5. SIGNALLARNI FURE QATORIGA YOYISH

5.1. Signallarning matematik modellari

Texnikada “signal” deganda fizik tizim holatini aks ettiruvchi qandaydir miqdor tushuniladi. Radiotexnikada signal deb kuchlanish (ko‘pincha) yoki tok o‘zgarishini ifodalovchi vaqt funksiyasi $s(t)$ ga aytiladi.

Berilgan analitik (*determinant* – har qanday vaqt momentida aniqlangan) funksiya $s(t)$ signalning abstrakt matematik modeli hisoblanadi.

Determinant radiotexnik signallarning matematik modellarini quyidagi turlarga ajratish mumkin:

- uzluksiz signal (garmonik tebranish):

$$s(t) = U\cos\omega_0 t\,, \quad s(t) = U\sin\omega_0 t. \tag{5.1}$$

Garmonik signalning aniqlanish sohasi $t \in (-\infty, \infty)$.

- uzluksiz signal (Gauss impulsi):

$$s(t) = Ue^{-\alpha^2 t^2}, \quad t \in (-\infty, \infty). \tag{5.2}$$

- uzluksiz signal (eksponensial impuls):

$$s(t) = \begin{cases} Ue^{-\alpha t}, & t \in [0, \infty), \\ 0, & t < 0. \end{cases} \tag{5.3}$$

- finit signal, ya’ni cheklangan vaqt intervalida noldan farqli qiymatlarni qabul qiluvchi signal (to‘g‘ri to‘rtburchakli videoimpuls):

$$s(t) = \begin{cases} U, & t \in [-\frac{T}{2}, \frac{T}{2}], \\ 0, & t \notin [-\frac{T}{2}, \frac{T}{2}]. \end{cases} \tag{5.4}$$

- finit signal (uchburchakli videoimpuls):

$$s(t) = \begin{cases} \frac{U}{T}(T-t), & t \in [0,T], \\ 0, & t \notin [0,T]. \end{cases} \quad (5.5)$$

- davriy signal:

$$s_r(t) = \sum_{k=-\infty}^{\infty} r(t-kT), \;\; k = 0, \pm 1, \pm 2, \dots, \quad (5.6)$$

bu yerda, $r(t) - T$ intervaldagi (ketma-ketlik davrida) finit signal.

- oniy qiymatlar ketma-ketligi hisoblanuvchi diskret signal:

$$s(kT) = e^{-\alpha kT}, \quad k = 0, 1, 2, \dots. \quad (5.7)$$

Sinov signallari. Signallarning matematik modellari orasida sinov, namunaviy, nazorat signallari alohida o‘rin egallaydi. Ushbu signallar nazariy tadqiqotlar olib borishda, ularga taqriban mos keluvchi fizik (radiotexnik) signallar eksperimental radiotexnika va amaliy radioo‘lchashlarda juda keng foydalaniladi.

Keng tarqalgan sinov signallaridan biri bu *birlik zinasimon funksiya, ulash funksiyasi* yoki *Xevisayd funksiyasi*:

$$\sigma(t) = 1(t) = \begin{cases} 1, & t > 0, \\ 1/2, & t = 0, \\ 0, & t < 0. \end{cases} \quad (5.8)$$

Eng muhim sinov radiotexnik signali *delta-funksiya* yoki *Dirak funksiyasi* $\delta(t)$ hisoblanadi va quyidagi ifodalar orqali aniqlanadi:

$$1.\, \delta(t) = \begin{cases} \infty, & t = 0, \\ 0, & t \neq 0. \end{cases} \quad 2. \int_{-\infty}^{\infty} \delta(t)dt = 1 \;\; (\delta - funksiya\, yuzasi). \quad (5.9)$$

(5.9) ifodaning birinchi qismidan kelib chiqadiki, $\delta(t)$ funksiya faqat $t = 0$ argumentdagina mavjud bo‘lganligi uchun quyidagi munosabatlar o‘rinli:

$$1.\ \delta(t-t_0)=\begin{cases}\infty, & t=t_0,\\ 0, & t\neq t_0.\end{cases} \quad 2.\ \int_{-\infty}^{\infty}\delta(t-t_0)dt=1. \qquad (5.10)$$

(5.9) ifodaning ikkinchi qismidan kelib chiqadiki, $\delta(t)$ funksiya o‘lchov birligi t argumenta o‘lchov birligiga teskari kattalik. Yana bir muhim xususiyati, bu δ-funksiyaning filtrlash xossasi hisoblanadi:

$$\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt = f(t_0)\int_{-\infty}^{\infty}\delta(t-t_0)dt = f(t_0), \qquad (5.11)$$

ya’ni, δ-funksiyaga ko‘paytiruvchi sifatidagi integral osti funksiyaning integrali nolga teng bo‘lmagan argumentli δ-funksiyaning qiymatiga teng.

$\delta(t)$ funksiya umumlashtirilgan, ramziy funksiyalar deb ataluvchi funksiyalar qatoriga kiradi. Uning yordamida, masalan, klassik ma’noda mavjud bo‘lmagan Xevisayd funksiyasining xosilasini aniqlash mumkin:

$$\frac{d\sigma(t)}{dt}=\delta(t). \qquad (5.12)$$

Xevisayd funksiyasi (5.8) o‘z navbatida (5.12) asosida quyidagicha ifodalanishi mumkin:

$$\sigma(t)=\int_{-\infty}^{t}\delta(t)d\lambda. \qquad (5.13)$$

Garmonik signal (5.1) va garmonik (kvazigarmonik) ulanish funksiyasi $s(t)=U\cos\omega_0 t,\ t\geq 0$ ni Xevisayd funksiyasidan foydalanib quyidagicha yozish mumkin $s(t)=U\sigma(t)\cos\omega_0 t$.

Radiosignal. Quyidagi model ko‘rinishidagi signallar radiosignallar deb ataladi.

$$u(t)=U(t)\cos\{\omega_0 t+\varphi(t)+\varphi_0\}=U(t)\cos\Psi(\mathrm{t}). \qquad (5.14)$$

Radiosignalning o‘rovchisi (egiluvchisi) $U(t)$, to‘liq fazasi $\Psi(\mathrm{t})$ va faza funksiya $\varphi(t)$ lari ajratiladi. $\omega_0 = 2\pi f_0$ chastota tashuvchi chastota deb ataladi. (5.14) model ko‘rinishidagi signalning $U(t)$ o‘rovchisi va $\varphi(t)$ faza funksiyasi $T_0 = 2\pi/\omega_0$ (tashuvchi chastota davri) vaqt oralig‘ida sezilarli o‘zgarmaydi. Ko‘pchilik signallarni (5.14) ifoda (signal)ning xususiy hollari sifatida ifodalash mumkin, masalan $U(t) = U = const$ bo‘lgan holat yoki $\omega_0 = 0$ bo‘lgan holat yoki $\varphi(t) = 0$ bo’lgan holat va h.k. Agar $\varphi(t) = 0$ bo‘lsa, u holda φ_0 boshlang‘ich faza deyiladi.

Eng sodda radiosignal garmonik funksiya (5.1) hisoblanadi.

Agar $U(t)$ o‘rovchi finit funksiya bo‘lsa, u holda radiosignal (5.14) radioimpuls deb ataladi. $U(t)$ o‘rovchi finit funksiyaga mos videoimpuls, ω_0 – radioimpulsning to‘ldiruvchi chastotasi ($\varphi(t) = \varphi_0$ holatda) hisoblanadi. O‘rovchi sifatida to‘g‘ri to‘rtburchakli videoimpuls (5.4) ni tanlasak va $\varphi(t) = \varphi_0 = 0$ deb olsak, u holda to‘g‘ri to‘rtburchakli radioimpuls shaklidagi radiosignalni hosil qilamiz, ya’ni

$$s(t) = \begin{cases} U\cos\omega_0 t, & t \in [-\frac{T}{2}, \frac{T}{2}], \\ 0, & t \notin [-\frac{T}{2}, \frac{T}{2}]. \end{cases} \tag{5.15}$$

Agar $U(t)$ o‘rovchi $t \in (-\infty, \infty)$ yoki $t \in [t, \infty)$ intervalda aniqlangan uzluksiz funksiya bo‘lsa, u holda ushbu signal (5.14) radiosignalga mos videosignal deb ataladi.

5.2. Davriy signallarning tahlili

Signallarni tahlil qilishda ularni funksional qatorlar yoyilmasi shaklida ifodalash juda muhim hisoblanadi. Funksional qatorlar fizika va matematikada ko‘pgina masalalarni yechishda juda keng ishlatiladi. Ayniqsa trigonometrik, garmonik qatorlar va Furye qatorlari alohida o‘rin egallashadi.

Furye trigonometrik qatori. Cheklanmagan interval $t \in (-\infty, \infty)$ da aniqlangan davriy signal $s_r(t) = \sum_{k=-\infty}^{\infty} r(t - kT)$, $k = 0, \pm 1, \pm 2, \ldots$ ni quyidagi Furye trigonometrik qatori ko‘rinishida ifodalash mumkin.

$$s_r(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} (a_k \cos k\omega_1 t + b_k \sin k\omega_1 t), \qquad (5.16)$$

bunda, $\omega_1 = \frac{2\pi}{T} = \frac{2\pi}{f_1}$, $f_1 = \frac{1}{T}$ va $k = 1, 2, \dots$.

Signalni bunday yoyilma (5.16) shaklida ifodalash uchun $r(t)$ signal ($r(t)$ – T oraliqdagi finit signal) T davr oralig‘ida *Dirixle shartini* qanoatlantirishi lozim, ya’ni

- 2-tur uzulishga ega bo‘lmasligi;
- chekli sondagi 1-tur uzulishlarga ega bo‘lishi;
- chekli sondagi ekstremumlarga ega bo‘lishi kerak.

a_k (shu jumladan a_0 ham) va b_k koeffisiyentlar quyidagi formulalar orqali aniqlanadi

$$a_k = \frac{2}{T}\int_0^T r(t) \cos k\omega_1 t\, dt, \quad b_k = \frac{2}{T}\int_0^T r(t) \sin k\omega_1 t\, dt. \qquad (5.17)$$

Ba’zan a_k koeffisiyentni hisoblash umumiy formulasidan a_0 koeffisiyent o‘rniga (5.17) formulaga $k = 0$ ni qo‘yib $a_0/2$ ni hisoblash qulay, ya’ni

$$\frac{a_0}{2} = \frac{1}{T}\int_0^T r(t)dt. \qquad (5.18)$$

Amaliyotda (5.16) qatorning ikkinchi ko‘rinishidan foydalanish qulay, ya’ni quyidagi o‘zgartirishni amalga oshirib

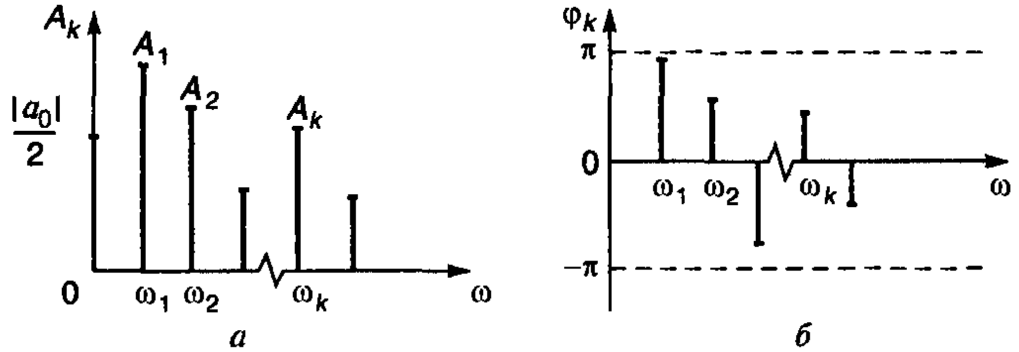
$$a_k \cos k\omega_1 t + b_k \sin k\omega_1 t =$$
$$= \sqrt{a_k^2 + b_k^2}\left(\frac{a_k^2}{\sqrt{a_k^2 + b_k^2}} \cos k\omega_1 t + \frac{b_k^2}{\sqrt{a_k^2 + b_k^2}} \sin k\omega_1 t\right) =$$
$$= \sqrt{a_k^2 + b_k^2}(\cos\varphi_k \cos k\omega_1 t - \sin\varphi_k \sin k\omega_1 t) =$$
$$= A_k \cos(k\omega_1 t + \varphi_k),$$

bunda, $\text{tg}\varphi_k = -\frac{b_k}{a_k}$, $A_k = \sqrt{a_k^2 + b_k^2}$, bo'lib, $s_r(t)$ signalning Furye qatori ikkinchi ko'rinishini hosil qilamiz

$$s_r(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} A_k \cos(k\omega_1 t + \varphi_k). \qquad (5.19)$$

Bu o'rinda $\omega_k = k\omega_1 = 2\pi k f_1 = 2\pi k/T$ belgilanishlar keng ishlatiladi.

(5.19) ifodadagi $a_0/2$ va A_k koeffisiyentlar majmui $s_r(t)$ signalning amplituda spektrini, φ_k koeffisiyentlar majmui – faza spektrini tashkil etadi. Davriy signalning amplituda va faza spektrlari 5.1-rasmda keltirilgan.



5.1-rasm. *Davriy signalning amplituda (a) va faza (b) spektrlari*

Fure kompleks qatori. Eyler formulalaridan foydalanib

$$\cos\alpha = \frac{1}{2}\left(e^{j\alpha} + e^{-j\alpha}\right), \quad \sin\alpha = \frac{1}{j2}\left(e^{j\alpha} - e^{-j\alpha}\right),$$

(5.16) qatorni quyidagi ko'rinishda ifodalash mumkin

$$s_r(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty}\left[\frac{a_k}{2}\left(e^{jk\omega_1 t} + e^{-jk\omega_1 t}\right) + \frac{b_k}{j2}\left(e^{jk\omega_1 t} - e^{-jk\omega_1 t}\right)\right] =$$

$$= \frac{a_0}{2} + \sum_{k=1}^{\infty}\frac{1}{2}(a_k - jb_k)e^{jk\omega_1 t} + \sum_{k=1}^{\infty}\frac{1}{2}(a_k + jb_k)e^{-jk\omega_1 t}.$$

Kompleks amplitudani

$$\frac{1}{2}(a_k - jb_k) = \dot{C}_k, \quad \frac{1}{2}(a_k + jb_k) = \dot{C}_{-k} \tag{5.20}$$

va “manfiy” chastota $\omega_{-k} = -k\omega_1 = -\omega_k$ ya’ni k ning o‘zgarish oralig‘iga $k < 0$ qiymatlarni kiritib, (5.16) ifodani quyidagi ko‘rinishga keltiramiz

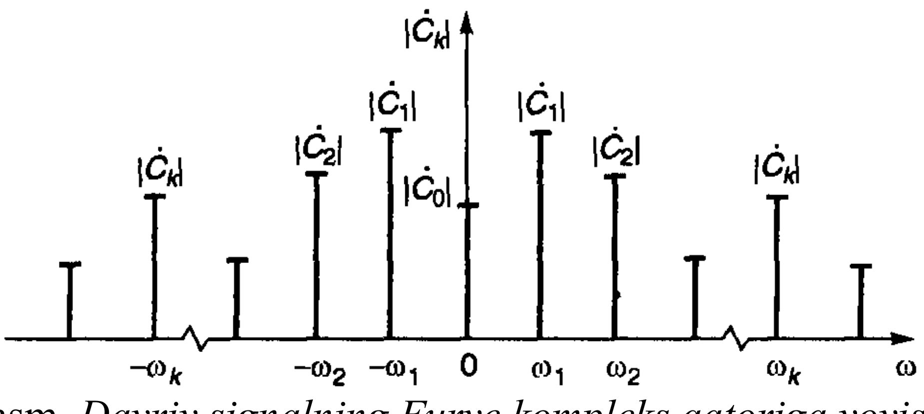
$$s_r(t) = \frac{a_0}{2} + \sum_{k=-\infty}^{\infty} \dot{C}_k e^{j\omega_k t}, \qquad k \neq 0.$$

Ushbu ifoda Furye qatorining kompleks shakli deb ataladi. Agar quyidagi qo‘shimcha o‘zgartirishni kiritsak, ya’ni $\dot{C}_0 = C_0 = a_0/2$, Furye kompleks qatorini quyidagicha ixcham ko‘rinishda yozish mumkin

$$s_r(t) = \sum_{k=-\infty}^{\infty} \dot{C}_k e^{j\omega_k t}. \tag{5.21}$$

Furye qatorining kompleks ko‘rinishi matematik o‘zgartirishlar (almashtirishlar)ni bajarishda qulaylik yaratishi bilan ahamiyatga ega.

(5.21) qatorning $\dot{C}_k$ koeffisiyentlari $s_r(t)$ davriy signalning ω_k, $k = 0, \pm 1, \pm 2, ...$ chastotaning hamma qiymatlarida $|\dot{C}_k|$ amplituda va $\varphi_k = arg\dot{C}_k$ faza spektrlari bilan aniqlangan diskret kompleks spektrini ifodalaydi. 5.2-rasmda $|\dot{C}_k|$ amplituda spektri keltirilgan.



5.2-rasm. *Davriy signalning Furye kompleks qatoriga yoyishdan foydalanilgandagi amplituda spektri*

$|\dot{C}_k| = |\dot{C}_{-k}| = C_k = A_k/2$ ekanligini inobatga olib, (5.21) qatorni kengroq ko‘rib chiqamiz

$$s_r(t) = \sum_{k=-\infty}^{\infty} \dot{C}_k e^{j\omega_k t}$$
$$= \cdots + \dot{C}_{-k} e^{-jk\omega_1 t} + \cdots + C_0 + \cdots + \dot{C}_k e^{jk\omega_1 t} + \cdots;$$

va yana Eyler formulalaridan foydalanib, yig‘indini quyidagicha o‘zgartiramiz

$$\dot{C}_{-k} e^{-jk\omega_1 t} + \dot{C}_k e^{jk\omega_1 t} = 2C_k \cos\varphi_k \cos k\omega_1 t - 2C_k \sin\varphi_k \sin k\omega_1 t =$$
$$= a_k \sin k\omega_1 t + b_k \sin k\omega_1 t = 2C_k \cos(k\omega_1 t + \varphi_k).$$

Bunda $a_k = 2C_k \cos\varphi_k$, $b_k = -2C_k \sin\varphi_k$ ekanligi e’tiborga olingan.

(5.17) ifodani (5.20) ga qo‘yib, quyidagini olamiz

$$\dot{C}_k = \frac{1}{2}(a_k - jb_k) = \frac{1}{T}\int_0^T r(t)\cos k\omega_1 t\,dt - j\frac{1}{T}\int_0^T r(t)\sin k\omega_1 t\,dt =$$
$$= \frac{1}{T}\int_0^T r(t)e^{-jk\omega_1 t}dt. \qquad (5.22)$$

(5.22) formula bevosita $\dot{C}_k$, $k = 0, \pm 1, \pm 2, ...$ qiymatlarni hisoblashda ishlatiladi.

Eslatma 1.

(5.17) va (5.22) ifodalardagi integral chegaralarini o‘zgartirish mumkin, faqat integrallash oralig‘i butun davrga mos bo‘lishi kerak, ya’ni $-T/2$ dan $T/2$ gacha yoki $-T$ dan 0 gacha va h.k. Bu holat T davrli $f(t)$ davriy funksiya uchun $\int_\lambda^{\lambda+T} f(t)dt$ integralning qiymati λ ga bog‘liq emasligidan kelib chiqadi. Ushbu munosabat amaliy masalalarni yechishda qulay hisoblanadi. Masalan (5.17) ifodada integral chegaralarini simmetrik, ya’ni $-T/2$ dan $T/2$ gacha deb olsak, (5.17) qator quyidagilardan tashkil topganligini payqash qiyin emas, ya'ni agar $s_r(t)$ funksiya juft bo'lsa, faqat a_k koeffisiyentli kosinusoidal garmonikalardan, agar $s_r(t)$ funksiya toq bo'lsa, faqat b_k koeffisiyentli sinusoidal garmonikalardan iborat, bunda integrallash chegaralari a_k va b_k koeffisiyentlarni hisoblashda qanday olinishiga bog'liq emas.

Eslatma 2.

Furye qatorlari bo'yicha topilgan spektrlar ekvidistantligini ta'kidlab o'tamiz, ya'ni qator koeffisiyentlari albatta (zaruriy) $\omega = 0$ tashkil etuvchi va $\omega_1 = 2\pi/T$ qadam bilan joylashuvchi ekvidistant ketma-ketlik $(\dots, -2\omega_1, -\omega_1, 0, \omega_1, 2\omega_1, \dots)$ asosida hosil bo'ladi. Koeffisiyentlarning o'zi esa har qanday qiymatni, shu jumladan nol qiymatni ham qabul qilishi mumkin.

Eslatma 3.

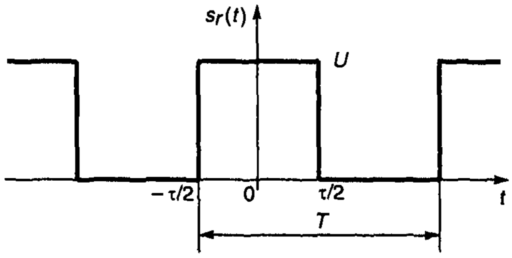
Davriy signallarni Furye qatorlariga yoyishda funksiyalar ortogonalligi to'g'risida so'z boradi. Shuning uchun funksiyalarning ortogonalligi ta'rifini eslatib o'tamiz: $\dot{\alpha}_0(t), \dot{\alpha}_1(t), \dot{\alpha}_2(t), \dots, \dot{\alpha}_m(t), \dots$ kompleks funksiyalar $[a, b]$intervalda ortogonal hisoblanadi, agar quyidagi shart bajarilsa

$$\int_a^b \dot{\alpha}_m(t)\dot{\alpha}_n(t)dt = 0 \text{ agar } m \neq n \text{ va } \int_a^b |\dot{\alpha}_m(t)|^2 dt \neq 0. \quad (5.23)$$

Ko'rib chiqilgan garmonik Furye qatorlari uchun $[a, b]$ ortogonallik intervali $T/2\pi/\omega_1$ hisoblanadi, $\dot{\alpha}_0(t), \dot{\alpha}_1(t), \dot{\alpha}_2(t), \dots, \dot{\alpha}_m(t), \dots$ funksiyalarni esa $e^{\pm jk\omega_1 t}$ kompleks eksponentalar yoki $\cos k\omega_1 t$, $\sin k\omega_1 t$ funksiyalar tashkil etadi (buni (5.23) ifoda orqali bevosita tekshirib ko'rish mumkin).

5.3. Ba'zi davriy signallarning spektrlari

To'g'ri to'rtburchakli videoimpulslar ketma-ketligi. 5.3-rasmda keltirilgan signalning spektrini ko'rib chiqamiz. Ushbu signal turli radiotexnik jarayonlarda, uning modeli esa nazariy radiotexnikada juda keng ishlatiladi.



5.3-rasm. *To‘g‘ri to‘rtburchakli videoimpulslar ketma-ketligi*

Signalning T intervaldagi analitik ifodasi quyidagicha:

$$r(t) = \begin{cases} U, & t \in [-\frac{\tau}{2}, \frac{\tau}{2}], \\ 0, & t \notin [-\frac{\tau}{2}, \frac{\tau}{2}]. \end{cases} \tag{5.24}$$

To‘g‘ri to‘rtburchakli impulsning davomiyligi τ tushunchasini kiritamiz. Furye kompleks qatori (5.21) dan foydalanamiz

$$\dot{C}_k = \frac{1}{T}\int_{(T)} r(t)e^{-jk\omega_1 t}dt = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} Ue^{-jk\omega_1 t}dt =$$

$$= -\frac{U}{jk\omega_1 T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} e^{-jk\omega_1 t}d(-jk\omega_1 t) = -\frac{U}{jk\omega_1 T}e^{-jk\omega_1 t}\Big|_{-\tau/2}^{\tau/2} =$$

$$= -\frac{U}{jk\omega_1 T}\left\{e^{-\frac{jk\omega_1\tau}{2}} - e^{\frac{jk\omega_1\tau}{2}}\right\} = \frac{2U}{k\omega_1 T}\frac{e^{\frac{jk\omega_1\tau}{2}} - e^{-\frac{jk\omega_1\tau}{2}}}{j2} =$$

$$= \frac{2U}{k\omega_1 T}\sin\frac{k\omega_1\tau}{2} = \frac{U\tau}{T}\frac{\sin\frac{k\omega_1\tau}{2}}{\frac{k\omega_1\tau}{2}} = \frac{U\tau}{T}\frac{\sin\frac{k\tau}{T}\pi}{\frac{k\tau}{T}\pi} = \frac{U}{q}\frac{\sin\frac{k}{q}\pi}{\frac{k}{q}\pi}. \tag{5.25}$$

Bunda integrallash chegaralari 0 va T lar o’rnida T interval bo‘yicha zarur integrallashni ko‘rsatuvchi (T) belgilashdan foydalanilgan (1-eslatmaga qarang). Qulay integrallash chegaralari birorta aniq $r(t)$ signal (funksiya)ning ifodasini integral ostiga olish jarayonida yuzaga keladi.

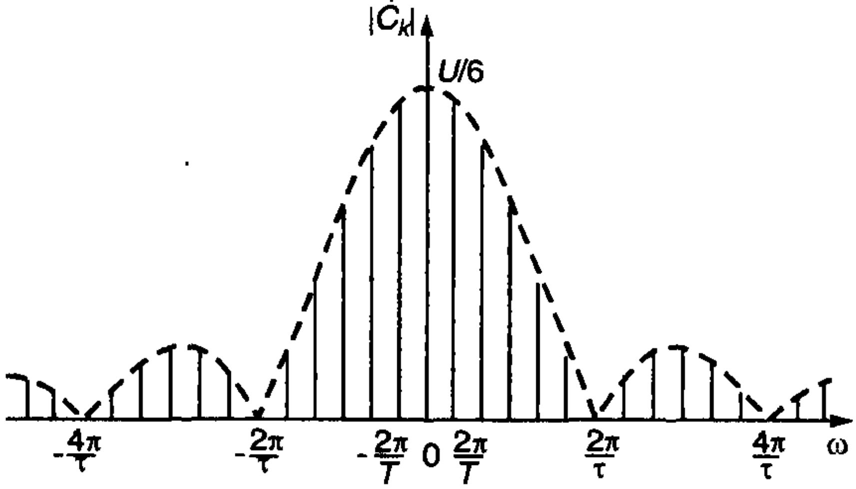
$$\lim_{k\to 0}\frac{\sin\frac{k}{q}\pi}{\frac{k}{q}\pi}=1,\ \ C_0=\frac{U}{q}=\frac{U\tau}{T}.$$

C_0 qiymati va $\dot{C}_k$ koeffisiyentlari ketma-ketlik kovakligi deb ataluvchi $\frac{T}{\tau}=q$ ning aniq qiymatlari uchun (5.25) ifoda orqali aniqlanadi. $k=q,2q,3q,\ldots$ tartib raqamli koeffisiyentlarning qiymati nolga teng. Amplituda diskret spektri $\left|\dot{C}_k\right|$ o‘rovchisining shaklini tahlil qilib, uni $\left|\frac{U}{q}\frac{\sin x}{x}\right|$ funksiya sifatida ekanligini payqash qiyin emas (bunda, sinusning diskret argumenti $\frac{k\pi}{q}$ ni uzluksiz argument x ga almashtiriladi). Amplituda $U=1$ va kovaklik $q=6$ ga teng bo‘lgan signalning $\left|\dot{C}_k\right|$ spektri va spektr o‘rovchisi (punktir chiziq) 5.4-rasmda keltirilgan.

To‘g‘ri to‘rtburchakli videoimpulslar ketma-ketligining kovakligi $q=2$ bo‘lgan holat uchun Furye qatoriga yoyish kompleks koeffisiyentlari quyidagicha bo‘ladi

$$\dot{C}_k=\frac{U}{2}\frac{\sin k\frac{\pi}{2}}{k\frac{\pi}{2}}, \tag{5.26}$$

demak, $C_0=\frac{U}{2},\ \dot{C}_1=\frac{U}{\pi},\ \dot{C}_{2,4,\ldots}=0,\ \dot{C}_3=-\frac{U}{3\pi},\ \dot{C}_5=\frac{U}{5\pi},\ldots$.



5.4-rasm. *To‘g‘ri to‘rtburchakli videoimpulslar ketma-ketligining amplituda spektri* ($q=6$)

Furye qatoriga yoyilmasi quyidagi ko‘rinishga ega bo‘ladi

$$s_r(t) = \cdots + \frac{U}{5\pi}e^{-j5\omega_1 t} - \frac{U}{3\pi}e^{-j3\omega_1 t} + \frac{U}{\pi}e^{-j\omega_1 t} + \frac{U}{2} + \\ + \frac{U}{\pi}e^{j\omega_1 t} - \frac{U}{3\pi}e^{j3\omega_1 t} + \cdots \qquad (5.27)$$

Har bir juft $\frac{U}{k\pi}\left(e^{-jk\omega_1 t} + e^{jk\omega_1 t}\right)$ tashkil etuvchini Eyler formulasi asosida quyidagi ko‘rinishda yozish mumkin:

$$\frac{U}{k\pi}\left(e^{-jk\omega_1 t} + e^{jk\omega_1 t}\right) = \frac{2U}{k\pi}\cos k\omega_1 t,$$

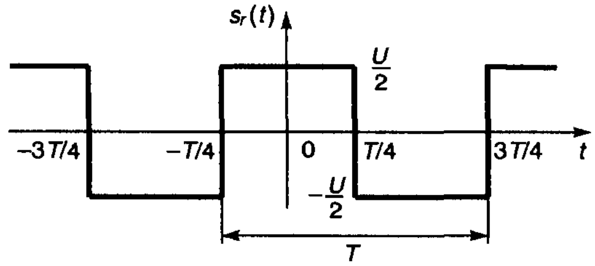
va natijada (5.27) qator quyidagi soddaroq ko‘rinishga ega bo‘ladi:

$$s_r(t) = \frac{2U}{\pi}\left\{\frac{\pi}{4} + \cos\omega_1 t - \frac{1}{3}\cos 3\omega_1 t + \frac{1}{5}\cos 5\omega_1 t - \frac{1}{7}\cos 7\omega_1 t \\ + \cdots\right\}. \qquad (5.28)$$

5.3-rasmda keltirilgan ketma-ketlik juft signal bo‘lganligi uchun, (5.28) ifodani (5.16) shakldagi $b_k = 0$ koeffisiyentli Furye qatori sifatida ham yoki (5.19) shakldagi Furye qatori sifatida ham qarash mumkin. Keyingi holatda, ya’ni Furye qatorining ikkinchi ko‘rinishi sifatida qaralganda faza spektri φ_k yoyilma garmonikalari oldidagi tegishli belgilarni “ta’minlaydi”, shuning uchun ham $\varphi_1 = 0,\ \varphi_3 = -\pi,$ $\varphi_5 = -2\pi,\ \varphi_7 = -3\pi,$ ... qiymatlarni qabul qiladi, va natijada

$$s_r(t) = \frac{2U}{\pi}\left\{\frac{\pi}{4} + \cos\omega_1 t + \frac{1}{3}\cos(3\omega_1 t - \pi) + \frac{1}{5}\cos(5\omega_1 t - 2\pi) + \cdots\right\}.$$

O‘quvchilarga mashq sifatida quyidagi 5.5-rasmda keltirilgan signalning Furye qatoriga yoyilmasini topishni tavsiya etamiz. Chunki ushbu signal turli radiotexnik jarayonlarda ko‘plab ishlatiladi va bu turdagi signal meandr deb nomlanadi.



5.5-rasm. *Meandr*

$s_r(t)$ analitik meandr signalini shakllantiruvchi T intervaldagi $r(t)$ ketma-ketlik quyidagicha yoziladi:

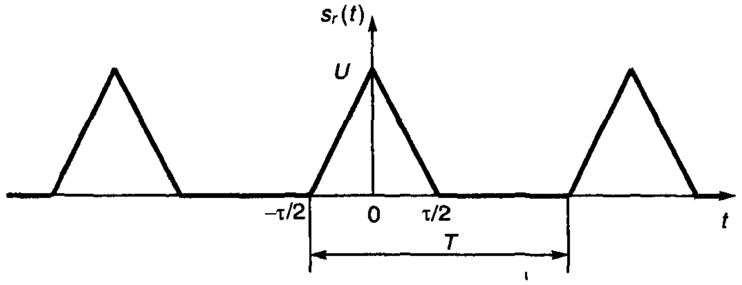
$$r(t) = \begin{cases} \frac{U}{2}, & t \in [-\frac{T}{4}, \frac{T}{4}], \\ -\frac{U}{2}, & t \in [\frac{T}{4}, \frac{3T}{4}]. \end{cases}$$

Yuqorida ko‘rib chiqilgan to‘g‘ri to‘rtburchakli videoimpulslar ketma-ketligi meandr va $U/2$ doimiy tashkil etuvchining yig‘indisidan iborat ekanligini payqash qiyin emas, va (5.27) yoyilmada $C_0 = U/2$ ning mavjudligi esa bunga dalil bo‘ladi.

Meandr va to‘g‘ri to‘rtburchakli videoimpulslar ketma-ketligining yoyilmasidan ($q = 2$ bo‘lgan hol uchun) shuni kuzatish mumkinki, yoyish koeffisiyentlarining qiymati $1/k$ qonuniga mos ravishda kamayib boradi.

Uchburchakli videoimpulslar ketma-ketligi. Uchburchakli videoimpulslar ketma-ketligidan iborat bo‘lgan davriy signalni ko‘rib chiqamiz (5.6-rasm). Impulslar ketma-ketligi uchun analitik ifoda quyidagi ko’rinishga ega:

$$r(t) = \begin{cases} U\left(1 - \frac{2}{\tau}|t|\right), & |t| \in [0, \frac{\tau}{2}], \\ 0, & |t| \notin [0, \frac{\tau}{2}]. \end{cases} \qquad (5.29)$$



5.6-rasm. *Uchburchakli videoimpulslar ketma-ketligi*

(5.21) ifodada keltirilgan Furye kompleks qatoridan foydalanib, $\dot{C}_k$ koeffisiyentlari uchun quyidagi ifodaga ega bo'lamiz:

$$\dot{C}_k = \frac{1}{T}\int\limits_{(T)} r(t)e^{-jk\omega_1 t}dt =$$

$$= \frac{1}{T}\int\limits_{-\frac{\tau}{2}}^{0} U\left(1+\frac{2}{\tau}t\right)e^{-jk\omega_1 t}dt + \frac{1}{T}\int\limits_{0}^{\frac{\tau}{2}} U\left(1-\frac{2}{\tau}t\right)e^{-jk\omega_1 t}dt =$$

$$= \frac{U}{T}\left\{\frac{2}{\tau}\int\limits_{-\frac{\tau}{2}}^{0} te^{-jk\omega_1 t}dt + \int\limits_{-\frac{\tau}{2}}^{0} e^{-jk\omega_1 t}dt - \frac{2}{\tau}\int\limits_{0}^{\frac{\tau}{2}} te^{-jk\omega_1 t}dt + \right.$$

$$\left. + \int\limits_{0}^{\frac{\tau}{2}} e^{-jk\omega_1 t}dt\right\}.$$

Ifodadagi integrallarni hisoblash va uncha murakkab bo'lmagan amallarni bajarib (kitobxonga ushbu amallarni mustaqil bajarish tasviya etiladi), quyidagini hosil qilamiz:

$$\dot{C}_k = \frac{U\tau}{2T}\left\{\frac{\sin\frac{k\omega_1\tau}{4}}{\frac{k\omega_1\tau}{4}}\right\}^2 = \frac{U\tau}{2T}\left\{\frac{\sin\frac{k\pi\tau}{2T}}{\frac{k\pi\tau}{2T}}\right\}^2 = \frac{U}{2q}\left\{\frac{\sin\frac{k}{q}\frac{\pi}{2}}{\frac{k}{q}\frac{\pi}{2}}\right\}^2. \quad (5.30)$$

$q = 1$ deb, ya'ni (5.29) ifodadagi uchburchakli videoimpulsning davomiyligi τ takrorlanish davri T bilan teng deb olsak, (5.21) qator koeffisiantlari uchun quyidagi ifodani olamiz:

$$\dot{C}_k = \frac{U}{2}\left\{\frac{\sin k\frac{\pi}{2}}{k\frac{\pi}{2}}\right\}^2.$$

(5.24) va (5.29) ifodalardagi signallar spektrlari orasida qandaydir bog'liqlik mavjud, ammo (5.28) ifodaga o'xshash shaklda aniqlangan uchburchakli videoimpulslar ketma-ketligini Furye kompleks qatoriga yoyilmasi

$$s_r(t) = \frac{2U}{\pi}\left\{\frac{\pi}{4} + \cos\omega_1 t + \frac{1}{3^2}\cos 3\omega_1 t + \frac{1}{5^2}\cos 5\omega_1 t + \right.$$
$$\left. + \frac{1}{7^2}\cos 7\omega_1 t + \cdots\right\},$$

ko'rinishida bo'lib, yoyilma koeffisiyentlari $1/k^2$ qonuniga mos ravishda kamayib boradi, ya'ni koeffisiyentlar tezroq kamayadi. Bu uchburchakli videoimpulsning shakli bilan bog'liq: unda "sakrashlar" yoki 1-tur uzilishlar mavjud emas.

5.4. Furye almashtirishi

Nodavriy signallar spektrlarini tahlil qilish asosini Furye to'g'ri

$$F\{s(t)\} = \dot{S}(\omega) = \int_{-\infty}^{\infty} s(t)e^{-j\omega t}dt \qquad (5.31)$$

va teskari

$$F^{-1}\{\dot{S}(\omega)\} = s(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \dot{S}(\omega)e^{j\omega t}d\omega \qquad (5.32)$$

almashtirishlari tashkil qiladi.

$\dot{S}(\omega)$ funksiya $s(t)$ signalning spektral funksiyasi, spektr zichligi yoki oddiygina spektri deb ataladi. Agar $s(t)$ signal Dirixle shartini hamda quyidagi absolyut integrallanish shartini qanoatlantirsa (5.31) va (5.32) almashtirishlarini amalga oshirish mumkin bo'ladi:

$$\int_{-\infty}^{\infty} |s(t)| dt < \infty.$$

$\dot{S}(\omega)$ spektral funksiya umumiy holda kompleks funksiya bo'lib, Eyler formulasi $e^{\pm j\alpha} = \cos\alpha \pm j\sin\alpha$ ni e'tiborga olib, ushbu funksiyani quyidagi ko'rinishga keltirish mumkin:

$$\dot{S}(\omega) = \int_{-\infty}^{\infty} s(t)\cos\omega t dt - j\int_{-\infty}^{\infty} s(t)\sin\omega t dt =$$
$$= \mathrm{Re}\dot{S}(\omega) + j\mathrm{Im}\dot{S}(\omega) = A(\omega) - jB(\omega). \qquad (5.33)$$

Toq funksiyadan simmetrik chegaralarda olingan aniq integral nolga teng. (5.33) ifodadagi $s(t)$ signalni juft va toq signallar yig'indisidan iborat $s(t) = s_{juft}(t) + s_{toq}(t)$ deb qarasak, Furye kosinusoidal almashtirishi $A(\omega)$ – $s(t)$ signalning juft va Furye sinusoidal almashtirishi $B(\omega)$ – $s(t)$ signalning toq qismlari orqali aniqlanishini kuzatish mumkin. Bundan foydali amaliy xulosa kelib chiqadi, ya'ni $s(t)$ juft funksiyaning Furye almashtirishi chastota ω ning haqiqiy funksiyasi, $s(t)$ toq funksiyaning Furye almashtirishi chastota ω ning mavhum funksiyasi hisoblanadi.

Furye teskari almashtirishi $F^{-1}\{A(\omega) - jB(\omega)\}$ ni kuzatib, $A(\omega)$ – chastota ω ning juft, $B(\omega)$ – esa toq funksiyasi ekanligini aytish mumkin:

$$A(\omega) = A(-\omega), \;\; B(\omega) = -B(-\omega).$$

Kitobxonga ushbu fikrni mustaqil ravishda isbot qilish tavsiya etiladi (bunda shuni e'tiborga olish kerakki, $\dot{S}(\omega)$ ning teskari Furye almashtirishi vaqtning haqiqiy funksiyasi hisoblanadi). Bundan $\dot{S}(\omega)$ ning yana bir muhim xossasi kelib chiqadi:

$$\dot{S}^*(\omega) = \{A(\omega) - jB(\omega)\}^* = A(\omega) + jB(\omega) =$$
$$= A(-\omega) - jB(-\omega) = \dot{S}(-\omega), \qquad (5.34)$$

ya'ni, dastlabki spektral funksiyaga kompleks bog'langan funksiyani topish uchun argument ω belgisini o'zgartirish yetarli hisoblanadi.

Spektral funksiyani quyidagi namunaviy ko'rinishda ifodalash mumkin:

$$\dot{S}(\omega) = \left|\dot{S}(\omega)\right| \exp j\varphi(\omega). \qquad (5.35)$$

Bunda

$$\left|\dot{S}(\omega)\right| = \sqrt{A^2(\omega) + B^2(\omega)} \geq 0$$

ifoda spektral amplituda spektri funksiyasi (ko'pincha amplituda spektri) deb,

$$\varphi(\omega) = arg\dot{S}(\omega) = arctg\frac{\mathrm{Im}\dot{S}(\omega)}{\mathrm{Re}\dot{S}(\omega)}$$

ifoda esa spektral faza funksiyasi (ko'pincha faza spektri) deb ataladi. Bundan amplituda spektri $\left|\dot{S}(\omega)\right|$ juft, faza spektri $\varphi(\omega)$ esa toq funksiya ekanligini ko'rish mumkin. Ushbuni e'tiborga olib va (5.35) ifodani (5.32) ifodaga qo'ysak, quyidagiga ega bo'lamiz

$$s(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\dot{S}(\omega)\right| e^{j\varphi(\omega)} e^{j\omega t} d\omega =$$
$$= \int_{0}^{\infty}\frac{\left|\dot{S}(\omega)\right|}{\pi}\cos[\omega t + \varphi(\omega)]\, d\omega, \qquad (5.36)$$

spektral funksiyaning fizik ma'nosi: $s(t)$ signal juda kichik $\left|\dot{S}(\omega)\right|\frac{d\omega}{\pi}$ amplitudali, chastotalar intervali 0 dan ∞ gacha uzluksiz to'ldiriluvchi cheksiz ko'p sonli garmonik tashkil etuvchilarning yig'indisidan iborat; ushbu tashkil etuvchilarning boshlang'ich fazalari $\varphi(\omega)$ funksiyasi orqali, cheksiz kichik amplitudalarning chastotaga bog'liqligi "zichligi"

$\left|\dot{S}(\omega)\right|$ funksiyasi orqali ifodalanadi. (5.36) ifodadagi ikkinchi integral "manfiy" chastotalarning yuzaga kelishini izohlaydi: manfiy chastotalarning yuzaga kelishi Furye to'g'ri va teskari almashtirishlarining matematik operasiya sifatidagi xarakteri bilan bog'liq bo'lib, fizik jihatdan noreal hisoblanadi. Ushbu mulohazani 5.2 va 5.3 bandlardagi natijalar bilan taqqoslash foydalidir.

Spektral funksiya $\dot{S}(\omega)$ ning o'lchov birligi signalning o'lchov birligining vaqtga ko'paytmasi kabidir: ya'ni agar $s(t)$ signalning o'lchov birligi – voltlarda bo'lsa, u holda spektral funksiyaning o'lchov birligi $\left[\dot{S}(\omega)\right] = V \cdot s = V/Gs$.

Furye almashtirishining simmetrikligi. Faraz qilaylik, $s(t)$ juft signalning haqiqiy spektri $\dot{S}(\omega) = S(\omega)$ ga teng bo'lsin, ma'lumki spektral funksiya ham chastota ω ning juft funksiyasi bo'ladi. U holda $S(t)$ signal $2\pi s(\omega)$ spektrga ega bo'lishi kerak. Aynan $\exp(\pm j\omega t)$ yadrosiga kiruvchi argumentlar ω va t larning "o'zaro almashinuvi" (5.31) va (5.32) ifodalar juftligining simmetrikligidan dalolat beradi.

Davriy ketma-ketlikning spektri va yakka impulsning spektral funksiyasi orasidagi bog'liqlik. Furye kompleks qatori koeffisiyentlarini hisoblash formulasi, ya'ni (5.22) ifoda

$$\dot{C}_k = \frac{1}{T}\int_{0}^{T} r(t)e^{-jk\omega_1 t}dt$$

va (5.31) ifoda, ya'ni Furye to'g'ri almashtirishi yoki $r(t)$ davriy ketma-ketlik impulsini tasvirlovchi impulsning spektral funksiyasi

$$\dot{R}(\omega) = \int_{-\infty}^{\infty} r(t)e^{-jk\omega_1 t}dt,$$

ni taqqoslab, ular orasida juda sodda bog'lanish mavjudligini ko'rishimiz mumkin

$$\dot{C}_k = \frac{1}{T}\dot{R}(k\omega_1). \tag{5.37}$$

5.5. Fure almashtirishning xossalari

Signal $s(t)$ va uning spektri $\dot{S}(\omega)$ oralig‘ida yagona bog‘liqlik mavjud. Signal shakliga o‘zgartirish kiritish natijasida uning spektri ham o‘zgaradi. Quyida signallarga ishlov berishda yuz beradigan asosiy o‘zgarishlar va ularga mos ravishda signal spektrining o‘zgarishlarini ko‘rib chiqamiz.

5.5.1. Signalni vaqt bo‘yicha surish

Misol uchun $s_1(t)$ signal $t_1 < t < t_2$ vaqt orasida mavjud bo‘lib, $\dot{S}_1(\omega)$ spektr zichligiga ega bo‘lsin. Ushbu signal $s_1(t)$ ni shaklini saqlagan holda uni t_0 ga kechiktirsak, u holda vaqtning yangi funksiyasi $s_2(t)$ ni olamiz, ya’ni $s_2(t) = s_1(t - t_0)$ bo‘lib, endi bu signal $t_1 + t_0$ dan $t_2 + t_0$ gacha vaqt oralig‘ida mavjud bo‘ladi. (5.31) ifodaga asosan $s_2(t)$ signalning spektri zichligi quyidagi ifoda orqali aniqlanadi:

$$\dot{S}_2(\omega) = \int_{t_1+t_0}^{t_2+t_0} s_2(t)e^{-j\omega t}dt = \int_{t_1+t_0}^{t_2+t_0} s_1(t - t_0)e^{-j\omega t}dt. \quad (5.38)$$

Yangi o‘zgaruvchi $\tau = t - t_0$ ni kiritib (5.38) ifoda o‘rniga quyidagi ifodani olamiz:

$$\dot{S}_2(\omega) = e^{-j\omega t_0}\int_{t_1}^{t_2} s_1(\tau)e^{-j\omega\tau}d\tau = e^{-j\omega t_0}\dot{S}_1(\omega). \quad (5.39)$$

(5.39) ifodadan ko‘rinadiki signal $s(t)$ ni $\pm t_0$ ga siljitish natijasida uning spektri $\dot{S}(\omega)$ ning faza xarakteristikasi $\pm\omega t_0$ ga o‘zgaradi. Aksincha, agar signal $s(t)$ spektral tashkil etuvchilarini $\varphi = \pm\omega t_0$ ga o‘zgartirsak, u holda u bilan chiziqli bog‘liq ravishda har bir spektr tashkil etuvchisi $\pm\omega t_0$ ga o‘zgaradi va signal $\pm t_0$ vaqtga kechikadi yoki ilgarilaydi. Signal spektri amplituda-chastota xarakteristikasi ushbu signalning vaqt o‘qida egallagan joyiga bog‘liq emas.

Fure almashtirishning yuqorida keltirilgan xossasi chiziqli radiotexnik tizimlardan signallar buzilishsiz o‘tishlarini ta’minlashi uchun qo‘yiladigan talabni keltirib chiqaradi: chiziqli RTTning