1. Coin is flipped five times

$X =$ # of heads

$X_1 = 1$ if I get H on 1^{st} toss, $1 = 1$
$\quad 0$ o/w

$X_2 = 1$ " " " " " 2^{nd} toss

$X_3 = 1$ " " " " 3^{rd} toss

Binomial $= 0, 1, 2, 3, 4 \ldots \infty$ play a game n times

geometric = play till you win

$X_4 =$ " " " " 4^{th} toss

$X_5 =$ " " " " " 5^{th} toss

$E[X] = ?$

P(H on 2^{nd} toss) $= \frac{2^4}{2^5} = \frac{1}{2}$; $3^{rd} = \frac{1}{2}$; $4^{th} = \frac{1}{2}$; $5^{th} = \frac{1}{2}$

$\Omega = 2^5$

P(H on 1^{st} toss) $= \frac{2^4}{2^5} = 2^{-1} = \frac{1}{2}$

$$\sum_{i=1}^{5} X(i) P(\{i\}) = X(1) P(\{1\}) + X(2) P(2) + \ldots + X(5) P(5)$$

$$= 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{5}{2} = 2.5$$

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play a game n times

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P(H on 2nd toss) $= \frac{2^4}{2^5} = \frac{1}{2}$

3rd $= \frac{1}{2}$

4th $= \frac{1}{2}$

5th $= \frac{1}{2}$

$$\sum_{i=1}^{5} X(i)\,P(\{i\}) = X(1)P(\{1\}) + X(2)P(2) + \dots + X(5)P(5)$$

$$= 1\cdot\frac{1}{2} + 1\cdot\frac{1}{2} + 1\cdot\frac{1}{2} + 1\cdot\frac{1}{2} + 1\cdot\frac{1}{2} = \frac{5}{2} = 2.5$$

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Binomial = 0, 1, 2, 3, 4 ... ∞ play a game n times

geometric = play till you win

3. Roll a die 10 times $\Omega = 6$

$X =$ # of times a 6 appears

$X_1 = 1$ if 6 appears on dice #1
0 o/w

$X_2 = 1$ " " " " dice #2
0 o/w

$\vdots$

$X_{10} = 1$ " " " " dice #10

$P(X=1) = \frac{1}{6}$

$E[X] = ?$

* Probability of 1/6 appearing on one dice

$$= \sum_{i=1}^{10} X(i) P(\{i\}) = \left(1 \times \frac{1}{6}\right) + 0 \times \frac{1}{6} + \left(\left(1 \times \frac{1}{6}\right) + \left(0 \times \frac{1}{6}\right)\right) \cdots\cdots \left(\left(1 \times \frac{1}{6}\right) + \left(0 \times \frac{1}{6}\right)\right)$$

$$\frac{10}{6} = \frac{5}{3}$$

5. Roll two dice

-3 comes twice as any other number

$P(3) = \frac{1}{7} \times 2 = \frac{2}{7}$ $= P(3)$

prob of any other number

This makes our distribution/mass uniform :) !

X = Sum of numbers that appear on the two dice

* the other 5 dice have Prob of $\frac{1}{7}$

$P(X=4) =$

$X = \{2,3,4,5,6,7,8,9,10,11,12\}$

Two dice add up to 2

$X=2 \longrightarrow (1,1) = \frac{1}{49}$

$X=3$: $(2,1)\ \frac{2}{49}$, $(1,2)\ \frac{2}{49}$

$X=4$: $\frac{2}{49}(3,1)$, $\frac{2}{49}(1,3)$, $\frac{1}{49}(2,2)$

$X=5$: $\frac{2}{49}(2,3)$, $\frac{2}{49}(3,2)$, $\frac{1}{49}(4,1)$, $\frac{1}{49}(1,4)$

$X=6$: $\frac{4}{49}(3,3)$, $\frac{1}{49}(4,2)$, $\frac{1}{49}(2,4)$, $\frac{1}{49}(5,1)$, $\frac{1}{49}(1,5)$

$X=7$: $\frac{1}{49}(1,6)$, $\frac{1}{49}(6,1)$, $\frac{2}{49}(2,5)$, $\frac{1}{49}(5,2)$, $\frac{2}{49}(3,4)$, $\frac{2}{49}(4,3)$

$X=8$: $(6,2)$ $(4,4)$ $(5,3)$ $(3,5)$ $(2,6)$ $\frac{2}{49} + \frac{2}{49} + \frac{2}{49} + \frac{1}{49}$

$X=9$: $(3,6) = \frac{2}{49}$, $(6,3) = \frac{2}{49}$, $(4,5) = \frac{1}{49}$, $(5,4) = \frac{1}{49}$

$X=10$: $(5,5) = \frac{1}{49}$, $(6,4) = \frac{1}{49}$, $(4,6) = \frac{1}{49}$

$X=11$: $(6,5) = \frac{1}{49}$, $(5,6) = \frac{1}{49}$

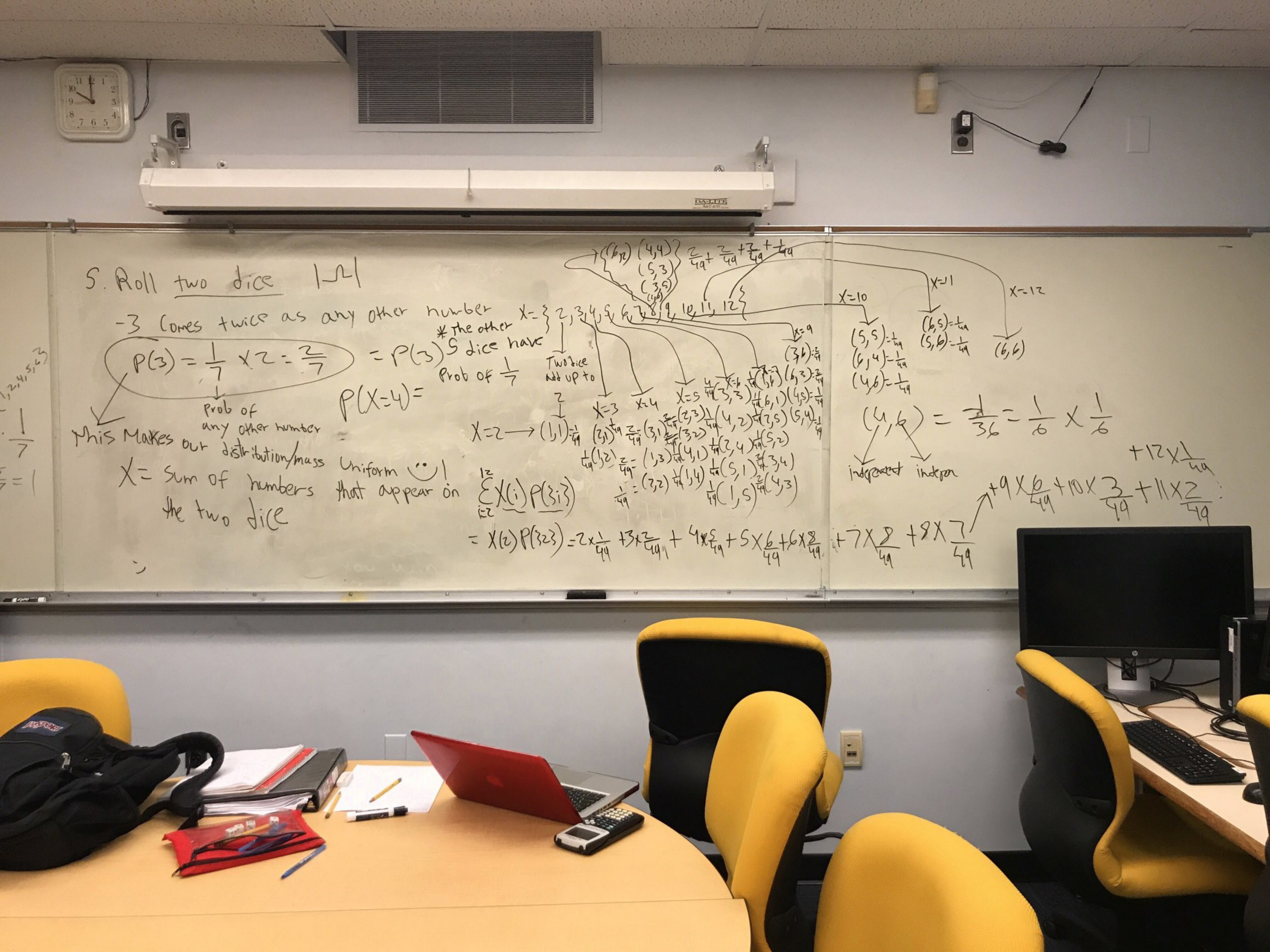
$X=12$: $(6,6)$

$(4,6) = \frac{1}{36} = \frac{1}{6} \times \frac{1}{6}$

independent   indepen

$\sum_{i=2}^{12} X(i)\, P(\{i\})$

$= X(2)P(\{2\}) = 2\times\frac{1}{49} + 3\times\frac{2}{49} + 4\times\frac{5}{49} + 5\times\frac{6}{49} + 6\times\frac{8}{49} + 7\times\frac{8}{49} + 8\times\frac{7}{49} + 9\times\frac{6}{49} + 10\times\frac{3}{49} + 11\times\frac{2}{49} + 12\times\frac{1}{4}$


5. Roll two dice
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= P(3)
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Prob of any other number
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X = Sum of numbers that appear on the two dice
P(X=4) =
X = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
Two dice add up to
X=2 ⟶ (1,1) = 1/49
X=3
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(5,5) = 1/49
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(4,6) = 1/36 = 1/6 x 1/6
independent
indepen
X(2)P({2}) = 2x1/49 + 3x2/49 + 4x5/49 + 5x6/49 + 6x8/49 + 7x8/49 + 8x7/49
+9x6/49 + 10x3/49 + 11x2/49
+12x1/49