Your Paper

You

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Abstract

Your abstract.

1 Introduction

Your introduction goes here! Simply start writing your document and use the Recompile button to view the updated PDF preview. Examples of commonly used commands and features are listed below, to help you get started.

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2 Some examples to get started

Let $(\mathcal{X}, \mathcal{A}, \mathbb{P}_{\mathcal{A}})$ be a probability space. Let $X : \mathcal{X} \to \mathbb{R}$ be a discrete random variable on the space with probability density function $f_X : \mathcal{X} \to \mathbb{R}_+$. We use the shorthand notation $p(x) = \mathbb{P}_{\mathcal{A}}[X = x]$. Let $(\mathcal{Y}, \mathcal{B}, \mathbb{P}_{\mathcal{B}})$ be a probability space. Let $Y : \mathcal{Y} \to \mathbb{R}$ be a discrete random variable on the space with probability density function $f_Y : \mathcal{Y} \to \mathbb{R}_+$. We use the shorthand notation $p(y) = \mathbb{P}_{\mathcal{B}}[X = x]$.

Convention: $0 \log(0) = 0$.

Definition of Entropy: $H_q(X) = \sum_{x \in \mathcal{X}} -p(x) \log_q(p(x))$.

TODO: Maybe mention axiomatic definition.

Typical base $q = 2$: $H(X) := H_2(X)$ and $\log(p) = \log_2(p)$.

Existence of Entropy: If $\mathcal{X}$ is finite, $H_q(X)$ exists.

Example, when Entropy does not exist: Let $X =$,

Using definition of expected value: $H_p(X) = \mathbb{E}(-\log_q(p(X)) = \mathbb{E}(\frac{1}{\log_q(p(X))})$.

Properties of Entropy:

Theorem 1. $H(X) \geq 0$.

Proof. Note that $\log(\frac{1}{[0,1]}) = \log([1, \infty]) = [0, \infty]$ and $p(X)(\mathcal{X}) \geq 0$.
Using the monotonicity of the expected value, we obtain

$$0 \leq \mathbb{E}\left(\log\left(\frac{1}{p(X)}\right)\right) = \mathbb{E}\left(-\log\left(p(X)\right)\right) = H(X)$$

□

Theorem 2. *The entropy function H(X) is concave.*

Proof. □

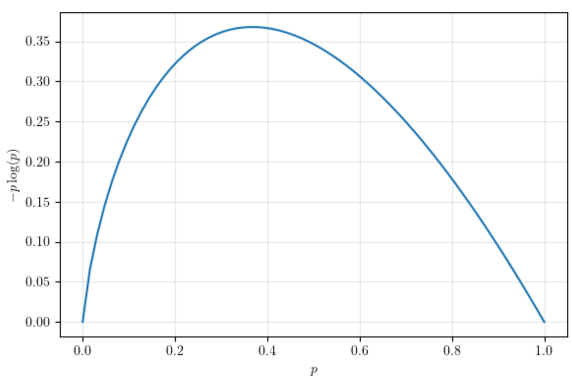

Figure 1: The entropy function is a concave function.