Homework 2

CS 206: Discrete Structures II
Summer 2018

Due: 5:00 PM EDT, Thursday, July 12th, 2018

Total points: 60

Name:

NetID:

INSTRUCTIONS:

1. Print all the pages in this document and make sure you write the solutions in the space provided for each problem. This is very important! Even if you are using LaTeX, make sure your solutions fit into the given space.
2. Make sure you write your name and NetID in the space provided above.
3. After you are done writing, scan the sheets in the correct order into a PDF, and upload the PDF to Gradescope before the deadline mentioned above. No late submissions barring exceptional circumstances! The submitted PDF should have all the seven pages in the correct order.
4. As mentioned in the class, you may discuss with others but my suggestion would be that you try the problems on your own first. Even if you do end up discussing, make sure you understand the solution and write it in your own words. If we suspect that you have copied verbatim, you may be called to explain the solution.

Part I

Problem 1. [10 pts]
How many integer solutions are there to the following system of equations?

$$x \times y \times z = 256$$

$$x, y, z \geq 2.$$

Justify your answer. You can leave your answer in terms of factorials and/or binominal coefficients.

Problem 2. [10 pts]
Let A and B be finite sets such that $|A| = 15$ and $|B| = 5$. How many 3-to-1 functions with A as domain and B as codomain are there? Justify your anwer. You can leave your answer in terms of factorials and/or binomial coefficients.

Part II

Problem 3. [20 pts]
A robot is located at the coordinates (p, q, r) on a 3D maze. The robot needs to go to the origin, i.e. the point $(0, 0, 0)$. From any location (x, y, z) the valid moves for the robot are one of the following:

- $(x, y, z) \to (x - 1, y, z)$ if $x > 0$.
- $(x, y, z) \to (x, y - 1, z)$ if $y > 0$.
- $(x, y, z) \to (x, y, z - 1)$ if $z > 0$.

Count the number of different valid paths that the robot can take to go from (p, q, r) to $(0, 0, 0)$. A valid path is a path in which all the intermediate moves are valid. For example, for $p = q = r = 2$, a valid path from (p, q, r) to $(0, 0, 0)$ is the following:

$$(2, 2, 2) \to (2, 1, 2) \to (2, 1, 1) \to (1, 1, 1) \to (0, 1, 1) \to (0, 1, 0) \to (0, 0, 0).$$

You must justify your answer. Write you answer in terms of factorials and/or binomial coefficients. Also, you have this page and the next page for writing the solution.

More space for problem 3 solution:

Problem 4. [20 pts]
A driver and their racing team are taking part in a race that involves doing 80 laps around a circuit. They have 10 distinct sets of tires (say $T_1, T_2, \ldots, T_{10}$) and want to use each set of tires for at least one lap during the race. There are, however, some constraints:

- The tires cannot be changed during a lap, and can only be changed at the beginning or end of a lap. Obviously, the end of lap 1 is the same as the beginning of lap 2, and so on, and so there are exactly 80 opportunities for the team to possibly change the tires (It doesn't make sense to change tires after the last lap, i.e. after the race has ended).

- Once a set of tires is removed from the car, they can never be reused and must be discarded. For example, if at the beginning of lap 1, i.e., the beginning of the race, the team installs the set of tires T_5, and lets the driver race with those tires for 10 laps, and replaces the set T_5 with the set T_1 at the end of lap 10, then they can never re-use the set T_5 again for the rest of the race – that set of tires must be discarded.

A *tire change schedule* is a schedule that instructs the team when and how they should change tires, respecting the above constraints. For example, a valid tire change schedule is as follows: *Use set T_1 for the first 20 laps, then use set T_5 for the next* 10 *laps, then use set T_3 for the next* 15 *laps, then use set T_2 for the next* 15 *laps, and use set T_4 for the last* 20 *laps.*
Find the total number of possible tire change schedules that respect all the constraints. Justify your answer. You may leave your answer in terms of factorials and/or binomial coefficients. You have this and the next page for writing your solution.

More space for problem 4 solution: