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TABLE OF CONTENTS

Question **1**

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 1 & -2 & 7 \\ 3 & 8 & 4 \end{bmatrix}$$

$$a_{13} = 5$$

$$a_{21} = 1$$

$$a_{32} = 8$$

$$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = (-1)^{1+3} \cdot \begin{vmatrix} 1 & -2 \\ 3 & 8 \end{vmatrix} = \big(8 - (-6)\big) = 14$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = (-1)^{2+1} \cdot \begin{vmatrix} -3 & 5 \\ 8 & 4 \end{vmatrix} = -(-12 - 40) = 52$$

$$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = (-1)^{3+2} \cdot \begin{vmatrix} 2 & 5 \\ 1 & 7 \end{vmatrix} = -(14 - 5) = -9$$

Question **2**

$$A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

$a)$ $A + B \implies$ cannot be calculated, because the summation operation can only be performed in cases of same dimension matrices. However, that is not the case, as A is a $3x2$ matrix and B is a $2x2$ matrix.

$$b)\ C \cdot A = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3+16+4 & 9+8-2 \\ 2+16-6 & -6+8+3 \end{bmatrix} = \begin{bmatrix} -17 & 15 \\ 12 & 5 \end{bmatrix}$$

$$c)\ 5B - 2D = 5 \cdot \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} - 2 \cdot \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 10 & -15 \\ 20 & -10 \end{bmatrix} - \begin{bmatrix} 10 & -2 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10-10 & -15+2 \\ 20-4 & -10 \end{bmatrix} = \begin{bmatrix} 0 & -13 \\ 16 & -10 \end{bmatrix}$$

$$d)\ B \cdot D - D = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 10-6 & -2 \\ 20-4 & -4 \end{bmatrix} - \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 14 & -4 \end{bmatrix}$$

$e)$ $B \cdot C \cdot D \implies$ cannot be calculated. In order to perform matrix multiplication of two matrices A_{nxm} , B_{kxp} , m has to be equal to k ($m = k$) and the final matrix will be of dimension nxp .

In this example, B and C can be multiplied because they follow the aformentioned requirement, but their product of multiplication is a $2x3$ matrix, so it cannot be multiplied with $2x2$ matrix D.

Question **3**

$$A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

$$a) \ \left(A^T + C\right)^T = \left[\begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}^T + \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}\right]^T = \left[\begin{bmatrix} -1 & 4 & 2 \\ 3 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}\right]^T$$

$$= \begin{bmatrix} -1+3 & 4+4 & 2+2 \\ 3-2 & 2+4 & -1-3 \end{bmatrix}^T = \begin{bmatrix} 2 & 8 & 4 \\ 1 & 6 & -4 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ 8 & 6 \\ 4 & -4 \end{bmatrix}$$

$$b) \ -C^T - A = -\begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}^T - \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix} = -\begin{bmatrix} 3 & -2 \\ 4 & 4 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} -3+1 & -2-3 \\ -4-4 & -4-2 \\ -2-2 & 3+1 \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ -8 & -6 \\ -4 & 4 \end{bmatrix}$$

Question **4**

$$A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

$$a)\ B \cdot D = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 10-6 & -2 \\ 20-4 & -4 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 16 & -4 \end{bmatrix}$$

$$b)\ D \cdot B = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 10-4 & -15+2 \\ 4 & -6 \end{bmatrix} = \begin{bmatrix} 6 & -13 \\ 4 & -6 \end{bmatrix}$$

$$c)\ B^2 = B \cdot B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 4-12 & -6+6 \\ 8-8 & -12+4 \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix} =$$

$$= -8 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -8 \cdot I_2$$

Question **5**

$$B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$$

$a)$ Find B^{-1}

$b)$ Verify $B \cdot B^{-1} = B^{-1} \cdot B = I$

Following the formula of the inverse matrix:

$$a)\ B^{-1} = \frac{1}{|B|} \cdot adj(B) = \frac{1}{|B|} \cdot \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}^T = \frac{1}{|B|} \cdot \begin{bmatrix} B_{11} & B_{21} \\ B_{12} & B_{22} \end{bmatrix}$$

$$\det(B) = |B| = \begin{vmatrix} 2 & -3 \\ 4 & -2 \end{vmatrix} = 2 \cdot (-2) - (4 \cdot (-3)) = -4 + 12 = 8 \qquad (1)$$

$$adj(B) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}^T = \begin{bmatrix} B_{11} & B_{21} \\ B_{12} & B_{22} \end{bmatrix} = \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -4 & 2 \end{bmatrix} \qquad (2)$$

$$\overset{(1),\,(2)}{\Longrightarrow} B^{-1} = \frac{1}{|B|} \cdot adj(B) = \frac{1}{8} \cdot \begin{bmatrix} -2 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{8} & \frac{3}{8} \\ -\frac{4}{8} & \frac{2}{8} \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} & \frac{3}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} \qquad (3)$$

$$b)\ B \cdot B^{-1} = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{4} & \frac{3}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} -\frac{2}{4} + \frac{3}{2} & \frac{6}{8} - \frac{3}{4} \\ -\frac{4}{4} + \frac{2}{2} & \frac{12}{8} - \frac{2}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$B^{-1} \cdot B = \begin{bmatrix} -\frac{1}{4} & \frac{3}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{4} + \frac{12}{8} & \frac{3}{4} - \frac{6}{8} \\ -\frac{2}{2} + \frac{4}{4} & \frac{3}{2} - \frac{2}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

Question **6**

Let $A = \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, the unknown vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and the dot products vector

$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 9 \end{bmatrix}$.

This system of linear equations in its condensed form, is written as follows:

$$A \cdot x = b$$

The unknown vector x is calculated by:

$$A \cdot x = b \implies A^{-1} \cdot A \cdot x = A^{-1} \cdot b \implies I \cdot x = A^{-1} \cdot b \implies x = A^{-1} \cdot b \qquad (1)$$

$$A^{-1} = \frac{1}{|A|}\, adj(A) \qquad (2)$$

Determinant of matrix A:

$$\det(A) = |A| = \begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} + (-1)^{1+2} \cdot 2 \cdot \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} + (-1)^{1+3} \cdot 0 \cdot \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= \; 1 \cdot 1 \cdot (0 \cdot 2 - (-1) \cdot 1) - 1 \cdot 2 \cdot (2 \cdot 2) + 0 = +1 - 8 = -7 \qquad (3)$$

Adjoint matrix (transpose of cofactor matrix) of A :

$$adj(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix},$$

where the $ij-th$ **cofactor** $cof(A)_{ij} = A_{ij} = (-1)^{i+j} \cdot \det(M_{ij}),\ i,\ j$ the rows and columns of matrix A respectively and M_{ij} the matrix made by removing row i and column j of matrix A. The proper signs of the cofactors in the **adjoint** (adjugate) matrix, are included in the calculation of A_{ij} .

$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 1, \; A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = -4, \; A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 2$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = -4, \; A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2, \; A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = -1$$

$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2, \; A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = 1, \; A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -4$$

$$adj(A) = \begin{bmatrix} 1 & -4 & -2 \\ -4 & 2 & 1 \\ 2 & -1 & -4 \end{bmatrix} \qquad (\ 4\ )$$

$$\overset{(2),\,(3),\,(4)}{\Longrightarrow} \quad A^{-1} = \frac{1}{-7} \cdot \begin{bmatrix} 1 & -4 & -2 \\ -4 & 2 & 1 \\ 2 & -1 & -4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{4}{7} & \frac{2}{7} \\ \frac{4}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} & \frac{4}{7} \end{bmatrix} \qquad (\ 5\ )$$

$$\overset{(1),\,(5)}{\Longrightarrow} \; x = A^{-1} \cdot b \;\Longrightarrow$$

$$x = \begin{bmatrix} -\frac{1}{7} & \frac{4}{7} & \frac{2}{7} \\ \frac{4}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} & \frac{4}{7} \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} \cdot 5 + \frac{4}{7} \cdot 8 + \frac{2}{7} \cdot 9 \\ \frac{4}{7} \cdot 5 - \frac{2}{7} \cdot 8 - \frac{1}{7} \cdot 9 \\ -\frac{2}{7} \cdot 5 + \frac{1}{7} \cdot 8 + \frac{4}{7} \cdot 9 \end{bmatrix} = \begin{bmatrix} \frac{45}{7} \\ -\frac{5}{7} \\ \frac{34}{7} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Longrightarrow \; x_1 = \frac{45}{7}, \; x_2 = -\frac{5}{7}, \; x_3 = \frac{34}{7}$$

(coordinates of the previously-but-not-still-unknown vector x)

Question **7**

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$$

$a)$ Find A^2
$b)$ Find A^{-1}

$$a)\ A^2 = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix} = \begin{bmatrix} \cos^2\theta + sin^2\theta & \cos\theta\sin\theta - \sin\theta\cos\theta \\ \sin\theta\cos\theta - \sin\theta\cos\theta & \sin^2\theta + \cos^2\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

where I_2, the identity matrix of dimension $2x2$.

$$b)\ A^{-1} = \frac{1}{|A|} \cdot adj(A) = \frac{1}{|A|} \cdot \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T = \frac{1}{|A|} \cdot \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} \qquad (1)$$

$$\det(A) = |A| = \begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{vmatrix} = -\cos^2\theta - \sin^2\theta = -(\cos^2\theta + \sin^2\theta) = -1 \qquad (2)$$

$$adj(A) = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} -cos\theta & -sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \qquad (3)$$

$$\overset{(1),\,(2),\,(3)}{\Longrightarrow} A^{-1} = \frac{1}{|A|} \cdot adj(A) = \frac{1}{|A|} \cdot \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = -1 \cdot \begin{bmatrix} -cos\theta & -sin\theta \\ -sin\theta & cos\theta \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ sin\theta & -cos\theta \end{bmatrix} = A$$

From questions 7.a and 7.b both, we extract the information that A is an **involutory** matrix, which means it is **its own reverse**, as it complies with the following:

$$A^2 = I_2$$

$$A^{-1} = A$$

Question **8**

Vectors: $u = [3,4],\ v = [4,3]$

The included angle θ of two vectors, is dependent on their respective magnitudes and inner product and is calculated as follows:

$$u \cdot v = \|u\| \cdot \|v\| \cdot \cos\theta_{uv} \implies \cos\theta_{uv} = \frac{u \cdot v}{\|u\| \cdot \|v\|} \quad (1)$$

$$\overset{(1)}{\implies} \cos\theta_{uv} = \frac{u \cdot v}{\|u\| \cdot \|v\|} = \frac{[3,4] \cdot [4,3]}{\|[3,4]\| \cdot \|[4,3]\|} = \frac{12 + 12}{(\sqrt{9+16}) \cdot (\sqrt{16+9})} = \frac{24}{5 \cdot 5}$$

$$= \frac{24}{25} = 0.96 \implies \cos^{-1}(0.96) = 16.26^\circ \implies \theta_{uv} = 16.26^\circ$$

Vectors u and v are **not collinear**, as their included angle θ is neither 0° nor 180° ($\cos\theta = 1$ and $\cos\theta = -1$ respectively). They are also **not orthogonal**, as in that case, their included angle θ would be 90° ($\cos\theta = 0$).

Question **9**

Vectors: $u = [a, b],\ v = 2u$

As described in **Question 8**, the included angle θ of two vectors, is dependent on their respective magnitudes and inner product, calculated by using formula **(1)** :

$$u \cdot v = \ \|u\| \cdot \|v\| \cdot \cos\theta_{uv} \implies \cos\theta_{uv} = \frac{u \cdot v}{\|u\| \cdot \|v\|} \quad (1)$$

$$\overset{(1)}{\implies} \cos\theta_{uv} = \frac{u \cdot v}{\|u\| \cdot \|v\|} = \cos\theta_{uv} = \frac{u \cdot 2u}{\|u\| \cdot \|2u\|} = \frac{[a, b] \cdot [2a, 2b]}{\|[a, b]\| \cdot \|[2a, 2b]\|} =$$

$$\frac{2 \cdot (a^2 + b^2)}{(\sqrt{a^2 + b^2}) \cdot (\sqrt{4a^2 + 4b^2})} = \frac{2 \cdot (a^2 + b^2)}{2 \cdot \left(\sqrt{a^2 + b^2}\right)^2} = \frac{a^2 + b^2}{a^2 + b^2} = \ 1$$

$$\implies \ \cos^{-1}(1) = 0° \implies \ \theta_{uv} = 0°$$

In this case, the two vectors are **collinear**, as vector v is a **scalar multiple** of vector u ($v = k \cdot u$), and have the **same direction** as the **scalar** k is a **positive** number ($k = 2$), therefore their included angle θ is $0°$ (validated by the calculations above).

Question **10**

In order to find the similar documents based on the 3-dimensional vector representations, $d_n \in \mathbb{R}^3, n = 1, \ldots, 5$, a 5x5 matrix was constructed, with each row being each of the 3-D vectors d_1, d_2, d_3, d_4, d_5 , normalised as follows:

$$d_{ij\_normalized} = \frac{d_{ij}}{||d_i||}$$

where d_{ij} the $j-th$ element of the $i-th$ vector and $||d_i|| = \sqrt{d_{i1}^2 + d_{i2}^2 + d_{i3}^2}$ the norm of the respective vector i:

$$D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} \Longrightarrow D_{5x3} = \begin{bmatrix} 8 & 6 & 0 \\ 0 & 6 & 8 \\ 6 & 0 & 8 \\ 2 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \Longrightarrow D_{normalized} = \begin{bmatrix} \frac{8}{10} & \frac{6}{10} & \frac{0}{10} \\ \frac{0}{10} & \frac{6}{10} & \frac{8}{10} \\ \frac{6}{10} & \frac{0}{10} & \frac{8}{10} \\ \frac{2}{3.6055} & \frac{3}{3.6055} & \frac{0}{3.6055} \\ \frac{9}{10.8166} & \frac{6}{10.8166} & \frac{0}{10.8166} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.6 & 0 \\ 0 & 0.6 & 0.8 \\ 0.6 & 0 & 0.8 \\ 0.5547 & 0.8320 & 0 \\ 0.8320 & 0.5547 & 0 \end{bmatrix}$$

Finally, we multiply **normalized** matrix **D** with its **transpose**, to get the inner product of every pair of vectors and perform the comparisons between the document representations:

$$D_{normalized} \cdot D^T_{normalized} = \begin{bmatrix} 0.8 & 0.6 & 0 \\ 0 & 0.6 & 0.8 \\ 0.6 & 0 & 0.8 \\ 0.5547 & 0.8320 & 0 \\ 0.8320 & 0.5547 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0 & 0.6 & 0.5547 & 0.8320 \\ 0.6 & 0.6 & 0 & 0.8320 & 0.5547 \\ 0 & 0.8 & 0.8 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.36 & 0.48 & 0.9429 & 0.9984 \\ \mathbf{0.36} & 1 & 0.64 & 0.4992 & 0.3328 \\ \mathbf{0.48} & \mathbf{0.64} & 1 & 0.3328 & 0.4992 \\ \mathbf{0.9429} & \mathbf{0.4992} & \mathbf{0.3328} & 1 & 0.9230 \\ \mathbf{0.9984} & \mathbf{0.3328} & \mathbf{0.4992} & \mathbf{0.9230} & 1 \end{bmatrix} = D_{normalized\_dot\_product}$$

We keep only the **lower triangular matrix**, where each value represents the dot product of the $j-th$ vector (d_j) with the $i-th$ vector (d_j), where i, j the rows and the columns of the matrix $D_{normalized\_dot\_product}$ respectively.

The formula of the dot product of two vectors, using the included angle θ , is:

$$d_i \cdot d_j = \|d_i\| \cdot \|d_j\| \cdot \cos\theta \implies \cos\theta = \frac{d_i \cdot d_j}{\|d_i\| \cdot \|d_j\|} = \cos\theta_{d_i,d_j}$$

The process we followed above, calculates exactly the value of the cosine of the included angle θ, giving information on the relation between those vectors. If the angle's value is close to 0, then the vectors are **orthogonal**, if it is close to 1 or -1 they are **collinear**.
(The correlation matrix (experimented with in the .py file) might be able to give more information about their relation, capturing strong negative, strong positive and weak / no corellation between the vectors - not yet explicitly examined).

Therefore, the results are the follwing:

Inner Product (of normalized vectors)	Notes	Similarity
$\cos\theta_{d_1,d_2} = 0.36$		Low
$\cos\theta_{d_1,d_3} = 0.48$		
$\cos\theta_{d_1,d_4} = 0.9429$	$> 0.94 \implies \theta_{d_1,d_4} < 19^\circ$	**High**
$\cos\theta_{d_1,d_5} = 0.9984$	$\sim 1 \implies \theta_{d_1,d_5} \sim 0^\circ$	**High**
$\cos\theta_{d_2,d_3} = 0.64$		
$\cos\theta_{d_2,d_4} = 0.4492$		
$\cos\theta_{d_2,d_5} = 0.3328$		Low
$\cos\theta_{d_3,d_4} = 0.3328$		Low
$\cos\theta_{d_3,d_5} = 0.4992$		
$\cos\theta_{d_4,d_5} = 0.9230$	$> 0.92 \implies \theta_{d_4,d_5} < 23^\circ$	**High**

Table 3.10

In conclusion, **Documents 1**, **4** and **5** are **very similar**, especially **1** and **5**.

Notes

For some questions of project 3, some **code** was developed to **experiment** with the given tasks and **validate** the manually calculated results, the file (M902_Project_3_CTKylafi_LT1200012**.py**) of which is included in the uploaded .zip file (M902_Project3_CTKylafi_LT1200012**.zip**).