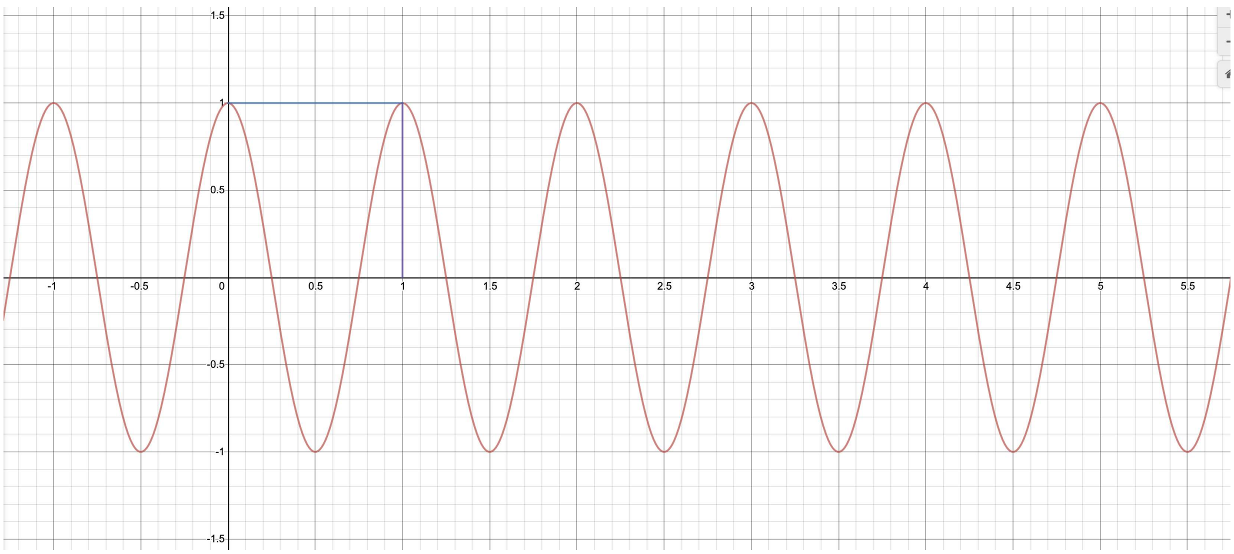
1. Calculate the composite function $S(f(x))$, where $f(x) = ax + b$ and $S(x)$ the sigmoid function is given by

$$S(x) = \frac{1}{1 + e^{-x}}$$

2. Find the inverse of the following functions

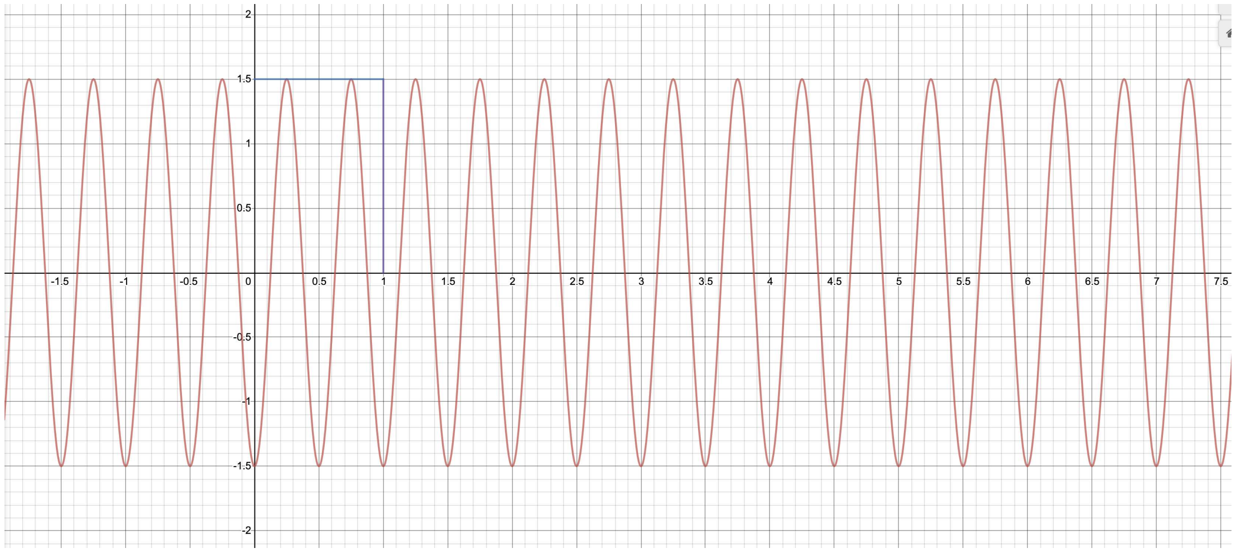
$$\text{(i) } f(x) = \sqrt{x - 3}$$
$$\text{(ii) } f(x) = \log(x - 2)$$

3. Write the mathematical expression of the function $Acos(2\pi fx + \theta)$ for the graphs (a-d) using the corresponding values of the parameters A, f and θ.

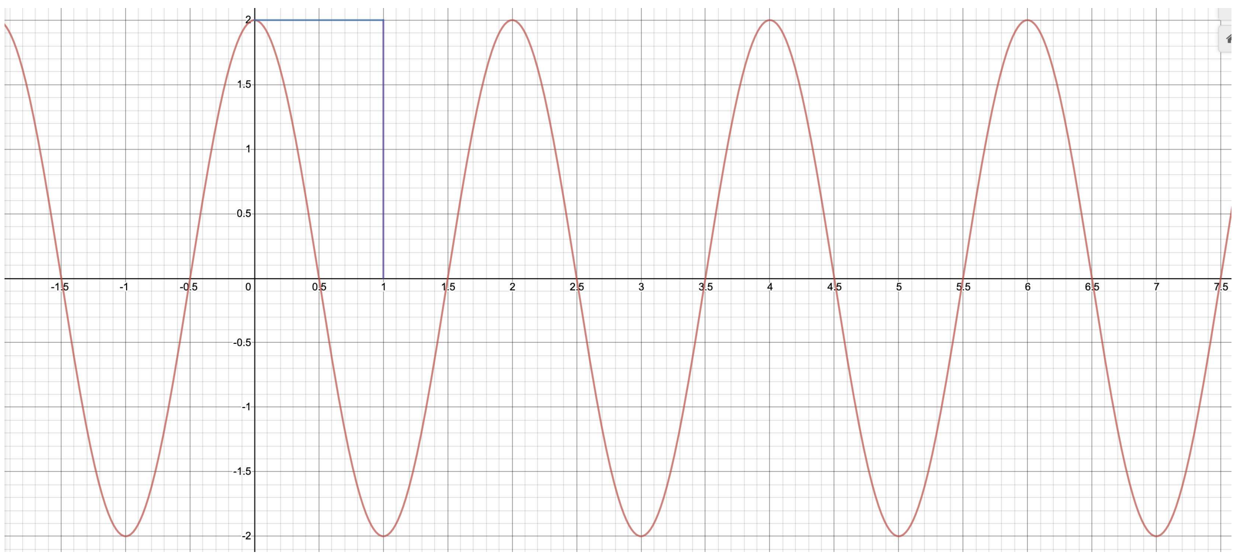


(a)

(b)

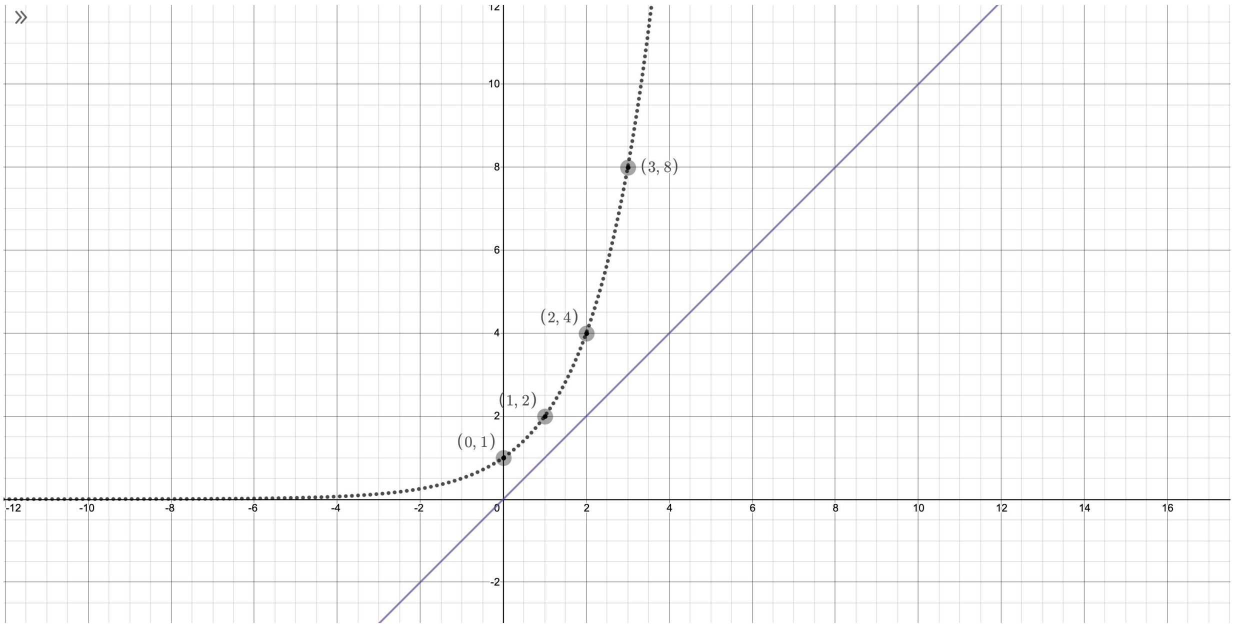


(c)

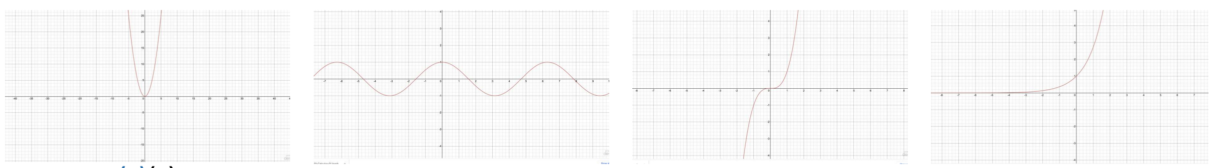


(d)

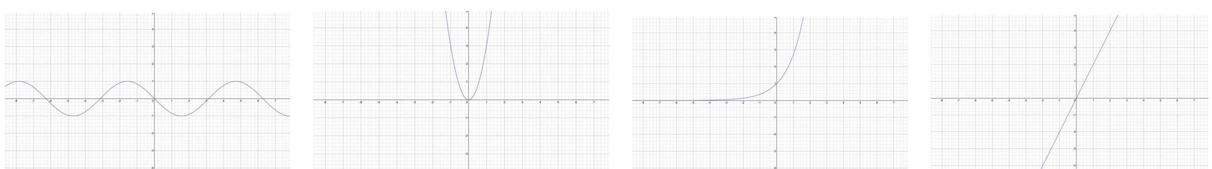
4. In the following graph we have shown the function $f(x) = 2^x$ together with the values of four points on this curve. Draw the inverse function $f^{-1}(x)$ and the respective points.



5. Find the corresponding graphs of the first derivatives of the functions depicted in the first row (graphs a-d).



(a)**(a)** (b) **(b)** (c)**(c)** (d)**(d)**



(a)(1) (b)(2) (c) (3) (c) (4)

6. The sigmoid function is one of the activation functions used in many neural networks and it is given by the expression:

$$S(x) = \frac{1}{1 + e^{-x}}.$$

Prove that the derivate of the sigmoid function that is used in the back-propagation stage of the learning procedure of a neural network is given by:

$$S'(x) = S(x)\,(1 - S(x))$$

7. Calculate the derivative of $S(f(x))$, where $S(x)$ is the sigmoid given in the previous question and $f(x) = ax + b$ with $a, b \in \mathbb{R}$.

8. Calculate the derivative with respect to μ of the following mathematical expressions:

(a)

$$\log \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x_1-\mu)^2}{2\sigma^2}}$$

(b)

$$\log \frac{1}{\sigma\sqrt{2\pi}}\left(e^{\frac{-(x_1-\mu)^2}{2\sigma^2}} + e^{\frac{-(x_2-\mu)^2}{2\sigma^2}}\right)$$

Note that the above terms correspond to the log-likelihood propability of a sample of n points that follow the normal distribution with mean value of μ and variance of σ^2] given by:

$$\log \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x_i-\mu)^2}{2\sigma^2}}$$

9. Calculate the derivatives with respect to σ of the mathematical expressions of the previous question.

10. The uniform probability density function of a random variable X is given by

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & x < a \text{ or } x > b \end{cases}$$

Calculate the mean value of X using the formula:

$$E[X] = \int_a^b x f_X(x) dx$$

and the second moment of X using the formula:

$$E[X^2] = \int_a^b x^2 f_X(x) dx$$