ΤΜΗΜΑ ΠΛΗΡΟΦΟΡΙΚΗΣ & ΤΗΛΕΠΙΚΟΙΝΩΝΙΩΝ

ΕΛΛΗΝΙΚΗ ΔΗΜΟΚΡΑΤΙΑ
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Βασικές Μαθηματικές Έννοιες στη Γλωσσική Τεχνολογία

Project 2

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Question **1**

The composite function $S(f(x))$, where $S(x) = \dfrac{1}{1+e^{-x}}$ and $f(x) = ax + b$, is calculated as follows:

$$S(f(x)) = \frac{1}{1+e^{-f(x)}} = \frac{1}{1+e^{-(ax+b)}} = \frac{1}{1+\frac{1}{e^{(ax+b)}}} = \frac{1}{\frac{e^{(ax+b)}+1}{e^{(ax+b)}}} = \frac{e^{(ax+b)}}{e^{(ax+b)}+1} = \frac{e^{ax}e^{b}}{e^{ax}e^{b}+1}$$

DOMAINS AND RANGES!!

Question 2

A function is **invertible** only if each input has a unique output, which means each output is paired with exactly one input. That way, when the mapping is reversed, it will still be a function.

Definition

Let f be a function whose domain is the set A and whose codomain is the set B ($f : A \to B$). Then we say that f is invertible if f is **one-to-one** mapping and there is a function g with domain $Im(f) \subseteq B$ and image (range) A ($g : Im(f) \to A$) such that:

$$f(x) = y \iff g(y) = x$$

In this case, we call g the inverse of f and denote it by f^{-1}.

i) $\mathbf{f(x) = \sqrt{x - 3}}$

For f :

- $x - 3 \geq 0 \iff x \geq 3 \iff Dom(f) = [\ 3,\ +\infty\ )$ $\qquad$ (**1**)
- f is **continuous** on $Dom(f) = [\ 3,\ +\infty\ )$ [1] $\qquad$ (**2**)
- f is **differentiable** on $A = \{\ Dom(f) - \{0\}\ \} = \{\ [\ 3,\ +\infty\ ) - \{0\}\ \}$
  $= \{\ (\ 3,\ +\infty\ )\ \}$ [2] $\qquad$ (**3**)

First, we have to prove that f is **one-to-one** mapping:

$$\overset{(2),\ (3)}{\Longrightarrow} f'(x) = (\ \sqrt{x-3}\ )' = \frac{1}{2\sqrt{x-3}} > 0,\ \forall x \in A = (3,\ +\infty) \qquad (\ \mathbf{4}\ )$$

$\overset{(4)}{\Longrightarrow} f$ is **strictly monotonic** (strictly inscreasing), therefore it is **one-to-one** mapping

One approach to finding a formula for f^{-1} is to solve $f(x) = y$ for x.

$$f(x) = \sqrt{x-3} = y \overset{y\geq 0,\ x\geq 3}{\iff} x - 3 = y^2 \iff x = y^2 + 3 = f^{-1}(y)$$

$$\overset{(1)}{\Longrightarrow} x \geq 3 \iff y^2 + 3 \geq 3 \iff y^2 \geq 0$$

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[1] as a composition of continuous functions

[2] as a composition of differential functions

- $f(x) \geq 0 \iff y \geq 0 \iff Im(f) = [\ 0,\ +\infty\ ]$
- $f : [\ 3,\ +\infty\ ] \to [\ 0,\ +\infty\ ]$

$f^{-1}(x) = x^2 + 3,\ x \in Dom(f^{-1}) \cap Im(f),\ Dom(f^{-1}) = Dom(f^{-1}) \cap Im(f))$

$f^{-1}(x) = x^2 + 3,\ x \in \mathbb{R} \cap [\ 0,\ +\infty\ ]$

$f^{-1}(x) = x^2 + 3,\ x \in \mathbb{R} \cap [\ 0,\ +\infty\ ]$

$ii)\ f(x) = log(x - 2)$

For f :

-
-
- $f^{-1}(x) = x^2 + 3,\ x \in \mathbb{R} \cap [\ 0,\ +\infty\ ]$

Question **3**

$s = A\ cos(\ 2\pi ft\ )$

(a) $A_1 = 1, \quad f = 1, \quad \theta = 0, \quad s_1 = cos(\ 2\pi t\ )$
(b) $A_2 = 2, \quad f = 3, \quad \theta = 0, \quad s_2 = 2\ cos(\ 6\pi t\ )$
(c) $A_1 = 1.5, \quad f = 2, \quad \theta = \pi, \quad s_3 = 1.5\ cos(\ 4\pi t\ +\ \pi)$
(d) $A_1 = 2, \quad f = 0.5, \quad \theta = 0, \quad s_4 = 2\ cos(\ \pi t\ )$

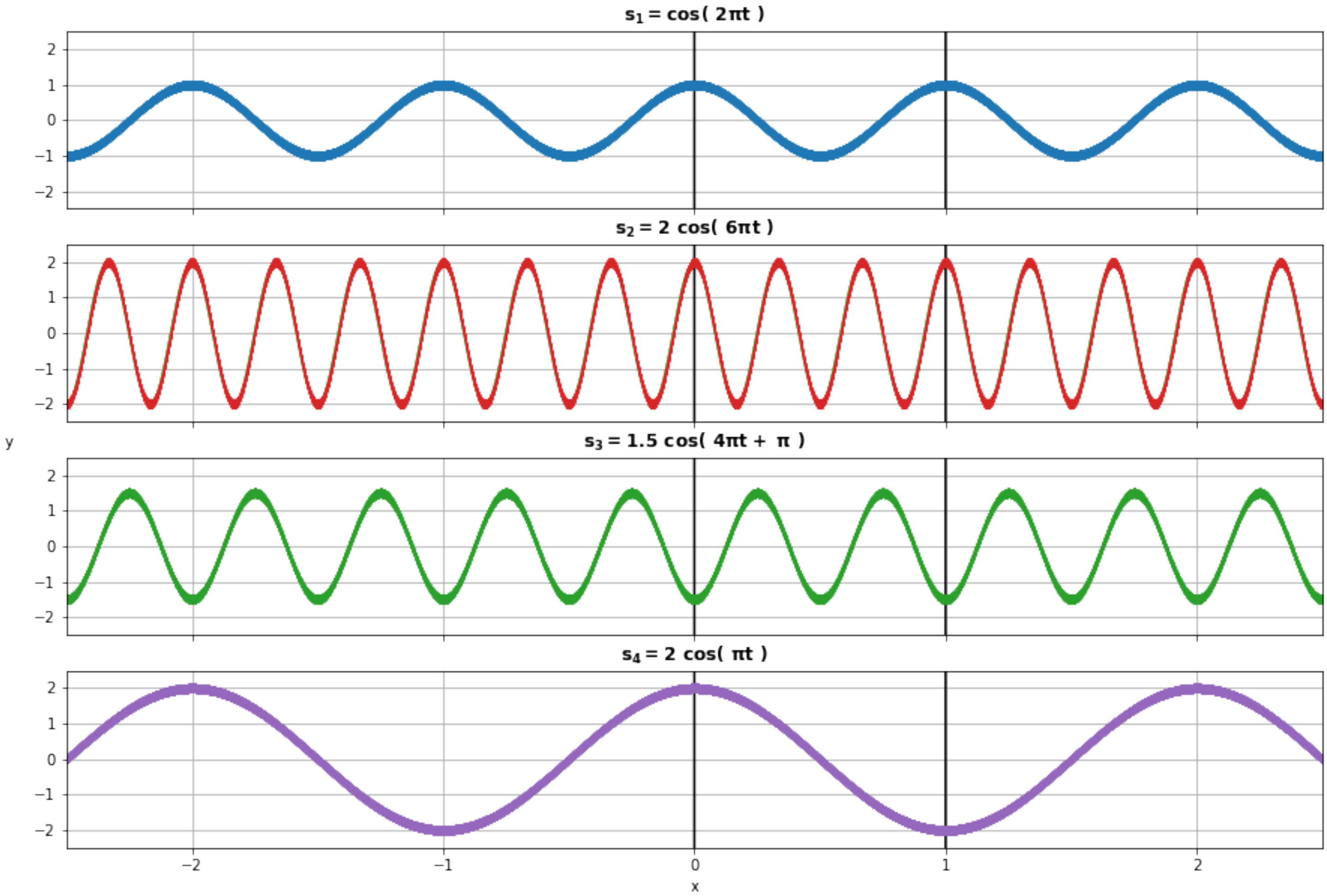
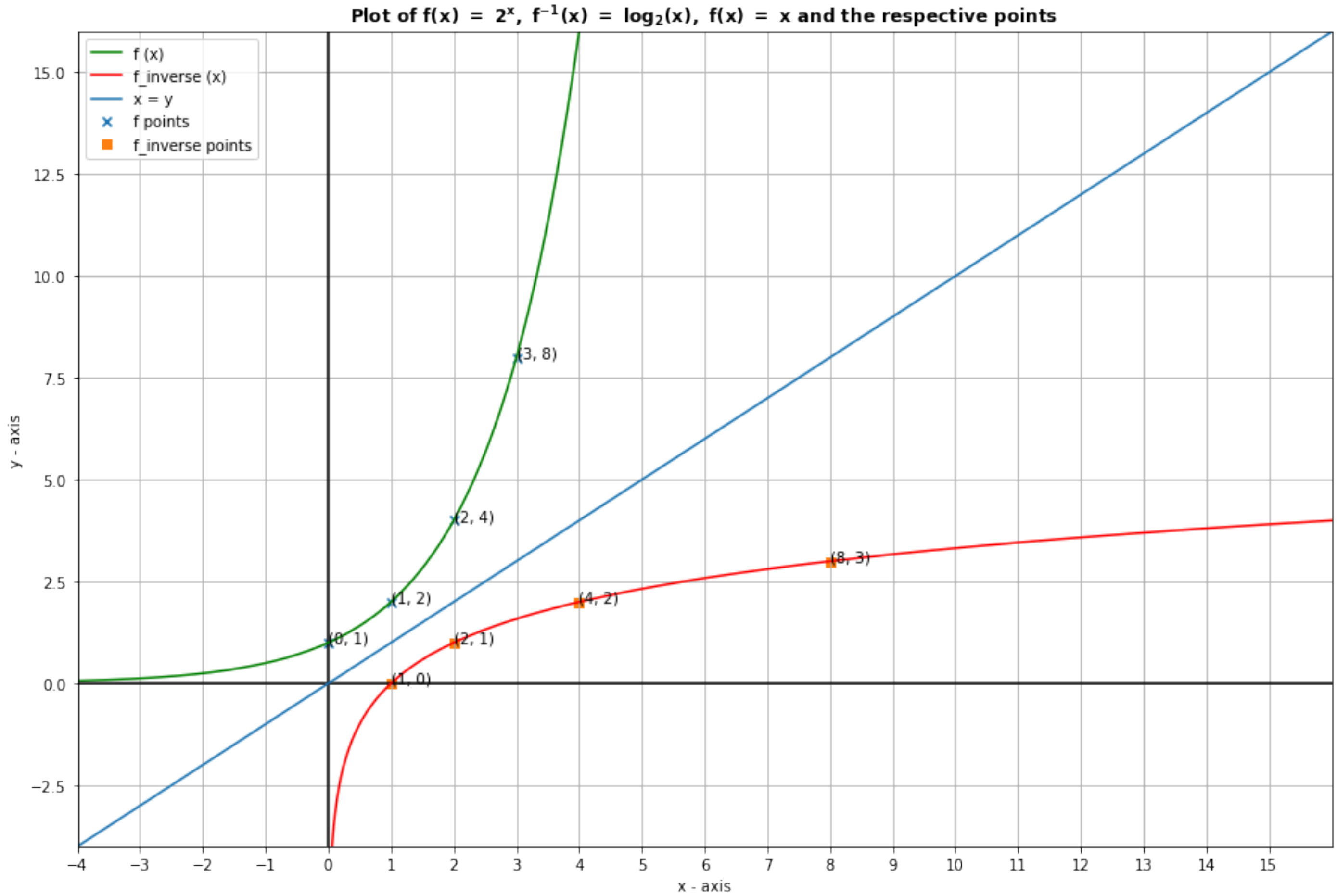


Figure 1

Question **4**



Plot of f(x) = 2^x, f^-1(x) = log_2(x), f(x) = x and the respective points
f (x)
f_inverse (x)
x = y
f points
f_inverse points
(3, 8)
(2, 4)
(1, 2)
(0, 1)
(8, 3)
(4, 2)
(2, 1)
(1, 0)
y - axis
x - axis

Question **5**

(a) The derivative of function $f(x) = ax^2$ is $f'(x) = 2ax$ ($\mathbf{a} \rightarrow \mathbf{4}$)

(b) The derivative of function $f(x) = cos(2\pi ft)$ is $f'(x) = -sin(2\pi ft)$ ($\mathbf{b} \rightarrow \mathbf{1}$)

(c) The derivative of function $f(x) = bx^3$ is $f'(x) = 3bx^2$ ($\mathbf{c} \rightarrow \mathbf{2}$)

(d) The derivative of function $f(x) = e^{cx}$ is $f'(x) = ce^{cx}$ ($\mathbf{d} \rightarrow \mathbf{3}$)

Question **6**

$$S'(x) = (\frac{1}{1+e^{-x}})' = [(1+e^{-x})^{-1}]' = (-1)(1+e^{-x})^{-2}(1+e^{-x})' = -\frac{(e^{-x})'}{(1+e^{-x})^2}$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} \quad (\mathbf{1})$$

$$S(x)(1-S(x)) = (\frac{1}{1+e^{-x}})(1-\frac{1}{1+e^{-x}}) = (\frac{1}{1+e^{-x}})(\frac{1+e^{-x}-1}{1+e^{-x}}) = (\frac{1}{1+e^{-x}})(\frac{e^{-x}}{1+e^{-x}})$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} \quad (\mathbf{2})$$

(1) , (2) $\implies S'(x) = S(x)\ (\ 1 - S(x)\ )$

Question **7**

$$S(x) = \frac{1}{1 + e^{-x}}, \quad f(x) = ax + b, \quad S(f(x)) = \frac{1}{1 + e^{-f(x)}}$$

$$S'(f(x)) = (\frac{1}{1 + e^{-f(x)}})' \overset{Q_1}{=} (\frac{e^{ax}e^{b}}{e^{ax}e^{b} + 1})' = \frac{(e^{ax}e^{b})'(e^{ax}e^{b} + 1) \ - \ (e^{ax}e^{b})(e^{ax}e^{b} + 1)'}{(e^{ax}e^{b} + 1)^2}$$

$$= \frac{(ae^{ax}e^{b})(e^{ax}e^{b} + 1) \ - \ (e^{ax}e^{b})(ae^{ax}e^{b})}{(e^{ax}e^{b} + 1)^2} = \frac{(ae^{ax}e^{b})(e^{ax}e^{b} + 1 \ - \ e^{ax}e^{b})}{(e^{ax}e^{b} + 1)^2}$$

$$= \frac{ae^{ax+b}}{(e^{ax}e^{b} + 1)^2}$$

Question **8**

Lala

Question **9**

Lala

Question **10**

Lala