

PHYSICS EXPERIMENT PROJECT

CLASS XII A

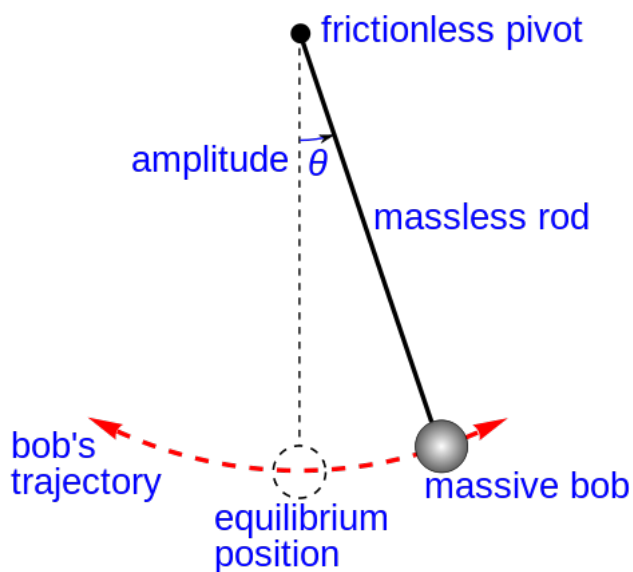
2017-18

Simple Harmonic Motion

Conducted by:

Rishit BANSAL 34

Apurva KULKARNI 5



Index

1	Introduction	2
2	Experiments	3
1	Error in Small Angle Approximation of a Pendulum	3
1.1	Aim	3
1.2	Materials Required	3
1.3	Theory	3
1.4	Procedure	6
1.5	Observations	6
1.6	Graphs	6
1.7	Result	7
1.8	Precautions	7
3	Conclusion	8
4	Acknowledgements	9
5	Bibliography	10

1. Introduction

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Vestibulum a ex dui. Duis at bibendum arcu. Curabitur gravida id ante ac pharetra. Pellentesque vel arcu rutrum, pulvinar ante ac, blandit urna. Etiam efficitur tincidunt lobortis. Donec facilisis lacus arcu. Praesent scelerisque, odio tristique tristique molestie, arcu lorem tempor dolor, a pretium nisl tortor id nisl. In ut consectetur nisi. Pellentesque consequat eros et quam venenatis ullamcorper. Donec non tristique ex. Etiam et tincidunt lorem. Integer lacinia eros eu placerat posuere. Aliquam ullamcorper quam ut sem lacinia, non tincidunt tortor interdum. Nunc porta massa sed nibh ultricies venenatis. Interdum et malesuada fames ac ante ipsum primis in faucibus. Quisque nec odio ac odio egestas eleifend ut sit amet odio.

2. Experiments

Exp. 1

Error in Small Angle Approximation of a Pendulum

Aim

- (a) To predict the angle of release of pendulum projectile by measuring range of the projectile.
- (b) To use the results of (a) to compare the predictions of SAA with a more accurate analysis considering the exact forces of the system.

Materials Required

Metallic bob, Thread, Iron Stand, Protractor, Metre Scale, Scissors

Theory

Simple harmonic motion is a type of periodic motion where the restoring force is directly proportional to the displacement and acts in the direction opposite to that of displacement. The restoring force is given by:

$$\mathbf{F} = -kx$$

where \mathbf{F} is the restoring force, k is a constant, and x is the displacement from the equilibrium position.

A pendulum is a weight suspended from a frictionless pivot so that it can swing freely. For a pendulum to act as an SHM oscillator, the force on the weight must be proportional to the displacement of the weight from the equilibrium position. But from *fig.1* it can be clearly seen that the magnitude of the restoring torque of mg is given by:

$$|\mathbf{\Gamma}| = -mgl \sin \theta$$

It is clearly seen that the restoring torque is not directly proportional to the angular displacement, which means that the pendulum is not an SHM oscillator. But for small values of θ , $\sin \theta$ is approximately equal to θ , and thus for this case alone does the

pendulum act as an SHM. The displacement of the weight for small angles can thus be given by:

$$\theta = \phi \sin \omega t \quad (1.1)$$

where ϕ is the amplitude (in this case, the initial *angle of release* of the pendulum), ω is the angular frequency, and t is a time instant.

Differentiating with respect to time, we obtain:

$$\frac{d\theta}{dt} = \vec{\omega} = \phi\omega \cos \omega t$$

and since the tangential velocity $v = r\omega$, in this case,

$$v = l\phi\omega \cos \omega t$$

where l is the length of the pendulum.

Further,

$$\begin{aligned} v &= l\phi\omega \sqrt{1 - \sin^2 \omega t} \\ &= l\omega \sqrt{\phi^2(1 - \sin^2 \omega t)} \\ &= l\omega \sqrt{\phi^2 - \phi^2 \sin^2 \omega t} \\ &= l \cdot \sqrt{\frac{g}{l}} \left(\sqrt{\phi^2 - \theta^2} \right) \quad (\because \omega = \sqrt{\frac{g}{l}} \text{ and 1.1}) \\ \implies \boxed{v = \sqrt{g(\phi^2 - \theta^2)}} \end{aligned} \quad (1.2)$$

We can avoid the above approximations, by considering the $\sin \theta$ term and adopting another approach through calculus to find v .

From *fig.1*, the tangential acceleration of the weight is given by:

$$a_t = -g \sin \theta$$

We know that $a_t = \frac{dv}{dt}$, thus,

$$\frac{dv}{ds} \frac{ds}{dt} = -g \sin \theta$$

Furthermore, $\frac{ds}{dt} = v$, from which we obtain

$$v dv = -g \sin \theta ds$$

Integrating on both sides,

$$\begin{aligned} \int_0^v v dv &= - \int_\phi^\theta gl \sin \theta d\theta \quad (\because ds = l d\theta) \\ \implies \frac{v^2}{2} &= gl [\cos \theta]_\phi^\theta = gl (\cos \theta - \cos \phi) \\ \implies \boxed{v = \sqrt{2gl(\cos \theta - \cos \phi)}} \end{aligned} \quad (1.3)$$

(1.3) is the general expression for the velocity of the weight in a pendulum as a function of θ without any *small angle approximations*.

Now, let's assume the string of the pendulum is cut. The velocity of the bob v at the instant the string is cut is given by (1.2) and (1.3). From fig.2 and projectile equations the time taken for the bob to reach max height is given by:

$$t_1 = \frac{v \sin \theta}{g}$$

At the time instant t_1 , the bob is at the max height in its parabolic trajectory. Thus, its total height at this moment is given by (fig.2):

$$d = \frac{v^2 \sin^2 \theta}{2g} + l(1 - \cos \theta) + h$$

We know that an object at height d will fall to the ground in time:

$$t_2 = \sqrt{\frac{2d}{g}}$$

Thus the total time taken for the bob to reach the ground after the string is cut is:

$$\begin{aligned} t &= (\text{Time taken to reach max height}) + (\text{Time taken to fall to ground from max height}) \\ &= t_1 + t_2 \end{aligned}$$

Thus the range r of the projectile (bob) is given by:

$$r = (v \cos \theta)t$$

But, this range r is relative to the position the bob was cut in. This quantity cannot be measured easily in an experimental environment. Instead, we can find the range relative to the equilibrium position of the bob, by adding $l \sin \theta$ to the range:

$$\begin{aligned} R &= l \sin \theta + r \\ &= l \sin \theta + (v \cos \theta)t \\ \Rightarrow R &= l \sin \theta + v \cos \theta \left(\frac{v \sin \theta}{g} + \sqrt{\frac{2\left(\frac{v^2 \sin^2 \theta}{2g} + l(1 - \cos \theta) + h\right)}{g}} \right) \end{aligned} \quad (1.4)$$

The first aim of this experiment is to observe readings of R , and predict the values of θ by looking at the graph of R vs θ for measured values of l , h and ϕ . The second aim is to compare and analyse the graphs of R vs θ for the two expressions for v obtained above (1.2 and 1.3).

Procedure

1. Create a pendulum by tying a cork to one end of a string, and a bob to the other end, and then attaching the cork to an iron stand.
2. Put a metre scale on the ground, aligning its zero with the bob's equilibrium position when seen from above the bob. Ensure the metre scale is parallel to the pendulum motion of the bob.
3. Ensure l (length of string from the point of suspension to the centre of the bob) is 1m.
4. Measure h and note it down (length from the centre of the bob to the ground, when bob is at equilibrium)
5. Hold the bob away from its equilibrium position, and measure the angle of release ϕ as shown in *fig.3*.
6. Release the bob and cut the string with a pair of scissors at any arbitrary time in the pendulum's motion.
7. Note the range R where the bob falls on the metre scale.
8. Repeat steps 3-6 for 2 more different values of the initial angle, setting up the pendulum again each time.

Observations

Length of pendulum (l) = 1m

Observed range for different initial angles			
No.	Initial Angle (ϕ) (Degrees)	Range (R) (cm)	Height (h) (cm)
1	20°	47	39
2	25°	54	35.5
3	30°	57	38

Graphs

The Graphs 1, 2 and 3 correspond to the 3 observations above, each graphing the range R on the x-axis, and angle of cutting θ on the y-axis.

Since in our proof in the theory section, we had assumed that after cutting the thread, the bob reaches maximum height first, and then falls down to the ground, we had limited our domain of observation to positive values of θ , and accordingly, the given graphs should be interpreted only in the first quadrant, as the other quadrants might not relay any useful information with regard to the experiment.

The vertical blue line in each graph corresponds to the range found during the experiment. This line is seen to intersect for two values of θ . Note that for uniformity in readings, we have arbitrarily considered the lower intersection in each case.

Approximate corresponds to the graph when v is taken as given by (1.2), and Real corresponds to the graph when v is taken as given by (1.3).

Graph 1: For $\phi = 20^\circ$, $R = 0.47\text{m}$, the following values of θ are obtained:

- Approximate: $0.2028^c = 11.6196^\circ$
- Real: $0.2087^c = 11.9575^\circ$

Graph 2: For $\phi = 25^\circ$, $R = 0.54\text{m}$, the following values of θ are obtained:

- Approximate: $0.1763^c = 10.1011^\circ$
- Real: $0.1814^c = 10.3934^\circ$

Graph 3: For $\phi = 30^\circ$, $R = 0.57\text{m}$, the following values of θ are obtained:

- Approximate: $0.0949^c = 5.4374^\circ$
- Real: $0.1003^c = 5.7468^\circ$

Result

[To be planned out]

Precautions

1. Ensure that the string has no knots, and its weight is negligible.
2. Do not cut the pendulum too late, as the amplitude might get damped, due to energy loss from friction in the air.
3. Ensure that string is cut carefully, without disturbing the oscillations in the cutting process.
4. Avoid any parallax while measuring angles/lengths.

3. Conclusion

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Vestibulum a ex dui. Duis at bibendum arcu. Curabitur gravida id ante ac pharetra. Pellentesque vel arcu rutrum, pulvinar ante ac, blandit urna. Etiam efficitur tincidunt lobortis. Donec facilisis lacus arcu. Praesent scelerisque, odio tristique tristique molestie, arcu lorem tempor dolor, a pretium nisl tortor id nisl. In ut consectetur nisi. Pellentesque consequat eros et quam venenatis ullamcorper. Donec non tristique ex. Etiam et tincidunt lorem. Integer lacinia eros eu placerat posuere. Aliquam ullamcorper quam ut sem lacinia, non tincidunt tortor interdum. Nunc porta massa sed nibh ultricies venenatis. Interdum et malesuada fames ac ante ipsum primis in faucibus. Quisque nec odio ac odio egestas eleifend ut sit amet odio.

4. Acknowledgements

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Vestibulum a ex dui. Duis at bibendum arcu. Curabitur gravida id ante ac pharetra. Pellentesque vel arcu rutrum, pulvinar ante ac, blandit urna. Etiam efficitur tincidunt lobortis. Donec facilisis lacus arcu. Praesent scelerisque, odio tristique tristique molestie, arcu lorem tempor dolor, a pretium nisl tortor id nisl. In ut consectetur nisi. Pellentesque consequat eros et quam venenatis ullamcorper. Donec non tristique ex. Etiam et tincidunt lorem. Integer lacinia eros eu placerat posuere. Aliquam ullamcorper quam ut sem lacinia, non tincidunt tortor interdum. Nunc porta massa sed nibh ultricies venenatis. Interdum et malesuada fames ac ante ipsum primis in faucibus. Quisque nec odio ac odio egestas eleifend ut sit amet odio.

5. Bibliography

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Vestibulum a ex dui. Duis at bibendum arcu. Curabitur gravida id ante ac pharetra. Pellentesque vel arcu rutrum, pulvinar ante ac, blandit urna. Etiam efficitur tincidunt lobortis. Donec facilisis lacus arcu. Praesent scelerisque, odio tristique tristique molestie, arcu lorem tempor dolor, a pretium nisl tortor id nisl. In ut consectetur nisi. Pellentesque consequat eros et quam venenatis ullamcorper. Donec non tristique ex. Etiam et tincidunt lorem. Integer lacinia eros eu placerat posuere. Aliquam ullamcorper quam ut sem lacinia, non tincidunt tortor interdum. Nunc porta massa sed nibh ultricies venenatis. Interdum et malesuada fames ac ante ipsum primis in faucibus. Quisque nec odio ac odio egestas eleifend ut sit amet odio.