Introduction to Entropy

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Abstract

This seminar paper provides a foundational overview of Dual-/Adjunct-Operators in functional analysis.

Contents

1 Revision

Definition 1. The following basic definitions:

i) Banach Space

ii) Hilbert Space

Definition 2. Let $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ be normed vector spaces on $\mathbb{R}$ or $\mathbb{C}$.
Let $T : X \to Y$ be linear.
T is a *linear operator*. T is *bounded*, if $\exists C > 0, \forall x \in X : \|Tx\|_Y \leq C\|x\|_X$.

Definition 3. i) Algebraic dual space: $X' := \mathcal{L}$.

ii) (Topological) Dual space: $X* := \mathcal{L}$.

Theorem 1. *The topological dual space $\mathcal{L}(X, Y)$ is a Banach space.*

Proof Idea. □

Theorem 2. *For linear operators, continous and bounded are equivalent.*

Proof Idea. TOOD □

Theorem 3. *Let $p \in [1, \infty)$.*

Proof Idea. TOOD □

Theorem 4.

Proof. See functional analysis lecture of SS/25. □

2 The adjunct operator

Definition 4.

Theorem 5.