



M902

Βασικές Μαθηματικές Έννοιες στη Γλωσσική Τεχνολογία

Project 2

Κυλάφη Χριστίνα-Θεανώ

LT1200012

TABLE OF CONTENTS

Question 1	3
Question 2	3
Question 3	4
Question 4	5
Question 5	5
Question 6	5
Question 7	6
Question 8	6
Question 9	7
Question 10	7

Question 1

The composite function $S(f(x))$ where $S(x) = \frac{1}{1 + e^{-x}}$ and $f(x) = ax + b$, is calculated as follows:

$$S(f(x)) = \frac{1}{1 + e^{-f(x)}} = \frac{1}{1 + e^{-(ax+b)}} = \frac{1}{1 + \frac{1}{e^{(ax+b)}}} = \frac{1}{\frac{e^{(ax+b)} + 1}{e^{(ax+b)}}} = \frac{e^{(ax+b)}}{e^{(ax+b)} + 1} = \frac{e^{ax}e^b}{e^{ax}e^b + 1}$$

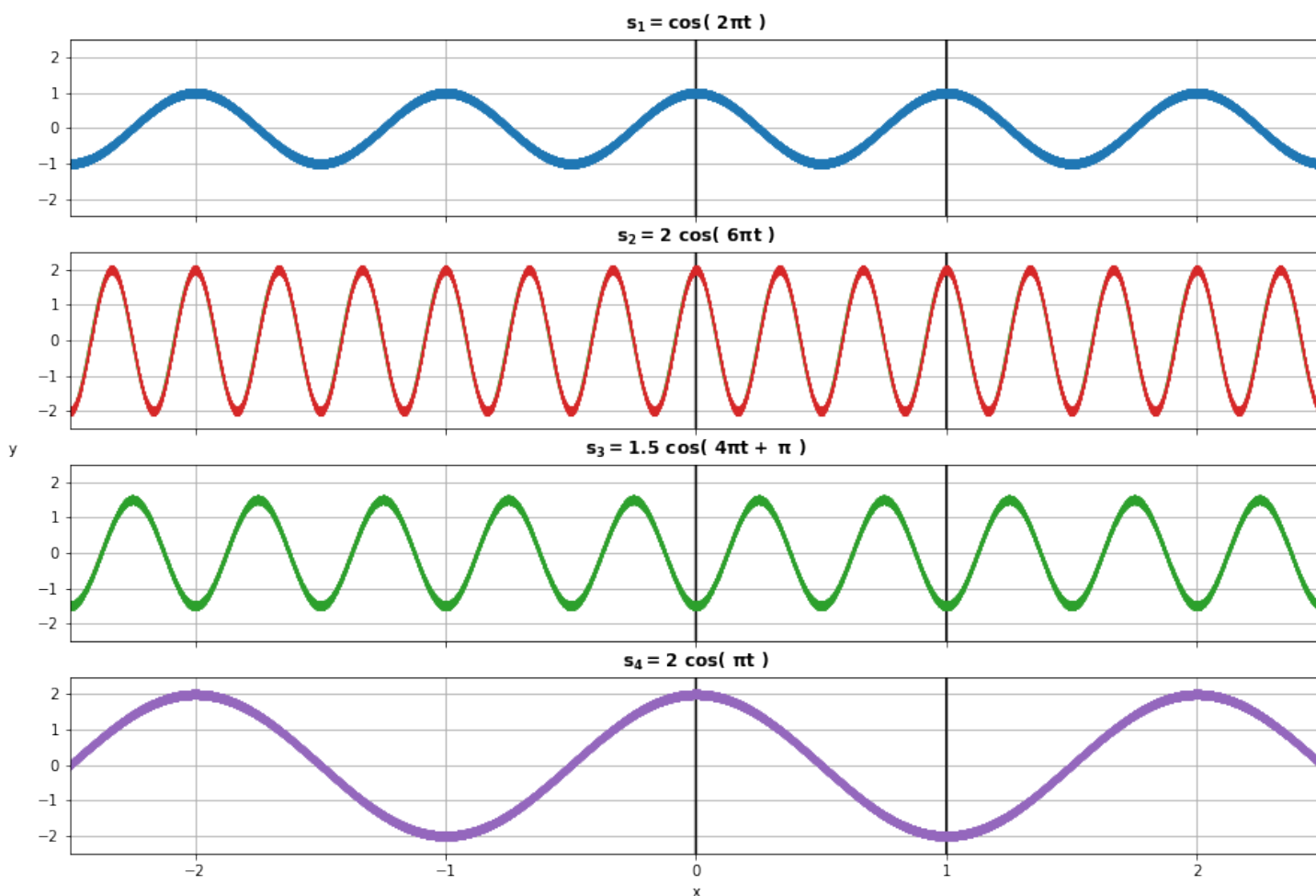
Question 2

Lalala

Question 3

$$s = A \cos(2\pi f t)$$

- (a) $A_1 = 1, \quad f = 1, \quad \theta = 0, \quad s_1 = \cos(2\pi t)$
(b) $A_2 = 2, \quad f = 3, \quad \theta = 0, \quad s_2 = 2 \cos(6\pi t)$
(c) $A_3 = 1.5, \quad f = 2, \quad \theta = \pi, \quad s_3 = 1.5 \cos(4\pi t + \pi)$
(d) $A_4 = 2, \quad f = 0.5, \quad \theta = 0, \quad s_4 = 2 \cos(\pi t)$



Question 4

Question 5

- (a) The derivative of function $f(x) = ax^2$ is $f'(x) = 2ax$ (**a** → **4**)
- (b) The derivative of function $f(x) = \cos(2\pi ft)$ is $f'(x) = -\sin(2\pi ft)$ (**b** → **1**)
- (c) The derivative of function $f(x) = bx^3$ is $f'(x) = 3bx^2$ (**c** → **2**)
- (d) The derivative of function $f(x) = e^{cx}$ is $f'(x) = ce^{cx}$ (**d** → **3**)

Question 6

$$\begin{aligned} S'(x) &= \left(\frac{1}{1+e^{-x}} \right)' = [(1+e^{-x})^{-1}]' = (-1)(1+e^{-x})^{-2}(1+e^{-x})' = -\frac{(e^{-x})'}{(1+e^{-x})^2} \\ &= \frac{e^{-x}}{(1+e^{-x})^2} \quad (\mathbf{1}) \end{aligned}$$

$$S(x)(1-S(x)) = \left(\frac{1}{1+e^{-x}} \right) \left(1 - \frac{1}{1+e^{-x}} \right) = \left(\frac{1}{1+e^{-x}} \right) \left(\frac{1+e^{-x}-1}{1+e^{-x}} \right) = \left(\frac{1}{1+e^{-x}} \right) \left(\frac{e^{-x}}{1+e^{-x}} \right)$$

$$= \frac{e^{-x}}{(1 + e^{-x})^2} \quad (2)$$

$$(1), (2) \implies S'(x) = S(x) (1 - S(x))$$

Question 7

$$S(x) = \frac{1}{1 + e^{-x}}, \quad f(x) = ax + b, \quad S(f(x)) = \frac{1}{1 + e^{-f(x)}}$$

$$\begin{aligned} S'(f(x)) &= \left(\frac{1}{1 + e^{-f(x)}} \right)' \stackrel{Q1}{=} \left(\frac{e^{ax}e^b}{e^{ax}e^b + 1} \right)' = \frac{(e^{ax}e^b)'(e^{ax}e^b + 1) - (e^{ax}e^b)(e^{ax}e^b + 1)'}{(e^{ax}e^b + 1)^2} \\ &= \frac{(ae^{ax}e^b)(e^{ax}e^b + 1) - (e^{ax}e^b)(ae^{ax}e^b)}{(e^{ax}e^b + 1)^2} = \frac{(ae^{ax}e^b)(e^{ax}e^b + 1 - e^{ax}e^b)}{(e^{ax}e^b + 1)^2} \\ &= \frac{ae^{ax+b}}{(e^{ax}e^b + 1)^2} \end{aligned}$$

Question 8

Lala

Question **9**

Lala

Question **10**

Lala