

Midterm exam

CS 206: Discrete Structures II
Summer 2018

(Sample)

Total points: 130

Duration: 2 hours

Name:

NetID:

INSTRUCTIONS:

1. You have to solve 13 problems in 2 hours. Each problem is worth 10 points. To get full points for a problem, you must give details for all the steps involved in solving the problem AND arrive at the correct answer. Giving partial details or arriving at the wrong answer will result in a partial score.
2. You may leave your answer in terms of factorials, binomial coefficients, and/or power of numbers.
3. Make sure you write your solutions ONLY in the space provided below each problem. There is plenty of space for each problem. You can use the back of the sheets for scratchwork.
4. You may refer to physical copies of any books or lecture notes you want to during the exam. However, the use of any electronic devices will lead to cancellation of your exam and a zero score, with the possibility of the authorities getting involved.
5. Make sure you write your name and NetID in the space provided above.
6. If we catch you cheating, or later suspect that your answers were copied from someone else, you will be given a zero on the exam, and might even be reported to the authorities!

Problem 1. [10 pts]

How many 11 letter words contain only vowels?

Problem 2. [10 pts]

How many 11 letter words contain only consonants?

Problem 3. [10 pts]

How many 11 letter words contain BOTH vowels and consonants?

Problem 4. [10 pts]

In how many ways can you distribute 1000 identical M&M's among 200 kids?

Problem 5. [10 pts]

In how many ways can you distribute 1000 identical M&M's among 200 kids, if you are allowed to keep some of the M&M's for yourself?

Problem 6. [10 pts]

What is the number of integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 30$ with $x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 3$, and $x_4 \leq 4$?

Problem 7. [10 pts]

What is the coefficient of $x^8y^6z^1$ in $(-12x + 3y + 4z)^{15}$?

Problem 8. [10 pts]

How many different strings of length 12 can you make using the letters of the word “**vicissitudes**”?

Problem 9. [10 pts]

Consider $S = \{1, \dots, 100\}$. You want to pick a subset of S that contains at least one number from each of the following sets:

- Even numbers.
- Multiples of three.
- Multiples of five.

In how many ways can you do so?

More space for Problem 9

Problem 10. [10 pts]

How many integer solutions are there to the equation $x_1 \times x_2 \times \dots \times x_n = -1$ if for every $1 \leq i \leq n$ we have that $-1 \leq x_i \leq 1$?

Hint: Surely none of the x_i s can be equal to zero since otherwise their product will become 0. This means each x_i is either 1 or -1 . Think about the scenarios in which the product of all the x_i s can be -1 ? Can it be -1 if exactly 4 of the x_i s are -1 and the rest are 1?

More space for Problem 10

Problem 11. [10 pts]

Prove using induction that $n! > 2^n$ for $n \geq 4$.

More space for Problem 11

Problem 12. [10 pts]

A group consists of the following people: a boy and a girl from NJ, a boy and a girl from Delaware, a boy and a girl from Texas, and a boy and a girl from California. We want to match every girl to a boy so that the following conditions are satisfied:

- No two girls are matched to the same boy.
- A girl is not matched to the boy from her state.

In how many ways can you do the matching?

More space for Problem 12

Problem 13. [10 pts]

Show that:

$$100 \cdot 2^{99} = \sum_{k=1}^{100} k \cdot \binom{100}{k}.$$

Hint: Use the binomial theorem with y set to 1, differentiate both sides with respect to x , and then set the value of x appropriately.

More space for Problem 13