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Project 4

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TABLE OF CONTENTS

Question **1**

$i\,.\ n = 3$ independent experiments (coin flips)

$$\Omega = \left\{ \begin{array}{c} \text{ΚΚΚ, ΚΚΓ, ΚΓΚ, ΚΓΓ} \\ \text{ΓΚΚ, ΓΚΓ, ΓΓΚ, ΓΓΓ} \end{array} \right\}$$

$ii\,.\ A_1 = \{$ ΓΚΚ, ΚΓΚ, ΚΚΓ, ΚΚΚ $\}$
$A_2 = \{$ ΓΚΚ, ΚΓΚ, ΚΚΓ $\}$
$A_3 = \{$ ΓΚΚ, ΚΓΚ, ΚΚΓ, ΚΚΚ $\} = A_1$
$A_4 = \{$ ΚΚΚ, ΓΓΓ $\}$
$A_5 = \{$ ΚΚΚ, ΚΚΓ, ΚΓΚ, ΚΓΓ $\}$

Let X a random variable expressing the number of successes (coin flip result → **Κ**), following **Binomial Distribution** (spoilers for iii below), $X \sim B(\ 3,\ 0.5)$. Then:

$$P(X = k) = \begin{pmatrix} n \\ k \end{pmatrix} p^k\ (1-p)^{n-k}\ ,$$

where
p is the **probability** of “success” outcome,
k is the number of **successes**,
n the total number of independent **experiments** performed.

$$P(A_1) = P(X=2) + P(X=3) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + \begin{pmatrix} 3 \\ 3 \end{pmatrix}\left(\frac{1}{2}\right)^3 = \frac{3!}{2!1!}\ \frac{1}{8} + \frac{3!}{3!0!}\ \frac{1}{8} == \frac{3}{8} + \frac{1}{8}$$

$$= \frac{N(A_1)}{N(\Omega)} = \frac{4}{8} = 0.5$$

$$P(A_2) = P(X=2) = \begin{pmatrix} 3 \\ 2 \end{pmatrix}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{3!}{2!1!}\ \frac{1}{8} = \frac{3}{8} = \frac{N(A_2)}{N(\Omega)} = 0.375$$

$P(A_3) = P(A_1) = 0.375$

$$P(A_4) = P(X=0) + P(X=3) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}\left(\frac{1}{2}\right)^3 + \begin{pmatrix} 3 \\ 3 \end{pmatrix}\left(\frac{1}{2}\right)^3 = \frac{3!}{0!3!}\ \frac{1}{8} + \frac{3!}{3!0!}\ \frac{1}{8} == \frac{1}{8} + \frac{1}{8}$$

$$= \frac{N(A_1)}{N(\Omega)} = \frac{2}{8} = 0.25$$

Event A_5 concerns only the first coin flip, which is independent of the overall number of experiments. Therefore, the probability of a sole coin flip (the first one) resulting in K, is always $P(K) = \frac{1}{2} = 0.5$.

$iii.\ n$ independent experiments (coin flips)

Here, for event A_2 we apply the same formula as in ii, with $X \sim B(\,n,\,0.5)$:

$$P(A_2) = P(X=2) = \binom{n}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2} = \frac{n!}{2!(n-2)!}\left(\frac{1}{2}\right)^n = \frac{n\ (n-1)}{2}\left(\frac{1}{2}\right)^n$$

As also mentioned in ii , the probability of A_5 is always the same and equals the probability of a single coin flip resulting in K, $P(K) = \frac{1}{2} = 0.5$.

Question **2**

Let X a random variable following normal distribution $X \sim N(\ 60,\ 5^2\ )$ expressing the student weights. Then:

$$a)\ \ P(X > 70) = P\left(\frac{X-\mu}{\sigma} > \frac{70-\mu}{\sigma}\right) = P\left(Z > \frac{70-60}{5}\right) = P(Z > 2) = 1 - P(Z < 2)$$

$$= 1 - \Phi(2) = 1 - 0.9772 = 0.0228$$

$$\beta)\ \ P(55 < X < 65) = P(X < 65) - P(X < 55) = P\left(\frac{X-\mu}{\sigma} < \frac{65-\mu}{\sigma}\right) - P\left(\frac{X-\mu}{\sigma} < \frac{55-\mu}{\sigma}\right)$$

$$= P\left(\frac{X-\mu}{\sigma} < \frac{65-60}{5}\right) - P\left(\frac{X-\mu}{\sigma} < \frac{55-60}{5}\right) = P(Z < 1) - P(Z < -1)$$

$$= \Phi(1) - \Phi(-1) = \Phi(1) - (1 - \Phi(1)) = 2\ \Phi(1) - 1 = 2 * 0.8413 - 1 = 0.6826$$

Question **3**

$P($ ανοσ $) = 0.7 = p$

The problem can be modelled as a binary outcome (rabbit immunised or not) experiment, executed n times (selecting n rabbits). Then, X is a random variable expressing the number of immunised rabbits picked, with $X \sim B(\ n,\ 0.7)$, where $n\ = 5$:

i . $P(X=3) = \binom{5}{3} * 0.7^3 * (1-0.7)^{5-3} = \frac{5!}{3!2!} * 0.7^3 * 0.3^2 = 10 * 0.7^3 * 0.3^2 = 0.3087$

ii . Here, two explanations of the question are going to be followed. However, the resulted probabilities are equal.

1. The probability of picking 3 non-immunised (failure) rabbits and then 1 immunised (success). The task can be modelled as the calculation of the probability that the first success (immunised rabbit) requires k independent trials, thus we calculate the probability of $k-1$ failures and 1 success (k_{th} trial). In this particular case, X is following the **Geometric Distribution**, $X \sim Geo(0.7)$:

$$P(X=k) = (1-p)^{k-1}p$$

Then:

$$P(X=4) = (1-0.7)^{4-1}0.7 = 0.3^3 * 0.7 = 0.0189$$

2. The probability of the first rabbit to be the only immunised one, out of 4 rabbits picked in total.

$$P(1_{st} \text{ rabbit immunised }) = 0.7 * (1-0.7)^{4-1} = 0.7 * 0.3^3 = 0.0189$$

Question **4**

Question **5**

Question **6**

Question **7**

Question **8**

Question **9**

Question **10**