2.3: Rate of Change and the Slope of a Line

Graphs, Equations of Lines, and Functions

MATH 30 - Spring 2017

California State University, San Marcos

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Success Tip #3

How to read a mathematics book:

- Spend time reading the book before trying the assigned problems
- Read the course text actively, not passively
  - As you read the examples, have paper and pencil at hand, and write out the steps one-by-one, making certain that you understand how to get from each step to the next
  - As we get further into the book, more and more of the details get omitted from the examples, and it is only by working through these methodically that you will realize where there are extra steps that are critical for the solution

Objectives

1. Calculate an average rate of change
2. Find the slope of a line
   - From its graph
   - Given two points
   - For horizontal and vertical lines
3. Solve applications of slope
4. Determine parallel or perpendicular lines using slope

Average Rate of Change

- Describes the change in one quantity with respect to the change in another

Ratios and Rates

A ratio is a comparison of two numbers using a quotient. In symbols, if a and b are two numbers, the ratio of a to b is $\frac{a}{b}$. Ratios that are used to compare quantities with different units are called rates.

- Units are included
- Can be negative or positive

Average Rate of Change
Example

- Find the ratio of the change in the number of newspapers to the length of time in which that change took place and attach the appropriate units

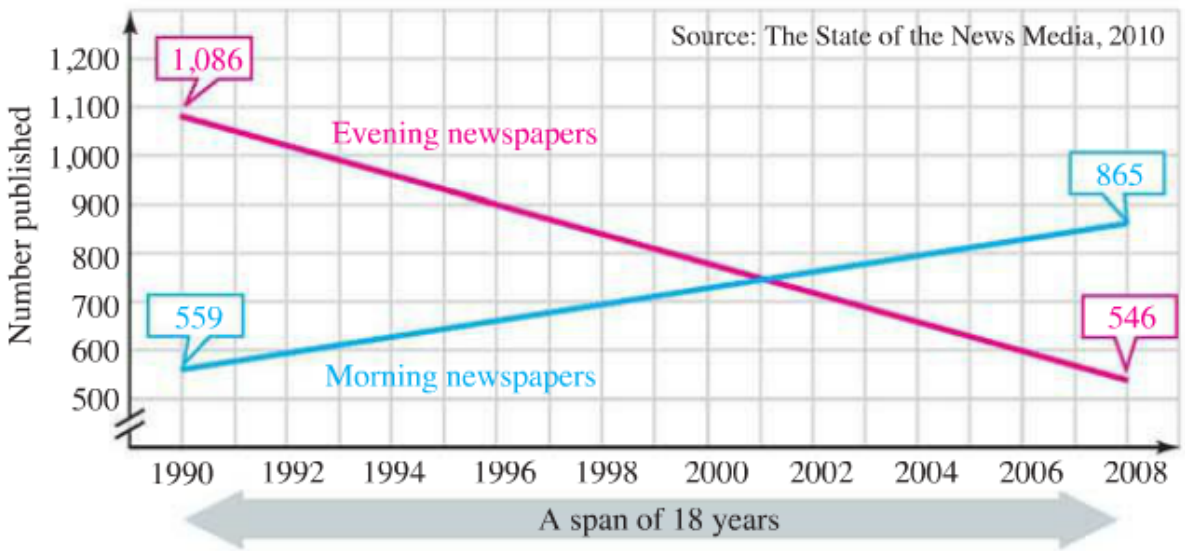

Figure: USA Newspapers Example

$$\text{Average Rate of Change} = \frac{\text{Change in number of newspapers}}{\text{Change in time}}.$$

Slope of a Line

- We measured the steepness of the lines in the graph to determine the average rates of change
- In doing so, we found the slope of each line

Slope of a Line

The slope of a line is a ratio that compares the vertical change to the corresponding horizontal change as we move along the line from one point to another.

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Slope of a Line
From its Graph

- To determine the slope of a line from its graph, we first pick two points on the line
- Then we write the ratio of the vertical change, called the rise, to the corresponding horizontal change, called the run

$$m = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{rise}}{\text{run}}$$

- Draw a slope triangle!

Slope of a Line
Example

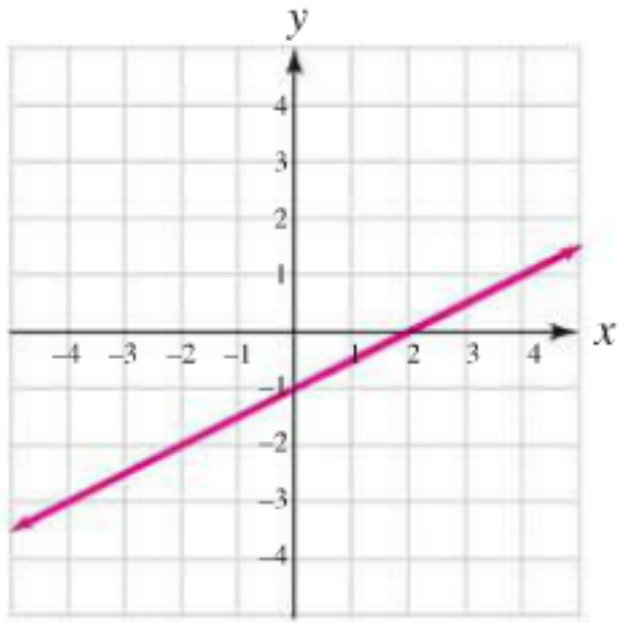

Figure: Find the Slope Using (4,1) and (-4,-3)

- The same value will be obtained no matter which two points on a line that are used to find its slope

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Slope of a Line
Given two Points

Slope of a Line

The slope m of a line passing through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

if $x_2 \neq x_1$.

Slope of a Line
Practice

Find the slope for the following pairs of coordinates:

1. (4,1) and (-4,-3)
2. (-2,4) and (3,-4)
3. (-3,6) and (4,-8)

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Slope of a Line
Horizontal Lines

If (x_1, y_2) and (x_2, y_2) are distinct points on a horizontal line, then $y_1 = y_2$, and so the numerator is 0. Therefore, the slope of a horizontal line is 0.

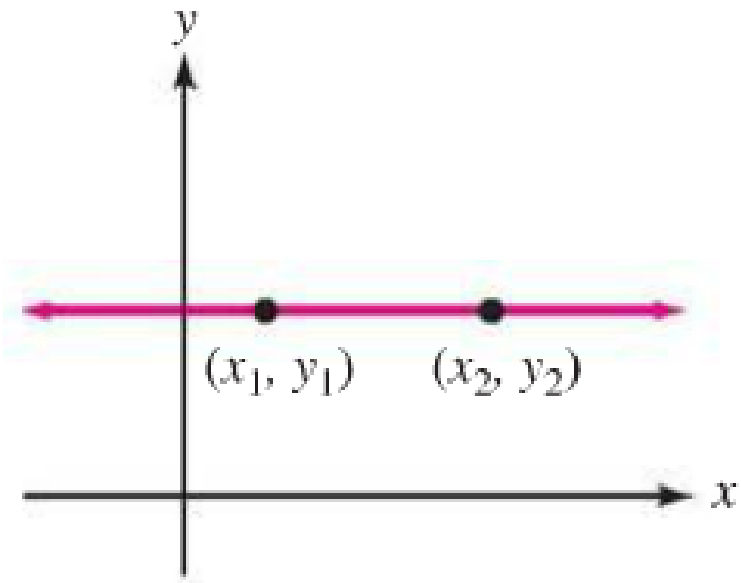


Figure: Horizontal Line

Slope of a Line
Vertical Lines

If (x_1, y_1) and (x_2, y_2) are distinct points on a vertical line then $x_1 = x_2$, and so the denominator is 0. Since a denominator of a fraction cannot be 0, then a vertical line has no defined slope.

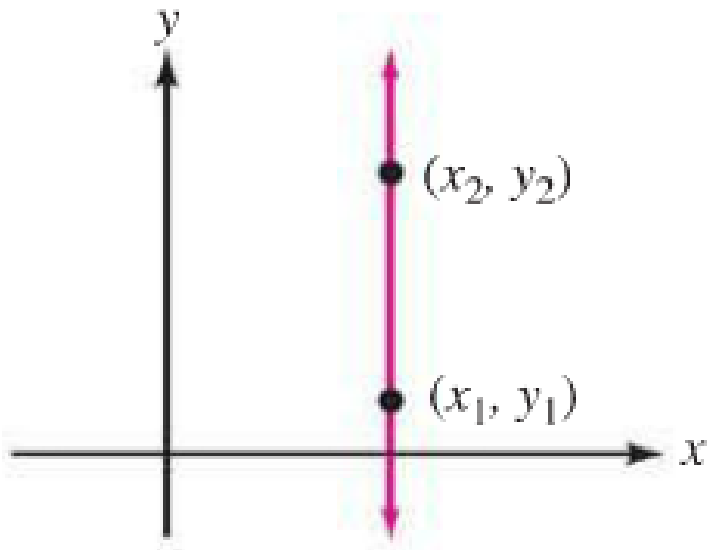
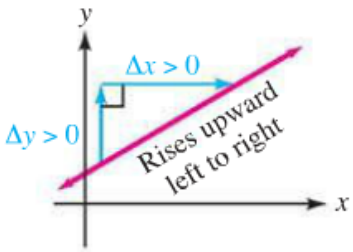


Figure: Vertical Line

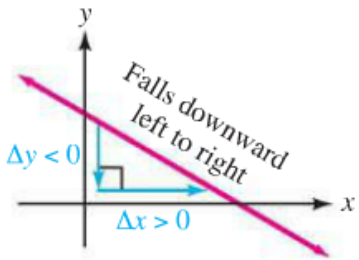
Slope of a Line
Summary

Slopes of Horizontal and Vertical Lines

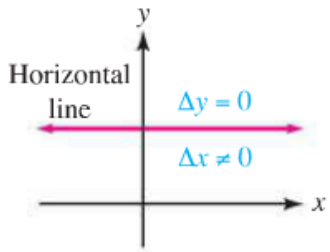
- Horizontal lines of the form $y = b$ have a slope of 0.
- Vertical lines of the form $x = a$ have no defined slope.



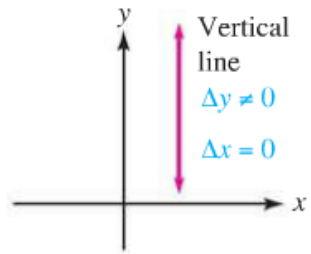
Positive slope: $m > 0$



Negative slope: $m < 0$



Zero slope: $m = 0$



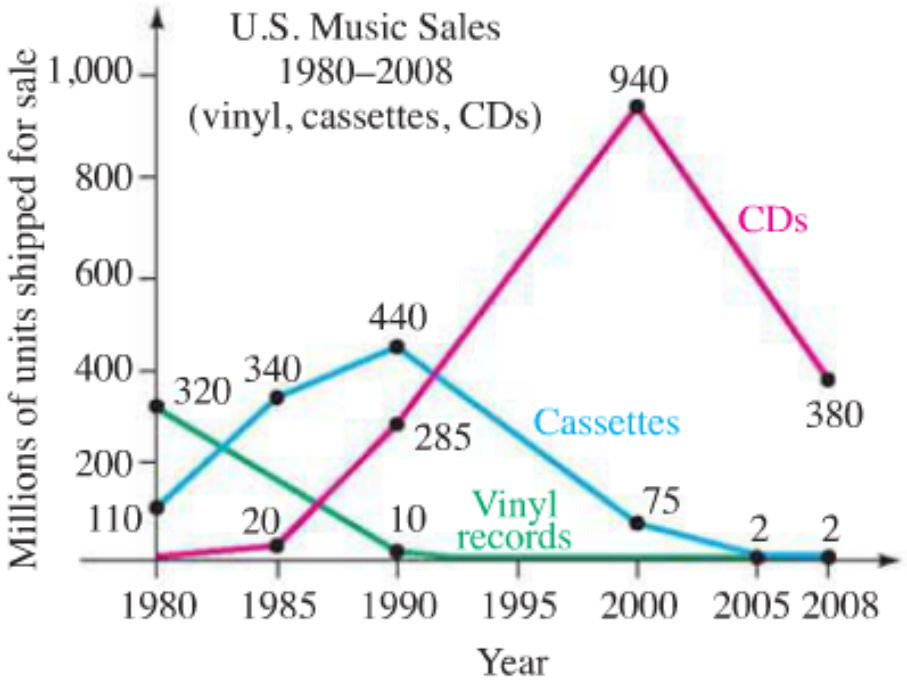
Undefined slope

Applications of Slope

53. U.S. Music Sales. The following line graph models the approximate number of CDs that were shipped for sale in the United States from 1980 through 2008.

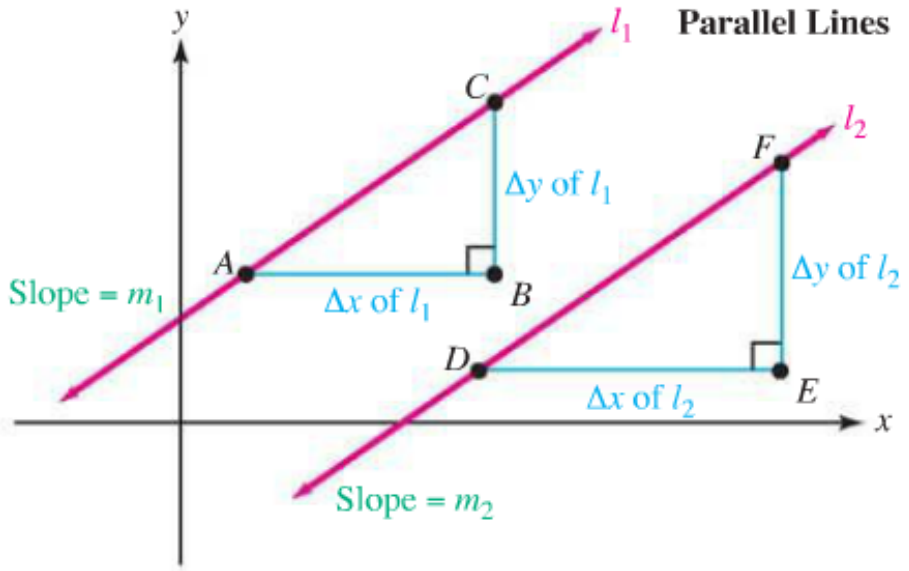
a. Find the rate of increase in the number of CDs shipped from 1990 to 2000.

b. Find the rate of decrease in the number of CDs shipped from 2000 to 2008.



Source: World Almanac 2010

Parallel Lines



Slopes of Parallel Lines

Nonvertical parallel lines have the same slope, and different lines having the same slope are parallel.

Parallel Lines
Example

Determine whether the line that passes through the points $(-6, 2)$ and $(3, -1)$ is parallel to a line with a slope of $-\frac{1}{3}$.

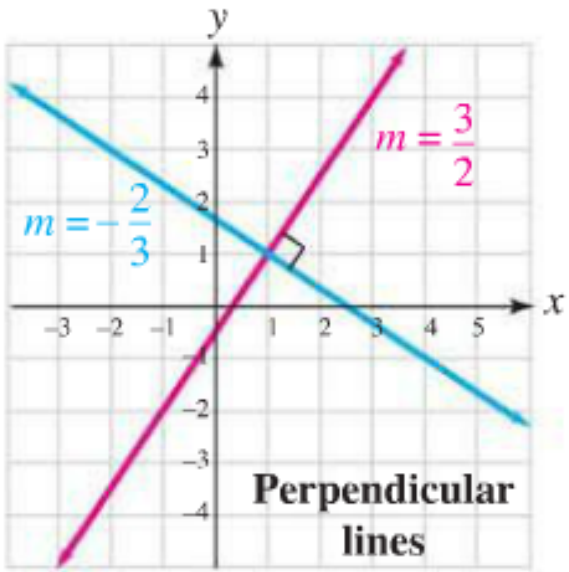
- Compare the slopes of the lines
- If the slopes are equal, the lines are parallel
- If the slopes are not equal, the lines are not parallel

Parallel Lines
Practice

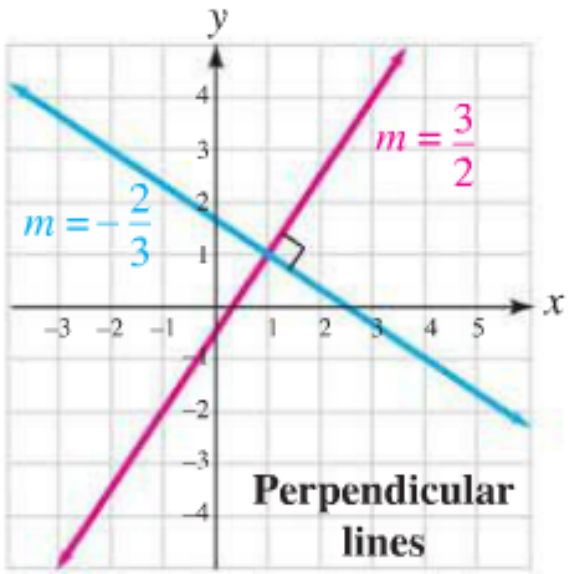
Determine whether the line that passes through the points $(4, -8)$ and $(1, -2)$ is parallel to a line with slope 2.

Perpendicular Lines

- Two lines that meet at right angles are called perpendicular lines
- Each of the four angles formed have a measure of 90°.



Perpendicular Lines

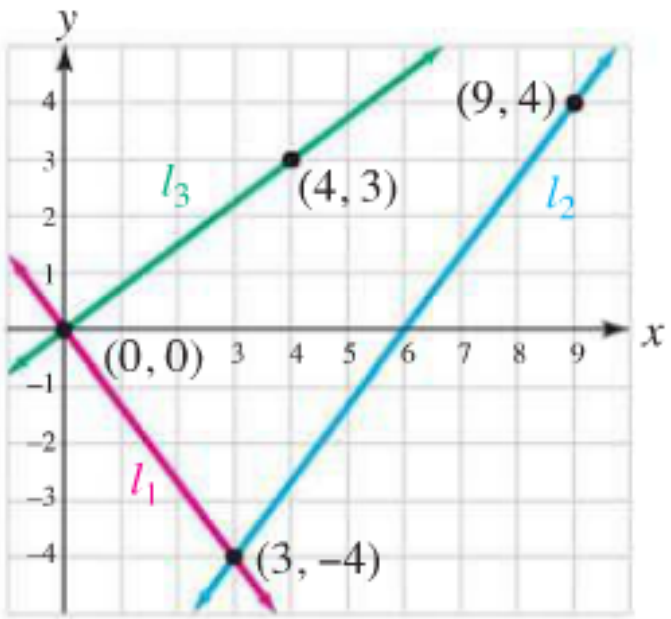


Slopes of Perpendicular Lines

- If two nonvertical lines are perpendicular, their slopes are negative reciprocals.
- If the slopes of two lines are negative reciprocals, the lines are perpendicular.

Perpendicular Lines
Practice

Are the lines ℓ_1 and ℓ_2 perpendicular?



Are the lines ℓ_1 and ℓ_3 perpendicular?