CS 206: Practice problems - Set I
Summer 2018
(Based on lectures 1-4)

Problem 1

Answer the following questions. Explain in one line how you got the answer.

1. Consider the word "OHMYGODIMONFIRE". How many distinguishable ways are there to rearrange the letters?
2. How many sequences of 1s and -1s of length 10 are there that sum up to 2?
3. How many 4-letter words can be obtained using any of the 26 letters of the alphabet, if repetition of letters is allowed?
4. How many 4-letter words can be obtained using any of the 26 letters of the alphabet, if repetition is not allowed?
5. How many 4-letter words contain at least one repeated letter?
6. How many 4-letter words contain the letter X?
7. How many 4-letter words consist of only the letters X and/or Y? (The words XXXX and YYYY are included in this count.)
8. How many ways are there of forming a committee of 10 members that has at least 4 women if you have to choose from 30 men and 20 women?
9. A group of n friends are all about to go separate ways in life and so they all hug each other. If one hug involves exactly 2 people and every pair of friends hugged exactly once, how many hugs were there in total?
10. A group of n friends are all about to go separate ways in life and so they all give each other gifts. If every pair of friends exchanged gifts exactly once, how many gifts were there in total?

Problem 2

Given a binary string S, let $\Delta(S) =$ number of 1's in S - number of 0's in S. S is balanced if $-2 \leq \Delta(S) \leq 2$. How many balanced binary strings of length n are there. Provide a formal proof for your answer.

Problem 3

In each of the following cases, how many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 17$ where x_1, x_2, x_3, x_4, x_5 are non-negative integers.

1. $x_1 \geq 1$.
2. $x_i \geq 2$ for $i = 1, 2, \ldots, 5$.
3. $0 \leq x_1 \leq 10$.
4. $0 \leq x_1 \leq 3$, $1 \leq x_2 \leq 4$, $x_3 \geq 15$.
5. $x_1 \geq x_2$.

Justify your answers.

Problem 4

What is the number of binary strings of length 100 that contain exactly three occurences of the strings 01?

Problem 5

Suppose you have 10 bins, all of different color (they are identical except for their distinct colors), and 30 balls, each ball having a unique number painted on it (they are identical except for the distinct numbers on them).

1. In how many ways can you distribute the balls into the bins?
2. Suppose you only want to put exactly one ball in every bin (and discard the rest). How many ways of doing that are there?
3. Now suppose that all the bins are painted with the same color so that they all look the same. What will your answer to the second question be?
4. Now suppose the numbers painted on the balls get erased and the balls look identical (but the bins still have their colors): what will your answer to the first question be?

5. Now suppose that the numbers on the balls are erased, and someone paints a third of the balls white, a third of the balls black, and the remaining third blue, while the bins still retain their original color: what's your answer to the first question? What about the second question?

6. Assuming the setting of the previous part, how many ways are there of distributing all the balls so that every bin contains at least one ball of every color?

7. Now suppose that the distinct numbers are painted back onto the balls while still retaining their color (as in the previous two parts), and the bins still have their color. How many ways of distributing the balls are there so that every bin gets at least one ball of every color?

8. Assume that the balls still have their colors and the distinct numbers on them from the previous part, however, now, the bins are all painted with the same color so that they become identical. What is your answer to the second question if you want to ensure that at least 8 white balls and at least 7 blue balls are discarded while distributing?