



M902

Βασικές Μαθηματικές Έννοιες στη Γλωσσική Τεχνολογία

Project 2

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Question 1

The composite function $S(f(x))$, where $S(x) = \frac{1}{1 + e^{-x}}$ and $f(x) = ax + b$, is calculated as follows:

$$S(f(x)) = \frac{1}{1 + e^{-f(x)}} = \frac{1}{1 + e^{-(ax+b)}} = \frac{1}{1 + \frac{1}{e^{(ax+b)}}} = \frac{1}{\frac{e^{(ax+b)} + 1}{e^{(ax+b)}}} = \frac{e^{(ax+b)}}{e^{(ax+b)} + 1} = \frac{e^{ax}e^b}{e^{ax}e^b + 1}$$

DOMAINS AND RANGES!!

Question 2

A function is **invertible** only if each input has a unique output, which means each output is paired with exactly one input. That way, when the mapping is reversed, it will still be a function.

Definition

Let f be a function whose domain is the set A and whose codomain is the set B ($f : A \rightarrow B$). Then we say that f is invertible if f is **one-to-one** mapping and there is a function g with domain $Im(f) \subseteq B$ and image (range) A ($g : Im(f) \rightarrow A$) such that:

$$f(x) = y \iff g(y) = x$$

In this case, we call g the inverse of f and denote it by f^{-1} .

i) $f(x) = \sqrt{x-3}$

$$\bullet x - 3 \geq 0 \iff x \geq 3 \iff Dom(f) = [3, +\infty) \quad (1)$$

$$\bullet f \text{ is } \mathbf{continuous} \text{ on } Dom(f) = [3, +\infty) ^1 \quad (2)$$

$$\bullet f \text{ is } \mathbf{differentiable} \text{ on } A = \{ Dom(f) - \{0\} \} = \{ [3, +\infty) - \{0\} \} \\ = \{ (3, +\infty) \} ^2 \quad (3)$$

First, we have to prove that f is **one-to-one** mapping:

$$\stackrel{(2),(3)}{\implies} f'(x) = (\sqrt{x-3})' = \frac{1}{2\sqrt{x-3}} > 0, \forall x \in A = (3, +\infty) \quad (4)$$

$$\stackrel{(4)}{\implies} f \text{ is } \mathbf{strictly monotonic} \text{ (strictly inscreasing), therefore it is } \mathbf{one-to-one} \text{ mapping}$$

One approach to finding a formula for f^{-1} is to solve $f(x) = y$ for x .

$$f(x) = \sqrt{x-3} = y \stackrel{y \geq 0, x \geq 3}{\iff} x - 3 = y^2 \iff x = y^2 + 3 = f^{-1}(y) \quad (5)$$

$$\stackrel{(1)}{\implies} x \geq 3 \iff y^2 + 3 \geq 3 \iff y^2 \geq 0, \text{ true } \forall y \in \mathbb{R} \quad (6)$$

$$\stackrel{(5),(6)}{\implies} Im(f) = \{ \mathbb{R} \cap [0, +\infty) \} = [0, +\infty) = Dom(f')$$

¹ as a composition of continuous functions

² as a composition of differential functions

$$\bullet f : [3, +\infty] \rightarrow \mathbb{R}$$

$$f(x) = \sqrt{x-3}$$

$$\bullet f^{-1} : [0, +\infty] \rightarrow \mathbb{R}$$

$$f^{-1} = x^2 + 3$$

$$\text{ii) } f(x) = \log(x-2)$$

$$\bullet x-2 > 0 \iff x > 2 \iff \text{Dom}(f) = (2, +\infty) \quad (7)$$

First, we have to prove that f is **one-to-one** mapping:

$$\text{Let } x_1, x_2 \in \text{Dom}(f) = (0, +\infty), x_1 \neq x_2$$

$$f(x_1) = f(x_2) \iff \log(x_1-2) = \log(x_2-2) \iff x_1-2 = x_2-2 \iff x_1 = x_2$$

$$\iff x_1 = x_2 \iff f \text{ is an } \mathbf{one-to-one} \text{ mapping of } \text{Dom}(f) \text{ to } \text{Im}(f)$$

As in i above, we will solve $f(x) = y$ for x .

$$f(x) = \log_2(x-2) = y \iff 2^{\log_2(x-2)} = 2^y \iff x-2 = 2^y$$

$$\iff x = 2^y + 2 = f^{-1}(y) \quad (8)$$

$$\stackrel{(7)}{\implies} x > 2 \iff 2^y + 2 > 2 \iff 2^y > 0, \text{ true } \forall y \in \mathbb{R} \quad (9)$$

$$\stackrel{(9)}{\implies} \text{Im}(f) = \mathbb{R} = \text{Dom}(f')$$

$$\bullet f : (2, +\infty] \rightarrow \mathbb{R}$$

$$f(x) = \log(x-2)$$

$$\bullet f^{-1} : \mathbb{R} \rightarrow \mathbb{R}$$

$$f^{-1} = 2^y + 2$$

Question 3

$$s = A \cos(2\pi f t)$$

- (a) $A_1 = 1, \quad f = 1, \quad \theta = 0, \quad s_1 = \cos(2\pi t)$
(b) $A_2 = 2, \quad f = 3, \quad \theta = 0, \quad s_2 = 2 \cos(6\pi t)$
(c) $A_1 = 1.5, \quad f = 2, \quad \theta = \pi, \quad s_3 = 1.5 \cos(4\pi t + \pi)$
(d) $A_1 = 2, \quad f = 0.5, \quad \theta = 0, \quad s_4 = 2 \cos(\pi t)$

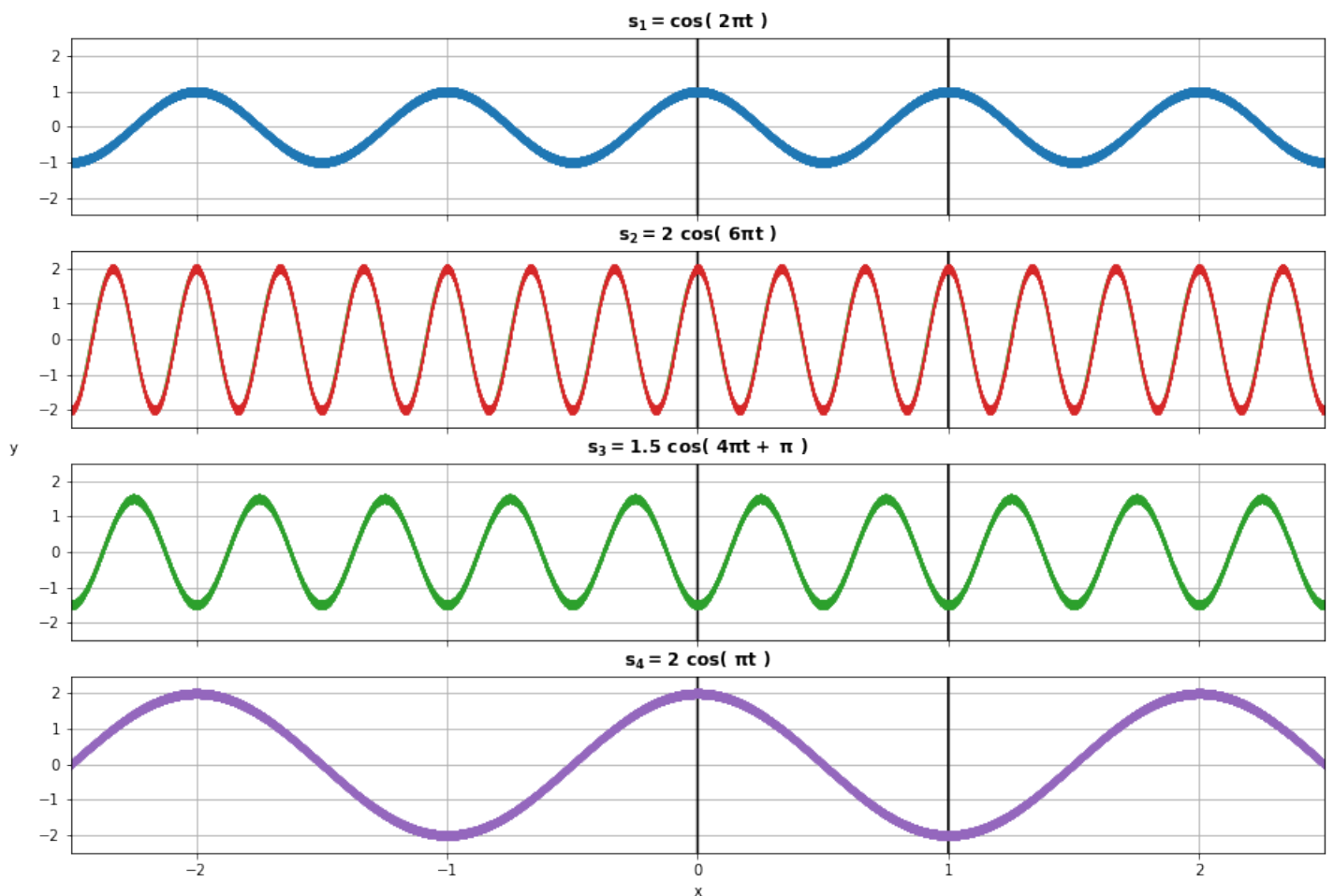
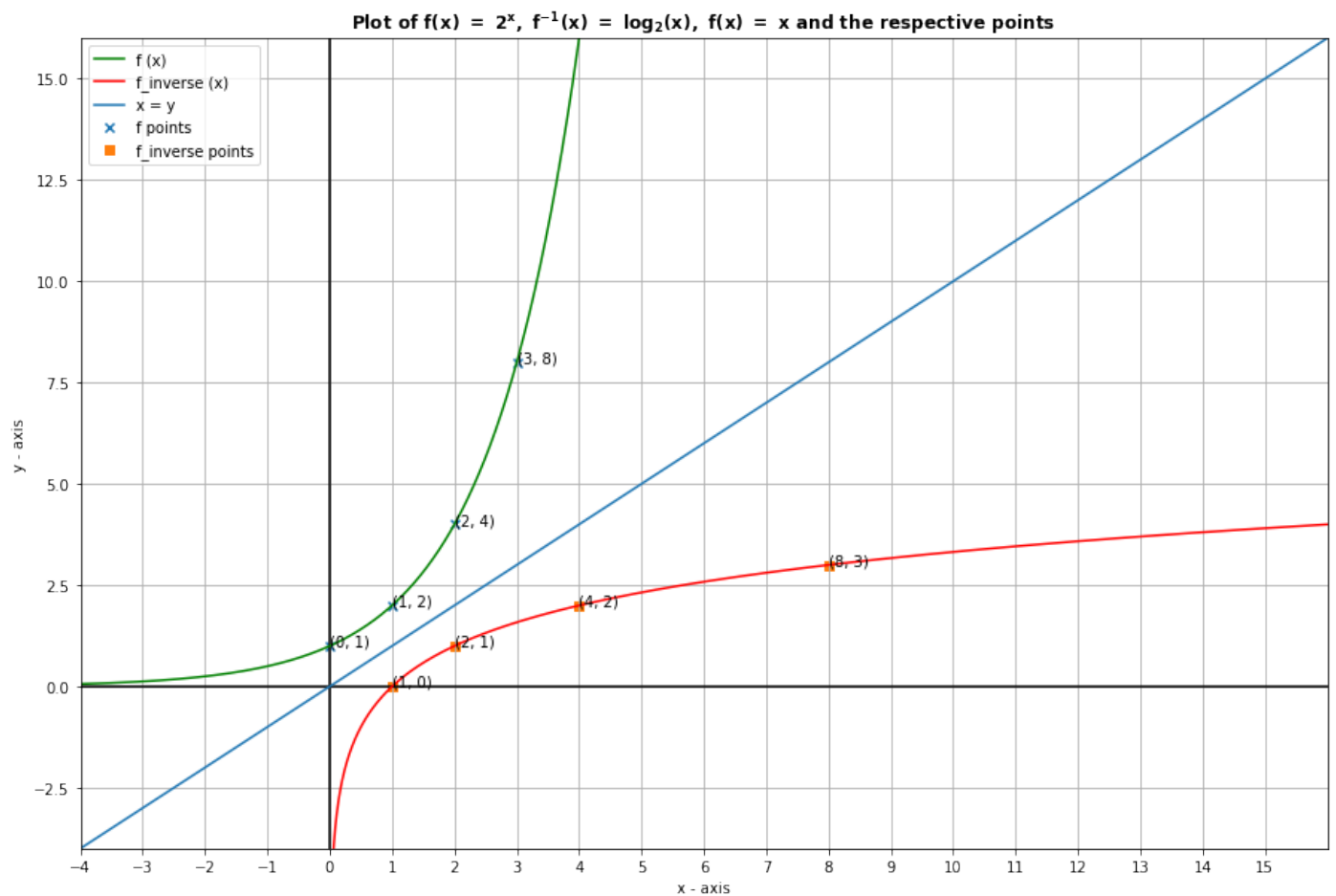


Figure 1

Question 4



Question 5

- (a) The derivative of function $f(x) = ax^2$ is $f'(x) = 2ax$ (**a** → **4**)
- (b) The derivative of function $f(x) = \cos(2\pi ft)$ is $f'(x) = -\sin(2\pi ft)$ (**b** → **1**)
- (c) The derivative of function $f(x) = bx^3$ is $f'(x) = 3bx^2$ (**c** → **2**)
- (d) The derivative of function $f(x) = e^{cx}$ is $f'(x) = ce^{cx}$ (**d** → **3**)

Question 6

$$\begin{aligned} S'(x) &= \left(\frac{1}{1+e^{-x}} \right)' = [(1+e^{-x})^{-1}]' = (-1)(1+e^{-x})^{-2}(1+e^{-x})' = -\frac{(e^{-x})'}{(1+e^{-x})^2} \\ &= \frac{e^{-x}}{(1+e^{-x})^2} \quad (1) \end{aligned}$$

$$\begin{aligned} S(x)(1-S(x)) &= \left(\frac{1}{1+e^{-x}} \right) \left(1 - \frac{1}{1+e^{-x}} \right) = \left(\frac{1}{1+e^{-x}} \right) \left(\frac{1+e^{-x}-1}{1+e^{-x}} \right) = \left(\frac{1}{1+e^{-x}} \right) \left(\frac{e^{-x}}{1+e^{-x}} \right) \\ &= \frac{e^{-x}}{(1+e^{-x})^2} \quad (2) \end{aligned}$$

$$(1), (2) \implies S'(x) = S(x)(1-S(x))$$

Question 7

$$S(x) = \frac{1}{1 + e^{-x}}, \quad f(x) = ax + b, \quad S(f(x)) = \frac{1}{1 + e^{-f(x)}}$$

$$\begin{aligned} S'(f(x)) &= \left(\frac{1}{1 + e^{-f(x)}} \right)' \stackrel{Q_1}{=} \left(\frac{e^{ax}e^b}{e^{ax}e^b + 1} \right)' = \frac{(e^{ax}e^b)'(e^{ax}e^b + 1) - (e^{ax}e^b)(e^{ax}e^b + 1)'}{(e^{ax}e^b + 1)^2} \\ &= \frac{(ae^{ax}e^b)(e^{ax}e^b + 1) - (e^{ax}e^b)(ae^{ax}e^b)}{(e^{ax}e^b + 1)^2} = \frac{(ae^{ax}e^b)(e^{ax}e^b + 1 - e^{ax}e^b)}{(e^{ax}e^b + 1)^2} \\ &= \frac{ae^{ax+b}}{(e^{ax}e^b + 1)^2} \end{aligned}$$

Question **8**

Lala

Question 9

Lala

Question **10**

Lala