



## **M902**

# Βασικές Μαθηματικές Έννοιες στη Γλωσσική Τεχνολογία

## **Project 3**

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November, 2020

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## Question 1

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$$A = \begin{bmatrix} 2 & -3 & 5 \\ 1 & -2 & 7 \\ 3 & 8 & 4 \end{bmatrix}$$

$$a_{13} = 5$$

$$a_{21} = 1$$

$$a_{32} = 8$$

$$A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} = (-1)^{1+3} \cdot \begin{vmatrix} 1 & -2 \\ 3 & 8 \end{vmatrix} = (8 - (-6)) = 14$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} = (-1)^{2+1} \cdot \begin{vmatrix} -3 & 5 \\ 8 & 4 \end{vmatrix} = -(-12 - 40) = 52$$

$$A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} = (-1)^{3+2} \cdot \begin{vmatrix} 2 & 5 \\ 1 & 7 \end{vmatrix} = -(14 - 5) = -9$$

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## Question 2

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$$A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

a)  $A + B \implies$  cannot be calculated, because the summation operation can only be performed in cases of same dimension matrices. However, that is not the case, as  $A$  is a  $3 \times 2$  matrix and  $B$  is a  $2 \times 2$  matrix.

$$b) C \cdot A = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -3 + 16 + 4 & 9 + 8 - 2 \\ 2 + 16 - 6 & -6 + 8 + 3 \end{bmatrix} = \begin{bmatrix} -17 & 15 \\ 12 & 5 \end{bmatrix}$$

$$c) 5B - 2D = 5 \cdot \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} - 2 \cdot \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 10 & -15 \\ 20 & -10 \end{bmatrix} - \begin{bmatrix} 10 & -2 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 - 10 & -15 + 2 \\ 20 - 4 & -10 \end{bmatrix} = \begin{bmatrix} 0 & -13 \\ 16 & -10 \end{bmatrix}$$

$$d) B \cdot D - D = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} - \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 10 - 6 & -2 \\ 20 - 4 & -4 \end{bmatrix} - \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 14 & -4 \end{bmatrix}$$

e)  $B \cdot C \cdot D \implies$  cannot be calculated. In order to perform matrix multiplication of two matrices  $A_{n \times m}$ ,  $B_{k \times p}$ ,  $m$  has to be equal to  $k$  ( $m = k$ ) and the final matrix will be of dimension  $n \times p$ .

In this example,  $B$  and  $C$  can be multiplied because they follow the aforementioned requirement, but their product of multiplication is a  $2 \times 3$  matrix, so it cannot be multiplied with  $2 \times 2$  matrix  $D$ .

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## Question 3

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$$A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

$$\begin{aligned} a) (A^T + C)^T &= \left[ \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}^T + \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix} \right]^T = \left[ \begin{bmatrix} -1 & 4 & 2 \\ 3 & 2 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix} \right]^T \\ &= \begin{bmatrix} -1+3 & 4+4 & 2+2 \\ 3-2 & 2+4 & -1-3 \end{bmatrix}^T = \begin{bmatrix} 2 & 8 & 4 \\ 1 & 6 & -4 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 \\ 8 & 6 \\ 4 & -4 \end{bmatrix} \end{aligned}$$

$$b) -C^T - A = -\begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}^T - \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix} = -\begin{bmatrix} 3 & -2 \\ 4 & 4 \\ 2 & -3 \end{bmatrix} - \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix} =$$

$$\begin{bmatrix} -3+1 & -2-3 \\ -4-4 & -4-2 \\ -2-2 & 3+1 \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ -8 & -6 \\ -4 & 4 \end{bmatrix}$$

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## Question 4

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$$A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}, \quad C = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix}, \quad D = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

$$a) B \cdot D = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 10 - 6 & -2 \\ 20 - 4 & -4 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ 16 & -4 \end{bmatrix}$$

$$b) D \cdot B = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 10 - 4 & -15 + 2 \\ 4 & -6 \end{bmatrix} = \begin{bmatrix} 6 & -13 \\ 4 & -6 \end{bmatrix}$$

$$c) B^2 = B \cdot B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 4 - 12 & -6 + 6 \\ 8 - 8 & -12 + 4 \end{bmatrix} = \begin{bmatrix} -8 & 0 \\ 0 & -8 \end{bmatrix} =$$

$$= -8 \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -8 \cdot I_2$$

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## Question 5

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$$B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$$

a) Find  $B^{-1}$

b) Verify  $B \cdot B^{-1} = B^{-1} \cdot B = I$

Following the formula of the inverse matrix:

$$a) B^{-1} = \frac{1}{|B|} \cdot adj(B) = \frac{1}{|B|} \cdot \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}^T = \frac{1}{|B|} \cdot \begin{bmatrix} B_{11} & B_{21} \\ B_{12} & B_{22} \end{bmatrix}$$

$$\det(B) = |B| = \begin{vmatrix} 2 & -3 \\ 4 & -2 \end{vmatrix} = 2 \cdot (-2) - (4 \cdot (-3)) = -4 + 12 = 8 \quad (1)$$

$$adj(B) = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}^T = \begin{bmatrix} B_{11} & B_{21} \\ B_{12} & B_{22} \end{bmatrix} = \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ -4 & 2 \end{bmatrix} \quad (2)$$

$$\stackrel{(1), (2)}{\implies} B^{-1} = \frac{1}{|B|} \cdot adj(B) = \frac{1}{8} \cdot \begin{bmatrix} -2 & 3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{8} & \frac{3}{8} \\ -\frac{4}{8} & \frac{2}{8} \end{bmatrix} = \begin{bmatrix} -\frac{1}{4} & \frac{3}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} \quad (3)$$

$$b) B \cdot B^{-1} = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{4} & \frac{3}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} = \begin{bmatrix} -\frac{2}{4} + \frac{3}{2} & \frac{6}{8} - \frac{3}{4} \\ -\frac{4}{4} + \frac{2}{2} & \frac{12}{8} - \frac{2}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$B^{-1} \cdot B = \begin{bmatrix} -\frac{1}{4} & \frac{3}{8} \\ -\frac{1}{2} & \frac{1}{4} \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{4} + \frac{12}{8} & \frac{3}{4} - \frac{6}{8} \\ -\frac{2}{2} + \frac{4}{4} & \frac{3}{2} - \frac{2}{4} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

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## Question 6

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Let  $A = \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ , the unknown vector  $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$  and the dot products vector

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 9 \end{bmatrix}.$$

This system of linear equations in its condensed form, is written as follows:

$$A \cdot x = b$$

The unknown vector  $x$  is calculated by:

$$A \cdot x = b \implies A^{-1} \cdot A \cdot x = A^{-1} \cdot b \implies I \cdot x = A^{-1} \cdot b \implies x = A^{-1} \cdot b \quad (1)$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A) \quad (2)$$

**Determinant** of matrix  $A$ :

$$\begin{aligned} \det(A) = |A| &= \begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} + (-1)^{1+2} \cdot 2 \cdot \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} + (-1)^{1+3} \cdot 0 \cdot \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \\ &= 1 \cdot 1 \cdot (0 \cdot 2 - (-1) \cdot 1) - 1 \cdot 2 \cdot (2 \cdot 2) + 0 = +1 - 8 = -7 \quad (3) \end{aligned}$$

**Adjoint** matrix (transpose of cofactor matrix) of  $A$  :

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix},$$

where the  $ij$ -th **cofactor**  $\text{cof}(A)_{ij} = A_{ij} = (-1)^{i+j} \cdot \det(M_{ij})$ ,  $i, j$  the rows and columns of matrix  $A$  respectively and  $M_{ij}$  the matrix made by removing row  $i$  and column  $j$  of matrix  $A$ . The proper signs of the cofactors in the **adjoint** (adjugate) matrix, are included in the calculation of  $A_{ij}$ .



$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 1, \quad A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = -4, \quad A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 2$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = -4, \quad A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2, \quad A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = -1$$

$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2, \quad A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = 1, \quad A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -4$$

$$\text{adj}(A) = \begin{bmatrix} 1 & -4 & -2 \\ -4 & 2 & 1 \\ 2 & -1 & -4 \end{bmatrix} \quad (4)$$

$$\stackrel{(2), (3), (4)}{\implies} A^{-1} = \frac{1}{-7} \cdot \begin{bmatrix} 1 & -4 & -2 \\ -4 & 2 & 1 \\ 2 & -1 & -4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{4}{7} & \frac{2}{7} \\ \frac{4}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} & \frac{4}{7} \end{bmatrix} \quad (5)$$

$$\stackrel{(1), (5)}{\implies} x = A^{-1} \cdot b \implies$$

$$x = \begin{bmatrix} -\frac{1}{7} & \frac{4}{7} & \frac{2}{7} \\ \frac{4}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} & \frac{4}{7} \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} \cdot 5 + \frac{4}{7} \cdot 8 + \frac{2}{7} \cdot 9 \\ \frac{4}{7} \cdot 5 - \frac{2}{7} \cdot 8 - \frac{1}{7} \cdot 9 \\ -\frac{2}{7} \cdot 5 + \frac{1}{7} \cdot 8 + \frac{4}{7} \cdot 9 \end{bmatrix} = \begin{bmatrix} \frac{45}{7} \\ -\frac{5}{7} \\ \frac{34}{7} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\implies x_1 = \frac{45}{7}, x_2 = -\frac{5}{7}, x_3 = \frac{34}{7}$$

( coordinates of the previously-but-not-still-known vector  $x$  )

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## Question 7

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$$A = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}$$

a) Find  $A^2$

b) Find  $A^{-1}$

$$a) A^2 = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix} \cdot \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix} = \begin{bmatrix} \cos^2 \theta + \sin^2 \theta & \cos \theta \sin \theta - \sin \theta \cos \theta \\ \sin \theta \cos \theta - \sin \theta \cos \theta & \sin^2 \theta + \cos^2 \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

where  $I_2$ , the identity matrix of dimension  $2 \times 2$ .

$$b) A^{-1} = \frac{1}{|A|} \cdot adj(A) = \frac{1}{|A|} \cdot \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T = \frac{1}{|A|} \cdot \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} \quad (1)$$

$$\det(A) = |A| = \begin{vmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{vmatrix} = -\cos^2 \theta - \sin^2 \theta = -(\cos^2 \theta + \sin^2 \theta) = -1 \quad (2)$$

$$adj(A) = \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix} = \begin{bmatrix} -\cos \theta & -\sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \quad (3)$$

$$\stackrel{(1), (2), (3)}{\Rightarrow} A^{-1} = \frac{1}{|A|} \cdot adj(A) = \frac{1}{|A|} \cdot \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} = -1 \cdot \begin{bmatrix} -\cos \theta & -\sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix} = A$$

From questions 7.a and 7.b both, we extract the information that  $A$  is an **involutory** matrix, which means it is **its own reverse**, as it complies with the following:

$$A^2 = I_2$$

$$A^{-1} = A$$

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## Question 8

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**Vectors:**  $u = [3,4]$ ,  $v = [4,3]$

The included angle  $\theta$  of two vectors, is dependent on their respective magnitudes and inner product and is calculated as follows:

$$u \cdot v = \|u\| \cdot \|v\| \cdot \cos \theta_{uv} \implies \cos \theta_{uv} = \frac{u \cdot v}{\|u\| \cdot \|v\|} \quad (1)$$

$$\stackrel{(1)}{\implies} \cos \theta_{uv} = \frac{u \cdot v}{\|u\| \cdot \|v\|} = \frac{[3,4] \cdot [4,3]}{\|[3,4]\| \cdot \|[4,3]\|} = \frac{12 + 12}{(\sqrt{9 + 16}) \cdot (\sqrt{16 + 9})} = \frac{24}{5 \cdot 5}$$

$$= \frac{24}{25} = 0.96 \implies \cos^{-1}(0.96) = 16.26^\circ \implies \theta_{uv} = 16.26^\circ$$

Vectors  $u$  and  $v$  are **not collinear**, as their included angle  $\theta$  is neither  $0^\circ$  nor  $180^\circ$  (  $\cos \theta = 1$  and  $\cos \theta = -1$  respectively ). They are also **not orthogonal**, as in that case, their included angle  $\theta$  would be  $90^\circ$  (  $\cos \theta = 0$  ).

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## Question 9

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**Vectors:**  $u = [a, b]$ ,  $v = 2u$

As described in **Question 8**, the included angle  $\theta$  of two vectors, is dependent on their respective magnitudes and inner product, calculated by using formula **(1)** :

$$u \cdot v = \|u\| \cdot \|v\| \cdot \cos \theta_{uv} \implies \cos \theta_{uv} = \frac{u \cdot v}{\|u\| \cdot \|v\|} \quad (1)$$

$$\stackrel{(1)}{\implies} \cos \theta_{uv} = \frac{u \cdot v}{\|u\| \cdot \|v\|} = \cos \theta_{uv} = \frac{u \cdot 2u}{\|u\| \cdot \|2u\|} = \frac{[a, b] \cdot [2a, 2b]}{\|[a, b]\| \cdot \|[2a, 2b]\|} =$$

$$\frac{2 \cdot (a^2 + b^2)}{(\sqrt{a^2 + b^2}) \cdot (\sqrt{4a^2 + 4b^2})} = \frac{2 \cdot (a^2 + b^2)}{2 \cdot \left(\sqrt{a^2 + b^2}\right)^2} = \frac{a^2 + b^2}{a^2 + b^2} = 1$$

$$\implies \cos^{-1}(1) = 0^\circ \implies \theta_{uv} = 0^\circ$$

In this case, the two vectors are **collinear**, as vector  $v$  is a **scalar multiple** of vector  $u$  ( $v = k \cdot u$ ), and have the **same direction** as the **scalar**  $k$  is a **positive** number ( $k = 2$ ), therefore their included angle  $\theta$  is  $0^\circ$  (validated by the calculations above).

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## Question 10

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In order to find the similar documents based on the 3-dimensional vector representations,  $d_n \in \mathbb{R}^3$ ,  $n = 1, \dots, 5$ , a 5x5 matrix was constructed, with each row being each of the 3-D vectors  $d_1, d_2, d_3, d_4, d_5$ , normalised as follows:

$$\sum_{i=1}^5 \sum_{j=1}^3 \frac{d_{ij}}{\|d_i\|}$$

where  $d_{ij}$  the  $j$ -th element of the  $i$ -th vector and  $\|d_i\| = \sqrt{d_{i1}^2 + d_{i2}^2 + d_{i3}^2}$  the norm of the respective vector  $i$ :

$$D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} \Rightarrow D_{5 \times 3} = \begin{bmatrix} 8 & 6 & 0 \\ 0 & 6 & 8 \\ 6 & 0 & 8 \\ 2 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \Rightarrow D_{normal} = \begin{bmatrix} \frac{8}{10} & \frac{6}{10} & \frac{0}{10} \\ \frac{0}{10} & \frac{6}{10} & \frac{8}{10} \\ \frac{6}{10} & \frac{0}{10} & \frac{8}{10} \\ \frac{2}{10} & \frac{3}{10} & \frac{0}{10} \\ \frac{9}{10.8166} & \frac{6}{10.8166} & \frac{0}{10.8166} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.6 & 0 \\ 0 & 0.6 & 0.8 \\ 0.6 & 0 & 0.8 \\ 0.5547 & 0.8320 & 0 \\ 0.8320 & 0.5547 & 0 \end{bmatrix}$$

Finally, we multiply **normalized** matrix **D** with its **transpose**, to get the inner product of every pair of vectors and perform the comparisons between the document representations:

$$D_{normal} \cdot D_{normal}^T = \begin{bmatrix} 0.8 & 0.6 & 0 \\ 0 & 0.6 & 0.8 \\ 0.6 & 0 & 0.8 \\ 0.5547 & 0.8320 & 0 \\ 0.8320 & 0.5547 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0 & 0.6 & 0.5547 & 0.8320 \\ 0.6 & 0.6 & 0 & 0.8320 & 0.5547 \\ 0 & 0.8 & 0.8 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0.36 & 0.48 & 0.9429 & 0.9984 \\ \mathbf{0.36} & 1 & 0.64 & 0.4992 & 0.3328 \\ \mathbf{0.48} & \mathbf{0.64} & 1 & 0.3328 & 0.4992 \\ \mathbf{0.9429} & \mathbf{0.4992} & \mathbf{0.3328} & 1 & 0.9230 \\ \mathbf{0.9984} & \mathbf{0.3328} & \mathbf{0.4992} & \mathbf{0.9230} & 1 \end{bmatrix} = D_{normal\_dot\_product}$$

We keep only the **lower triangular matrix**, where each value represents the dot product of the  $j$ -th vector ( $d_j$ ) with the  $i$ -th vector ( $d_i$ ), where  $i, j$  the rows and the columns of the matrix  $D_{dot\_product}$  respectively.

The formula of the dot product of two vectors, using the included angle  $\theta$  , is:

$$d_i \cdot d_j = \|d_i\| \cdot \|d_j\| \cdot \cos \theta \implies \cos \theta = \frac{d_i \cdot d_j}{\|d_i\| \cdot \|d_j\|} = \cos \theta_{d_i, d_j}$$

The process we followed above, calculates exactly the value of the cosine of the included angle  $\theta$ , giving information on the relation between those vectors. If the angle's value is close to 0, then the vectors are orthogonal, if it is close to 1 they are

Therefore, the results are the following:

Inner Product (of normalized vectors)	Notes	Similarity
$\cos \theta_{d_1, d_2} = 0.36$		Low
$\cos \theta_{d_1, d_3} = 0.48$		
$\cos \theta_{d_1, d_4} = 0.9429$	$> 0.94 \implies \theta_{d_1, d_4} < 19^\circ$	High
$\cos \theta_{d_1, d_5} = 0.9984$	$\sim 1 \implies \theta_{d_1, d_5} \sim 0^\circ$	High
$\cos \theta_{d_2, d_3} = 0.64$		
$\cos \theta_{d_2, d_4} = 0.4492$		
$\cos \theta_{d_2, d_5} = 0.3328$		Low
$\cos \theta_{d_3, d_4} = 0.3328$		Low
$\cos \theta_{d_3, d_5} = 0.4992$		
$\cos \theta_{d_4, d_5} = 0.9230$	$> 0.92 \implies \theta_{d_4, d_5} < 23^\circ$	High

Table 3.10

The **Documents 1, 4 and 5** are **very similar**.

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## Notes

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For some questions of project 3, some **code** was developed to **experiment** with the given tasks and **validate** the manually calculated results, the file ( M902\_Project\_3\_CTKylafi\_LT1200012.py ) of which is included in the uploaded .zip file ( M902\_Project3\_CTKylafi\_LT1200012.zip ).