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Project 4

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TABLE OF CONTENTS

Question **1**

$i\,.\ n = 3$ independent experiments (coin flips)

$$\Omega = \left\{ \begin{array}{c} \text{ΚΚΚ, ΚΚΓ, ΚΓΚ, ΚΓΓ} \\ \text{ΓΚΚ, ΓΚΓ, ΓΓΚ, ΓΓΓ} \end{array} \right\}$$

$ii\,.\ A_1 = \{$ ΓΚΚ, ΚΓΚ, ΚΚΓ, ΚΚΚ $\}$
$A_2 = \{$ ΓΚΚ, ΚΓΚ, ΚΚΓ $\}$
$A_3 = \{$ ΓΚΚ, ΚΓΚ, ΚΚΓ, ΚΚΚ $\} = A_1$
$A_4 = \{$ ΚΚΚ, ΓΓΓ $\}$
$A_5 = \{$ ΚΚΚ, ΚΚΓ, ΚΓΚ, ΚΓΓ $\}$

Let X a random variable expressing the number of successes (coin flip result → **Κ**), following **Binomial Distribution** (spoilers for iii below), $X \sim B(\ 3,\ 0.5)$. Then:

$$P(X = k) = \left(\begin{array}{c} n \\ k \end{array} \right) p^k\ (1 - p)^{n-k}\ ,$$

where
p is the **probability** of “success” outcome,
k is the number of **successes**,
n the total number of independent **experiments** performed.

$$P(A_1) = P(X = 2) + P(X = 3) = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) + \binom{3}{3}\left(\frac{1}{2}\right)^3 = \frac{3!}{2!1!}\ \frac{1}{8} + \frac{3!}{3!0!}\ \frac{1}{8} = = \frac{3}{8} + \frac{1}{8}$$

$$= \frac{N(A_1)}{N(\Omega)} = \frac{4}{8} = 0.5$$

$$P(A_2) = P(X = 2) = \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) = \frac{3!}{2!1!}\ \frac{1}{8} = \frac{3}{8} = \frac{N(A_2)}{N(\Omega)} = 0.375$$

$$P(A_3) = P(A_1) = 0.375$$

$$P(A_4) = P(X = 0) + P(X = 3) = \binom{3}{0}\left(\frac{1}{2}\right)^3 + \binom{3}{3}\left(\frac{1}{2}\right)^3 = \frac{3!}{0!3!}\ \frac{1}{8} + \frac{3!}{3!0!}\ \frac{1}{8} = = \frac{1}{8} + \frac{1}{8}$$

$$= \frac{N(A_1)}{N(\Omega)} = \frac{2}{8} = 0.25$$

Event A_5 concerns only the first coin flip, which is independent of the overall number of experiments. Therefore, the probability of a sole coin flip (the first one) resulting in K, is always $P(K) = \frac{1}{2} = 0.5$.

$iii.\ n$ independent experiments (coin flips)

Here, for event A_2 we apply the same formula as in ii, with $X \sim B(\ n,\ 0.5)$:

$$P(A_2) = P(X = 2) = \binom{n}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2} = \frac{n!}{2!(n-2)!}\left(\frac{1}{2}\right)^n = \frac{n\ (n-1)}{2}\left(\frac{1}{2}\right)^n$$

As also mentioned in ii , the probability of A_5 is always the same and equals the probability of a single coin flip resulting in K, $P(K) = \frac{1}{2} = 0.5$.

Question 2

Let X a random variable following Normal Distribution $X \sim N(\ 60,\ 5^2\ )$ expressing the student weights. Then:

$$\alpha)\ \ P(X > 70) = P\left(\frac{X-\mu}{\sigma} > \frac{70-\mu}{\sigma}\right) = P\left(Z > \frac{70-60}{5}\right) = P(Z > 2) = 1 - P(Z < 2)$$

$$= 1 - \Phi(2) = 1 - 0.9772 = 0.0228$$

$$\beta)\ \ P(55 < X < 65) = P(X < 65) - P(X < 55) = P\left(\frac{X-\mu}{\sigma} < \frac{65-\mu}{\sigma}\right) - P\left(\frac{X-\mu}{\sigma} < \frac{55-\mu}{\sigma}\right)$$

$$= P\left(\frac{X-\mu}{\sigma} < \frac{65-60}{5}\right) - P\left(\frac{X-\mu}{\sigma} < \frac{55-60}{5}\right) = P(Z < 1) - P(Z < -1)$$

$$= \Phi(1) - \Phi(-1) = \Phi(1) - (1 - \Phi(1)) = 2\ \Phi(1) - 1 = 2 * 0.8413 - 1 = 0.6826$$

Question **3**

$P(\text{ ανοσ }) = 0.7 = p$

The problem can be modelled as a binary outcome (rabbit immunised or not) experiment, executed n times (selecting n rabbits). Then, X is a random variable expressing the number of immunised rabbits picked, with $X \sim B(\ n,\ 0.7)$, where $n\ = 5$:

$i\ .\quad P(X=3) = \binom{5}{3} * 0.7^3 * (1-0.7)^{5-3} = \dfrac{5!}{3!2!} * 0.7^3 * 0.3^2 = 10 * 0.7^3 * 0.3^2 = 0.3087$

$ii\ .$ Here, two explanations of the question are going to be followed. However, the resulted probabilities are equal.

$1.$ The probability of picking 3 non-immunised (failure) rabbits and then 1 immunised (success). The task can be modelled as the calculation of the probability that the first success (immunised rabbit) requires k independent trials, thus we calculate the probability of $k-1$ failures and 1 success (k_{th} trial). In this particular case, X is following the **Geometric Distribution**, $X \sim Geo(0.7)$:

$$P(X=k) = (1-p)^{k-1}p$$

Then:

$$P(X=4) = (1-0.7)^{4-1}0.7 = 0.3^3 * 0.7 = 0.0189$$

$2.$ The probability of the first rabbit to be the only immunised one, out of 4 rabbits picked in total.

$$P(1_{st} \text{ rabbit immunised }) = 0.7 * (1-0.7)^{4-1} = 0.7 * 0.3^3 = 0.0189$$

Question **4**

The solution of this problem, was calculated through code developed in Python. The results are presented below:

		Class	Sentence
Training	**1**	**-**	μη χάσετε το χρόνο σας
	2	**+**	καταπληκτικές ερμηνείες σε ένα δύσκολο έργο
	3	**+**	η καλύτερη θεατρική παράσταση του χειμώνα
	4	**-**	δεν ήταν ευχάριστη
	5	**+**	μια ευχάριστη έκπληξη
Test	**1**	**?**	πέρασα μια ευχάριστη θεατρική βραδιά
	2	**?**	δεν πέρασα μια ευχάριστη θεατρική βραδιά

Word list of concatenated sentences of positive class “ + ” :
{ 'καταπληκτικές', 'ερμηνείες', 'σε', 'ένα', 'δύσκολο', 'έργο', 'η', 'καλύτερη', 'θεατρική', 'παράσταση', 'του', 'χειμώνα', 'μια', 'ευχάριστη', 'έκπληξη' }
Count: 15

Word list of concatenated sentences of negative class “ - ” :
{ 'μη', 'χάσετε', 'το', 'χρόνο', 'σας', 'δεν', 'ήταν', ‘ευχάριστη' }
Count: 8

Word set (union) of the above (all sentences):
{ 'παράσταση', 'σε', 'ερμηνείες', 'καταπληκτικές', 'το', 'δύσκολο', 'ήταν', 'καλύτερη', 'έκπληξη', 'ευχάριστη', 'έργο', 'ένα', 'μη', 'του', 'η', 'μια', 'χρόνο', 'χάσετε', 'δεν', 'θεατρική', 'σας', ‘χειμώνα' }
Count: 22

$$P(-) = \frac{N_{sentences\ of\ the\ class}}{N_{total\ sentences}} = \frac{2}{5} = 0.4$$

$$P(+) = \frac{N_{sentences\ of\ the\ class}}{N_{total\ sentences}} = \frac{3}{5} = 0.6$$

We then calculate the conditional probability of all the possible classes (negative / positive), given each test sentence. The maximum probability dictates the predicted class of the respective sentence, by the Naive Bayes classifier :

$$P(c\,|\,S_N) = P(c) \prod_{w \in W_{S_N}} P(w\,|\,c)\,,$$

where

W_{S_N} the words comprising test sentence S_N being examined, which are also included in the training set, otherwise they are omitted

$c \in C$, with C being the set of classes

$i\,.$ $P(-\,|\,S_1) = P(-) \prod_{w \in W_{S_1}} P(w\,|\,-) = P(-)\ P(\text{ μια } |\,-)\ P(\text{ ευχάριστη } |\,-)\ P(\text{ θεατρική } |\,-)$

$$= 0.4 * \frac{0+1}{8+22} * \frac{1+1}{8+22} * \frac{0+1}{8+22} = 0.4 * \frac{2}{30^3} = 2.\overline{962} * 10^{-5}$$

$$P(+\,|\,S_1) = P(+) \prod_{w \in W_{S_1}} P(w\,|\,+) = P(+)\ P(\text{ μια } |\,+)\ P(\text{ ευχάριστη } |\,+)\ P(\text{ θεατρική } |\,+)$$

$$= 0.6 * \frac{1+1}{15+22} * \frac{1+1}{15+22} * \frac{1+1}{15+22} = 0.6 * \frac{8}{37^3} = 9.476 * 10^{-5} > P(-\,|\,S_1)$$

Therefore, test sentence **1** is **classified as " + "**, which is **correct** !

$ii\,.$ The second sentence differs from the first one only on one word, "δεν", so we can calculate the respective probabilities by multiplying each of the previous probabilities with the term $P(\text{ δεν } |\,c\,)$, where c the class for which we examine the sentence.

$$P(-\,|\,S_2) = P(-) \prod_{w \in W_{S_2}} P(w\,|\,-) = P(-\,|\,S_1)\ P(\text{ δεν } |\,-) = 0.4 * \frac{2}{30^3} * \frac{2}{30}$$

$$= 0.4 * \frac{4}{30^4} = 1.975 * 10^{-6}$$

$$P(+\,|\,S_2) = P(+) \prod_{w \in W_{S_2}} P(w\,|\,+) = P(+\,|\,S_1)\ P(\text{ δεν } |\,+) = 0.6 * \frac{8}{37^3} * \frac{1}{37}$$

$$= 0.6 * \frac{8}{37^4} = 2.561 * 10^{-6} > P(-\,|\,S_2)$$

Test sentence **2** is also **classified as “ + ”**, which is **incorrect** !

However, it can be explained, mostly due to the nature of Naive Bayes classifier.

For instance, the aforementioned classifier, is largely **affected** by the amount of **training data** available. That means, less training data for a class may result in a bias towards the opposite class in a binary classification task like the one above (weight shrinking for classes with fewer examples). In fact, in our case, the training data for the true class of **sentence 2** (“ - ”) are less than the opposite class (“ + ”), in which it was falsely classified from NB classifier.

In this particular case, word “δεν”, which is the only difference between the two sentences that are tested, seems to have little contribution to the decision, as it is assigned a weight not capable of changing the classification result from “ + ” (**sentence 1**) to “ - ” (**sentence 2**) .

What is more, the classifier **assumes feature independency** in a manner that ignores possible relations between the words, such as “δεν”, which gives a completely different meaning to verbs when added before them. As a result, even as an important feature that changes the semantic frame of the sentence thus its class, it is not given the proper weight, leading to misclassification.

Question **5**

Question **6**

The general formula to make a transition from odds ($x : y$) in favour of an event to the probability of the event, is :

$$\frac{x}{x+y}$$

i . The odds for rain in Helsinki are 206:159

$$206 : 159 \rightarrow \frac{206}{206+159} = \frac{206}{365} = 0.564 = 56.4\,\%$$

ii . The odds for getting three of a kind in poker are about 1:46

$$1 : 46 \rightarrow \frac{1}{1+46} = \frac{1}{47} = 0.021 = 2.1\,\%$$

Question **7**

Let random variable X which expresses the daily product demand, following Normal Distribution, $X \sim N(\ 5000,\ 300^2)$.

Then:

$$\alpha)\ \ P(X < 5300) = P(\frac{X-\mu}{\sigma} < \frac{5300-5000}{300}) = P(Z < 1) = \Phi(1) = 0.8413$$

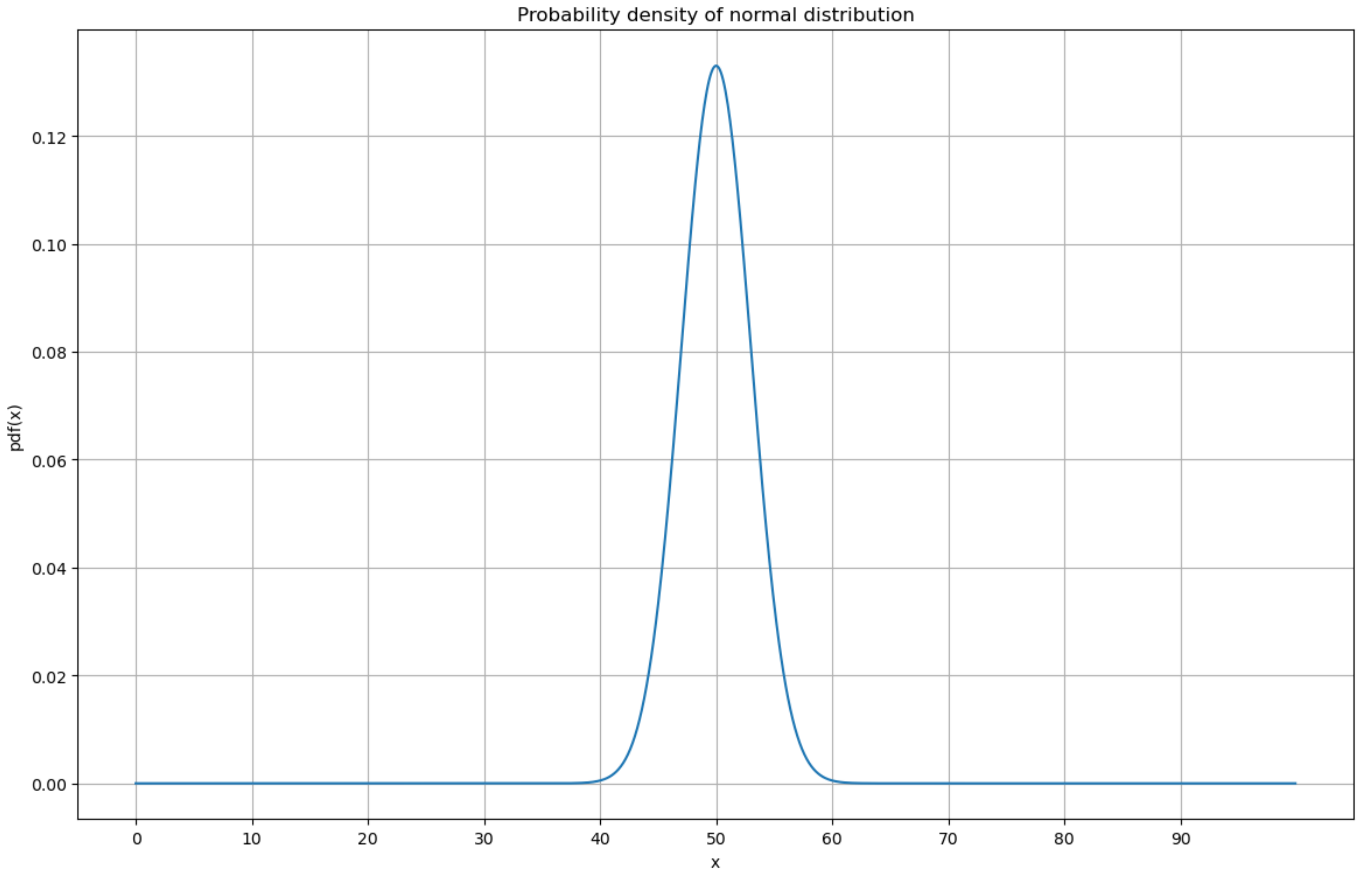
$$\beta)\ \ P(X < w) = P\left(\frac{X-\mu}{\sigma} < \frac{w-5000}{300}\right) = P\left(Z < \frac{w-5000}{300}\right) = \Phi\left(\frac{w-5000}{300}\right) = 0.9 \approx \Phi(1.28)$$

$$\Longrightarrow\ \Phi\left(\frac{w-5000}{300}\right) = \Phi(1.28)\ \Longrightarrow\ \frac{w-5000}{300} = 1.28$$

$$\Longrightarrow\ \frac{w-5000}{300} = 1.28\ \Longrightarrow\ w\ = 300 * 1.28 + 5000 = 5384\ \text{ products}$$

Question **8**

Question **9**



Probability density of normal distribution
0.12
0.10
0.08
0.06
0.04
0.02
0.00
pdf(x)
0
10
20
30
40
50
60
70
80
90
x

Question **10**