Homework 6

CS 206: Discrete Structures II
Summer 2018

Due: 11:59 PM EDT, Sunday, August 12th, 2018

Total points: 60

Name:

NetID:

INSTRUCTIONS:

1. Print all the pages in this document and make sure you write the solutions in the space provided for each problem. This is very important! Even if you are using LaTeX, make sure your solutions fit into the given space.

2. Make sure you write your name and NetID in the space provided above.

3. After you are done writing, scan the sheets in the correct order into a PDF, and upload the PDF to Gradescope before the deadline mentioned above. No late submissions barring exceptional circumstances! **The submitted PDF should have all the 13 pages in the correct order.**

4. As mentioned in the class, you may discuss with others but my suggestion would be that you try the problems on your own first. Even if you do end up discussing, make sure you understand the solution and write it in your own words. If we suspect that you have copied verbatim, you may be called to explain the solution.

Problem 1. [15 pts]
A red, a blue, and a green dice are rolled. All three dice are fair. Let X_R, X_B and X_G be random variables such that X_R is the number rolled by the red dice, X_B the number rolled by the blue dice, and X_G the number rolled by the green dice. Let X be the random variable defined as

$$X = \max(X_R, X_B, X_G).$$

Find $\mathbb{E}[X]$. Give details for all the steps involved in your solution.

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Problem 2. [15 pts]
Consider the experiment of tossing a fair coin 100 times, and let E be the event that you see an even number of heads during the 100 tosses, and X be the random variable that takes value 1 if the 10^{th} coin toss is heads and takes value 0 if the 10^{th} coin toss is tails. If I_E is the indicator random variable for the event E show that I_E and X are independent.

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Problem 3. [15 pts]
You are back at the casino table (from HW 4). It's the same game as before with the same rules: in each round a fair coin is tossed and if it comes up heads you win \$1, and if it comes up tails you lose \$1. The game consists of 50 such rounds. Your net gain at the end of the game is defined as the total amount of money won by you during the game minus the total amount of money lost by you during the game.

Having studied random variables and expecations in 206 recently, you are eager to apply this new knowledge to make the most of amount of money in the least amount of time, and so you come up with the following strategy: you will only play the game if your expected net gain is positive. Should you play the game? Why or why not?
Suppose that there is another table with the same game as above except that they use a biased coin there: the coin lands heads with probability $\frac{5}{8}$ and tails with probability $\frac{3}{8}$. Should you play at this table? Why or why not?
Give details for all the steps of your solution.

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Problem 4. [15 pts]
There are n processors in the system. m jobs arrive and each job is assigned to a randomly chosen processor such that each job is equally likely to be sent to any of the n processors. We call a processor *idle* if it is not assigned any jobs. What is the expected number of idle processors?

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