

OR  $\Rightarrow$

$\rightarrow B$  is const 1, so this group only depends on  $B$

$x_B$	00	01	11	10
$C$	00	11	11	00
0	00	11	11	00
1	00	11	11	00
$\downarrow$				

$A$  is always 1

$C$  is also always 1

connect into  
 $\Rightarrow$  both  $A$  and  $C$   
have to be true  
 $\Rightarrow AC$

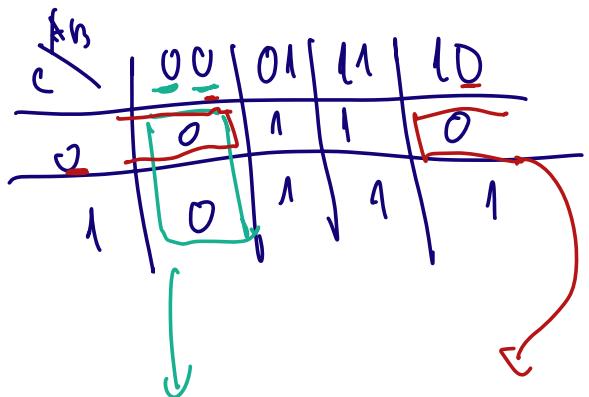
in conclusion we have

either  $AC$  or  $B$

$\Rightarrow AC + B$

or mathematically  $(A \wedge C) \vee B$

when we group by 1s we always use  
and ( $\wedge$ ) for same groups and or ( $\vee$ )  
to connect different groups



We can also group by 0s if that is more convenient, but we use  
opposite signs

$A$  and  $B$   
have to be 0

Maps can  
loop around,  
 $B$  and  $C$  have to be 0

$A + B$

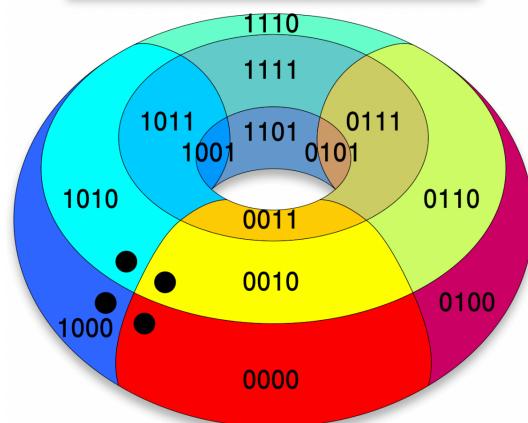
Now put together

$$(A + B) \cdot (B + C)$$

use distributive laws:

$$B + (A \cdot C)$$

$B + C$			
• 0000	0100	1100	1000
0001	0101	1101	1001
0011	0111	1111	1011
0010	0110	1110	1010



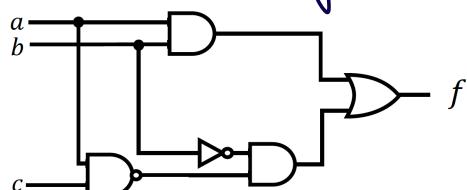
so we get equivalent circuit

## Laws of Boolean algebra

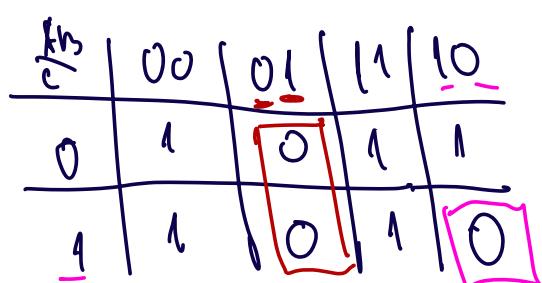
- **Identity :**  $A + 0 = A$        $A \cdot 1 = A$
- **Zero & One:**  $A + 1 = 1$        $A \cdot 0 = 0$
- **Inverse :**  $A + \bar{A} = 1$        $A \cdot \bar{A} = 0$
- **Commutative :**  $A + B = B + A$        $A \cdot B = B \cdot A$
- **Associative :**  $A + (B + C) = (A + B) + C$   
 $A \cdot (B \cdot C) = (A \cdot B) \cdot C$
- **Distributive :**  $A \cdot (B + C) = (A \cdot B) + (A \cdot C)$   
 $A + (B \cdot C) = (A + B) \cdot (A + C)$
- **Demorgan's**       $\overline{A + B} = \bar{A} \cdot \bar{B}$   
 $\overline{A \cdot B} = \bar{A} + \bar{B}$

20

example with negative inputs

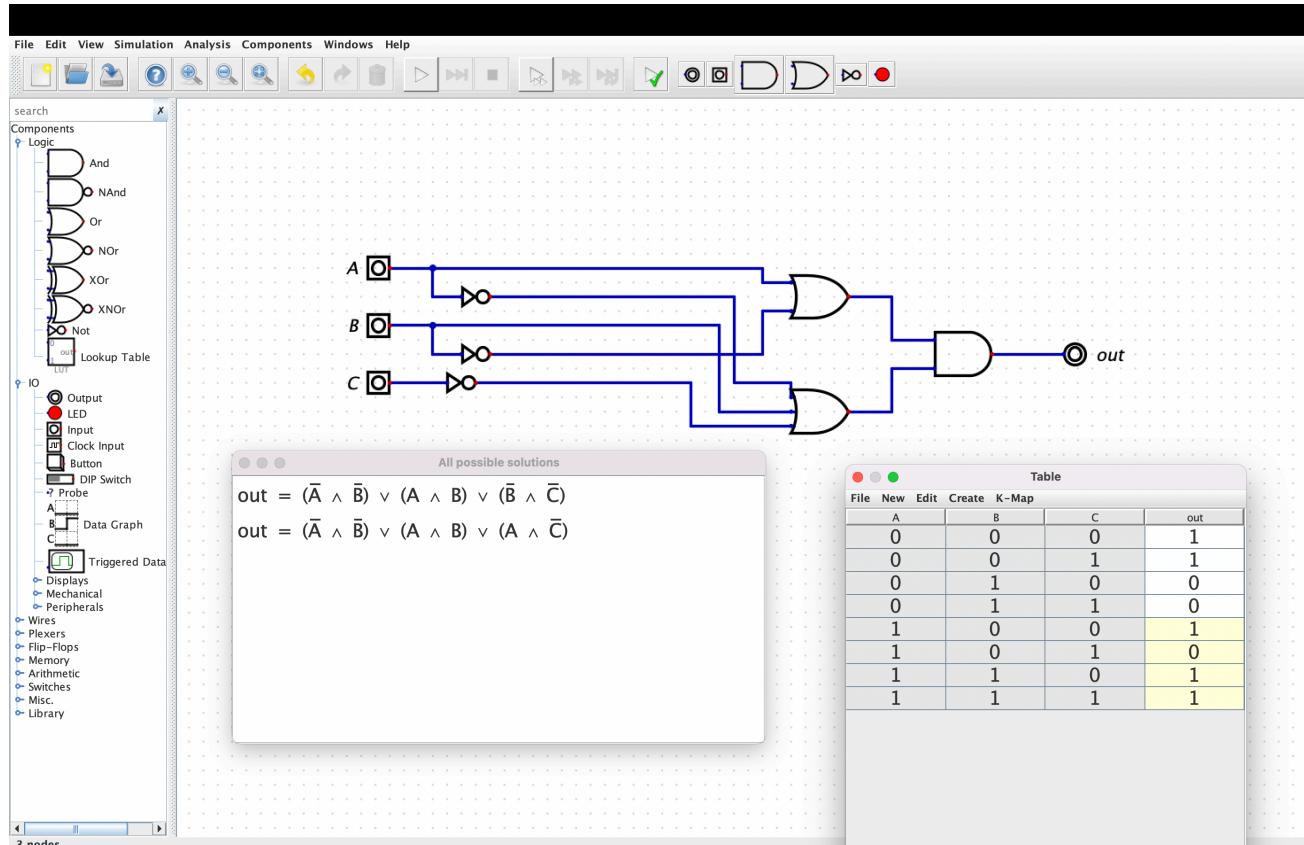


a	b	c	ab	ac	$\bar{a} \cdot \bar{c}$	$\bar{b}$	$\bar{a} \cdot \bar{c} \cdot \bar{b}$	f
0	0	0	0	0	1	1	1	1
0	0	1	0	0	1	1	1	1
0	1	0	0	0	1	0	0	0
0	1	1	0	0	1	0	0	0
1	0	0	0	0	1	1	1	1
1	0	1	0	1	0	1	0	0
1	1	0	1	0	1	0	0	1
1	1	1	1	1	0	0	0	1



gives more compact  
 $(A + \bar{B}) \cdot (\bar{A} + B + \bar{C})$

We get the same result



example with 4 inputs

a	b	c	d	w	x	y	z
0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	1
0	0	1	0	1	1	1	0
0	0	1	1	1	1	0	1
0	1	0	0	1	1	0	0
0	1	0	1	1	0	1	1
0	1	1	0	1	0	1	0
0	1	1	1	1	0	0	1
1	0	0	0	1	0	0	0
1	0	0	1	0	1	1	1
1	0	1	0	0	1	1	0
1	0	1	1	0	1	0	1
1	1	0	0	0	1	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	0	1	0
1	1	1	1	0	0	0	1

$\begin{matrix} \bar{A} \\ \bar{B} \end{matrix}$	00	01	11	10
CD	0	1	0	1
00	0	1	0	1
01	1	1	0	0
11	1	1	0	0
10	1	1	0	0

for w

$$\bar{A}C + AD + \bar{A}\bar{B} + A\bar{B}\bar{C}\bar{D}$$

simplify:

$$\bar{A} \cdot (C + D + B) + A \bar{B} \bar{C} \bar{D}$$