Exercises in Linear Algebra

Exercise 1

Let $A = \begin{bmatrix} 2 & -3 & 5 \\ 1 & -2 & 7 \\ 3 & 8 & 4 \end{bmatrix}$. Find a_{13} , a_{21} and a_{32} .

Exercise 2

Consider the following matrices:

$$A = \begin{bmatrix} -1 & 3 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}, C = \begin{bmatrix} 3 & 4 & 2 \\ -2 & 4 & -3 \end{bmatrix} \text{ and } D = \begin{bmatrix} 5 & -1 \\ 2 & 0 \end{bmatrix}$$

Compute the following (where possible).

a) $A + B$, b) CA , c) $5B - 2D$, d) $BD - D$ and e) BCD

Exercise 3

Using the matrices defined in Ex. 2, compute the following.

a) $(A^T + C)^T$, b) $-C^T - A$

Exercise 4

Using the matrices defined in Ex. 2, compute the following.

a) BD , b) DB , c) B^2

Exercise 5

Find the inverse matrix of $B = \begin{bmatrix} 2 & -3 \\ 4 & -2 \end{bmatrix}$

Verify that $BB^{-1} = B^{-1}B = I$

Exercise 6

Suppose the vectors $a_1 = [1, 2, 0]^T$, $a_2 = [2, 0, -1]^T$, $a_3 = [0, 1, 2]^T$. The dot products with an unknown vector x are 5 , 8 and 9 respectively. Find the unknown vector.

Exercise 7

Let $\mathbf{A} = \begin{bmatrix} cos\theta & sin\theta \\ sin\theta & -cos\theta \end{bmatrix}$.

a) Find $\mathbf{A}^2$

b) Find $\mathbf{A}^{-1}$

Exercise 8

Find the included angle θ of the vectors $\mathbf{u} = [3, 4]$ and $\mathbf{v} = [4, 3]$

Exercise 9

Find the included angle θ of the vectors $\mathbf{u} = [a, b]$ and $\mathbf{v} = 2\mathbf{u}$

Exercise 10

Suppose that we selected three features of a document (i.e. a document is represented by a 3-d vector $\mathbf{x} \in \mathbb{R}^3$). Five documents were examined and represented by the following vectors:

$\mathbf{d_1} = [8, 6, 0]$, $\mathbf{d_2} = [0, 6, 8]$, $\mathbf{d_3} = [6, 0, 8]$, $\mathbf{d_4} = [2, 3, 0]$, $\mathbf{d_5} = [9, 6, 0]$,

Find whether any similar documents (under this representation) exist.