

16-833: Robot Localization and Mapping, Spring 2023  
**Homework 2 - SLAM using Extended Kalman  
Filter (EKF-SLAM)**

Due: Wednesday Mar 17, 11:59pm, 2023

Your homework should be submitted as a **typeset PDF file** along with a folder including **code only(no data)**. The PDF and code must be submitted on **Gradescope**. In your implementation, please only fill in the functions marked by **TODO**, and play with parameters, and keep other parts unchanged. If you have questions, post them on Piazza or come to office hours. Please do not post solutions or codes on Piazza.

This homework must be done **individually**, and plagiarism will be taken seriously. You are free to discuss and troubleshoot with others, but the code and writeup must be your own. Note that you should list the name and Andrew ID of each student you have discussed with on the first page of your PDF file.

## 1 Theory (40 points)

In this part we are going to guide you through some steps of EKF-SLAM in math. This will be helpful in your implementation in the next problem.

A robot is moving on the 2D ground plane. In each time step  $t$ , the robot is controlled to move forward (the  $x$ -direction of the robot's coordinates)  $d_t$  meters, and then rotate  $\alpha_t$  radian. The pose of the robot in the global coordinates at time  $t$  is written as a vector  $\mathbf{p}_t = \begin{bmatrix} x_t & y_t & \theta_t \end{bmatrix}^\top$ , where  $x_t$  and  $y_t$  are the 2D coordinates of the robot's position, and  $\theta_t$  is the robot's orientation.

1. Based on the assumption that there is no noise or error in the control system, predict the next pose  $\mathbf{p}_{t+1}$  as a nonlinear function of the current pose  $\mathbf{p}_t$  and the control inputs  $d_t$  and  $\alpha_t$ . (5 points)
2. However, in reality there are some errors when the robot moves due to the mechanism and the terrain. Assume the errors follow Gaussian distributions:  $e_x \sim \mathcal{N}(0, \sigma_x^2)$  in  $x$ -direction,  $e_y \sim \mathcal{N}(0, \sigma_y^2)$  in  $y$ -direction, and  $e_\alpha \sim \mathcal{N}(0, \sigma_\alpha^2)$  in rotation respectively (all in robot's coordinates). For details, please see Fig. 1. Now if the uncertainty of the robot's pose at time  $t$  can be represented as a 3-dimensional Gaussian distribution  $\mathcal{N}(0, \Sigma_t)$ , what is the predicted uncertainty of the robot at time  $t+1$ ? Please express it as a Gaussian distribution with zero mean. (5 points)
3. Consider a landmark  $l$  being observed by the robot at time  $t$  with a laser sensor which gives a measurement of the bearing angle  $\beta$  (in the interval  $(-\pi, \pi]$ ) and the range  $r$ , with noise  $n_\beta \sim \mathcal{N}(0, \sigma_\beta^2)$  and  $n_r \sim \mathcal{N}(0, \sigma_r^2)$  respectively. Write

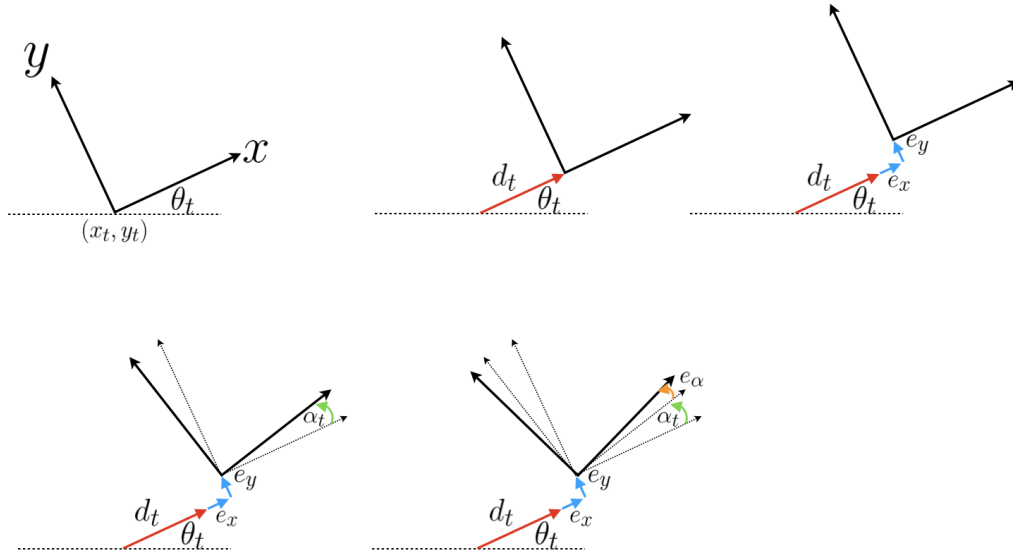


Figure 1: 0) Robot state; 1) robot moves  $d_t$  along its x-axis; 2) due to noise it shifted  $e_x$  and  $e_y$  along its x and y axes respectively; 3) robot rotates  $\alpha_t$  after shifting; 4) due to noise the rotation is disturbed by  $e_\alpha$ .

down the estimated position  $(l_x, l_y)$  of landmark  $l$  in global coordinates as a function of  $\mathbf{p}_t$ ,  $\beta$ ,  $r$ , and the noise terms. (5 points)

4. According to 1.3, if we now know that  $l$  is at  $(l_x, l_y)$  in the global coordinates, please predict the measurement of bearing and range based on  $l_x$ ,  $l_y$ ,  $\mathbf{p}_t$ , and the noise terms (use functions  $np.arctan2(.)$  and  $warp2pi(.)$  that warps an arbitrary angular value into the range  $(-\pi, \pi]$  if needed). (5 points)
5. An important step in EKF-SLAM is to find the measurement Jacobian  $H_p$  with respect to the robot pose. Please apply the results in 1.4 to derive the analytical form of  $H_p$  (Note:  $H_p$  should be a  $2 \times 3$  matrix represented by  $\mathbf{p}_t$  and  $l$ ). (10 points)
6. For each measurement of bearing and range, we also need a measurement Jacobian  $H_l$  with respect to its corresponding landmark  $l$ . Please again derive the analytical form of  $H_l$  (Note:  $H_l$  should be a  $2 \times 2$  matrix represented by  $\mathbf{p}_t$  and  $l$ ). Why do we not need to calculate the measurement Jacobian with respect to other landmarks except for itself (Hint: based on what assumption)? (10 points)

Note in this section, in addition to conventional linearization with additive noise (in the form of  $x_{t+1} = g(x_t, u_t) + \epsilon_t$ ), you may encounter non-additive noise (in the form of  $x_{t+1} = g(x_t, u_t, \epsilon_t)$ ). [Derivation of the linearization](#) for this formulation can be helpful. Also, all the noise reading  $(\epsilon, n)$  can be regarded as infinitesimal. Second and higher order terms comprised by their products can be ignored.

## 2 Implementation and Evaluation (45 points)

In this part you will implement your own 2D EKF-SLAM solver in Python. The robot in problem 1 starts with observing the surrounding environment and mea-

asuring some landmarks, then executes a control input to move. The measurement and control steps are repeated several times. For simplicity, we assume the robot observes the same set of landmarks in the same order at each pose. We want to use EKF-SLAM to recover the trajectory of the robot and the positions of the landmarks from the control input and measurements.

1. Find the data file `data/data.txt` that contains the control inputs and measurements. Open the file to take a look. The data is in the format of measurements =  $\begin{bmatrix} \beta_1 & r_1 & \beta_2 & r_2 & \cdots \end{bmatrix}$ , where  $\beta_i, r_i$  correspond to landmark  $l_i$ , and control =  $\begin{bmatrix} d & \alpha \end{bmatrix}$  (refer to problem 1 for the notations). The lines are in sequential time order.

What is the fixed number of landmarks being observed over the entire sequence? (5 points)

2. The `code/` folder contains the Python code file `ekf_slam.py`. Follow the comments and fill in the functions `warp2pi`, `init_landmarks`, `predict`, `update` to enable the system. The visualization will show the trajectory and the landmarks with their covariances in the shape of  $3\text{-}\sigma$  ellises. It will be updated sequentially, and you may process the next step by clicking your mouse or hitting any key.

Attach a clear figure of your visualization in the write up once all the steps are finished. (25 points)

3. In the output figure, the magenta and blue ellipses represent the predicted and updated uncertainties of the robot's position (orientation not shown here) at each time respectively. Also, the red and green ellipses represent the initial and all the updated uncertainties of the landmarks, respectively.

Describe how EKF-SLAM improves the estimation of both the trajectory and the map by comparing the uncertainty ellipses. (5 points)

4. Now let's evaluate our output map. Suppose we have the ground truth positions of all the landmarks:

$$\begin{bmatrix} l_1^\top & l_2^\top & \cdots & l_k^\top \end{bmatrix} = \begin{bmatrix} 3 & 6 & 3 & 12 & 7 & 8 & 7 & 14 & 11 & 6 & 11 & 12 \end{bmatrix}.$$

Plot the ground truth positions of the landmarks in the output figure (code already provided in function `evaluate`) and attach it below. Is each of them inside the smallest corresponding ellipse? What does that mean?

Compute and report the *Euclidean* and *Mahalanobis* distances of each landmark estimation with respect to the ground truth. What do the numbers tell you? (10 points)

### 3 Discussion (15 points)

1. Explain why the zero terms in the initial landmark covariance matrix become non-zero in the final state covariance matrix (print out the final  $P$  matrix to check it). Additionally, when setting the initial value for the full covariance matrix  $P$  (line 201 in `ekf_slam.py`) an assumption is made regarding certain cross-covariances that is not necessarily correct. Can you point out what that is? (5 points)

2. Play with the parameters (line 163-167). Modify each parameter  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_\alpha\sigma_b$ ,  $\sigma_r$  at a time and fix the others. In particular, you should set each parameter to be 10 times larger/smaller to the original parameters to discuss how each parameters influence the results. Attach figures for better explanation. (10 points)
3. With the same set of landmarks being observed all the time, the EKF-SLAM system runs in constant time in each iteration. However, for real-world problems, the total number of landmarks can grow unbounded if the robot keeps exploring new environments. This will slow down the EKF-SLAM system a lot and become a crucial problem for real-time applications. What changes can we make to the EKF-SLAM framework to achieve constant computation time in each cycle or at least speed up the process (list as many possible solutions as you can think of)? (Bonus 5 points)

## 4 Code Submission

Upload your `ekf_slam.py`. Please do not upload the `data/` folder or any other data or image.