3 Introduction

3.1 Large-scale Structure of the Universe

The large-scale structure refers to the vast hierarchies of matter in the universe, spanning distances from hundreds of light-years to billions.
Solar System: The Sun's radius is 6.96×10^5 km, about 109 times Earth's radius. The solar system's diameter is roughly 9.1 billion km (to Neptune).

Star Cluster: Groups of stars bound by gravity, ranging from a few hundred to a million stars.

- *Globular Clusters:* Dense, with millions of stars (e.g., Omega Centauri, 10 million stars, 150 light-years in diameter).
- *Open Clusters:* Loosely bound with hundreds to a thousand stars, spanning up to 30 light-years.

Galaxy: A gravitationally bound system containing stars, stellar remnants, gas, dust, and dark matter.

- Galaxies range from a few hundred million to 100 trillion stars, with an average of 100 million. They can span 3,000 to 300,000 light-years.
- The observable universe contains hundreds of billions of galaxies.

Milky Way: Our galaxy, with 100-400 billion stars, spanning 90,000 light-years in diameter.
Galaxy Group, Cluster, and Supercluster:

- *Galaxy Group:* A collection of up to 50 galaxies (e.g., the Local Group, containing 50-80 galaxies).
- *Galaxy Cluster:* A group of hundreds to a thousand galaxies, spanning 1-15 million light-years (e.g., Virgo Cluster).
- *Galaxy Supercluster:* A vast collection of clusters and groups, with voids between them. They expand due to Hubble's Law.

Galaxy Filaments: The largest structures, these thread-like formations of galaxies, gas, and dark matter stretch from millions to billions of light-years. Examples include the Super-Virgo Wall (100 million light-years) and Hercules–Corona Borealis Great Wall (10 billion light-years).

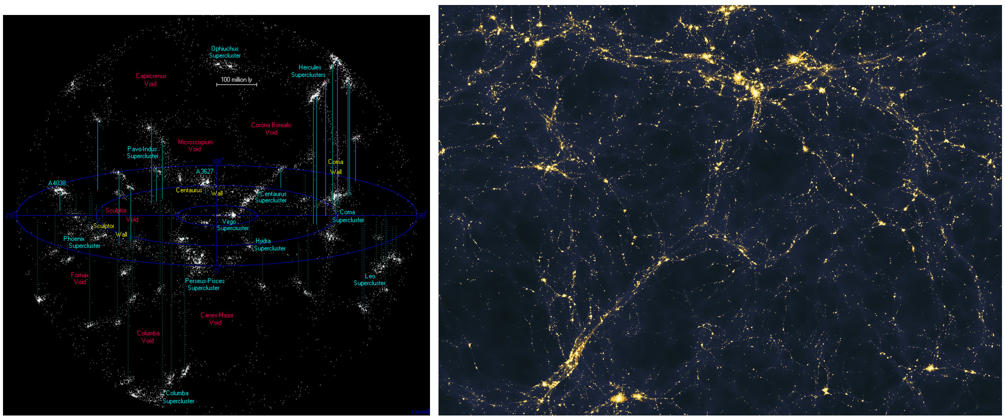

Figure 1: Galaxy superclusters and voids (left). Galaxy filaments (right).

3.2 Units

Due to the vast scales in cosmology and particle physics, common units like km and kg are impractical. Below are the key units used.

Cosmology Units:

- **Mass and Luminosity**:

$$M_\odot = 1.98 \times 10^{33}\,\text{g}, \quad L_\odot = 3.85 \times 10^{33}\,\text{erg/s}.$$

- **Distance Units**: - Astronomical Unit (AU):

$$1\,\text{AU} = 1.49 \times 10^{13}\,\text{cm}.$$

- Light Year (ly):

$$1\,\text{ly} = 9.46 \times 10^{17}\,\text{cm}.$$

- Parsecs (pc): defined as a distance at which a length of 1AU subtends an angle of 1arc-second or 1/3600 degrees, see figure .

$$1\,\text{pc} = 3.26\,\text{ly}.$$

- Mega-Parsec (Mpc):

$$1\,\text{Mpc} = 10^6\,\text{pc}.$$

The universe is considered "large" at the distance scales of 100 Mpc and beyond.

Natural Units:

The most commonly occurring fundamental constants in the world of sub-atomic particles are

- speed of light $c = 3 \times 10^8$m/s
- Planck constant $\hbar = \frac{h}{2\pi} = 1.0545 \times 10^{-34}$ J/s $= 6.582 \times 10^{-25}$ eV/s
- Boltzmann constant $k_B = 1.380 \times 10^{-23}$ J/K $= 8.617 \times 10^{-5}$ eV/K

where we have used 1eV = 1.602×10^{-19} J
- The natural units is a system of measurements by setting $c = 1$, $\hbar = 1$, and $k_B = 1$. This leads to energy units in eV:

$$1\,\text{s} = 1.52 \times 10^{15}\,\text{eV}^{-1}, \quad 1\,\text{cm} = 5.06 \times 10^{4}\,\text{eV}^{-1}.$$

- Temperature in eV:

$$1\,\text{K} = 8.62 \times 10^{-5}\,\text{eV}.$$

Planck Units:

Planck units are based on four fundamental constants: c, G, $\hbar$, and k_B. The key Planck units are:

$$t_{\text{Pl}} = 5.39 \times 10^{-44}\,\text{s}, \quad \ell_{\text{Pl}} = 1.16 \times 10^{-33}\,\text{cm}, \quad m_{\text{Pl}} = 2.17 \times 10^{-5}\,\text{g}, \quad T_{\text{Pl}} = 1.14 \times 10^{32}\,\text{K}.$$

In natural units, Planck mass is:

$$m_{\text{Pl}} = 1.22 \times 10^{28}\,\text{eV}.$$

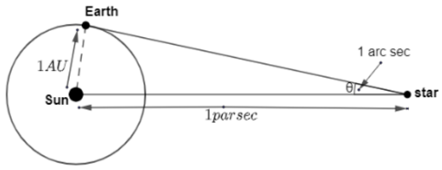

Figure 2: Definition of one parsec.

3.3 A brief History

The universe is expanding. The stars and the galaxies are moving away from each other. So there must have been a time in the past when the galaxies and stars were closer and universe was denser, and obviously hotter. The hot dense state of the early universe is called *big bang*. This plausible 'scenario' of the universe is called the 'big bang theory'. What happened during or before the big bang is beyond the reach of physics. There are a few misconceptions about big bang theoy

- "universe started from big bang" – not true, universe was there to begin with, the big bang was a hot dense state
- "big bang was an explosion at some point in space" – not true, big bang was not confined to a point in space, rather it was a moment of the universe – the big bang stage was everywhere. Space is itself part of the universe. Universe is infinitely large today, was infinitely large during the big bang. However, any finite region of the universe was extremely small during the big bang. For example, the observable universe is estimated to be about a few mili-meter during big bang.
- "universe is expanding into open space since the big bang" – no, universe is not expanding into anything, the space is itself a part of the universe.

Since anything during and before the big bang is beyond the reach of physics, time is usually taken as $t = 0$ at the big bang. The time t is not ideal for tracking history – temperature (and some other observable) is more convenient. Following is chronology of events that happened since $t = 0$

- Planck time: $t \sim 10^{-43}s$, $T = 10^{19}$ GeV: Physics as we know it fails at this high temperature. Quantum gravity, string theory, etc are expected to play a role at this stage. Active area of research – everything is speculative.
- Inflationary stage: between $t \sim (10^{-37}, 10^{-36})$s to $t \sim (10^{-33}, 10^{-32})$s $T = 10^{17}GeV$: Exponential expansion of the universe, led by an unstable "inflaton" field. The universe expands by a factor of 10^{60} to 10^{100} between during this period. This stage is still speculative – no experimental evidence of inflation exists.
- $t \sim 10^{-35} - 10^{-5}s$ Quark-gluon plasma: Universe dominated by the elementary subatomic particles, quarks, gluons, phots, gauge bosons etc. Temperature is $T \sim 10^{27}$K.
- QCD phase transition $t \sim 10^{-5}$s, $T \sim 100 - 300$MeV: At this stage quarks bind together to form hadrons (protons and the neutrons etc). We are certain that the universe passed through this phase – but the details are unclear.
- Big-bang nucleosynthesis, $t \sim$ few seconds to $\sim$ 3 minuits: The protons and the neutrons combine for the first time to form nuclei of the lightest elements in the periodic table, *i.e.,* D, ^{4}He, ^{3}He, ^{3}Li. Nuclei heavier than Be was not formed at this stage. Heavier elements like Fe etc where mostly synthesised in the stars that did not form until about 400 million years. The theory of BBN is very well known and is experimentally verified. BBN provides evidence for big bang.

- $t \sim 10^5$s (1.2days), $T \sim$ few KeV: Photons fall out of chemical equilibrium, *i.e.,* photon's number changing reactions became slower than the rate of the expansion of the universe. The number of photons would not change rapidly afterwards.

- $t \sim 10^{4-5}$yrs, $T \sim$ 3eV: Not much happens. Matter and radiation energy densities become equal. The universe starts to become matter dominated afterwards.

- Recombination $t \sim 400,000$ yrs, $T \sim$eV: Recombination is the stage when the first atom, the Hydrogen atom, is formed. As no free electrons are left, the universe becomes transparent to the photons, and the cosmic microwave background (CMB) is formed. The picture 3 shows how the universe looked after recombination. Anything before this is not visible to us.

- The first stars were born after 400 million years, and the galaxies took about a billion years to form. The solar system appeared after 9 billion years. Today, it is about 13.7 billion years since the time of the Big Bang – the temperature is $T = 2.7$K. We will calculate this temperature.

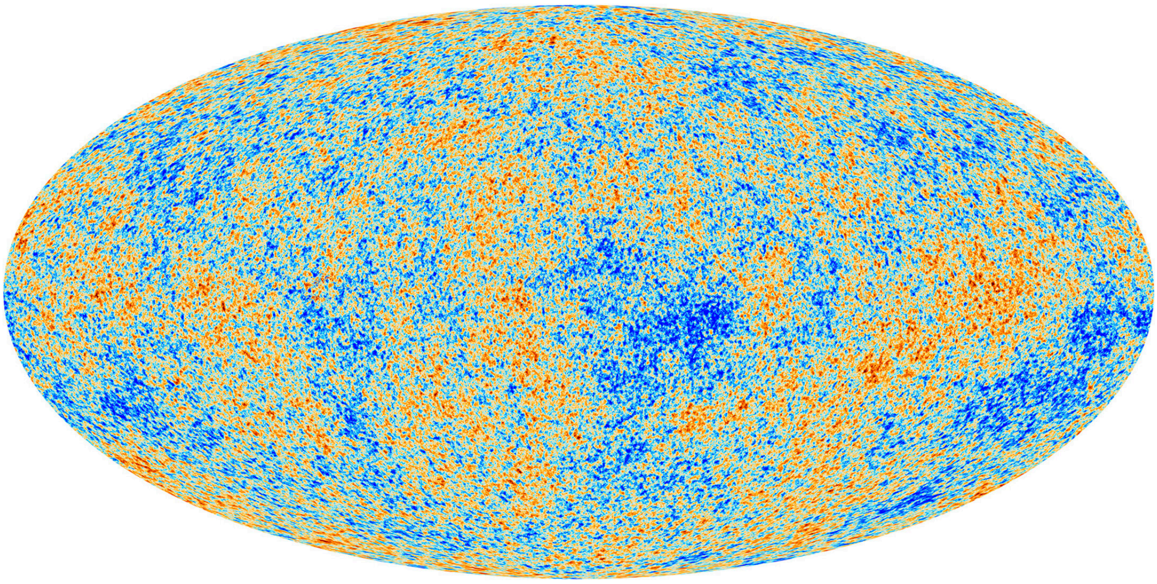

Figure 3: The cosmic microwave background (CMB) courtesy the ESA's Planck mission.

3.4 Elementary Particles

Elementary particles are the fundamental building blocks of matter and interaction – to the best of our knowledge, these particles can not be broken further into smaller pieces. Additionally, there are fundamental interactions like strong interaction, electromagnetic interaction, and weak interactions. Phenomenon like interactions and decay of elementary particles require Quantum Field Theory, a notorious difficulty subject. Here we briefly discuss the classifications of elementary particles based on their properties, like spin, and the interactions they take part in. Based on their spin, elementary particles are divided in to two groups, fermions having half-integer spin, and bosons having integer spin. There are two types of fermions, quarks and leptons, and there are six for each of them. The six quarks are grouped in three 'families'

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \tag{3}$$

where u is for 'up', d for 'down', c for 'charm', s for 'strange', t for 'top', and b for 'beauty'. The six leptons also grouped in three families

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_e \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \tag{4}$$

where e, μ, τ stands for electron, muon, and tau, respectively, and $\nu_{e,\mu,\tau}$ are electron-neutrino, muon-neutrino, and tau-neutrino, respectively. In addition to the fermions, we have the bosons: photons (γ), gluons (g), W-boson, Z-boson, and the Higgs.

Before we discuss why fermions are divided into quarks and leptons, lets take a look at their masses and charges. With reference to the figure 4 all the quarks have fractional charges. The masses of quark vary

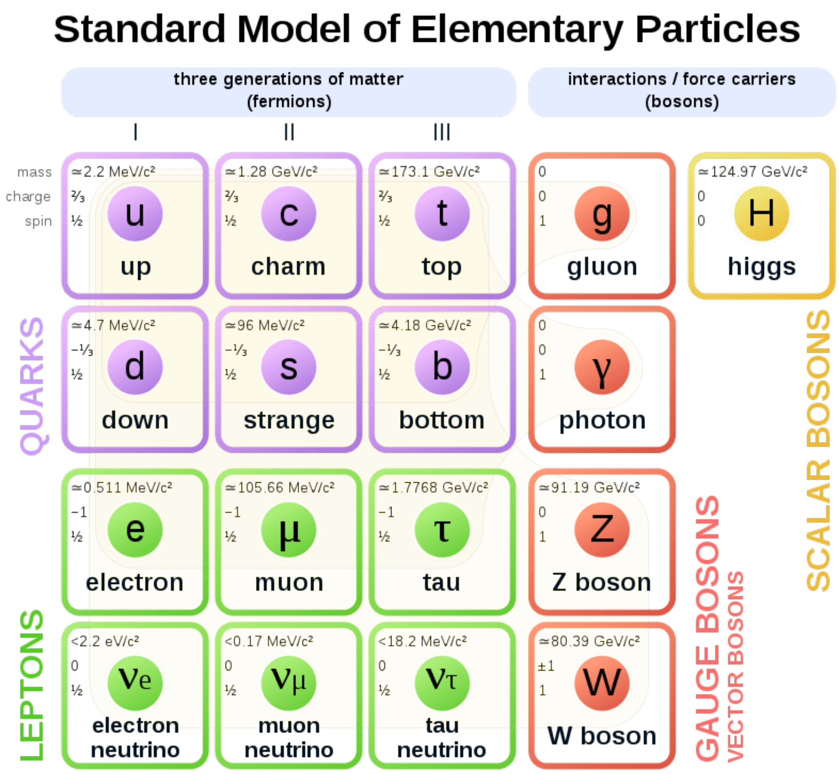

Figure 4: All the known (till now) particles of nature.

widely, ranging from 2.2 MeV for the up-quark to 173 GeV for the top quark. In the lepton sector, only the electrons, the muons, and the tauons are negatively charged, but all the neutrinos are charge neutral. But when it comes to mass of the leptons, things are very interesting. Only the charged leptons e, μ, τ have mass. For long time it was believed that the neutrinos are massless. But neutrinos emitted by the sun shows a phenomenon called the "oscillation' which is not possible if neutrinos are massless. This discovery was awarded by Nobel Prize in physics in 2015. In QFT, the origin of the masses of particles is explained by a mechanism called the Higgs mechanism which requires the existence of a scalar boson called the Higgs. The Higgs boson was discovered in 2012 at the LHC. It must be mentioned that Higgs mechanism described the masses of all the elementary particles apart from the neutrinos. The origin neutrino mass is yet to be discovered.

Fermions are the building-block of matter. They interact by three types of interactions, *strong interaction, weak interaction*, and *electromagnetic interaction*. There is also the gravitational interaction, but quantum theory of gravity does not exists. Only the quarks participate in strong interactions. This distinguishes the quarks from the leptons. Both quarks and leptons participate in the weak and the electromagnetic interaction.

According to QFT, interactions between fermions take place through the exchange of bosonic particles. The strong interaction takes place due to the exchange of massless bosons called *gluons*, the weak interaction through the exchange of heavy bosons called W and Z, and the electromagnetic interaction through the exchange of massless photons.

The elementary particles and their interactions is mathematically described by a quantum field theory framework called the Standard Model of Particle Physics, or simply the SM. The SM is the most fundamental theory of nature known to us, and as you can see from the SM Lagrangian given below, it is very simple !

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - \tfrac{1}{2}\partial_\mu\phi^0\partial_\mu\phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2 W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5){U^{lep}}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa {U^{lep}}^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda {U^{lep}}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda {U^{lep}}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1-\gamma^5)\nu^\kappa\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\underline{\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) +} \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\overline{\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa} + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \\
\partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM\left(\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2}\bar{X}^0 X^0 H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + igMs_w\left(\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0\right).
\end{gathered}
$$

Figure 5: The Standard Model of particle physics.

3.5 Olber's Paradox:

Until the early 20th century it was believed that the universe is infinitely large, eternally old, and static. These assumptions lead to a paradox. The night sky is dark with thousands of stars and galaxies randomly scattered. If the universe is infinite, the distributions of the stars and galaxies must also extend to infinity. If the universe is eternally old, so would be the stars and the galaxies. Hence, in a clear night sky if our line of sight is extended to infinity, it must end in a star or a galaxy. So we will not see the sky as dark. In fact, if we assume that all stars are as bright as the sun, calculations show that the sky should be as bright as the surface of the sun. But this is not what we observe. This paradoxical observation is called Olber's paradox.

One or more of the assumptions must be wrong. Firstly, the assumption that the line of sight can be infinitely extended before it ends in a star is not be true. Non-luminous objects may block the line of sight. Secondly, the assumption of an eternally old universe may be untrue. In a universe that has a beginning, the stars have existed for a finite time. As the speed of light is finite, the light from far away stars have not yet reached us since they started to shine. Finally, the assumption of infinitely large universe may be untrue. For a universe that have a finite size, only a fraction of the sky is covered by stars leaving a vast area that looks dark. Olber's paradox can be resolved in many ways. As it stands, the correct explanation is that the universe has finite age, *i.e.,* it had a beginning.

3.6 Isotropy and homogeneity

The study of the universe rests on the *cosmological principle* which states that *at sufficiently large scale the universe is homogeneous and isotropic.* Homogeneous means that there is no unique location in the universe – the universe looks the same from any location. Isotropy means all directions from a point in the universe are identical. There is no proof of the principle, but observations, for example the cosmic microwave spectrum spectrum, supports the idea. Furthermore, N-body simulation of the universe based on ΛCDM model gives rise to an isotropic and homogeneous universe.

3.7 Hubble's Law

In 1929 Edwin Hubble observed that light from distant galaxies are redshifted and the redshift is proportional to the distance to the galaxy. Red shift is define as

$$z = \frac{\lambda_{\rm obs} - \lambda_{\rm em}}{\lambda_{\rm em}}\,, \tag{5}$$

where $\lambda_{\rm em}$ is the wavelength of emitted light and $\lambda_{\rm obs}$ the wavelength observed at earth. If r is the distance to a galaxy, then according to Hubble's observation

$$z \propto r\,.$$

If the red-shift is attributed to Doppler effect, the shift must be proportional to the relative velocity between the observer and the galaxy

$$z \propto v\,.$$

Hence,

$$v \propto r\,, \quad \text{or} \quad \boxed{v = H_0 r} \tag{6}$$

where the constant of proportionality H_0 is called the Hubble constant. The equation states that galaxies are receding from us with velocities that are proportional to the distance to the galxy. This is called the Hubble's law. In figure 7 receding velocities are plotted against the distance.

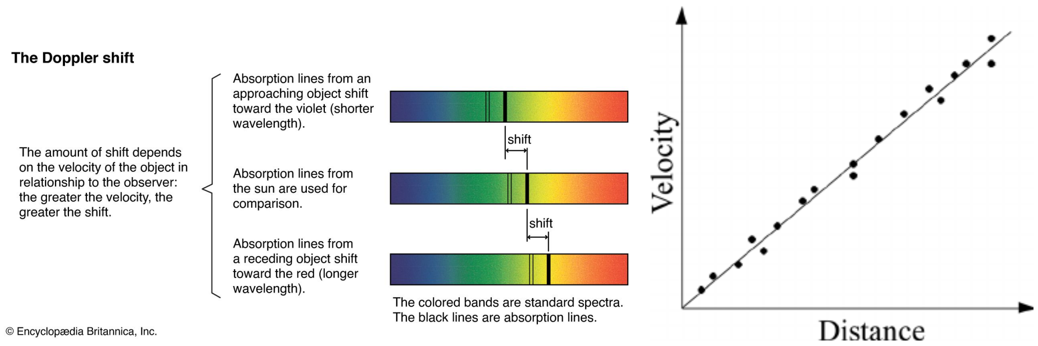

Figure 6: Hubble's observation: galaxies receding at velocities proportional to distance.

Hubbles' law combined with the cosmological principle implies that the universe is expanding. To understand how it works, consider any three points $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ in the universe forming a triangle. The relative distances between the points are $r_{12} = |\vec{r}_1 - \vec{r}_2|$, $r_{23} = |\vec{r}_2 - \vec{r}_3|$, $r_{31} = |\vec{r}_3 - \vec{r}_1|$. According to the cosmological principle, the points recede away from each other in a homogenous fashion such that the shape of the triangle remains the same. The relative distances at time t is related to those at t_0 as

$$r_{12}(t) = a(t) r_{12}(t_0)\,, \quad r_{23}(t) = a(t) r_{23}(t_0)\,, \quad r_{31}(t) = a(t) r_{31}(t_0)\,.$$

and receding speeds are

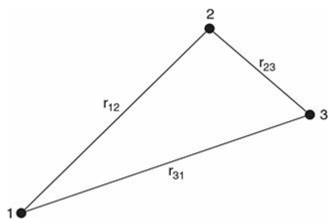

Figure 7: Hubble's observation: galaxies receding at velocities proportional to distance.

$$v_{12}(t) = \frac{\dot{a}}{a} r_{12}(t)\,, \quad r_{23}(t) = \frac{\dot{a}}{a} r_{23}(t)\,, \quad r_{31}(t) = \frac{\dot{a}}{a} r_{31}(t)\,. \tag{7}$$

So, any two points in the universe is receding away from each other with speed that is proportional to their distance of separation and the proportionality constant is

$$H_0 = \frac{\dot{a}}{a}$$

is universal. This was the first direct evidence of the expansion of the universe.

The Hubble constant H_0 has a dimension of t^{-1}. Hubble's own measurement of this constant was about seven times larger than present measurements using CMB data and using Supernova (explosion of a star) data. Though they yield slightly different values we will use

$$H_0 = (72 \pm 7)\ km\ s^{-1} Mpc^{-1}\,. \tag{8}$$

In literature a dimensionless Hubble constant h is more often used

$$H_0 = 100h\ km\ s^{-1} Mpc^{-1} \tag{9}$$

where $h = 0.72 \pm 0.007$.

The Hubble constant is not a constant in time – throughout the different stages of the universe it had different values. The value quoted above is the one at the present epoch – and thats why I have put 0 in the suffix.

As H_0 has the dimension of inverse time, H_0^{-1} is approximately the age of the universe. If the universe is expanding any two galaxies were closer in the past. Let's assume that there is no force in the universe to accelerate of decelerate the relative motion of galaxies and they recede at constant speed. So, two galaxies have taken a time

$$t_0 = \frac{r}{v} = \frac{r}{H_0 r} = H_0^{-1}\,,$$

to be separated by distance r. This is approximately the age of the universe. With $H_0 = 72 km/s/Mpc$ one gets $H_0^{-1} \approx 14$ billion years as the universe's age.