Physics Experiment Project

Class XII A

2017-18

Simple Harmonic Motion

Conducted by:

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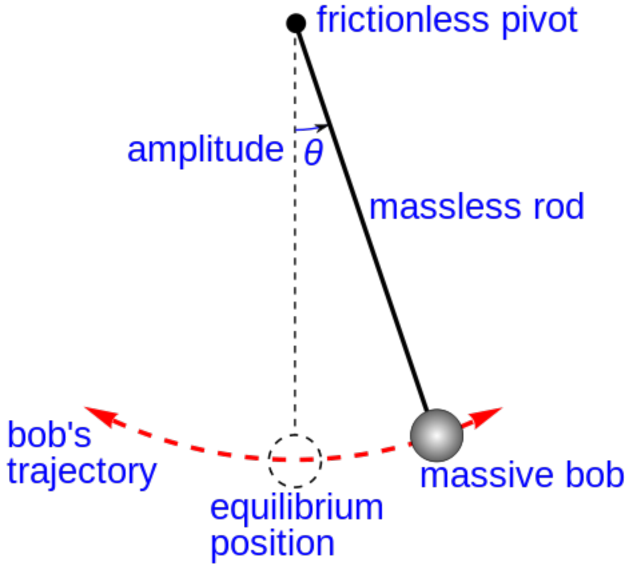

1. Acknowledgements

We would like to express our special thanks of gratitude to Radhika ma'am and Vinni ma'am who gave us the golden oppurtunity to perform our project on the topic Simple Harmonic Motion. We would also like to thank our laboratory coordinator, Lekha Ma'am for providing us with lab time to perform our experiment, and as well as Sujatha Ma'am for providing us with all the lab apparatus for our experiment.

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2. Introduction

Simple Harmonic Motion appears in many regions of physics, as it serves for a useful mathematical model, to describe a variety of motions. Last year, we had chosen this topic and performed an experiment to observe the phenonmenon of beats, in a dual pendulum system, where one pendulum held a light source, and the other held a convex lens. The objective of the experiment was to obtain a sharp image of the light source on a screen, during the simultaneous oscilattory motion of both the pendulums. The success of this experiment, and its interesting observations, prompted us to re-take the SHM topic this year, and perform another experiment on the same.

SHM describes the motion of a body to be sinusoidal in time, and this aspect of it often leads it to be used as an approximation of several motions in mechanics. One such example is the motion of a Simple Pendulum. The motion of a pendulum is assumed to follow an SHM for small amplitudes. The objective of this experiment is to determine how accurate this assumption is for bigger amplitudes. We do the same by predicting the angle of release of a body in pendulum motion, whose string has been cut during its oscillatory motion.

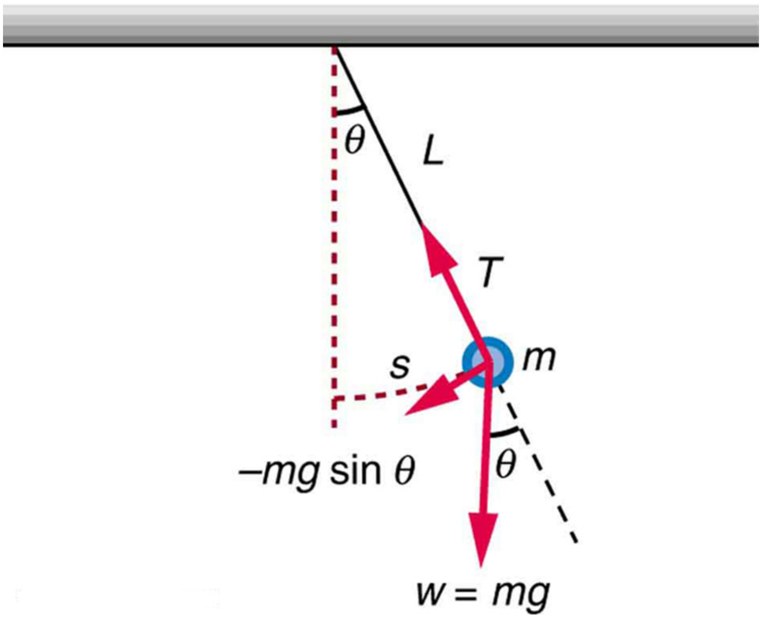


Fig.1 Forces in a pendulum system

3. Experimental details

Error in Small Angle Approximation of a Pendulum

Aim

(a) To predict the angle of release of pendulum projectile by measuring range of the projectile.

(b) To use the results of (a) to compare the predictions of SAA with a more accurate analysis considering the exact forces of the system.

Materials Required

Metallic bob, Thread, Iron Stand, Protractor, Metre Scale, Scissors

Theory

Simple harmonic motion is a type of periodic motion where the restoring force is directly proportional to the displacement and acts in the direction opposite to that of displacement. The restoring force is given by:

$$\mathbf{F} = -kx$$

where $\mathbf{F}$ is the restoring force, k is a constant, and x is the displacement from the equillibrium position.

A pendulum is a weight suspended from a frictionless pivot so that it can swing freely. For a pendulum to act as an SHM oscillator, the force on the weight must be proportional to the displacement of the weight from the equillibrium position. But from *fig.1* it can be clearly seen that the magnitude of the restoring torque of mg is given by:

$$|\mathbf{\Gamma}| = -mgl\sin\theta$$

It is clearly seen that the restoring torque is not directly proportional to the angular displacement, which means that the pendulum is not an SHM oscillator. But for small values of θ, $\sin\theta$ is approximately equal to θ, and thus for this case alone does the pendulum act as an SHM. The displacement of the weight for small angles can thus be given by:

$$\theta = \phi\sin\omega t \tag{1.1}$$

where ϕ is the amplitude (in this case, the initial *angle of release* of the pendulum), ω is the angular frequency, and t is a time instant.

Differentiating with respect to time, we obtain:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \vec{\omega} = \phi\omega\cos\omega t$$

and since the tangential velocity $v = r\omega$, in this case,

$$v = l\phi\omega\cos\omega t$$

where l is the length of the pendulum.
Further,

$$\begin{aligned}
v &= l\phi\omega\sqrt{1-\sin^2\omega t} \\
&= l\omega\sqrt{\phi^2(1-\sin^2\omega t)} \\
&= l\omega\sqrt{\phi^2-\phi^2\sin^2\omega t} \\
&= l.\sqrt{\frac{g}{l}}\left(\sqrt{\phi^2-\theta^2}\right) && (\because \omega = \sqrt{\frac{g}{l}} \text{ and } 1.1) \\
\implies & \boxed{v = \sqrt{g(\phi^2-\theta^2)}} && (1.2)
\end{aligned}$$

We can avoid the above approximations, by considering the $\sin\theta$ term and adopting another approach through calculus to find v.

From *fig.1*, the tangential acceleration of the weight is given by:

$$a_t = -g\sin\theta$$

We know that $a_t = \frac{\mathrm{d}v}{\mathrm{d}t}$, thus,

$$\frac{\mathrm{d}v}{\mathrm{d}s}\frac{\mathrm{d}s}{\mathrm{d}t} = -g\sin\theta$$

Furthermore, $\frac{\mathrm{d}s}{\mathrm{d}t} = v$,from which we obtain

$$v\mathrm{d}v = -g\sin\theta\mathrm{d}s$$

Integrating on both sides,

$$\begin{aligned}
\int_0^v v\mathrm{d}v &= -\int_\phi^\theta gl\sin\theta\mathrm{d}\theta && (\because \mathrm{d}s = l\mathrm{d}\theta) \\
\implies \frac{v^2}{2} &= gl[\cos\theta]_\phi^\theta = gl(\cos\theta-\cos\phi) \\
\implies & \boxed{v = \sqrt{2gl(\cos\theta-\cos\phi)}} && (1.3)
\end{aligned}$$

(1.3) is the general expression for the velocity of the weight in a pendulum as a function of θ without any *small angle approximations*.

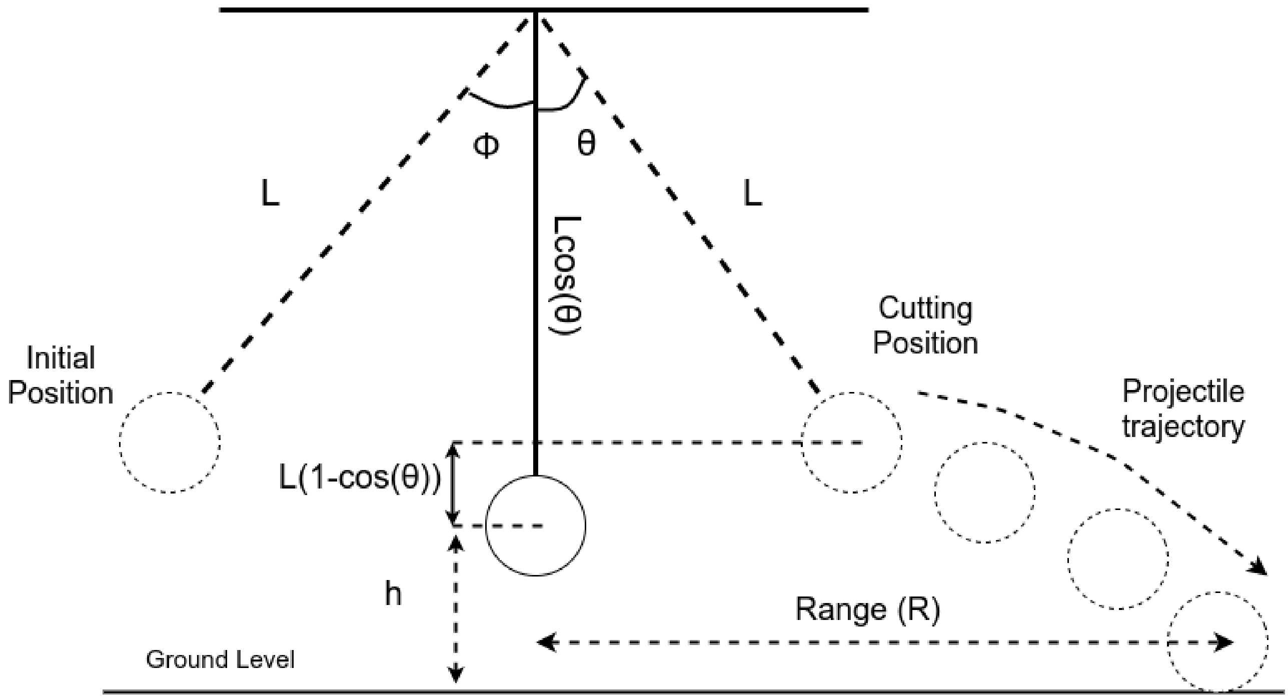


Fig.2 Experiment Diagram

Now, lets assume the string of the pendulum is cut. The velocity of the bob v at the instant the string is cut is given by *(1.2)* and *(1.3)*. From *fig.2* and projectile equations the time taken for the bob to reach max height is given by:

$$t_1 = \frac{v \sin\theta}{g}$$

At the time instant t_1, the bob is at the max height in its parabolic trajectory. Thus, its total height at this moment is given by (*fig.2*):

$$d = \frac{v^2 \sin^2\theta}{2g} + l(1 - \cos\theta) + h$$

We know that an object at height d will fall to the ground in time:

$$t_2 = \sqrt{\frac{2d}{g}}$$

Thus the total time taken for the bob to reach the ground after the string is cut is:

$$\begin{aligned} t &= \text{(Time taken to reach max height)} + \text{(Time taken to fall to ground from max height)} \\ &= t_1 + t_2 \end{aligned}$$

Thus the range r of the projectile (bob) is given by:

$$r = (v \cos\theta)t$$

But, this range r is relative to the position the bob was cut in. This quantity cannot be measured easily in an experimental environment. Instead, we can find the range relative to the equillibrium position of the bob, by adding $l \sin\theta$ to the range:

$$\begin{aligned} R &= l\sin\theta + r \\ &= l\sin\theta + (v\cos\theta)t \end{aligned}$$

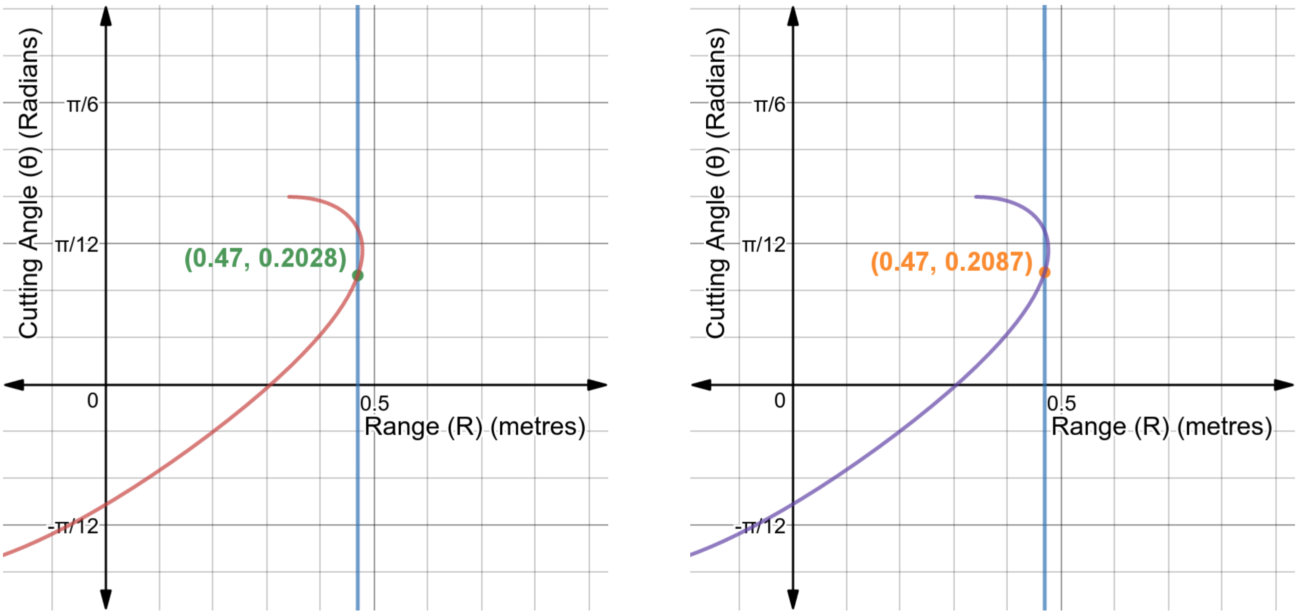
$$\implies \boxed{R = l\sin\theta + v\cos\theta\left(\frac{v\sin\theta}{g} + \sqrt{\frac{2(\frac{v^2\sin^2\theta}{2g} + l(1-\cos\theta) + h)}{g}}\right)} \tag{1.4}$$

The first aim of this experiment is to observe readings of R, and predict the values of θ by looking at the graph of R vs θ for measured values of l, h and ϕ. The second aim is to compare and analyse the graphs of R vs θ for the two expressions for v obtained above (1.2 and 1.3).

Procedure

1. Create a pendulum by tying a cork to one end of a string, and a bob to the other end, and then attaching the cork to an iron stand.

2. Put a metre scale on the ground, aligning its zero with the bob's equillibrium position when seen from above the bob. Ensure the metre scale is parallel to the pendulum motion of the bob.



(a) Approximate (b) Real

Graph 1: Graphs for $\phi = 20°$

3. Ensure l (length of string from the point of suspension to the centre of the bob) is 1m.
4. Measure h and note it down (length from the centre of the bob to the ground, when bob is at equllibrium)
5. Hold the bob away from its equillibrium position, and meausure the angle of release ϕ.
6. Release the bob and cut the string with a pair of scissors at any arbitrary time in the pendulum's motion.
7. Note the range R where the bob falls on the metre scale.
8. Repeat steps 3-6 for 2 more different values of the initial angle, setting up the pendulum again each time.

Observations

Length of pendulum (l) = 1m

Observed range for different initial angles			
No.	**Initial Angle** (ϕ) (Degrees)	**Range** (R) (cm)	**Height** (h) (cm)
1	20°	47	39
2	25°	54	35.5
3	30°	57	38

Graphs

The Graphs 1, 2 and 3 correspond to the 3 observations above, each graphing the range R on the x-axis, and angle of cutting θ on the y-axis.

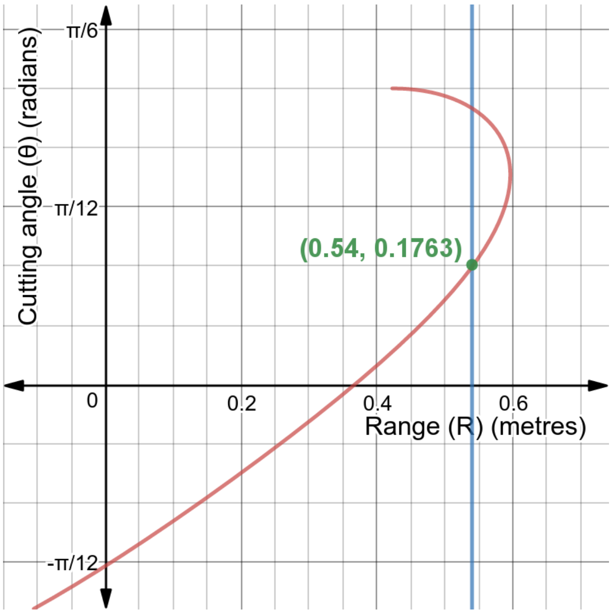
Since in our proof in the theory section, we had assumed that after cutting the thread, the bob reaches maximum height first, and then falls down to the ground, we had limited our domain of observation to positive values of θ, and accordingly, the given graphs should be interpreted only in the first quadrant, as the other quadrants might not relay any useful information with regard to the experiment.

The vertical blue line in each graph corresponds to the range found during the experiment. This line is seen to intersect for two values of θ. Note that for uniformity in readings, we have arbitrarily considered the lower intersection in each case.

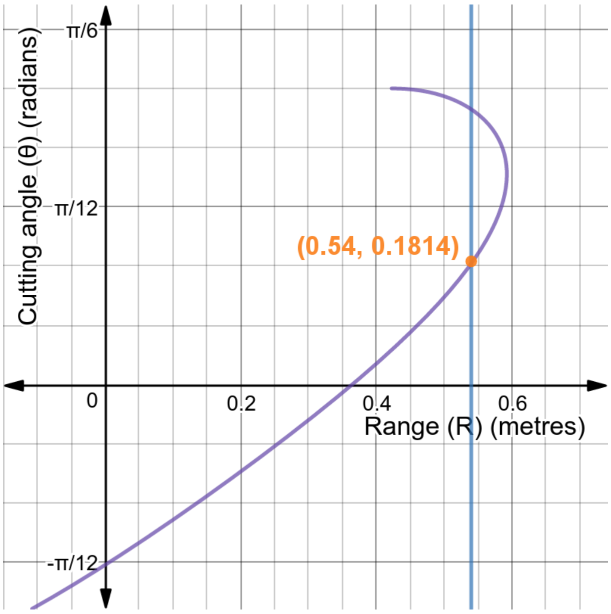
Approximate corresponds to the graph when v is taken as given by (1.2), and Real corresponds to the graph when v is taken as given by (1.3).

Graph 1: For $\phi = 20°, R = 0.47$m, the following values of θ are obtained:

- Approximate: $0.2028^c = 11.6196°$
- Real: $0.2087^c = 11.9575°$

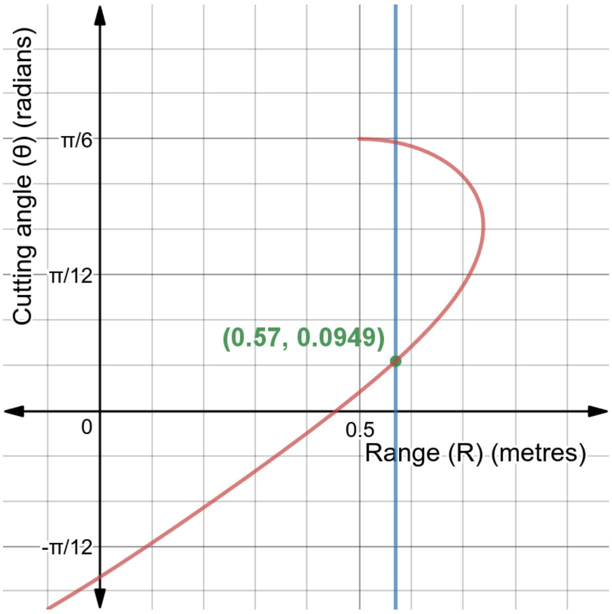


(a) Approximate

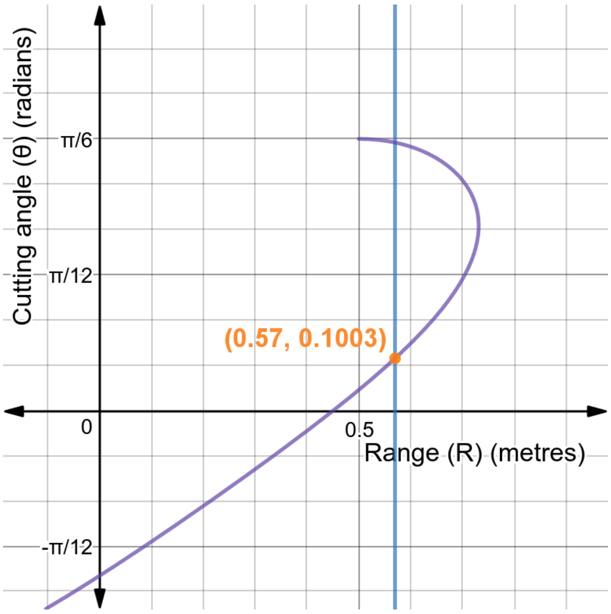


(b) Real

Graph 1: Graphs for $\phi = 25°$



(a) Approximate



(b) Real

Graph 1: Graphs for $\phi = 30°$

- Error: 0.3379
- Percentage Error: 2.826%

Graph 2: For $\phi = 25°, R = 0.54\text{m}$, the following values of θ are obtained:

- Approximate: $0.1763^c = 10.1011°$
- Real: $0.1814^c = 10.3934°$
- Error: 0.2923
- Percentage Error: 2.812%

Graph 3: For $\phi = 30°, R = 0.57\text{m}$, the following values of θ are obtained:

- Approximate: $0.0949^c = 5.4374°$
- Real: $0.1003^c = 5.7468°$
- Error: 0.3094
- Percentage Error: 5.384%

Result

It is clearly seen that there is a variation between the equation of a pendulum using SAA, and the more accurate equation. It is further also seen that the error seems to increase for larger amplitudes of the pendulum.

Precautions

1. Ensure that the string has no knots, and its weight is negligible.
2. Do not cut the pendulum too late, as the amplitude might get damped, due to energy loss from friction in the air.
3. Ensure that string is cut carefully, without disturbing the oscillations in the cutting process.
4. Avoid any parallax while measuring angles/lengths.

4. Conclusion

The results of this experiment clearly show that the as the ampltiude of a pendulum increases, so does the error of approximation of assuming it to follow an SHM motion. We alo conclude that using a more accurate proof through calculus considering the forces in the system, we can produce a more accurate equation for the motion of a pendulum, which satisfies all angles of release in the domain. We also conclude that by measuring the range of a pendulum projectile, we can accurately predict the angle where it had been cut.

Scope for Further Study

A more comprehensive analysis with this experimental procedure can probably be used to determine the limit at which the errors os Small Angle Approximation in a pendulum become too high too accurately model a system. Using a mounted camera in front of the system and motion capturing techniques, we can actually observe the angle of release when the pendulum was cut and verify the same with our calculated readings in this experiment.

Bibliography

Sources of Information:

- https://en.wikipedia.org/wiki/Simple_harmonic_motion
- Physics Part-2 Textbook Class XII

We would also like to thank our physics teachers, Radhika Ma'am and Vinni Ma'am for helping us with all the doubts we had during the course of this experiment.