Homework 3

CS 206: Discrete Structures II
Summer 2018

Due: 5:00 PM EDT, Thursday, July 19th, 2018

Total points: 60

Name:

NetID:

INSTRUCTIONS:

1. Print all the pages in this document and make sure you write the solutions in the space provided for each problem. This is very important! Even if you are using LaTeX, make sure your solutions fit into the given space.

2. Make sure you write your name and NetID in the space provided above.

3. After you are done writing, scan the sheets in the correct order into a PDF, and upload the PDF to Gradescope before the deadline mentioned above. No late submissions barring exceptional circumstances! **The submitted PDF should have all the 9 pages in the correct order.**

4. As mentioned in the class, you may discuss with others but my suggestion would be that you try the problems on your own first. Even if you do end up discussing, make sure you understand the solution and write it in your own words. If we suspect that you have copied verbatim, you may be called to explain the solution.

Problem 1. [10 pts]
How many strings of length 10 are there that consist only of lower case letters (a-z) and contain at least one occurrence of every vowel?

More space for Problem 1:

Problem 2. [20 pts]
There are 100 new teachers in the state of NJ who just finished their training and 21 high schools (say one from each of the 21 counties of NJ) in need of teachers. Of course, if the teachers are left to their own devices, they will all flock to the schools in the good counties, and so the Board of Education wants to make sure that every school gets at least one teacher assigned to it. The Board of Education wants to know in how many ways can they assign teachers to schools so that every school gets at least one new teacher assigned to it. Can you help them? You can leave you answer in the form of a sum if needed.

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Problem 3. [10 + 10 + 10 = 30 pts]
In one of the lectures we saw that when n is odd

$$\sum_{0 \le k < \frac{n}{2}} \binom{n}{k} = 2^{n-1}.$$

We want to understand how large can the sum on the left hand side be when n is even. In this problem you will show (in three steps) that:

$$\sum_{0 \le k < \frac{n}{2}} \binom{n}{k} \le \frac{2^{n-1}}{1 + \frac{1}{n}}$$

1. Prove that, when n is even,

$$\sum_{0 \le k < \frac{n}{2}} \binom{n}{k} = \frac{2^n - \binom{n}{n/2}}{2}.$$

2. Prove that, when n is even,

$$\binom{n}{n/2} \ge \frac{2^n}{n+1}.$$

   [Hint: Try to use the fact that, as function of k, $\binom{n}{k}$ has its maxima at $n/2$ when n is even.]

3. Combine the previous parts to prove the desired upper bound on $\sum_{0 \le k < \frac{n}{2}} \binom{n}{k}$.

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