



M902

Βασικές Μαθηματικές Έννοιες στη Γλωσσική Τεχνολογία

Project 3

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Question **1**

Question **2**

Question **3**

Question 4

Question **5**

Question 6

Let $A = \begin{bmatrix} a_1^T \\ a_2^T \\ a_3^T \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$, the unknown vector $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and the dot products vector

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 9 \end{bmatrix}.$$

This system of linear equations in each condensed form, is written as follows:

$$A \cdot x = b$$

The unknown vector x is calculated by:

$$A \cdot x = b \implies A^{-1} \cdot A \cdot x = A^{-1} \cdot b \implies I \cdot x = A^{-1} \cdot b \implies x = A^{-1} \cdot b \quad (1)$$

$$A^{-1} = \frac{1}{|A|} \text{adj}(A) \quad (2)$$

Determinant of matrix A :

$$\det(A) = |A| = \begin{vmatrix} 1 & 2 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = (-1)^{1+1} \cdot 1 \cdot \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} + (-1)^{1+2} \cdot 2 \cdot \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} + (-1)^{1+3} \cdot 0 \cdot \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} =$$

$$1 \cdot 1 \cdot (0 \cdot 2 - (-1) \cdot 1) - 1 \cdot 2 \cdot (2 \cdot 2) + 0 = +1 - 8 = -7 \quad (3)$$

Adjugate matrix (transpose of cofactor matrix) of A :

$$\text{adj}(A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = 1, \quad A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 2 & -1 \\ 0 & 2 \end{vmatrix} = -4, \quad A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} = 2$$

$$A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = -4, \quad A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 2, \quad A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = -1$$

$$A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 2 & 0 \\ 0 & -1 \end{vmatrix} = -2, \quad A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix} = 1, \quad A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 2 \\ 2 & 0 \end{vmatrix} = -4$$

$$\text{adj}(A) = \begin{bmatrix} 1 & -4 & -2 \\ -4 & 2 & 1 \\ 2 & -1 & -4 \end{bmatrix} \quad (4)$$

$$\stackrel{(2), (3), (4)}{\implies} A^{-1} = \frac{1}{-7} \cdot \begin{bmatrix} 1 & -4 & -2 \\ -4 & 2 & 1 \\ 2 & -1 & -4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{4}{7} & \frac{2}{7} \\ \frac{4}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} & \frac{4}{7} \end{bmatrix}$$

$$\stackrel{(1), (5)}{\implies} x = A^{-1} \cdot b \implies$$

$$x = \begin{bmatrix} -\frac{1}{7} & \frac{4}{7} & \frac{2}{7} \\ \frac{4}{7} & -\frac{2}{7} & -\frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} & \frac{4}{7} \end{bmatrix} \cdot \begin{bmatrix} 5 \\ 8 \\ 9 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} \cdot 5 + \frac{4}{7} \cdot 8 + \frac{2}{7} \cdot 9 \\ \frac{4}{7} \cdot 5 - \frac{2}{7} \cdot 8 - \frac{1}{7} \cdot 9 \\ -\frac{2}{7} \cdot 5 + \frac{1}{7} \cdot 8 + \frac{4}{7} \cdot 9 \end{bmatrix} = \begin{bmatrix} \frac{45}{7} \\ -\frac{5}{7} \\ \frac{34}{7} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \implies$$

$$x_1 = \frac{45}{7}, \quad x_2 = -\frac{5}{7}, \quad x_3 = \frac{34}{7}$$

Question **7**

Question **8**

Question 9

Question 10

In order to find the similar documents based on the 3-dimensional vector representations, $d_n \in \mathbb{R}^3$, $n = 1, \dots, 5$, a 5x5 matrix was constructed, with each row being each of the 3-D vectors d_1, d_2, d_3, d_4, d_5 , normalised as follows:

$$\sum_{i=1}^5 \sum_{j=1}^3 \frac{d_{ij}}{\|d_i\|}$$

where d_{ij} the j -th element of the i -th vector and $\|d_i\| = \sqrt{d_{i1}^2 + d_{i2}^2 + d_{i3}^2}$ the norm of the respective vector i :

$$D = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix} \Rightarrow D_{5 \times 3} = \begin{bmatrix} 8 & 6 & 0 \\ 0 & 6 & 8 \\ 6 & 0 & 8 \\ 2 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \Rightarrow D_{normal} = \begin{bmatrix} \frac{8}{10} & \frac{6}{10} & \frac{0}{10} \\ \frac{0}{10} & \frac{6}{10} & \frac{8}{10} \\ \frac{6}{10} & \frac{0}{10} & \frac{8}{10} \\ \frac{2}{10} & \frac{3}{10} & \frac{0}{10} \\ \frac{9}{10.8166} & \frac{6}{10.8166} & \frac{0}{10.8166} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.6 & 0 \\ 0 & 0.6 & 0.8 \\ 0.6 & 0 & 0.8 \\ 0.5547 & 0.8320 & 0 \\ 0.8320 & 0.5547 & 0 \end{bmatrix}$$

Finally, we multiply **normalized** matrix **D** with its **transpose**, to get the inner product of every pair of vectors and perform the comparisons between the document representations:

$$D_{normal} \cdot D_{normal}^T = \begin{bmatrix} 0.8 & 0.6 & 0 \\ 0 & 0.6 & 0.8 \\ 0.6 & 0 & 0.8 \\ 0.5547 & 0.8320 & 0 \\ 0.8320 & 0.5547 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & 0 & 0.6 & 0.5547 & 0.8320 \\ 0.6 & 0.6 & 0 & 0.8320 & 0.5547 \\ 0 & 0.8 & 0.8 & 0 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0.36 & 0.48 & 0.9429 & 0.9984 \\ 0.36 & 1 & 0.64 & 0.4992 & 0.3328 \\ 0.48 & 0.64 & 1 & 0.3328 & 0.4992 \\ 0.9429 & 0.4992 & 0.3328 & 1 & 0.9230 \\ 0.9984 & 0.3328 & 0.4992 & 0.9230 & 1 \end{bmatrix} = D_{normal_dot_product}$$

We keep only the **lower triangular matrix**, where each value represents the dot product of the j -th vector (d_j) with the i -th vector (d_i), where i, j the rows and the columns of the matrix $D_{dot_product}$ respectively.

The formula of the dot product of two vectors, using the included angle θ , is:

$$d_i \cdot d_j = \|d_i\| * \|d_j\| * \cos\theta \implies \cos\theta = \frac{d_i \cdot d_j}{\|d_i\| * \|d_j\|} = \cos\theta_{d_i, d_j}$$

The process we followed above, calculates exactly the value of the cosine of the included angle θ , giving information on the relation between those vectors. If the angle's value is close to 0, then the vectors are orthogonal, if it is close to 1 they are

Therefore, the results are the following:

Inner Product (of normalized vectors)	Value	Similarity
$\cos\theta_{d_1, d_2} = 0.36$		Low
$\cos\theta_{d_1, d_3} = 0.48$		
$\cos\theta_{d_1, d_4} = 0.9429$	$> 0.94 \implies \theta_{d_1, d_4} < 19^\circ$	High
$\cos\theta_{d_1, d_5} = 0.9984$	$\sim 1 \implies \theta_{d_1, d_5} \sim 0^\circ$	High
$\cos\theta_{d_2, d_3} = 0.64$		
$\cos\theta_{d_2, d_4} = 0.4492$		
$\cos\theta_{d_2, d_5} = 0.3328$		Low
$\cos\theta_{d_3, d_4} = 0.3328$		Low
$\cos\theta_{d_3, d_5} = 0.4992$		
$\cos\theta_{d_4, d_5} = 0.9230$	$> 0.92 \implies \theta_{d_4, d_5} < 23^\circ$	High

Table 3.10

The **Documents 1, 4 and 5** are **very similar**.

Also, for this exercise, some code was developed to validate the results (included in the uploaded .zip file).