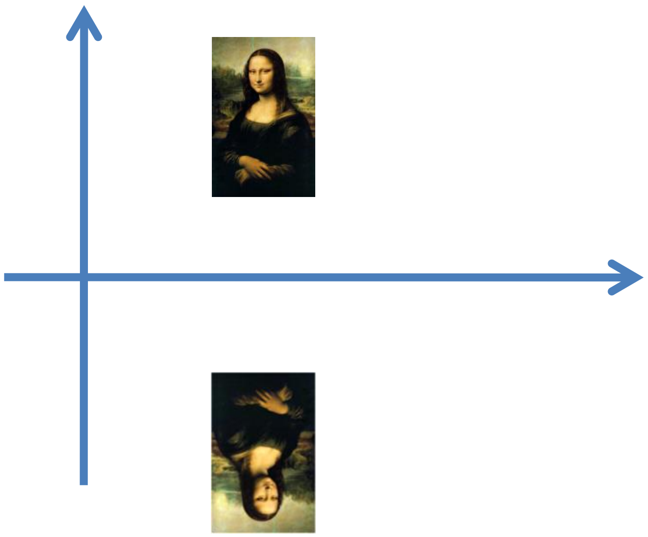
(Homogeneous Coordinates Representation)

Reflection ex.

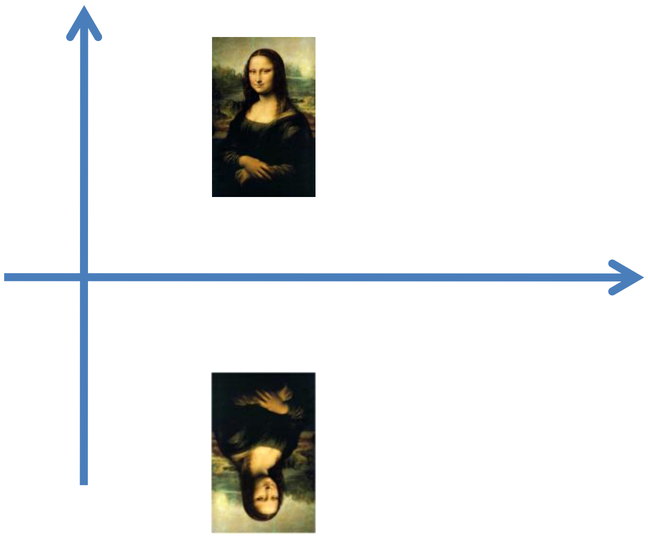
Reflection

Reflection

Reflection about X-axis



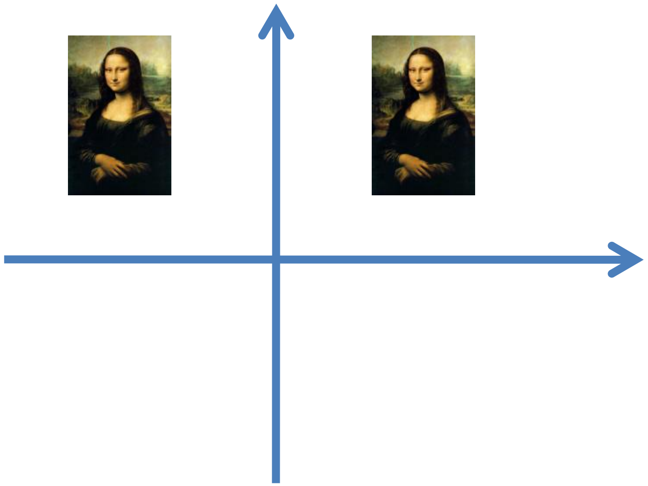
Reflection about X-axis



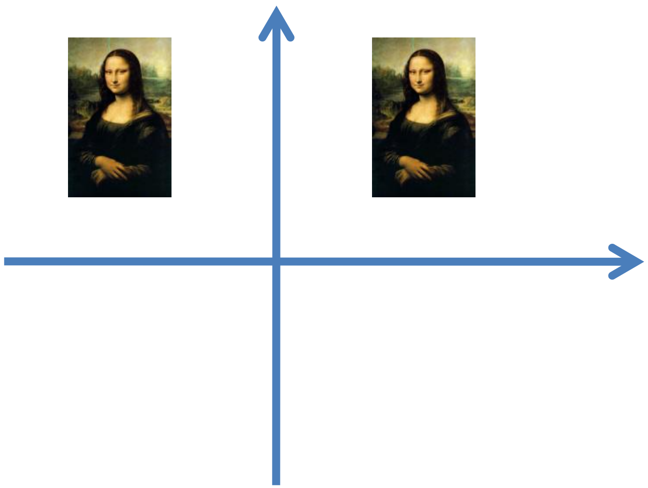
$$\begin{vmatrix} x \\ y \\ 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

Xnew=xold
Ynew=-yold

Reflection about Y-axis



Reflection about Y-axis



$$\begin{vmatrix} x \\ y \\ 1 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} * \begin{vmatrix} x \\ y \\ 1 \end{vmatrix}$$

Xnew=-xold
Ynew=Yold

Refer the link for problems

- http://csis.pace.edu/~marchese/CG/Lect6/cg_l6_part1.htm

- P5.js Documentation☹(must try) – All transformations
- https://p5js.org/reference/#/p5/translate

- https://www.tutorialandexample.com/2d-reflection/

Day 10
PROBLEMS

Probelms

- **Problem-01:**
- Given a circle C with radius 10 and center coordinates (1, 4). Apply the translation with distance 5 towards X axis and 1 towards Y axis. Obtain the new coordinates of C without changing its radius.

Solution-

Given-

- Old center coordinates of C = (X_{old}, Y_{old}) = (1, 4)
- Translation vector = (T_x, T_y) = (5, 1)

Let the new center coordinates of C = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 1 + 5 = 6$
- $Y_{new} = Y_{old} + T_y = 4 + 1 = 5$

Thus, New center coordinates of C = (6, 5).

In matrix form, the new center coordinates of C after translation may be obtained as-

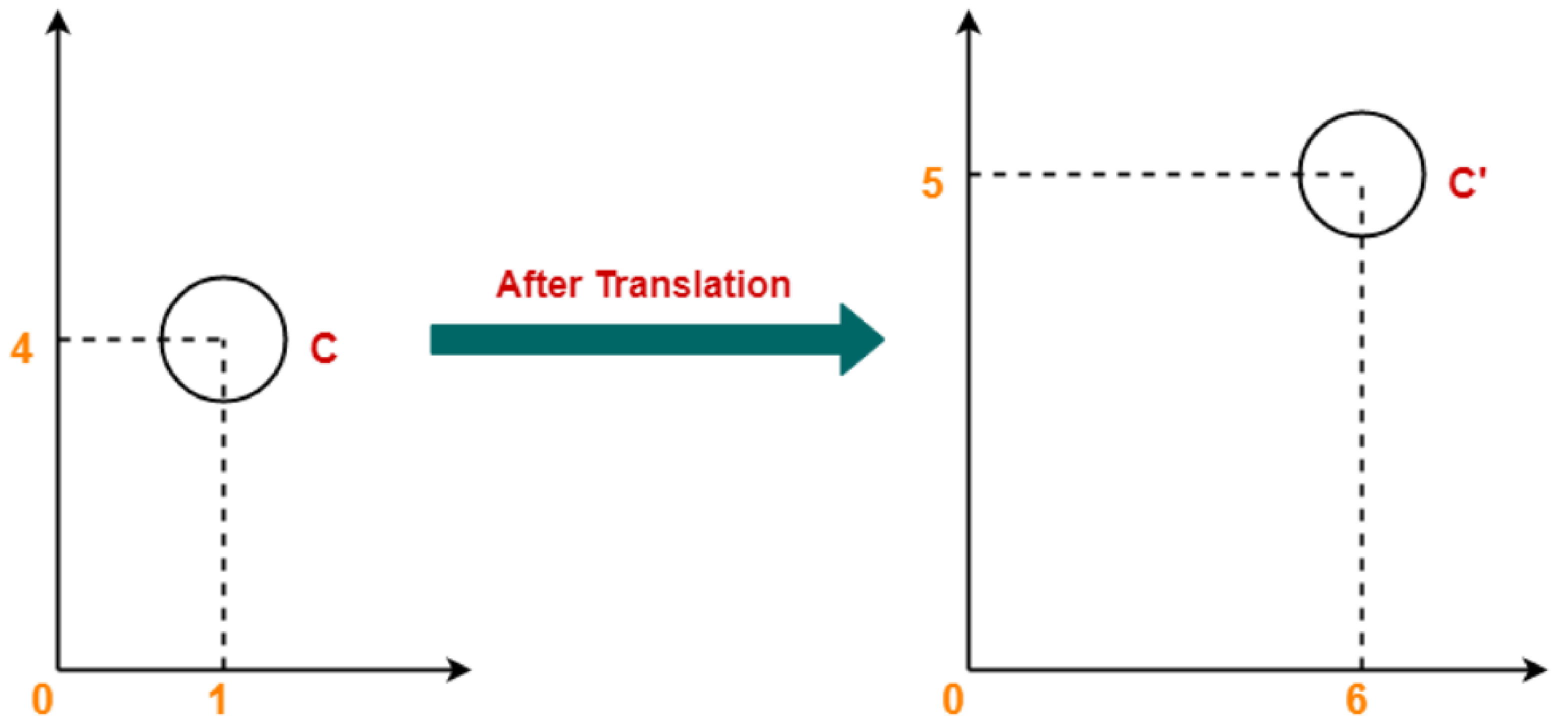
$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

Thus, New center coordinates of C = (6, 5).

Thus, New center coordinates of C = (6, 5).



Problem-02:

- Given a square with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the translation with distance 1 towards X axis and 1 towards Y axis. Obtain the new coordinates of the square.

Given-

- Old coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
- Translation vector = $(T_x, T_y) = (1, 1)$

For Coordinates A(0, 3)

Let the new coordinates of corner A = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

Thus, New coordinates of corner A = (1, 4).

For Coordinates B(3, 3)

Let the new coordinates of corner B = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 3 + 1 = 4$

Thus, New coordinates of corner B = (4, 4).

For Coordinates C(3, 0)

Let the new coordinates of corner C = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 3 + 1 = 4$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$

Thus, New coordinates of corner C = (4, 1).

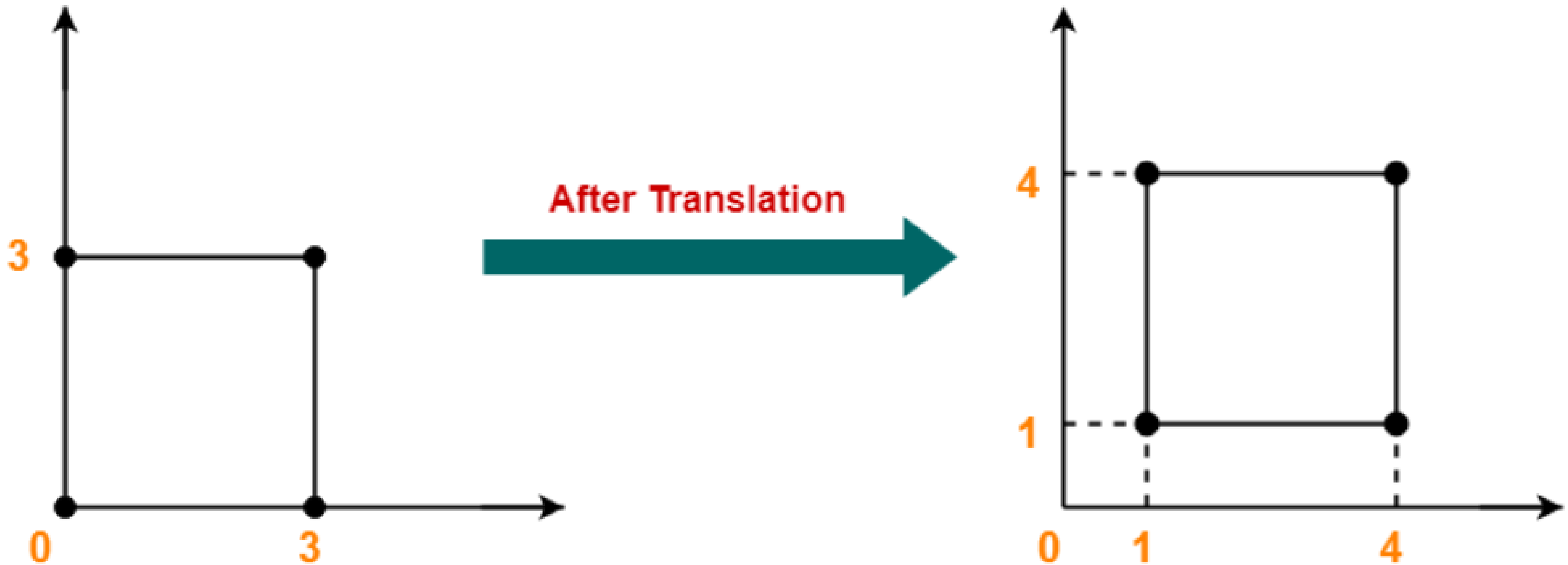
For Coordinates D(0, 0)

Let the new coordinates of corner D = (X_{new}, Y_{new}).

Applying the translation equations, we have-

- $X_{new} = X_{old} + T_x = 0 + 1 = 1$
- $Y_{new} = Y_{old} + T_y = 0 + 1 = 1$

Thus, New coordinates of corner D = (1, 1).


After Translation
3
0
3
4
1
0
1
4

Problem-03:

- Given a line segment with starting point as (0, 0) and ending point as (4, 4). Apply 30 degree rotation anticlockwise direction on the line segment and find out the new coordinates of the line.

Given-

- Old ending coordinates of the line = (X_{old}, Y_{old}) = (4, 4)
- Rotation angle = $\theta = 30^o$

We rotate a straight line by its end points with the same angle. Then, we re-draw a line between the new end points.

- Let new ending coordinates of the line after rotation = (X_{new}, Y_{new}).

- Applying the rotation equations, we have-

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \times \begin{bmatrix} X_{old} \\ Y_{old} \end{bmatrix}$$

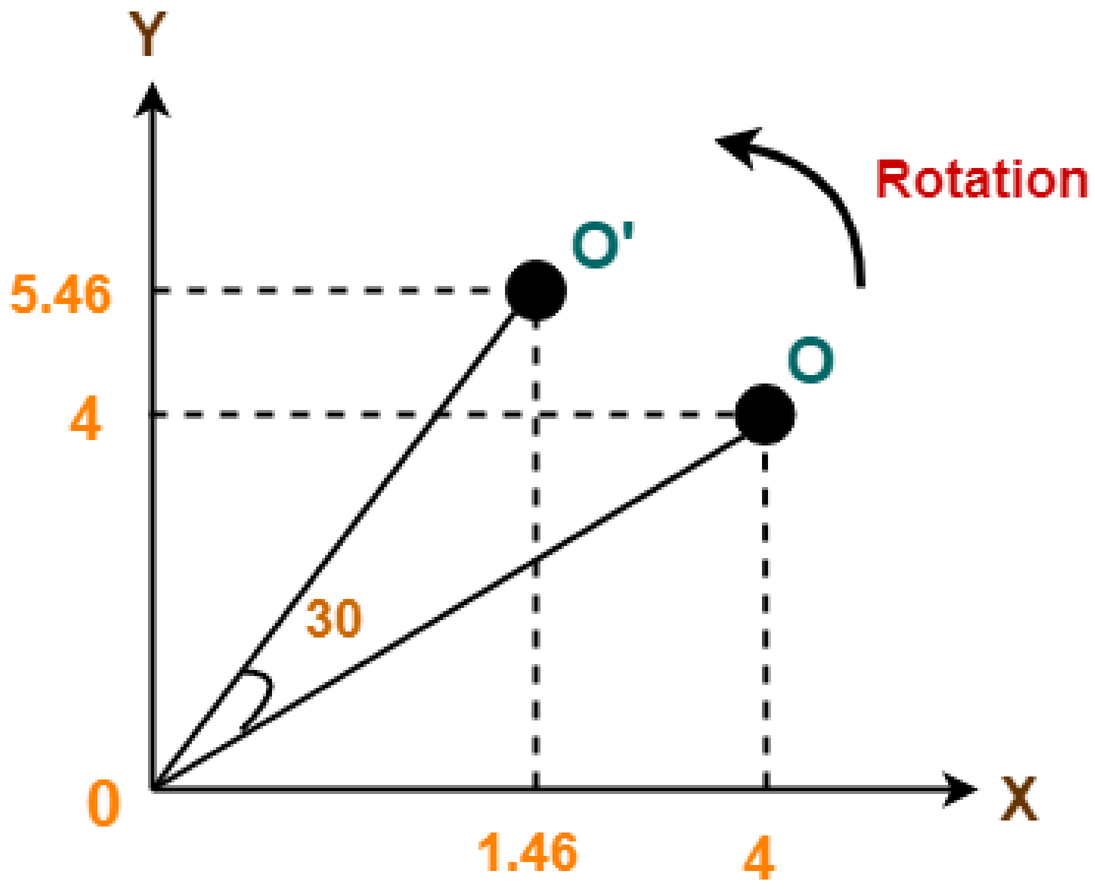
$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \times \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 4 \times \cos 30 - 4 \times \sin 30 \\ 4 \times \sin 30 + 4 \times \cos 30 \end{bmatrix}$$

$$\begin{bmatrix} X_{new} \\ Y_{new} \end{bmatrix} = \begin{bmatrix} 1.46 \\ 5.46 \end{bmatrix}$$

Thus, New ending coordinates of the line after rotation = (1.46, 5.46).



Problem 04:

Given a triangle with corner coordinates (0, 0), (1, 0) and (1, 1). Rotate the triangle by 90 degree anticlockwise direction and find out the new coordinates.

Given:

We rotate a polygon by rotating each vertex of it with the same rotation angle.

Given-

- Old corner coordinates of the triangle = A (0, 0), B(1, 0), C(1, 1)
- Rotation angle = $\theta = 90^o$

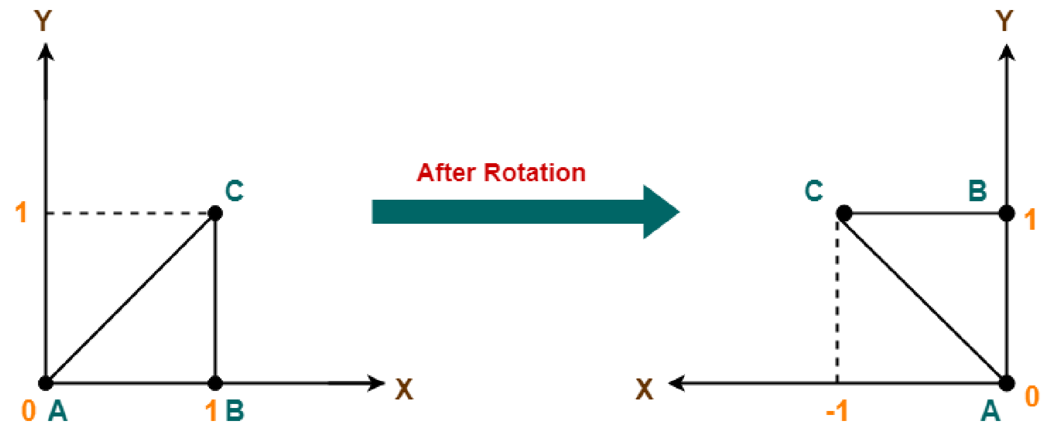
- $x' = x\cos\theta - y\sin\theta$
- $y' = x\sin\theta + y\cos\theta$

- **For Coordinates A(0, 0)**
- Let the new coordinates of corner A after rotation = (X_{new}, Y_{new}).
- Applying the rotation equations, we have-
- X_{new}
- $= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $= 0 \times \cos 90^o - 0 \times \sin 90^o$
- $= 0$
- Y_{new}
- $= X_{old} \times \sin\theta + Y_{old} \times \cos\theta$
- $= 0 \times \sin 90^o + 0 \times \cos 90^o$
- $= 0$
- Thus, New coordinates of corner A after rotation = (0, 0).

- **For Coordinates B(1, 0)**
- Let the new coordinates of corner B after rotation = (X_{new}, Y_{new}).
- X_{new}
- $= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $= 1 \times \cos 90^o - 0 \times \sin 90^o$
- $= 0$
- Y_{new}
- $= X_{old} \times \sin\theta + Y_{old} \times \cos\theta$
- $= 1 \times \sin 90^o + 0 \times \cos 90^o$
- $= 1 + 0$
- $= 1$
- Thus, New coordinates of corner B after rotation = (0, 1).

- **<u>For Coordinates C(1, 1)</u>**
- Let the new coordinates of corner C after rotation = (X_{new}, Y_{new}).
- X_{new}
- $= X_{old} \times \cos\theta - Y_{old} \times \sin\theta$
- $= 1 \times \cos 90^o - 1 \times \sin 90^o$
- $= 0 - 1$
- $= -1$
- Y_{new}
- $= X_{old} \times \sin\theta + Y_{old} \times \cos\theta$
- $= 1 \times \sin 90^o + 1 \times \cos 90^o$
- $= 1 + 0$
- $= 1$
- Thus, New coordinates of corner C after rotation = (-1, 1).

- Thus, New coordinates of the triangle after rotation = A (0, 0), B(0, 1), C(-1, 1).



Problem-05:

- Given a square object with coordinate points A(0, 3), B(3, 3), C(3, 0), D(0, 0). Apply the scaling parameter 2 towards X axis and 3 towards Y axis and obtain the new coordinates of the object.

Solution-

Given-

- Old corner coordinates of the square = A (0, 3), B(3, 3), C(3, 0), D(0, 0)
- Scaling factor along X axis = 2
- Scaling factor along Y axis = 3

For Coordinates A(0, 3)

Let the new coordinates of corner A after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner A after scaling = (0, 9).

For Coordinates B(3, 3)

Let the new coordinates of corner B after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 3 \times 3 = 9$

Thus, New coordinates of corner B after scaling = (6, 9).

For Coordinates C(3, 0)

Let the new coordinates of corner C after scaling = (X_{new}, Y_{new}).

Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 3 \times 2 = 6$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner C after scaling = (6, 0).

For Coordinates D(0, 0)

Let the new coordinates of corner D after scaling = (X_{new}, Y_{new}).

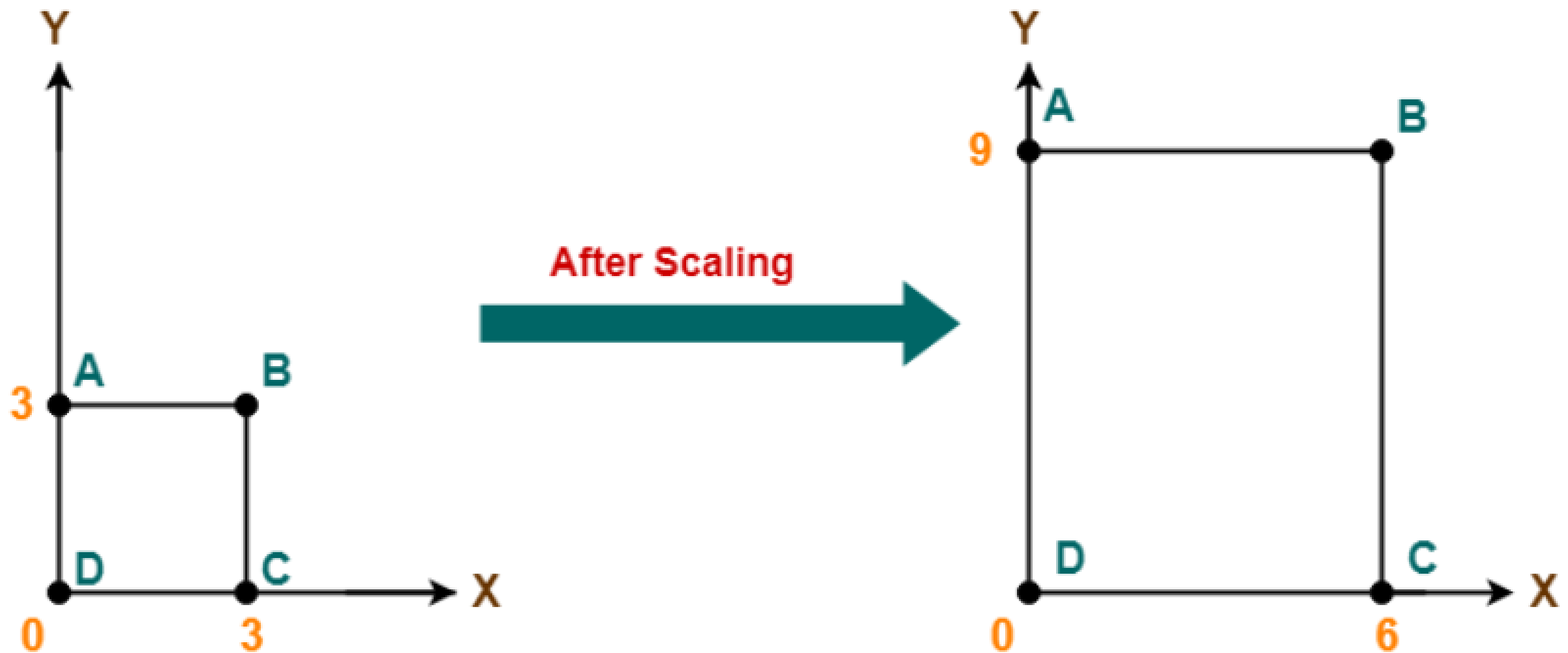
Applying the scaling equations, we have-

- $X_{new} = X_{old} \times S_x = 0 \times 2 = 0$
- $Y_{new} = Y_{old} \times S_y = 0 \times 3 = 0$

Thus, New coordinates of corner D after scaling = (0, 0).

Thus, New coordinates of the square after scaling = A (0, 9), B(6, 9), C(6, 0), D(0, 0).

Thus, New coordinates of the square after scaling = A (0, 9), B(6, 9), C(6, 0), D(0, 0).



Reflecion : Problem-06:

- Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the X axis and obtain the new coordinates of the object.

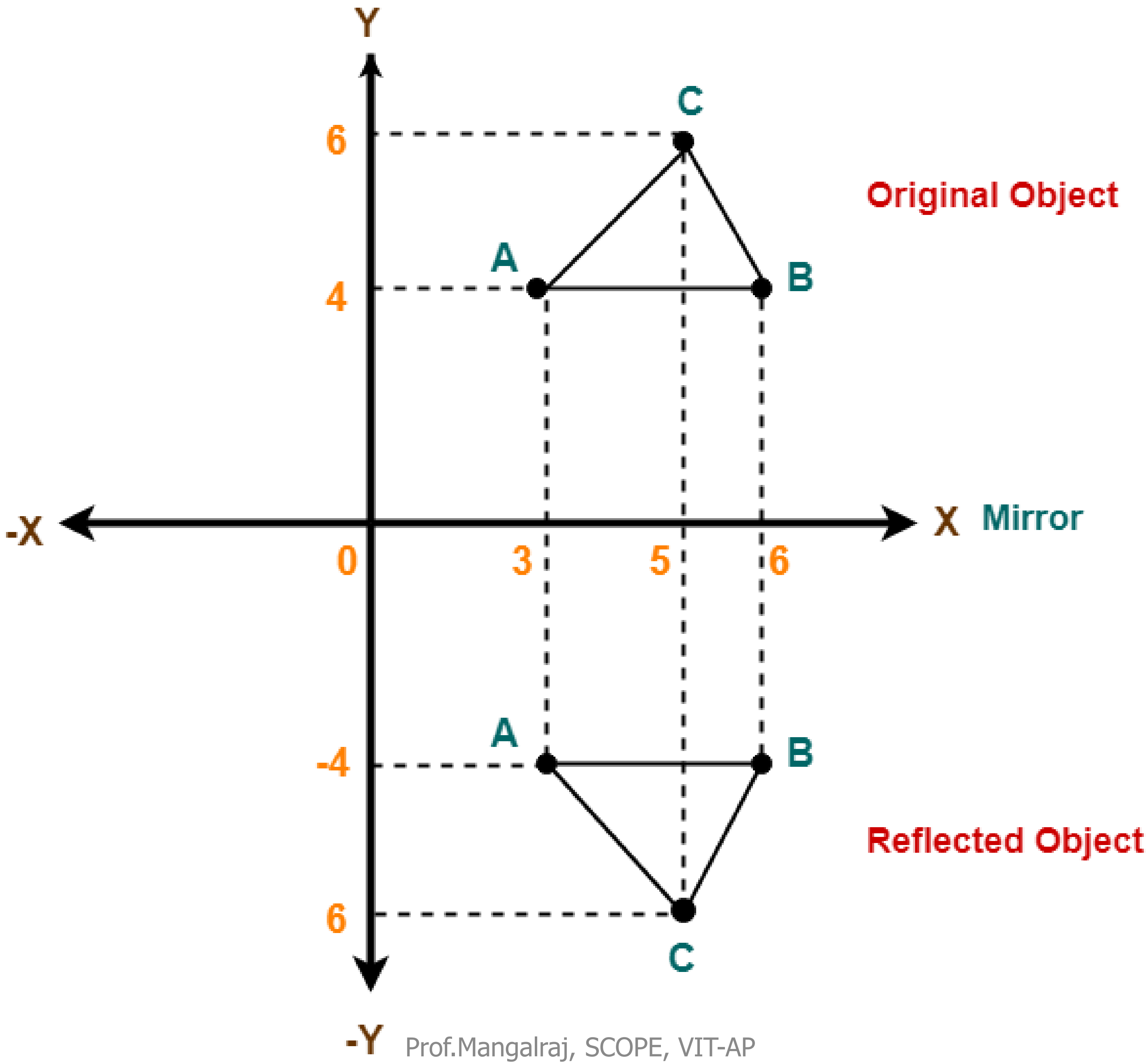
- **Solution-**

Given-

- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the X axis

- **For Coordinates A(3, 4)**
- Let the new coordinates of corner A after reflection = (X_{new}, Y_{new}).
- Applying the reflection equations, we have-
- $X_{new} = X_{old} = 3$
- $Y_{new} = -Y_{old} = -4$
- Thus, New coordinates of corner A after reflection = (3, -4).
- **For Coordinates B(6, 4)**
- Let the new coordinates of corner B after reflection = (X_{new}, Y_{new}).
- Applying the reflection equations, we have-
- $X_{new} = X_{old} = 6$
- $Y_{new} = -Y_{old} = -4$
- Thus, New coordinates of corner B after reflection = (6, -4).

- **For Coordinates C(5, 6)**
- Let the new coordinates of corner C after reflection = (X_{new}, Y_{new}).
- Applying the reflection equations, we have-
- $X_{new} = X_{old} = 5$
- $Y_{new} = -Y_{old} = -6$
- Thus, New coordinates of corner C after reflection = (5, -6).
- Thus, New coordinates of the triangle after reflection = A (3, -4), B(6, -4), C(5, -6).

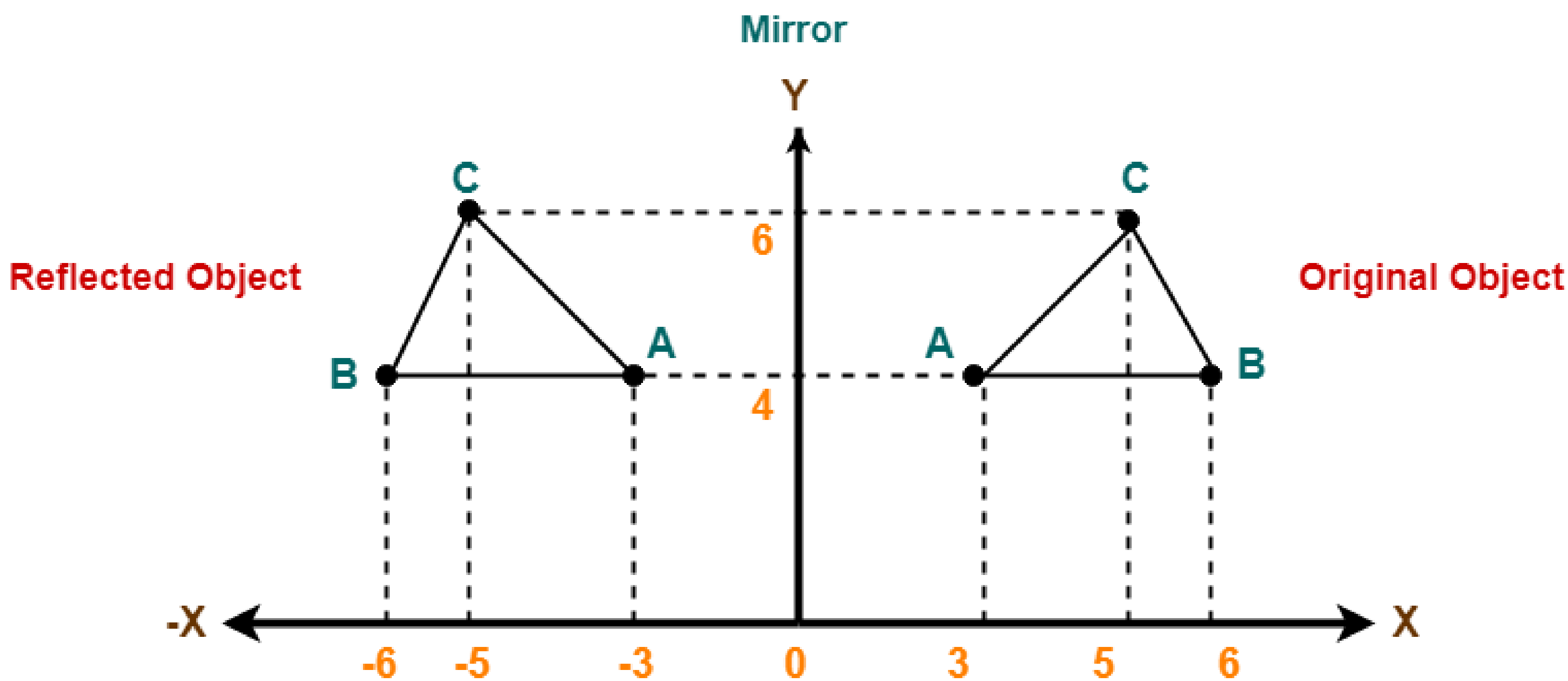

Y
C
6
Original Object
A
4
B
-X
X
Mirror
0
3
5
6
A
-4
B
Reflected Object
6
C
-Y

Problem-07:

- Given a triangle with coordinate points A(3, 4), B(6, 4), C(5, 6). Apply the reflection on the Y axis and obtain the new coordinates of the object.

- **Solution-**
- Given-
- Old corner coordinates of the triangle = A (3, 4), B(6, 4), C(5, 6)
- Reflection has to be taken on the Y axis

- Thus, New coordinates of the triangle after reflection = A (-3, 4), B(-6, 4), C(-5, 6).



Shearing: Problem-08:

- Given a triangle with points (1, 1), (0, 0) and (1, 0). Apply shear parameter 2 on X axis and 2 on Y axis and find out the new coordinates of the object.

- **Solution-**

Given-

- Old corner coordinates of the triangle = A (1, 1), B(0, 0), C(1, 0)
- Shearing parameter towards X direction (Sh_x) = 2
- Shearing parameter towards Y direction (Sh_y) = 2

Shearing in X Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 1 = 3$
- $Y_{new} = Y_{old} = 1$

Thus, New coordinates of corner A after shearing = (3, 1).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 0 + 2 \times 0 = 0$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} + Sh_x \times Y_{old} = 1 + 2 \times 0 = 1$
- $Y_{new} = Y_{old} = 0$

Thus, New coordinates of corner C after shearing = (1, 0).

Thus, New coordinates of the triangle after shearing in X axis = A (3, 1), B(0, 0), C(1, 0).

Shearing in Y Axis-

For Coordinates A(1, 1)

Let the new coordinates of corner A after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 1 + 2 \times 1 = 3$

Thus, New coordinates of corner A after shearing = (1, 3).

For Coordinates B(0, 0)

Let the new coordinates of corner B after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 0$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 0 = 0$

Thus, New coordinates of corner B after shearing = (0, 0).

For Coordinates C(1, 0)

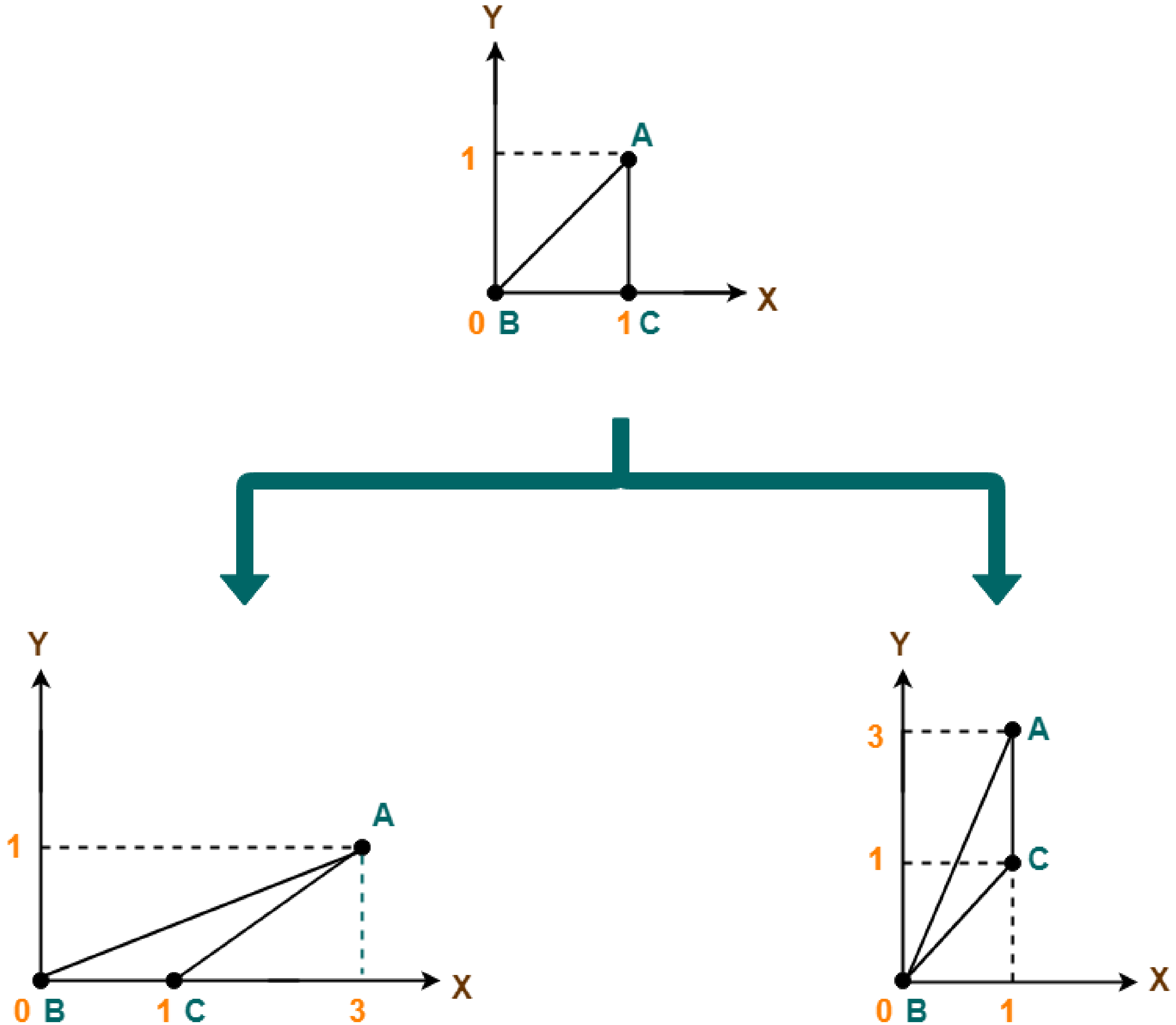
Let the new coordinates of corner C after shearing = (X_{new}, Y_{new}).

Applying the shearing equations, we have-

- $X_{new} = X_{old} = 1$
- $Y_{new} = Y_{old} + Sh_y \times X_{old} = 0 + 2 \times 1 = 2$

Thus, New coordinates of corner C after shearing = (1, 2).

Thus, New coordinates of the triangle after shearing in Y axis = A (1, 3), B(0, 0), C(1, 2).



Shearing in X Axis

Shearing in Y Axis

Processing example