CS 206: Practice problems - Set II

Summer 2018

(Based on lectures 7-9)

1. We pick 13 cards at random from a well-shuffled deck and deal them to a player X.
   (a) What is the probability that player X gets NO aces? What's that of getting no aces, but all the kings? What's the conditional probability that he/she/it gets no aces, given that they received all the kings?
   (b) Let A be the event that corresponds to player X getting ALL aces. What is the probability of getting all the aces but no jacks? Let B be the event which denotes getting all the aces and all the remaining card are spades. What is Pr(A) and Pr(B)? What is Pr($B|A$)?
   (c) What is the conditional probability of no aces, given that player X didn't get any spades?
   (d) What is the probability that player X get NO spades? What is the probability that player X gets no card higher than 9?
   (e) What is the probability that neither of the events in part (d) occur?
2. An ordinary deck of cards is shuffled and the cards are lined up. What is the probability that no aces are adjacent? What is the probability that no spades are adjacent? What is the probability that no aces or adjacent or separated by only one non-ace?
3. 13 players are dealt 4 cards each
   (a) Describe the sample space and calculate its size.
   (b) What is the probability of event A : { each player gets one card from each suit}?
   (c) What is the probability of event B " { each player has all four cards of the same value}
   (d) Are events A and B indenpendent?
   (e) What is the probability that players one, two and three have only been dealt with only aces, queens or kings?
   (f) What is the probability that one player has one card from each suit, but no one else has cards from more than one suit?
4. An experiment has two outcomes, one with probability $1 - p$, the other with probability $2p^2$. What is p?
5. TRUE or FALSE: "If $P(A \setminus B) + P(B \setminus C) = P((A \cup B) \setminus C)$ then $B \cap C = \emptyset$." Decide the issue and explain your answer.
6. TRUE or FALSE: "If $P(A) < P(B \setminus A)$ then $P(A) < \frac{1}{2}$." Decide the issue and explain your answer.
7. Prove $P[(A \cap B^c) \cup (A^c \cap B)] = P(A) + P(B) - 2P(A \cap B)$. Describe in English the event whose probaility is computed by the expression on the left-hand side of the equation.
8. (S,P) is the probability space of tossing two dice (one red and one blue) under equally likely probability.
   (a) Let $A = \{\text{red die is 3,4 or 5}\}$, $B = \{\text{blue die is 1 or 2}\}$, C=sum is 7. Are these pairwise independent? Are they mutually independent? Explain your answers.
   (b) Let $A = \{\text{red die is odd}\}$, $B = \{\text{blue die is even}\}$, C=sum is odd. Compute $P(B|A)$, $P(C|A)$ and $P(C|B)$. Are these pairwise independent? Are they mutually independent? Explain you answers.

(c) Repeat (b), but now both dice have TWO faces that say 5 and NO face that says 6.

(d) Let $A = \{\text{sum is } 7\}$ and B ={there is at least one 6}. Compute $P(A)$ and $P(A|B)$. Here is an apparent paradox: compute $P(A|C_x)$ where, for $x = 1, 2, 3, 4, 5or6$, C_x is the event that {there is at least one x}. Since you know that some C_x always occurs, how can $P(A) \neq P(A|C_x)$. Discuss whether this makes sense or not. How do you reconcile it?

9. A computer has printer (P), disk (D) and terminal (T) outputs. Sixty percent of all output characters are on D, thirty percent on P and the rest on T. The error rate for D is 1/2000, for P it is 2/1000, for T it its 1/1000. The experiment ε is that a character is output and we observe (i) which type of device made that output and (ii) whether the character was correct. Write down the sample space. What is the probability the character was written on the disk, given A={it was incorrect}?

10. Repeat previous question, but now we have a forth output device, a magnetic tape (M) with error rate of 1/500. And on this computer fifty five percent of all output characters are on D, twenty five percent are on D, fifteen percent are on T and five percent on M.

11. A box contains 100 balls. 20 are red, 30 are green, and the rest are yellow. 3/4 of the red balls are small (and the rest are big), 2/3 of the green balls are small and 1/2 of the yellow ones are small. The experiment is to choose a ball at random and to observe its color and its size.

    (a) Carefully describe the probability space for this experiment.

    (b) What is the probability of the event $A = \{\text{a small ball is chosen}\}$

    (c) You are told that A occurs. What is $P(red|A)$?

12. There are 4 envelopes, one of which contains 100 dollars, the other 3 being empty. You take one of the envelops at random.

    (a) what is the probability that when you open it, it will contain 100 dollars.

    (b) Now, before you open your envelop, somebody opens one the other 3 and shows you that it is empty. You are now offered the choice to (i) keep your original envelpe or (ii) change to one of the remaining 2. What is the probability that you win 100$ if you do (i)? What is the probability if you do (ii)? Explain in detail.

13. Decide if A and B can be independent if they are mutually exclusive, and explain your answer.

14. Decide if A and B can be independent if $A \subseteq B$. Explain your answer. You may assume both events have probability larger than zero and smaller than one.

15. When are the events $A \cap B$ and C independent? (a) Always (b) Never (c) Sometimes. If you say (c) then try to characterize when it occurs.