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## Key Cutting Algorithm Application to Measurement Placement for Power System State Estimation

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### Abstract

This paper proposes the application development of a new optimization algorithm called Key Cutting Algorithm (KCA) to solve the measurement placement for power system state estimation. The “0” and “1” in KCA are represented as measurement installation in power system. Similarity probability of a key to a key set is introduced for a new measurement installation. Observability and critical measurement are included in the objective function to guarantee the critical measurement free in measurement scheme. The numerical results show that the KCA with low probability factor yields the best answers in terms of percent of the best hit.

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### 1. Introduction

Several stochastic optimization algorithms have been introduced for solving the power system optimization problem. Key Cutting Algorithm (KCA) is proposed by Qin [1]. If we lost our key, the locksmith is almost asking to help. Normally, the locksmith uses the similar types of key and tries to open. This trying of locksmith is proceeded in KCA. “0” and “1” are representing as key tooth [1-2]. These representing are as in genetic algorithms (GA).

The measurement placement for power system state estimation is one importance problem for reducing investment cost. Also, the state estimation time is low when the number of measurement is low. Thus, the objective is to make the low investment cost for measurement system that the number of

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measurement should be low. In [3], GA is introduced to solve the phasor measurement unit (PMU) placement for harmonic state estimation in unbalance distribution system. However, PMU needs an efficient communication system. Also, GA [4] is applied for solving the optimal measurement system. The critical measurement identification [5] is included in the fitness function for guarantee the measurement scheme without critical measurement. GA in [3] and [4] use “0” and “1” for representing the PMU and measurement installation respectively. Thus, KCA should be also suitable applied to solve the measurement placement.

In this paper, the KCA is introduced to solve the measurement placement problem. Probability for cutting the key tooth to generate the new key set is concerned. Measurement placement with critical measurement free is considered in the objective function. 10-bus and IEEE-14 bus are test systems for KCA performance evaluation.

## 2. Methodologies

### 2.1. Key Cutting Algorithm

Key Cutting Algorithm (KCA) is a new optimization algorithm proposed by Qin [1]. In daily life, we may ever lose a key. If we have a backup key, it is easy to make a clone using the backup key. If we do not have a backup key, the locksmith is our helper. Locksmith will use multiple similar keys and tries to open the lock. Then, he will also make a new key set based on those similarities and repeat until a key opens the lock. These concepts can be shown as the following flowchart.

From Figure 1, the detail of each step is as followings.

Step 1: Generate the randomly initial key set; the  $2m$  keys are randomly generated. Each key includes several key teeth representing each solution. Each key tooth is represented in “0” or “1”.

Step 2: Calculate the fitness, the “0” and “1” might be converted to a real number according to the problem, the fitness of each key is calculated depending the objective function.

Step 3: Select a half of all keys with higher fitness, the first 50 percent of higher fitness of all keys are selected into next generation. This operation makes the best solution is copied into next generation.

Step 4: Calculate the probability factor of each tooth, the probability factor  $p_{ij}$  ( $i = 1, 2, m; j = 1, 2, n$ ) for each tooth  $S_{ij}$  is calculated by following equation:

$$p_{ij} = 1 - (\text{the number of } S_{ij} \text{ in column } j/m) \quad (1)$$

$$\mathbf{S} = \begin{bmatrix} S_{1n} & S_{1(n-1)} & \cdots & S_{12} & S_{11} \\ S_{2n} & S_{2(n-1)} & \cdots & S_{22} & S_{21} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ S_{(m-1)n} & S_{(m-1)(n-1)} & \cdots & S_{(m-1)2} & S_{(m-1)1} \\ S_{mn} & S_{m(n-1)} & \cdots & S_{m2} & S_{m1} \end{bmatrix} \quad (2)$$

Step 5: Generate the second half key set based on probability factor, these 50 percent keys are copied from the first half key set. Then, each key tooth may be cut according to the probability and calculated probability factor.

Step 6: After new keys are generated. The key fitness should be new calculated.

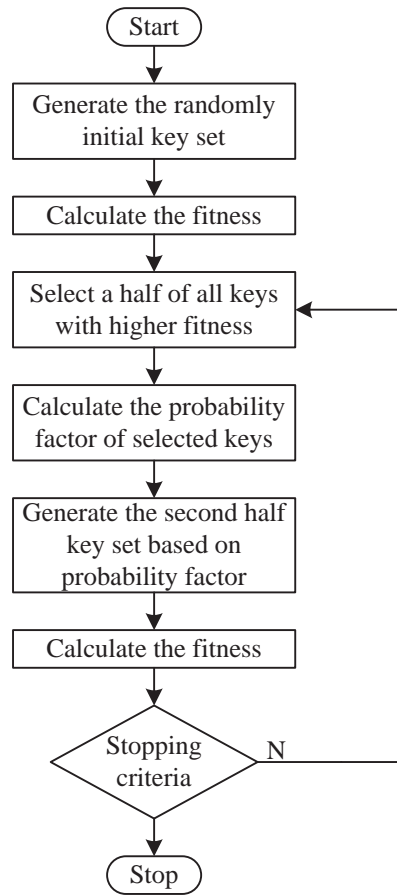


Fig. 1. KCA procedure flowchart

## 2.2. Measurement Placement for Power System State Estimation

Power system state estimation is required for online monitoring and controlling in power system. The power system data is transmitted into the control centre for power system state estimation via remote terminal unit (RTU). The power flow and power injection measurement pairs are mainly used for data preparation in power system state estimation process. The measurement scheme needs to make the system observable and should be without critical measurement. Since, critical measurement free can be detected a bad data in single measurement, critical measurement identification is required.

The power measurement input for state estimator in vector form is shown as following

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{w} \quad (3)$$

where  $\mathbf{z}$  is the measurement vector,  $\mathbf{h}(\bullet)$  is the  $(m \times 1)$  non-linear function vector,  $\mathbf{x}$  is the  $(2N \times 1)$  system state vector,  $\mathbf{w}$  is the  $(m \times 1)$  measurement error vector,  $m$  is the number of measurements, and  $N$  is the number of power system buses.

The state estimator needs a set of analog measurements and system topology to estimate the system state. In fact, the minimal measurements number is equal to  $(2N-1)$  state variables. Therefore, the critical number of real and reactive power measurement pairs is  $(N-1)$  [6] and additional one of bus voltage magnitude measurement at any bus.

### 2.2.1 Observability Analysis

In case of measurement pair (PQ) is used in power system then the  $\mathbf{H}$  matrix is block diagonal, the question of observability can be decoupled into  $P\delta$  observability and  $QV$  observability. A power system is defined to be  $P\delta$  ( $QV$ ) observable with respect to a measurement set if the  $\mathbf{H}_{P\delta}$  ( $\mathbf{H}_{QV}$ ) matrix is of rank  $N-1$  ( $N$ ). Moreover, the triangular factorization is performed to the measurement gain matrix  $\mathbf{G}_{P\delta}$  or  $\mathbf{H}_{P\delta}^T \mathbf{H}_{P\delta}$ . If the result of this factorization is only one zero pivot, the system is observable [6].

### 2.2.2 Critical Measurement Identification

Critical measurement means the lost of that measurement makes the system unobservable. The residual analysis is powerful used to identify the critical measurement.

The residual vector  $\mathbf{r}$  is defined as the difference between  $\mathbf{z}$  and the corresponding filtered quantities  $\hat{\mathbf{z}} = \mathbf{H}\hat{\mathbf{x}}$ . The residuals in terms of the elements of matrix  $\mathbf{E}$  as follows

$$\begin{aligned}\mathbf{r} &= \mathbf{z} - \hat{\mathbf{z}} = \mathbf{z} - \mathbf{H}\hat{\mathbf{x}} = \mathbf{z} - \mathbf{H}(\mathbf{G}^{-1}\mathbf{H}^T\mathbf{z}) \\ &= (\mathbf{I} - \mathbf{H}\mathbf{G}^{-1}\mathbf{H}^T)\mathbf{z} = \mathbf{E}\mathbf{z}\end{aligned}\quad (4)$$

where  $\mathbf{E} = \mathbf{I} - \mathbf{H}\mathbf{G}^{-1}\mathbf{H}^T$ ,  $\mathbf{z}$  is a unity vector (This simplification is based on the fact that critical measurement is independently of the measurement values) [5] and  $\mathbf{G} = \mathbf{H}^T\mathbf{H}$  is a gain matrix. Therefore, the  $i^{th}$  component of the residual vector is calculated by

$$r(i) = \sum_{k=1}^m E(i, k) \quad (5)$$

For each  $z(i)$  of the measurement set, if  $r(i)$  and  $E(i, i)$  are zero,  $z(i)$  is defined as a critical measurement [5].

## 3. KCA application to measurement placement

Key tooth of KCA represents either power injection at the bus or power flow measurement at the branch. Thus, the number of key tooth of one key is equal to number of buses plus branches. “1” of key tooth means measurement installing at those position and “0” is vice versa as in Fig. 2.

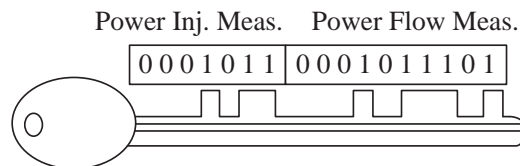


Fig. 2. Typical key tooth representing the measurement placement

The objective function of measurement placement problem is try to minimize the number of power measurement pair (  $M$  ) as followings

$$\text{Min} \quad \text{Number of } M \quad (6)$$

subject to observability constrain

$$\text{rank}(\mathbf{H}) = N - 1 \quad (7)$$

where  $\mathbf{H}$  (Abur, and. Magnago, 2001) is measurement matrix formulated from key tooth and  $N$  is system bus number. Minimum number of power measurement is  $N$  when considering critical measurement free. The observability constrain and critical measurement (  $CM$  ) [5, 7] can be included into objective function becomes the fitness function as following.

$$\text{fitness} = (N - \text{rank}(\mathbf{H})) + (M - N) + CM \times N \quad (8)$$

The best fitness in (8) is 1, thus 1 can be used for a stopping criteria.  $CM \times N$  is used to penalize when  $CM$  is present. Only the first part of fitness function is 1 when the measurement system is observable.

In key cutting method for new key generating, if the random number is greater than probability factor  $p_{ij}$ , the key tooth  $S_{ij}$  will be changed either from “0” to “1” or vice versa.

#### 4. Numerical results

The measurement placement of 10-bus and IEEE 14-bus systems are solved by KCA. In this research, KCA1 {KCA2} is the KCA using random number is greater than {less than} probability factor for changing key tooth. Typical power injection (Inj.) and power flow (Fl.) measurement pair placement of both KCA1 and KCA2 are shown in Table 1. Numerical results of KCA1 and KCA2 are shown in Table 2. The number of run is 20. Number of keys is 20 for 10-bus and 40 for IEEE 14-bus systems.

Table 1. Typical measurement placement of both KCA1 and KCA2

System	Typical measurement placement
10-bus [2]	Inj. 1, 2, 4, 6, 8, 9, 10 Fl. 2-3, 5-7, 5-9
IEEE 14-bus	Inj. 4, 6, 7, 10, 11, 12, 13, 14 Fl. 1-2, 1-5, 2-4, 3-4, 7-8, 12-13

In Table 1, all systems require  $N$  power measurement pair for critical measurement free. Therefore, any one power measurement pair can be lost while the measurement system remains observable. However, those measurement schemes are required additional one voltage magnitude measurement. In each running, the result of measurement placement is difference location.

Table 2. Numerical results and performance of KCA

System	Percent of the best hit		Average generation of the best hit	
	KCA1	KCA2	KCA1	KCA2
10-bus [2]	100.00	20.00	28.45	1.00
IEEE 14-bus	45.00	5.00	34.11	1.00

From Table 2, number of the best hit is number that hit the best fitness of all running. If the number of best hit is high, the performance of algorithm is also high in robustness. Here, KCA1 is high performance than KCA2 in terms of the best answer hit. Although, the average generation of the best hit of KCA1 is higher than KCA2, the KCA2 yields low percent of the best hit.

## 5. Conclusion

In this paper proposes the application development of KCA to solve power measurement pair placement for power system state estimation. KCA here is no need to change binary into real number since “0” and “1” represent the location of those measurement pair in the power system. Measurement placement problem is also considered critical measurement pair. In this development, KCA with low probability factor yields the high robustness of answer hit. As a result, if a key tooth is high similarity to a key tooth of all key set, the key tooth is high probability to change. This key tooth changing is tending to generate the fit key. However, KCA is still needed to develop for more efficiency.

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