ΤΜΗΜΑ ΠΛΗΡΟΦΟΡΙΚΗΣ & ΤΗΛΕΠΙΚΟΙΝΩΝΙΩΝ

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Βασικές Μαθηματικές Έννοιες στη Γλωσσική Τεχνολογία

Project 2

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Question **1**

The composite function $S(f(x))$, where $S(x) = \dfrac{1}{1+e^{-x}}$ and $f(x) = ax + b$, is calculated as follows:

$$S(f(x)) = \frac{1}{1+e^{-f(x)}} = \frac{1}{1+e^{-(ax+b)}} = \frac{1}{1+\frac{1}{e^{(ax+b)}}} = \frac{1}{\frac{e^{(ax+b)}+1}{e^{(ax+b)}}} = \frac{e^{(ax+b)}}{e^{(ax+b)}+1} = \frac{e^{ax}e^{b}}{e^{ax}e^{b}+1}$$

where:

- $x \in Dom(f) = \mathbb{R}$
- $f(x) \in Im(f) \subseteq Dom(S) = \mathbb{R}$
- $S(f(x)) \in Im(S)$

Question **2**

A function is **invertible** only if each input has a unique output, which means each output is paired with exactly one input. That way, when the mapping is reversed, it will still be a function.

Definition
Let f be a function whose domain is the set A and whose codomain is the set B ($f : A \to B$). Then we say that f is invertible if f is **one-to-one** mapping and there is a function g with domain $Im(f) \subseteq B$ and image (range) A ($g : Im(f) \to A$) such that:

$$f(x) = y \iff g(y) = x$$

In this case, we call g the inverse of f and denote it by f^{-1}.

i) $\mathbf{f(x) = \sqrt{x-3}}$

- $x - 3 \geq 0 \iff x \geq 3 \iff Dom(f) = [\ 3,\ +\infty\ )$ **(1)**
- f is **continuous** on $Dom(f) = [\ 3,\ +\infty\ )$ [1] **(2)**
- f is **differentiable** on $A = \{\ Dom(f) - \{0\}\ \} = \{\ [\ 3,\ +\infty\ ) - \{0\}\ \}$

$= \{\ (\ 3,\ +\infty\ )\ \}$ [2] **(3)**

First, we have to prove that f is **one-to-one** mapping:

$$\overset{(2),\ (3)}{\Longrightarrow} f'(x) = (\ \sqrt{x-3}\ )' = \frac{1}{2\sqrt{x-3}} > 0,\ \forall\ x \in A = (3,\ +\infty) \quad \textbf{(4)}$$

$\overset{(4)}{\Longrightarrow} f$ is **strictly monotonic** (strictly inscreasing), therefore it is **one-to-one** mapping

One approach for finding a formula for f^{-1} is to solve $f(x) = y$ for x.

$$f(x) = \sqrt{x-3} = y \overset{y \geq 0,\ x \geq 3}{\iff} x - 3 = y^2 \iff x = y^2 + 3 = f^{-1}(y) \quad \textbf{(5)}$$

$$\overset{(1)}{\Longrightarrow} x \geq 3 \iff y^2 + 3 \geq 3 \iff y^2 \geq 0,\ \ true\ \forall\ y \in \mathbb{R} \quad \textbf{(6)}$$

$$\overset{(5),\ (6)}{\Longrightarrow} Im(f) = \{\ \mathbb{R}\ \cap\ [\ 0,\ +\infty\ )\ \} = [\ 0,\ +\infty\ ) = Dom(f')$$

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[1] as a composition of continuous functions

[2] as a composition of differential functions

• $f : [\ 3,\ +\infty\ ] \to \mathbb{R}$
$f(x) = \sqrt{x-3}$

• $f^{-1} : [\ 0,\ +\infty\ ] \to \mathbb{R}$
$f^{-1} = x^2 + 3$

ii) f(x) = log(x − 2)

• $x - 2 > 0 \iff x > 2 \iff Dom(f) = (\ 2,\ +\infty\ )$ **(7)**

First, we have to prove that f is **one-to-one** mapping:

$Let\ x_1,\ x_2\ \in\ Dom(f) = (\ 0,\ +\infty\ ),\ x_1 \neq x_2$

$f(x_1) = f(x_2) \iff log(x_1 - 2) = log(x_2 - 2) \iff x_1 - 2 = x_2 - 2 \iff x_1 = x_2$

$\iff x_1 = x_2 \iff f$ is an **one-to-one** mapping of $Dom(f)$ to $Im(f)$

As in i above, we will solve $f(x) = y$ for x.

$f(x) = \log_2(x-2) = y \iff 2^{log_2(x-2)} = 2^y \iff x - 2 = 2^y$
$\iff x = 2^y + 2 = f^{-1}(y)$ **(8)**

$\overset{(7)}{\Longrightarrow} x > 2 \iff 2^y + 2 > 2 \iff 2^y > 0,\ \ true\ \forall\ y \in \mathbb{R}$ **(9)**
$\overset{(9)}{\Longrightarrow} Im(f) = \mathbb{R} = Dom(f')$

• $f : (\ 2,\ +\infty\ ] \to \mathbb{R}$
$f(x) = log(x-2)$

• $f^{-1} : \mathbb{R} \to \mathbb{R}$
$f^{-1} = 2^y + 2$

Question **3**

$s = A\ cos(\ 2\pi ft\ )$

(a) $A_1 = 1, \quad f = 1, \quad \theta = 0, \quad s_1 = cos(\ 2\pi t\ )$
(b) $A_2 = 2, \quad f = 3, \quad \theta = 0, \quad s_2 = 2\ cos(\ 6\pi t\ )$
(c) $A_1 = 1.5, \quad f = 2, \quad \theta = \pi, \quad s_3 = 1.5\ cos(\ 4\pi t\ +\ \pi)$
(d) $A_1 = 2, \quad f = 0.5, \quad \theta = 0, \quad s_4 = 2\ cos(\ \pi t\ )$

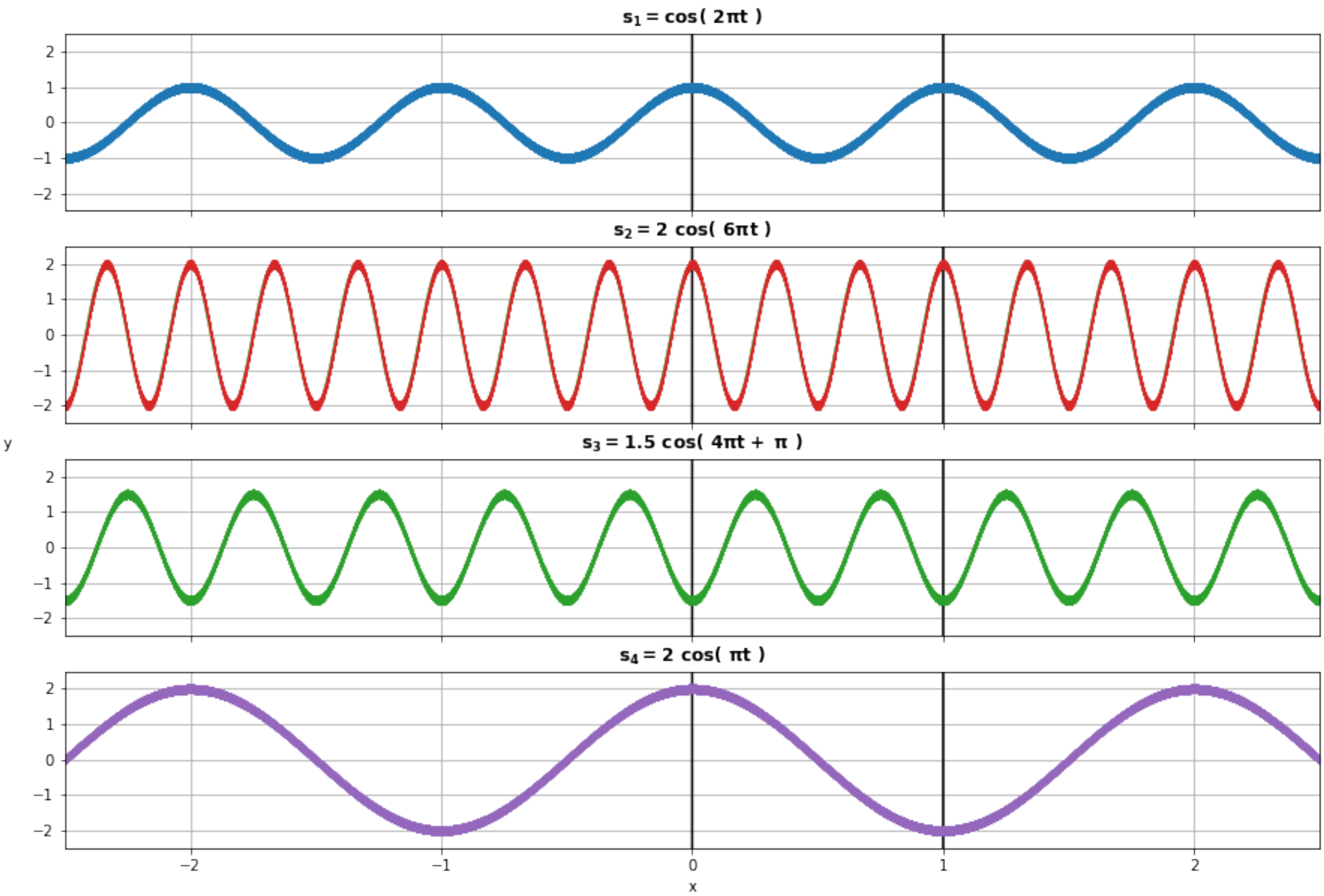
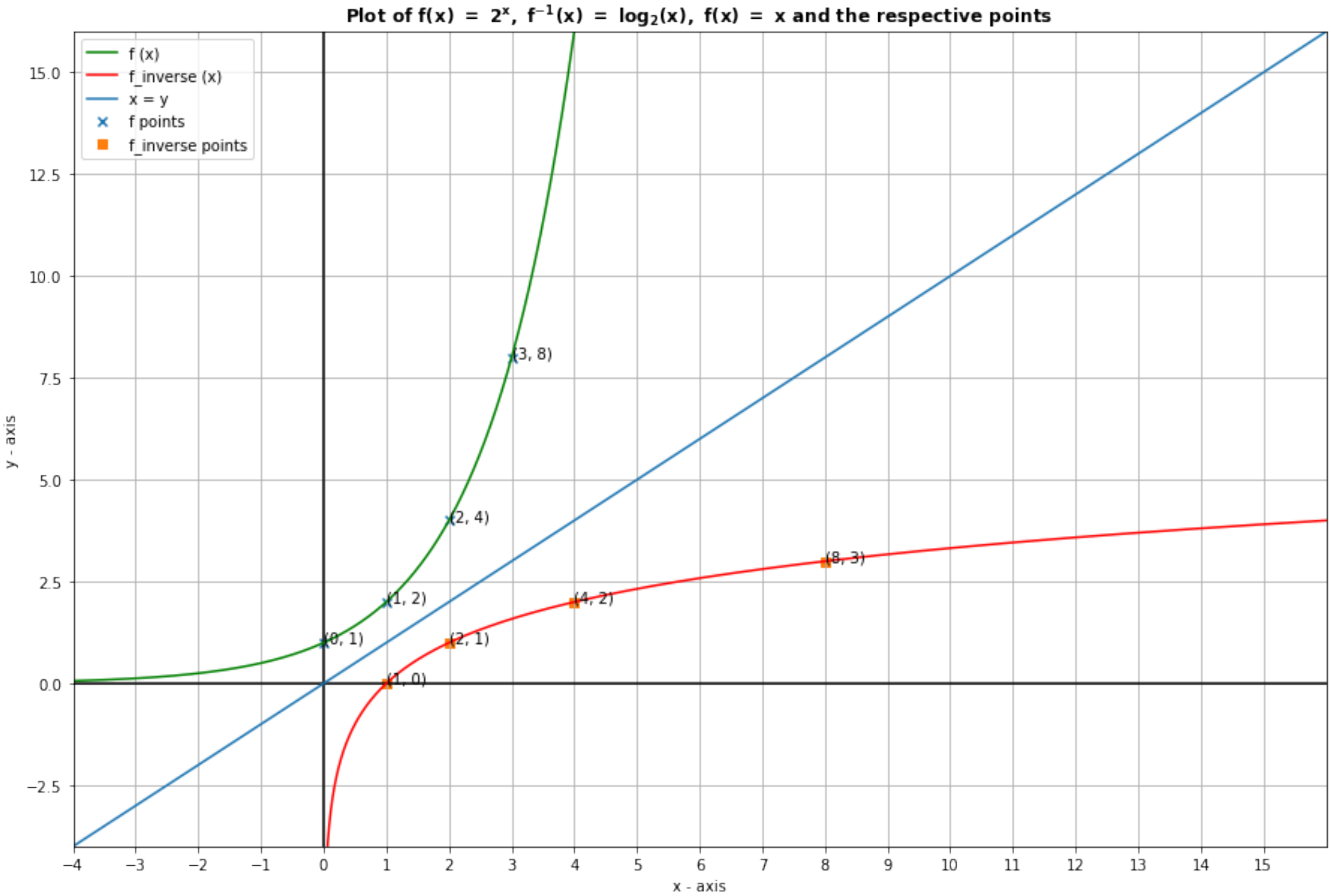


Figure 1

Question 4



Plot of f(x) = 2^x, f^-1(x) = log2(x), f(x) = x and the respective points
f (x)
f_inverse (x)
x = y
f points
f_inverse points
(3, 8)
(2, 4)
(1, 2)
(0, 1)
(8, 3)
(4, 2)
(2, 1)
(1, 0)
y - axis
x - axis

Question **5**

(a) The derivative of function $f(x) = ax^2$ is $f'(x) = 2ax$ ($\mathbf{a} \rightarrow \mathbf{4}$)

(b) The derivative of function $f(x) = cos(2\pi ft)$ is $f'(x) = -sin(2\pi ft)$ ($\mathbf{b} \rightarrow \mathbf{1}$)

(c) The derivative of function $f(x) = bx^3$ is $f'(x) = 3bx^2$ ($\mathbf{c} \rightarrow \mathbf{2}$)

(d) The derivative of function $f(x) = e^{cx}$ is $f'(x) = ce^{cx}$ ($\mathbf{d} \rightarrow \mathbf{3}$)

Question **6**

$$S'(x) = (\frac{1}{1+e^{-x}})' = [(1+e^{-x})^{-1}]' = (-1)(1+e^{-x})^{-2}(1+e^{-x})' = -\frac{(e^{-x})'}{(1+e^{-x})^2}$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} \quad (\mathbf{1})$$

$$S(x)(1-S(x)) = (\frac{1}{1+e^{-x}})(1-\frac{1}{1+e^{-x}}) = (\frac{1}{1+e^{-x}})(\frac{1+e^{-x}-1}{1+e^{-x}}) = (\frac{1}{1+e^{-x}})(\frac{e^{-x}}{1+e^{-x}})$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} \quad (\mathbf{2})$$

(1) , (2) $\implies S'(x) = S(x)\ (\ 1 - S(x)\ )$

Question **7**

$$S(x) = \frac{1}{1+e^{-x}}, \quad f(x) = ax + b, \quad S(f(x)) = \frac{1}{1+e^{-f(x)}}$$

$$S'(f(x)) = (\frac{1}{1+e^{-f(x)}})' \overset{Q_1}{=} (\frac{e^{ax}e^{b}}{e^{ax}e^{b}+1})' = \frac{(e^{ax}e^{b})'(e^{ax}e^{b}+1) \; - \; (e^{ax}e^{b})(e^{ax}e^{b}+1)'}{(e^{ax}e^{b}+1)^2}$$

$$= \frac{(ae^{ax}e^{b})(e^{ax}e^{b}+1) \; - \; (e^{ax}e^{b})(ae^{ax}e^{b})}{(e^{ax}e^{b}+1)^2} = \frac{(ae^{ax}e^{b})(e^{ax}e^{b}+1 \; - \; e^{ax}e^{b})}{(e^{ax}e^{b}+1)^2}$$

$$= \frac{ae^{ax+b}}{(e^{ax}e^{b}+1)^2}$$

Question **8**

Lala

Question **9**

Lala

Question **10**

Lala