

# Daily Class Slides

Geometry Spring 2022  
Chandru Narayan

# Introductions!

Chandru  
Narayan



Role at Bush: CS and Math teacher

What you were like in High School: Outgoing

Your first day of school tradition/superstition: Bowtie!

Who inspires you: Friendly People

Your interests outside of Bush: Bicycling, Astronomy

Something you are doing: Bicycling 110 miles to raise money for the Child Abuse Prevention dept at Mary Bridge Children's Hospital - My 15th year

A song you know all the words to: Katrinile Varum Geetham - A Tamil song about music in a light breeze

A talent I cherish: South Indian Cooking

Thursday, Jan 6th

# What's happening today?

## Check-in

Welcome new Students!

Reflections upon Fall Term

## Class Logistics

Ready to have fun! Be courteous, Participate, do lots of problems in class!

All Assignments in Portal and linked to Google Classroom. Do not be late in submitting them!

Bring fully charged laptop, geo instruments, notebook, toolbox & calculator

Dress Warmly Windows to be Open , Masks ON, No eating or drinks inside

Can you Access the textbook online?

## Today

Review Perimeter

Introduce Area & Volume - New Chapter 8 (Page 422 in book)!

5-minute Break

Area & Volume Investigation - Area of Rect, Parallelogram and any Triangle

## Reminder

Complete Investigation - Due today

Complete Homework - Due Jan 10th

# Introduce new Students!

Welcome **Luc, Charlotte, Cophine!**

State you name clearly pronouncing first and last names

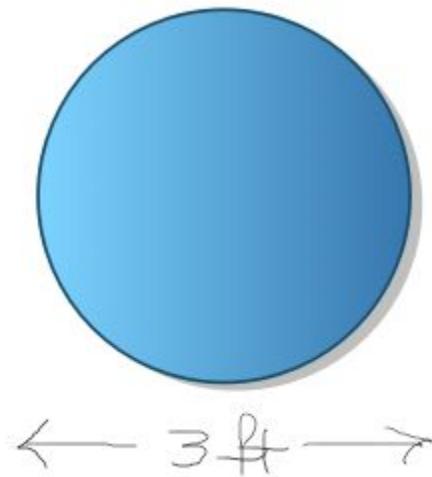
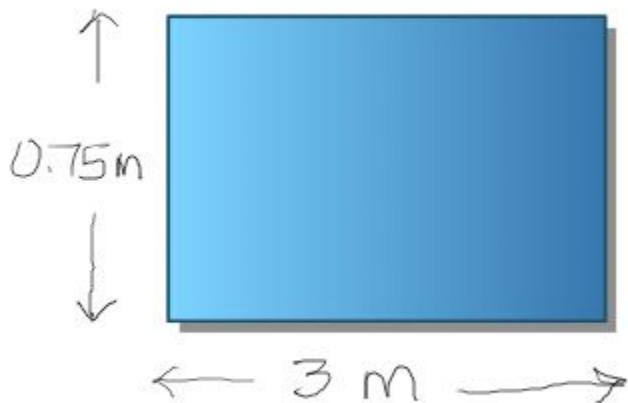
How would you like to be addressed?

Your personal pronouns

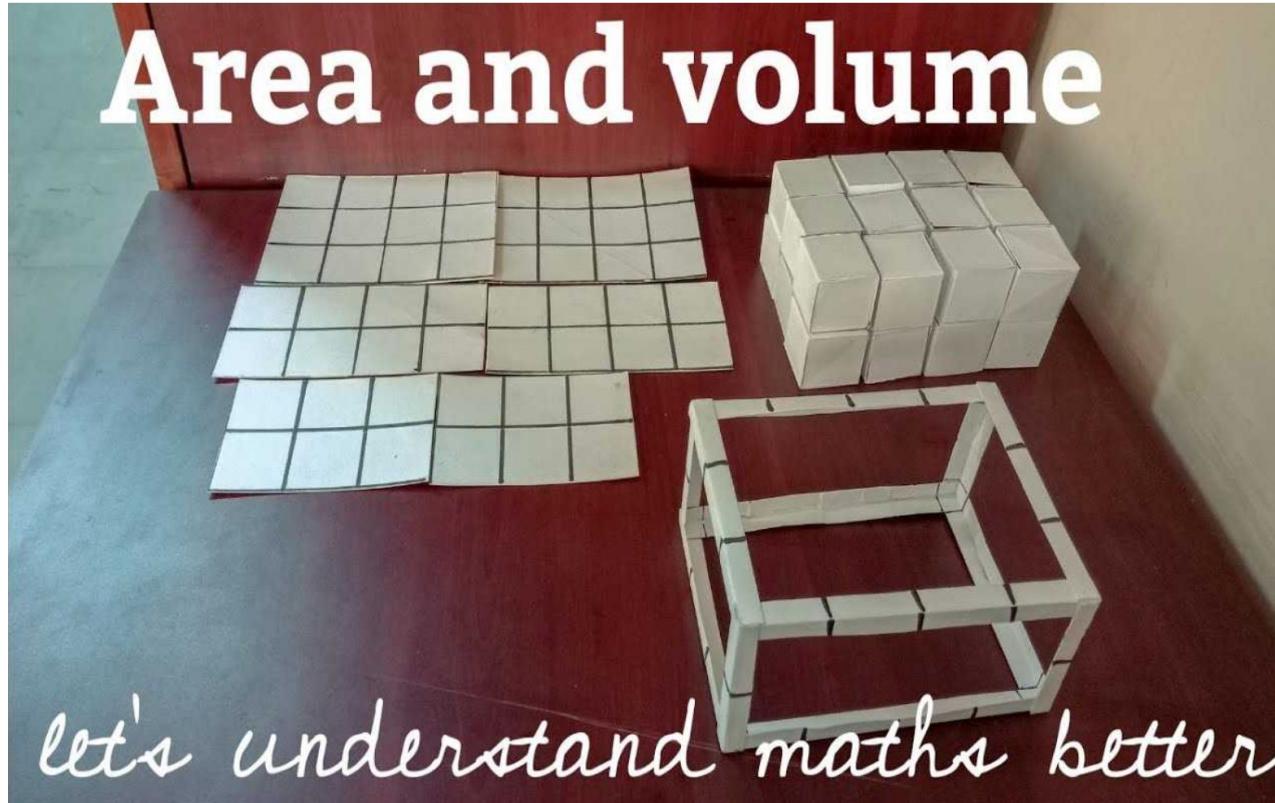
Something interesting or special/peculiar about you?

What are your expectations from this class?

# What is the Perimeter of these Shapes ? Units ?



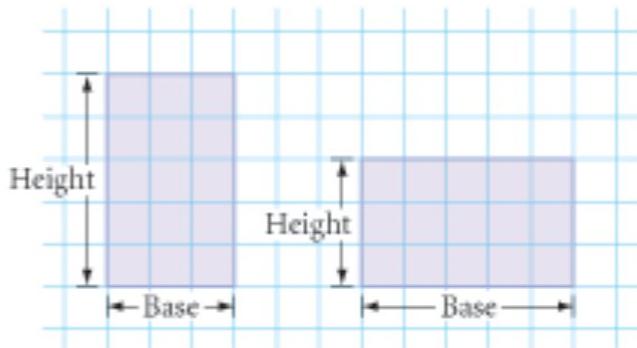
# Area & Volume - What are these? Units?



# How many tiles Investigation

- Get handout from Chandru or [print from GC](#)
- Hint for #5:
  - poster: 2x3ft, postcard: 4x6", queen bedsheet: 60x80", stamp: 1x1.5"
- Hint for #8:
  - Think of cutting out 1 triangle from one side of parallelogram and rearranging

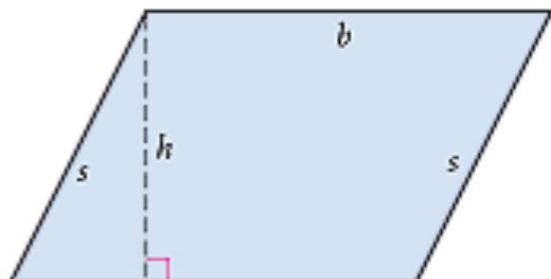
# Area of Rectangle & Parallelogram Conjectures



C-74

## Rectangle Area Conjecture

The area of a rectangle is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the rectangle.

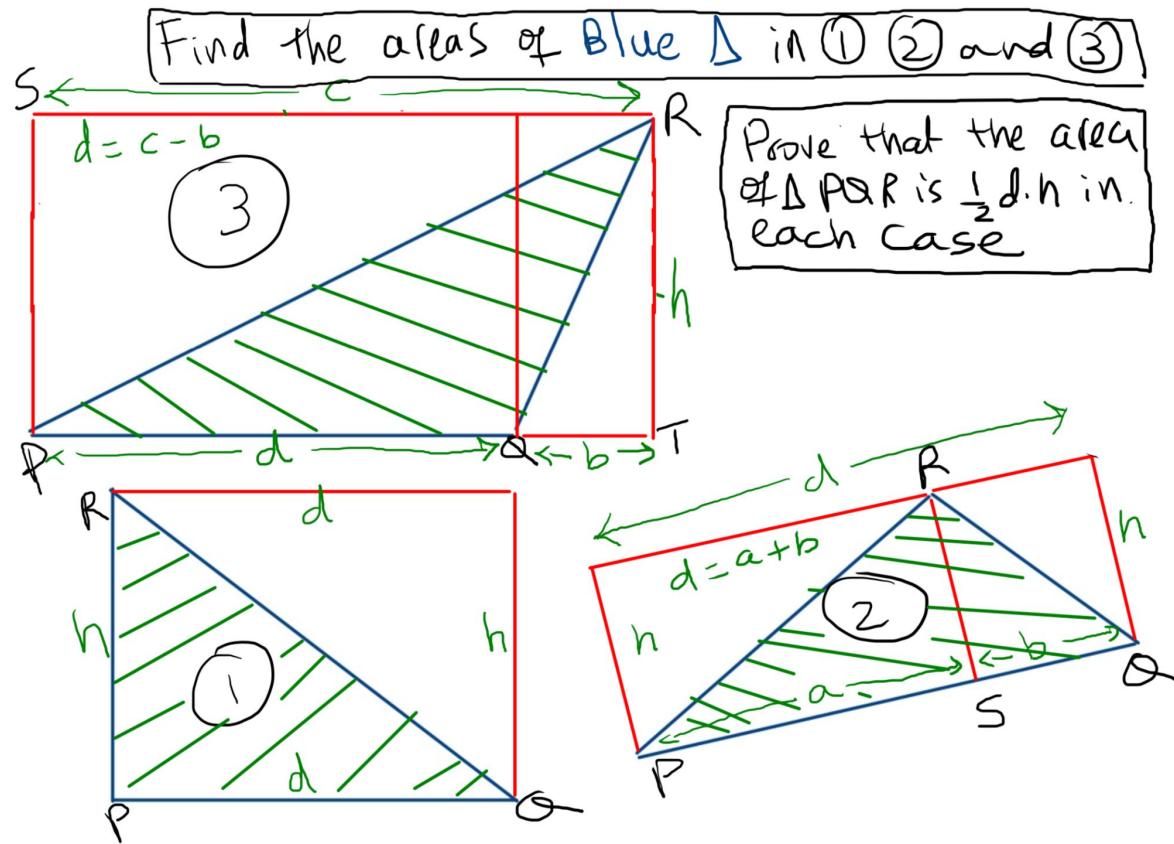


C-75

## Parallelogram Area Conjecture

The area of a parallelogram is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the parallelogram.

# Derive Area of any Triangle based on Rect area Conjectures

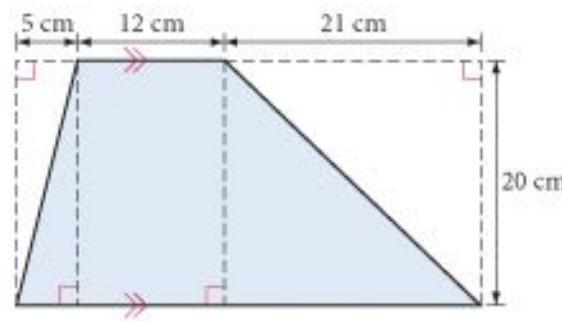


# Let's do a few problems

5.  $P = 40 \text{ ft}$   
 $A = ?$



23. Find the area of the trapezoid at right.



# Reminders!

## Reminders

[Complete How many Tiles Investigation](#) - Due today

[Complete Rect & Parallelogram Areas Homework](#) - Due Jan 10th

Monday, Jan 10th

# What's happening today?

## Check-in

How was your 1st week?

Did you complete the Fall Course Review. Please click here to complete

[Form Random Teams!](#)

Review Syllabus

## Today

Review Rectangle & Parallelogram Area Conjectures

Review formula for Area of any Triangle

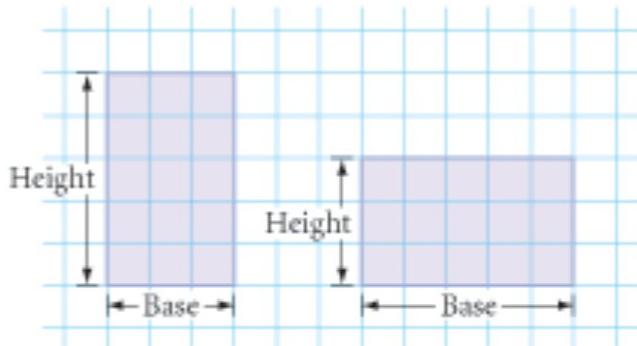
Review some Rect, Parallelogram Area Problems

A Plethora of Area Formulas to be Discovered!

## Reminder

Investigation & 8.1 Homework Due today

# Area of Rectangle & Parallelogram Conjectures

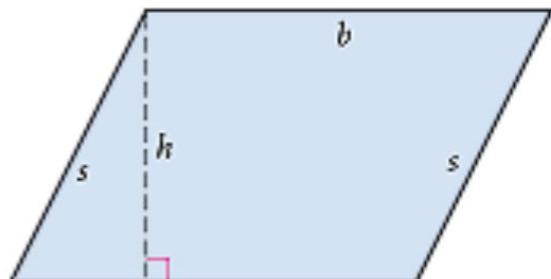


## Rectangle Area Conjecture

C-74

The area of a rectangle is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the rectangle.

$$A = \text{BASE} * \text{HEIGHT}$$



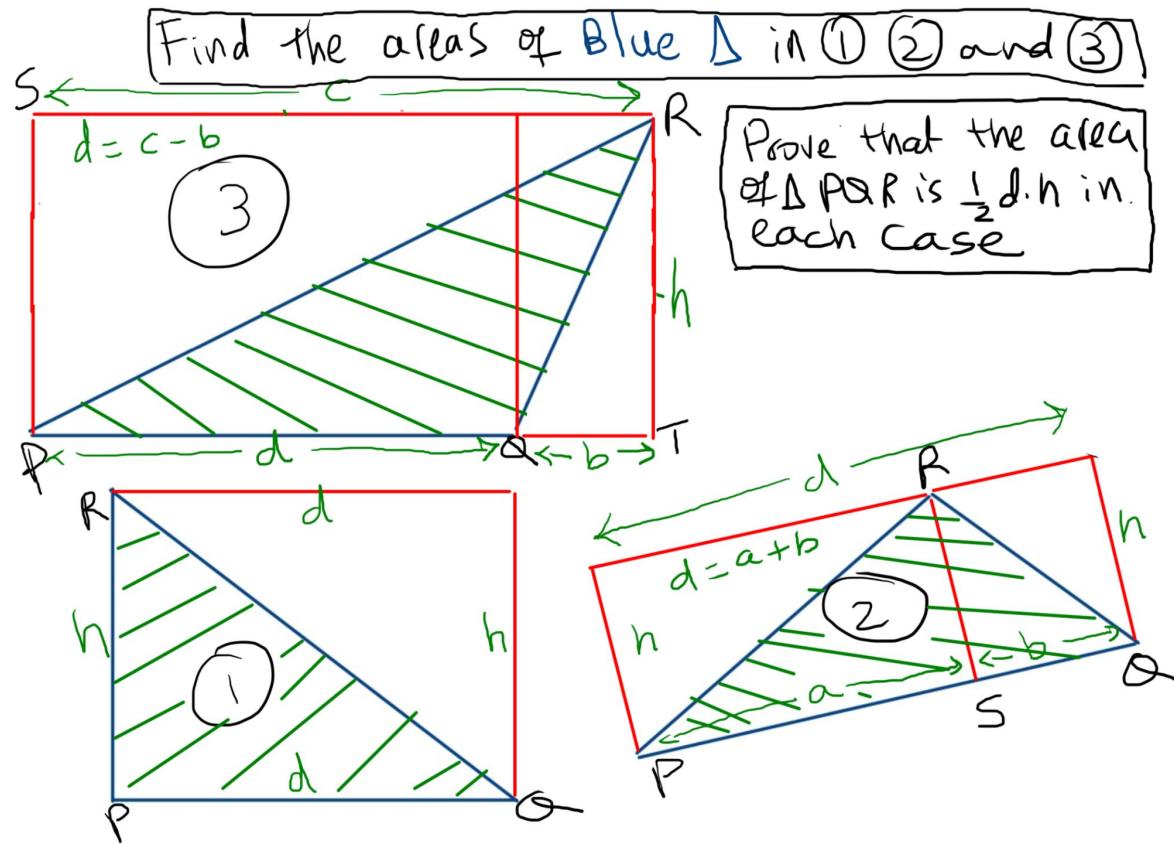
## Parallelogram Area Conjecture

C-75

The area of a parallelogram is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the parallelogram.

$$A = bh$$

# Derive Area of any Triangle based on Rect area Conjectures



# Area of Triangles Conjecture

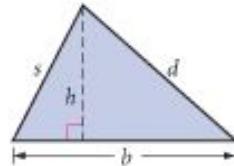


## Investigation 1

### Area Formula for Triangles

#### You will need

- heavy paper or cardboard



- Step 1 Cut out a triangle and label its parts as shown. Make and label a copy.
- Step 2 Arrange the triangles to form a figure for which you already have an area formula. Calculate the area of the figure.
- Step 3 What is the area of one of the triangles? Make a conjecture. Write a brief description in your notebook of how you arrived at the formula. Include an illustration.

#### Triangle Area Conjecture

C-76

The area of a triangle is given by the formula  $\frac{1}{2} b h$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the triangle.

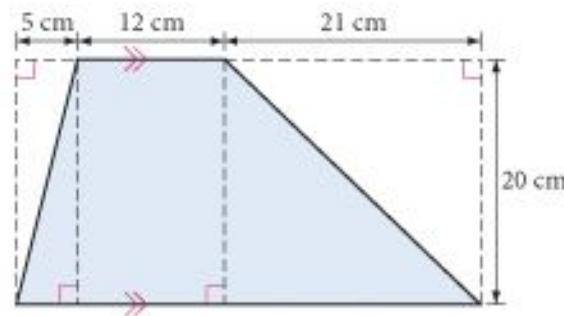
$$\frac{1}{2} b h$$

# Review Problems problems

5.  $P = 40 \text{ ft}$   
 $A = ?$



23. Find the area of the trapezoid at right.



Let's derive the Area Formula for a Trapezoid

# Area of Trapezoids Conjecture

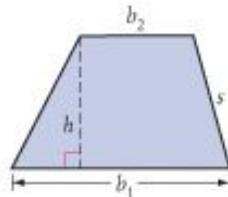


## Investigation 2

### Area Formula for Trapezoids

#### You will need

- heavy paper or cardboard



Step 1

Construct any trapezoid as shown. Label the trapezoid as shown. Label the height  $h$  as shown. Label the bases  $b_1$  and  $b_2$  as shown. Label the slanted sides  $s$  as shown. Label the height  $h$  as shown.

Step 2

Cut out the trapezoid and label a copy.

Step 3

Arrange the two trapezoids to form a figure for which you already have an area formula. What polygon is this? What is its area? What is the area of one trapezoid? Make a conjecture.

#### Trapezoid Area Conjecture

C-77

The area of a trapezoid is given by the formula  $\frac{1}{2} (b_1 + b_2) h$ , where  $A$  is the area,  $b_1$  and  $b_2$  are the lengths of the two bases, and  $h$  is the height of the trapezoid.

$$\frac{1}{2} (b_1 + b_2) h$$

# Area of Kites Conjecture



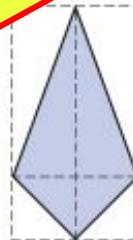
## Investigation 3

### Area Formula for Kites

Can you rearrange a kite into shapes that already have the area formula? Use the properties of a kite?

Create and carry out a plan of action to discover a formula for the area of a kite. Discuss your results with your group. State a conjecture.

Instead of following these steps, we are going to do these pure algebraic fashion!



#### Kite Area Conjecture

C-78

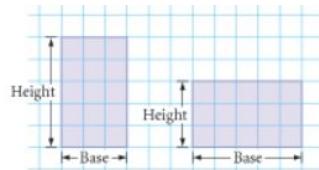
The area of a kite is given by the formula ?.

# Complete & Add Conjectures to Toolbox

## Rectangle Area Conjecture

C-74

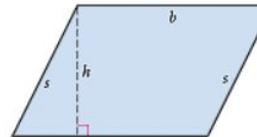
The area of a rectangle is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the rectangle.



## Parallelogram Area Conjecture

C-75

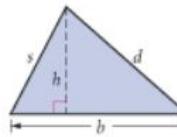
The area of a parallelogram is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the parallelogram.



## Triangle Area Conjecture

C-76

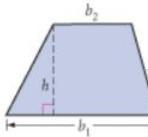
The area of a triangle is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $b$  is the length of the base, and  $h$  is the height of the triangle.



## Trapezoid Area Conjecture

C-77

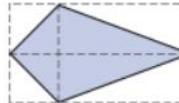
The area of a trapezoid is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $b_1$  and  $b_2$  are the lengths of the two bases, and  $h$  is the height of the trapezoid.



## Kite Area Conjecture

C-78

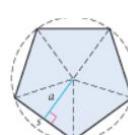
The area of a kite is given by the formula  $\underline{\hspace{2cm}}$ .



## Regular Polygon Area Conjecture

C-79

The area of a regular polygon is given by the formula  $\underline{\hspace{2cm}}$  or  $\underline{\hspace{2cm}}$ , where  $A$  is the area,  $P$  is the perimeter,  $a$  is the apothem,  $s$  is the length of each side, and  $n$  is the number of sides.



Wednesday, Jan 12th

# What's happening today?

## Check-in

### Form Random Teams!

Did you add conjectures to your toolbox?

Review Area formulas and how they work

Announce Areas Quiz 8.1-8.4 Wednesday Jan 19th

## Today

Do Area & Real-world problems from chapters 8.1-8.3

Cover Chapter 8.4 Areas of Regular Polygons

Do Polygon Area Problems

## Reminder

Complete Homework due by Friday to prepare for Quiz

AK will be posted regularly for review and prep

Update your Google Classroom Notification Settings

Check your emails - Comments will be made on Google classroom

Upload your work as a SINGLE PDF files (jpg not allowed)

- Use CamScanner or similar app

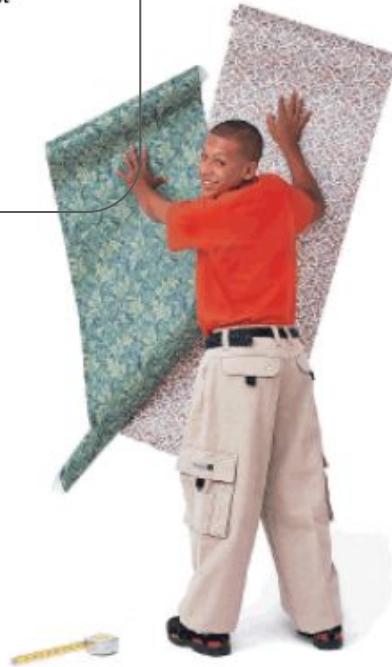
# Real-world Area Problems!

1. **Application** Tammy is estimating how much she should charge for painting 148 rooms in a new motel with one coat of base paint and one coat of finishing paint. The four walls and the ceiling of each room must be painted. Each room measures 14 ft by 16 ft by 10 ft high.

- Calculate the total area of all the surfaces to be painted with each coat. Ignore doors and windows.
- One gallon of base paint covers 500 square feet. One gallon of finishing paint covers 250 square feet. How many gallons of each will Tammy need for the job?

2. **Application** Rashad wants to wallpaper the four walls of his bedroom. The room is rectangular and measures 11 feet by 13 feet. The ceiling is 10 feet high. A roll of wallpaper at the store is 2.5 feet wide and 50 feet long. How many rolls should he buy? (Wallpaper is hung from ceiling to floor. Ignore doors and windows.)

3. **Application** It takes 65,000 solar cells, each 1.25 in. by 2.75 in., to power the Helios Prototype, shown below. How much surface area, in square feet, must be covered with the cells? The cells on Helios are 18% efficient. Suppose they were only 12% efficient, like solar cells used in homes. How much more surface area would need to be covered to deliver the same amount of power?

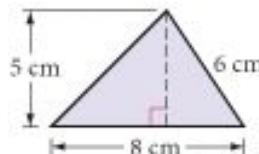


# Area Problems!

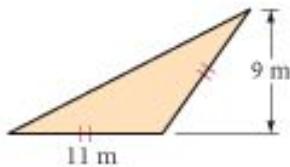
## EXERCISES

In Exercises 1–12, use your new area conjectures to solve for the unknown measures.

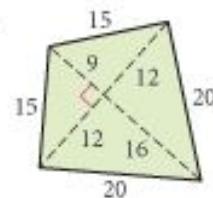
1.  $A = ?$



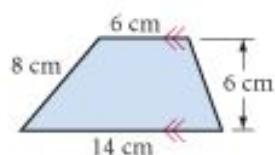
2.  $A = ?$



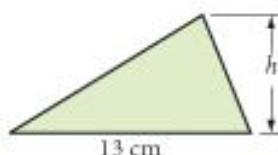
3.  $A = ?$



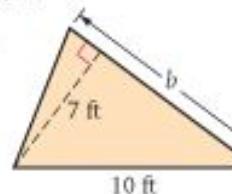
4.  $A = ?$



5.  $A = 39 \text{ cm}^2$   
 $h = ?$



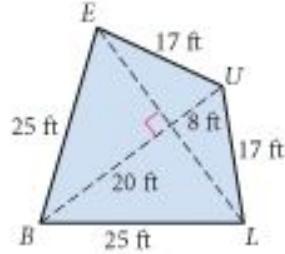
6.  $A = 31.5 \text{ ft}^2$   
 $b = ?$



# s'more Area Problems!

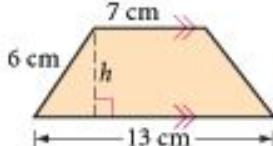
7.  $A = 420 \text{ ft}^2$

$LE = ?$



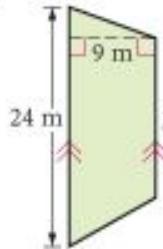
8.  $A = 50 \text{ cm}^2$  (h)

$h = ?$



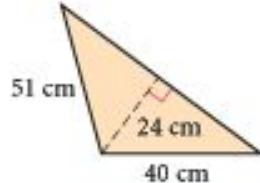
9.  $A = 180 \text{ m}^2$

$b = ?$



10.  $A = 924 \text{ cm}^2$

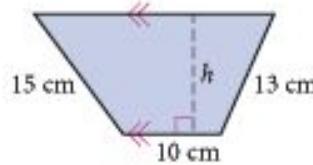
$P = ?$



11.  $A = 204 \text{ cm}^2$

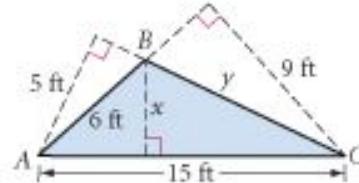
$P = 62 \text{ cm}$

$h = ?$

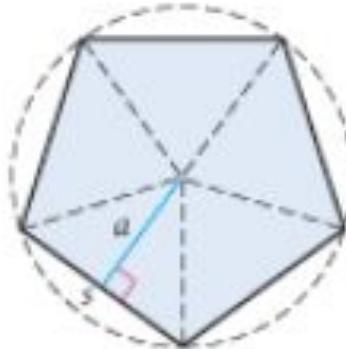
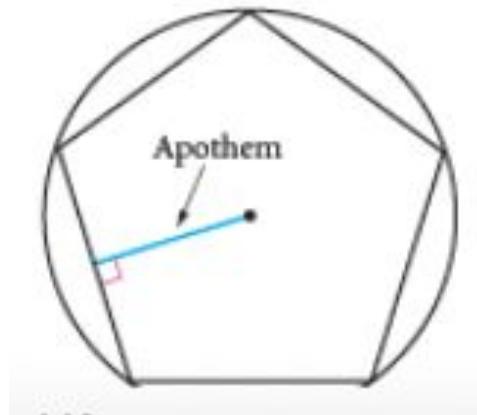
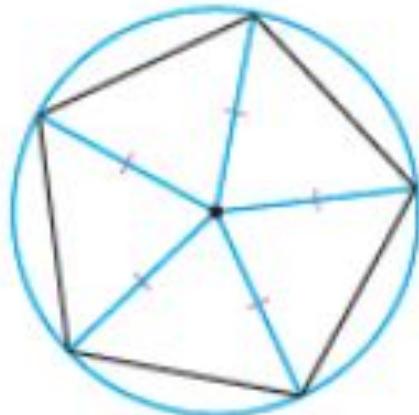


12.  $x = ?$  (h)

$y = ?$



# Chapter 8.4: Areas of Regular Polygons



Regular pentagon

## Regular Polygon Area Conjecture

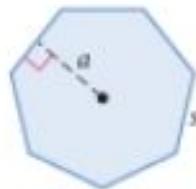
C-79

The area of a regular polygon is given by the formula  $\frac{1}{2} s n a$  or  $\frac{1}{2} a P$ , where  $A$  is the area,  $P$  is the perimeter,  $a$  is the apothem,  $s$  is the length of each side, and  $n$  is the number of sides.

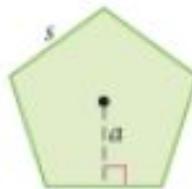
$$\frac{1}{2} s n a \quad \frac{1}{2} a P$$

# Regular Polygon Area Problems!

1.  $A \approx \frac{?}{s}$   
 $s = 24 \text{ cm}$   
 $a \approx 24.9 \text{ cm}$



2.  $a \approx \frac{?}{s}$   
 $s = 107.5 \text{ cm}$   
 $A \approx 19,887.5 \text{ cm}^2$



8. Find the approximate length of each side of a regular  $n$ -gon if  $a = 80$  feet,  $n = 20$ , and  $A \approx 20,000$  square feet.
9. **Construction** Draw a segment 4 cm long. Use a compass and straightedge to construct a regular hexagon with sides congruent to this segment. Use the Regular Polygon Area Conjecture and a centimeter ruler to approximate the hexagon's area. 

# Reminders

1. Complete Homework due by Friday to prepare for Quiz
2. AK will be posted regularly for review and prep
3. Update your Google Classroom Notification Settings
4. Check your emails - Comments will be made on Google classroom
5. Upload your work as a SINGLE PDF files (jpg not allowed)
  - Use CamScanner or similar app

Friday, Jan 14th

# What's happening today?

## Check-in

Complete 8.2-8.3 Area HW problems?

## Today

Work on Area Problems - regular polygons, real-world  
Algebra techniques - Linear Equations

## Reminders

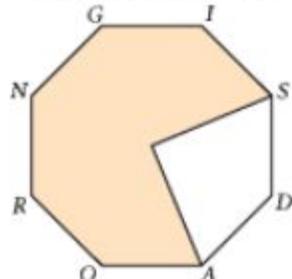
Areas Quiz 8.1-8.4 Wednesday - Jan 19th

Complete All Homework on-time to prepare for Quiz  
AK will be posted regularly for review and prep

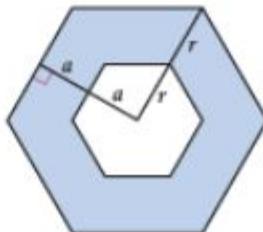
# Regular Polygon Area Problems!

11. A square is also a regular polygon. How is the apothem of a square related to the side length? Show that the Regular Polygon Area Conjecture simplifies to  $s^2$  for the area of a square.

13. Find the approximate area of the shaded region of the regular octagon *ROADSIGN*. The apothem measures 20 cm. Segment *GI* measures about 16.6 cm.



14. Find the approximate area of the shaded regular hexagonal donut. The apothem and sides of the smaller hexagon are half as long as the apothem and sides of the large hexagon.  
 $a \approx 6.9$  cm and  $r \approx 8$  cm 



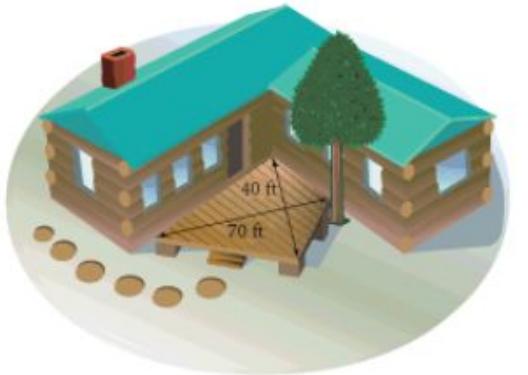
# Real-world Area Problems!

1. **Application** Tammy is estimating how much she should charge for painting 148 rooms in a new motel with one coat of base paint and one coat of finishing paint. The four walls and the ceiling of each room must be painted. Each room measures 14 ft by 16 ft by 10 ft high.
  - a. Calculate the total area of all the surfaces to be painted with each coat. Ignore doors and windows.
  - b. One gallon of base paint covers 500 square feet. One gallon of finishing paint covers 250 square feet. How many gallons of each will Tammy need for the job?

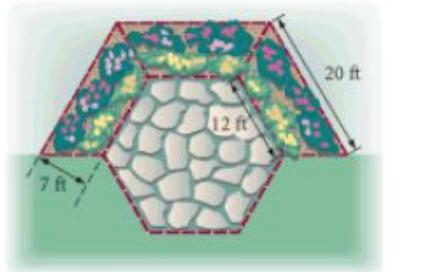


# s'more Real-world Area Problems!

4. **Application** Harold works at a state park. He needs to seal the redwood deck at the information center to protect the wood. He measures the deck and finds that it is a kite with diagonals 40 feet and 70 feet. Each gallon of sealant covers 400 square feet, and the sealant needs to be applied every six months. How many gallon containers should he buy to protect the deck for the next three years?



5. **Application** A landscape architect is designing three trapezoidal flowerbeds to wrap around three sides of a hexagonal flagstone patio, as shown. What is the area of the entire flowerbed? The landscape architect's fee is \$100 plus \$5 per square foot. What will the flowerbed cost?



# Linear Equations - pts of concurrency Chapter 6.6

$$\textcircled{1} \quad 3y = 12x - 21$$

$$\boxed{12x + 2y = 1}$$

$$\textcircled{2} \rightarrow 2y = -12x + 1$$

$$\textcircled{1} + \textcircled{2} \Rightarrow$$

$$5y = -20 \Rightarrow \boxed{y = -4} \textcircled{3}$$

$$\textcircled{3} \text{ in } \textcircled{1} \Rightarrow -12 = 12x - 21$$

$$\Rightarrow \underline{\underline{x = 9/12 = \frac{3}{4}}} \textcircled{4}$$

$$-4x + 3y = 3$$

$$\times 3 \quad \boxed{7x - 9y = 6} \textcircled{1}$$

$$-12x + 9y = 9 \textcircled{2}$$

$$\textcircled{1} + \textcircled{2} \Rightarrow$$

$$-5x = 15 \Rightarrow \boxed{x = -3} \textcircled{3}$$

$$\textcircled{3} \text{ in } \textcircled{1} \Rightarrow -21 - 9y = 6$$

$$\Rightarrow \boxed{y = 3} \textcircled{4}$$

System of 2 equations  
with 2 unknowns

Plot these lines to show approximates slopes,  
Y-intercepts & pt of intersection

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Wed, Jan 19th

# What's happening today?

## Check-in

Completed Area HW problems?

Have Calculator, Conjectures Toolbox, Geo instruments for Quiz?

Starting next week - **Phones will not be allowed in class** - Bring your calculators!

## Today

30-60-90 triangles

Radical Expressions

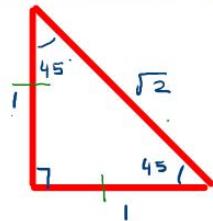
Areas Quiz 8.1-8.4

## Reminders

Project on Radical Expressions & Linear Equations due 1/25

# Special Triangles Chapter 9.2

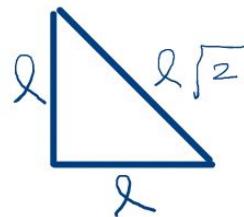
Isosceles Right-Triangle



$$\rightarrow \sqrt{3^2 + 3^2} = \sqrt{3^2(1+1)} = 3\sqrt{2}$$

Q.E.D.

A right-angled triangle with two legs of length 3. The angle between the legs is 90 degrees, and the angle at the vertex opposite the hypotenuse is 45 degrees. The hypotenuse is labeled  $3\sqrt{2}$ .

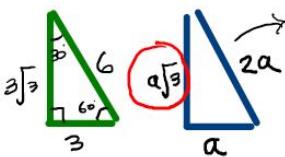
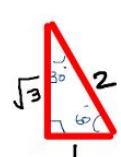


**Isosceles Right Triangle Conjecture**

C-83

In an isosceles right triangle, if the legs have length  $l$ , then the hypotenuse has length  $\underline{\quad}$ .

$30^\circ - 60^\circ - 90^\circ$  Triangle



$$\sqrt{(2a)^2 - a^2} = \sqrt{4a^2 - a^2} = \sqrt{3a^2} = a\sqrt{3}$$

Q.E.D!

**30°-60°-90° Triangle Conjecture**

C-84

In a  $30^\circ-60^\circ-90^\circ$  triangle, if the shorter leg has length  $a$ , then the longer leg has length  $\underline{\quad}$  and the hypotenuse has length  $\underline{\quad}$ .

# Radical Expressions

## Simplifying Radical Expressions

$$\sqrt{4 \cdot 2} = \sqrt{2 \cdot 2 \cdot 2} = \sqrt{2 \cdot 2} = 2\sqrt{2}$$

Reduce Number inside a radical to its Prime factors

OR

$$\sqrt{10 \cdot 5} = \sqrt{2 \cdot 5 \cdot 5} = \sqrt{2 \cdot 5^2} = 5\sqrt{2}$$

(2)  $\sqrt{25 \cdot 2} = \sqrt{5^2 \cdot 2} = 5\sqrt{2}$  look for perfect squares

What is  $(3\sqrt{6})(2\sqrt{3})$ ?

$$= 3 \cdot 2 \cdot \sqrt{6} \sqrt{3} = 6 \cdot \sqrt{3 \cdot 2} \sqrt{3} = 6 \sqrt{3 \cdot 2 \cdot 3} = 18\sqrt{2}$$

What is  $\sqrt{784}$ ?

$$= \sqrt{2 \cdot 392} = \sqrt{2 \cdot 2 \cdot 196} = 2\sqrt{196}$$

What is  $\sqrt{720}$ ?

$$= \sqrt{2 \cdot 2 \cdot 180} = \sqrt{2 \cdot 2 \cdot 90} = \sqrt{2 \cdot 2 \cdot 45} = 2\sqrt{45} = 4\sqrt{5} = 12\sqrt{5}$$

Fri Jan 21st

# What's happening today?

## Check-in

Submit Quiz Corrections with explanations for 100% marks

You need to explain your mistake and how you corrected it in your submission or verbally

Project on Radical Expressions & Special right-triangle conjectures due today

[Homework assigned on 8.5 & 8.6](#)

## Today

Questions on Project?

## Reminders

# Complete Investigation - Do Problems (Teams)



## Investigation

### Area Formula for Circles

Circles do not have straight sides like polygons do. However, the area of a circle can be rearranged. Let's investigate.

Step 1

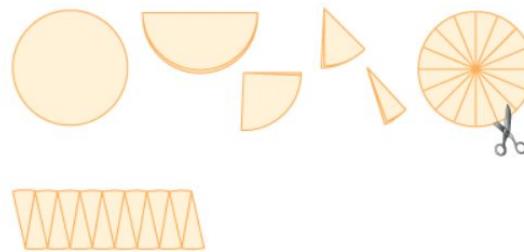
Use your compass to make a large circle. Cut out the circular region.

Step 2

Fold the circular region in half. Fold it in half a second time, then a third time and a fourth time. Unfold your circle and cut it along the folds into 16 wedges.

Step 3

Arrange the wedges in a row, alternating the tips up and down to form a shape that resembles a parallelogram.



If you cut the circle into more wedges, you could rearrange these thinner wedges to look even more like a rectangle, with fewer bumps. You would not lose or gain any area in this change, so the area of this new "rectangle," skimming off the bumps as you measure its length, would be closer to the area of the original circle.

If you could cut infinitely many wedges, you'd actually have a rectangle with smooth sides. What would its base length be? What would its height be in terms of  $C$ , the circumference of the circle?

Step 4

The radius of the original circle is  $r$  and the circumference is  $2\pi r$ . Give the base and the height of a rectangle made of a circle cut into infinitely many wedges. Find its area in terms of  $r$ . State your next conjecture.

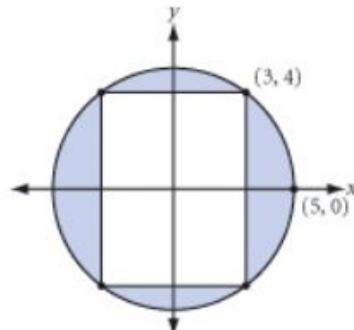
### Circle Area Conjecture

$$\cdot \pi r^2$$

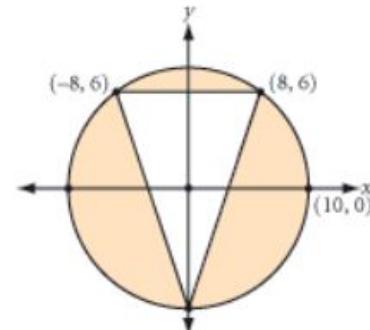
C-80

The area of a circle is given by the formula  $\underline{\hspace{2cm}}$ , where  $A$  is the area and  $r$  is the radius of the circle.

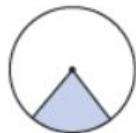
9. What is the area of the shaded region between the circle and the rectangle?



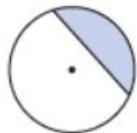
10. What is the area of the shaded region between the circle and the triangle?



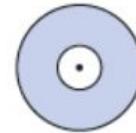
# Sectors - Segments - Annuli



Sector of a circle



Segment of a circle



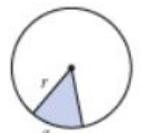
Annulus

A **sector of a circle** is the region between two radii and an arc of the circle.

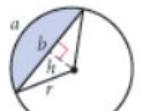
A **segment of a circle** is the region between a chord and an arc of the circle.

An **annulus** is the region between two concentric circles.

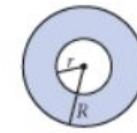
“Picture equations” are helpful when you try to visualize the areas of these regions. The picture equations below show you how to find the area of a sector of a circle, the area of a segment of a circle, and the area of an annulus.



$$\frac{a}{360} \cdot \pi r^2 = A_{\text{sector}}$$



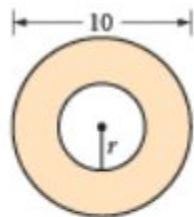
$$\frac{a}{360} \pi r^2 - \frac{1}{2}bh = A_{\text{segment}}$$



$$\pi R^2 - \pi r^2 = A_{\text{annulus}}$$

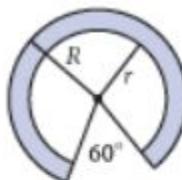
# Do some Problems

7.  $r = 2 \text{ cm}$



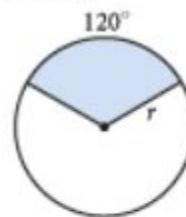
8.  $R = 12 \text{ cm}$

$r = 9 \text{ cm}$



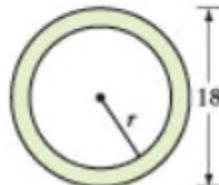
9. The shaded area is  $12\pi \text{ cm}^2$ .

Find  $r$ .

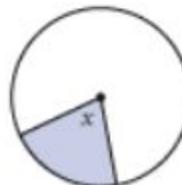


10. The shaded area is  $32\pi \text{ cm}^2$ . Find  $r$ .

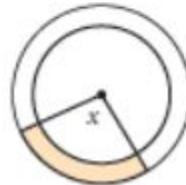
$32\pi \text{ cm}^2$



11. The shaded area is  $120\pi \text{ cm}^2$ , and the radius is 24 cm. Find  $x$ .



12. The shaded area is  $10\pi \text{ cm}^2$ .  
The radius of the large circle is  
10 cm, and the radius of the  
small circle is 8 cm. Find  $x$ . (h)



Tue Jan 25th

# What's happening today?

## Check-in

Submit Quiz Corrections for 100% marks

You need to explain your mistake and how you corrected it in your submission or verbally

Project on Radical Expressions & Special right-triangles due today (this weekend)

Homework assigned 8.5 & 8.6

Conjectures Notebook Review and Reflection Assigned

## Today

Points of trouble - Power Rules, Simplifying Expressions, PEMDAS, Units, Bases & Heights

Project Hints

## Reminders

# Power Rules & Radicals

## Power Rules & Radicals

$$\frac{1}{a} = a^{-1}, \sqrt{a} = a^{\frac{1}{2}}, a^0 = 1$$

$$\frac{1}{3} = 3^{-1}, \sqrt{5} = 5^{\frac{1}{2}}, 17^0 = 1$$

$$a^p + a^p = a^p(1+1) = 2a^p$$

$$\frac{2}{3} + \frac{2}{3} = 2 \cdot 3 = 18$$

$$a^p + a^q \quad (\text{cannot be simplified further})$$

$$\frac{2}{3} + \frac{3}{3} = 9 + 27 = 36$$

$$a^p \cdot a^q = a^{(p+q)}$$

$$\frac{3}{2} \cdot \frac{4}{2} = (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$$

$$\frac{a^p}{a^q} = a^{(p-q)}$$

$$\frac{2^3}{2^4} = \frac{\cancel{2 \cdot 2 \cdot 2}}{\cancel{2 \cdot 2 \cdot 2 \cdot 2}} = \frac{1}{2} \text{ or } 2^{(3-4)-1} = 2^{-1} = \frac{1}{2}$$

$$(a^p)^q = a^{pq}$$

$$(2^3)^4 = 2^{\frac{3 \cdot 4}{2}} = 2^{\frac{12}{2}} = (2 \cdot 2 \cdot 2)^4 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)$$

$$\sqrt{a} \cdot \sqrt{a} = \sqrt{a^2} = a$$

$$\sqrt{5} \cdot \sqrt{5} = \sqrt{5^2} = \sqrt{25} = 5$$

$$\sqrt{a} \sqrt{b} = \sqrt{ab}$$

$$\sqrt{3} \cdot \sqrt{5} = \sqrt{3 \cdot 5} = \sqrt{15}$$

$$\sqrt{a} + \sqrt{b} \quad (\text{cannot be simplified further})$$

$$\sqrt{3} + \sqrt{5}$$

$$\sqrt{a} + \sqrt{a} = \sqrt{a}(1+1) = 2\sqrt{a}$$

$$\sqrt{5} + \sqrt{5} = 2\sqrt{5}$$

# Simplifying Expressions

$$1. -x^2 = a^2 - c^2$$

Calculate  $x$

$$a=3, b=4, c=5$$

$$x = \sqrt{\frac{c^2 - a^2}{7}} = \sqrt{\frac{16}{7}}$$

$$2. \frac{b^2}{2} = c^2 - x^2$$

Calculate  $x$

$$x = \sqrt{\frac{c^2 - b^2}{2}} = \sqrt{17}$$

(Taking  $\sqrt$  is same as taking to power  $\frac{1}{2}$ )

$$3. \sqrt{a} + \sqrt{a}, \sqrt{a} + \sqrt{b}, \sqrt{a} \cdot \sqrt{b}$$

Simplify ③ & ④

$$4. (a^2)^3, (\sqrt{a} \sqrt{a})^2, c(a+b)^2$$

(Dividing by 2 is same as multiplying by  $\frac{1}{2}$ )

$$5. -4x + 3 = -2y \text{ what is slope of } y-\text{int}$$

$$2\sqrt{a}, \sqrt{a} + \sqrt{b}, \sqrt{ab}$$

$$2\sqrt{3}, \sqrt{3} + \sqrt{4}, 2\sqrt{3}$$

$$a^6, a^2, c(a^2 + 2ab + b^2)$$

$$729, 9, 245$$

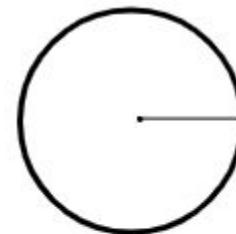
$$y = 2x - \frac{3}{2} \quad \begin{matrix} \text{slope: 2} \\ y-\text{int: } -\frac{3}{2} \end{matrix}$$

# How do Units work?

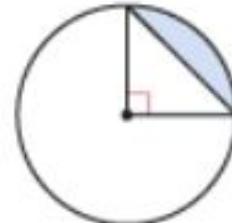
How do units work in these formulas?



The above fig is a Trapezoid of Area 500 sq-ft & height 8 meters. Calculate the length of the small base in meters if the large base is 10 meters in length.



4.  $r = 2 \text{ cm}$



Area of circle is 72 sq. ft  
What is the radius in meters?

Calculate the area of the shaded segment

# Soln to Units problems

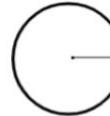
## How do Units work?

How do units work in these formulas?



The above fig is a Trapezoid of Area 500 sq-ft & height 8 meters. Calculate the length of the small base in meters if the large base is 10 meters in length.

$$A_{\text{TRA}} = \frac{72 \text{ ft}^2}{r} = \pi r^2$$
$$r = \sqrt{\frac{72 \text{ ft}^2}{\pi}} = 4.79 \text{ ft}$$
$$= 4.79 \text{ ft} \times \frac{1 \text{ m}}{3.28 \text{ ft}}$$



Area of circle is 72 sq. ft  
What is the radius in meters?

$$= 1.46 \text{ m}$$

$$4. r = 2 \text{ cm}$$
$$\text{Area} = \frac{\alpha \times \pi r^2 - \frac{1}{2} b h}{360}$$
$$= \frac{90}{360} \times \pi \times 2^2 - \frac{1}{2} \times 2 \times 2$$

Calculate the area of the shaded segment

$$= \frac{1}{4} \pi \cdot 4 - 2$$
$$A = \pi - 2 \text{ cm}$$

$$500 \text{ ft}^2 \times \frac{\text{m}^2}{\text{ft}^2} = \frac{1}{2} (10 + b_2) \text{ m} \times 8 \text{ m}$$

there are 3.28 ft in a meter

$$500 \times \cancel{\text{ft}^2} \times \frac{\text{m}^2}{3.28^2 \cancel{\text{ft}^2}} = \frac{1}{2} (10 + b_2) \times 8^{\cancel{4} \text{ m}^2}$$

$$4b.4 = 40 + 4b_2$$

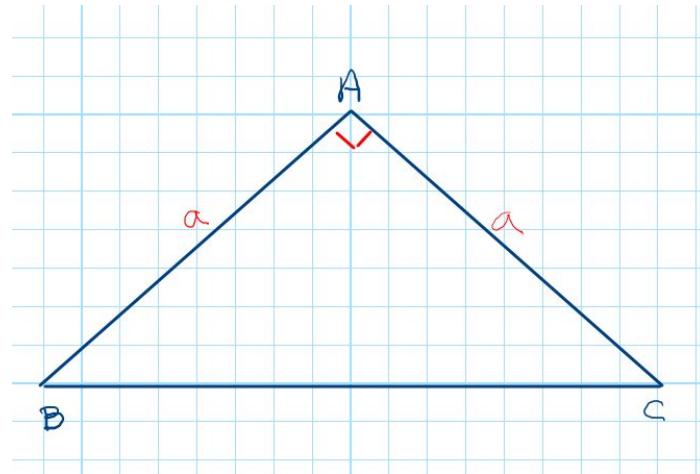
$$b_2 = 6.4 / 4 = 1.61 \text{ m}$$

Validate:  $\text{Area} = \frac{1}{2} (10 + 1.61) \times 8 = 46.4 \text{ m}^2$

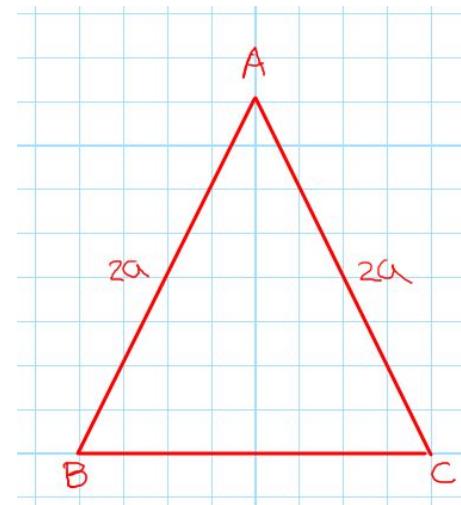
or  $46.4 \text{ m}^2 \times 3.28^2 \text{ ft}^2 / \cancel{\text{m}^2} = 499.6 \text{ ft}^2 \checkmark$

# Project Hints

Find the hypotenuse of an Isosceles Right-Triangle of side ' $a$ '



Find the Height of an Equilateral Triangle of side ' $2a$ '



Thu Jan 27th

# What's happening today?

Check-in

Today

Going 3-D today - Surface Areas Chapter 8.7!

Use Geogebra to build 3D Shapes

Reminders

Submit Quiz Corrections for 100% marks

You need to explain your mistake and how you corrected it in your submission or verbally

Project on Radical Expressions & Special right-triangles due today (this weekend)

[Homework assigned 8.7 Surface Area](#)

Conjectures Notebook Review and Reflection Assigned

Start studying for your next quiz! (8.4 - 8.7) (regular polygons, circles, pieces of circles, and surface area)

# Some 3-D Shapes and Vocabulary

## 3-Dimensional Shapes



Cylinder



Prism



Sphere



Cone

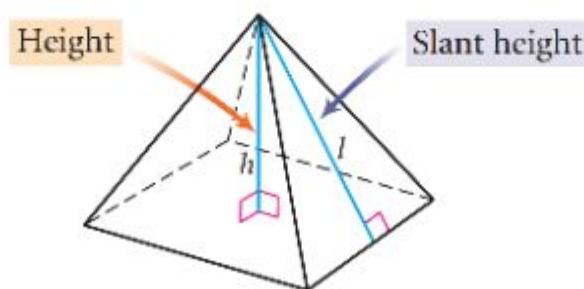
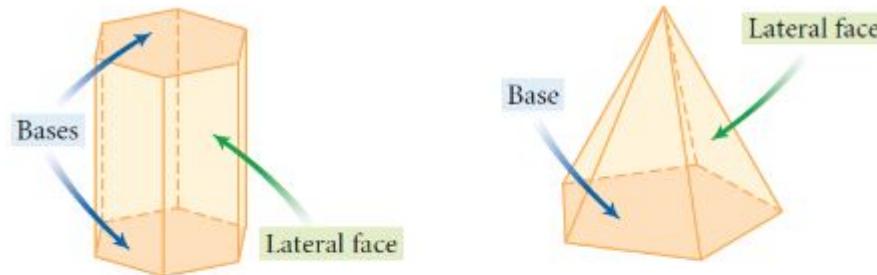


Pyramid

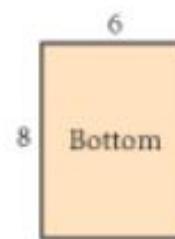
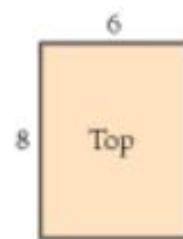
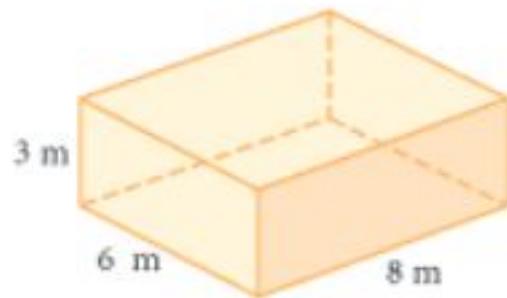


Hemisphere

# Some 3-D Shapes and Vocabulary

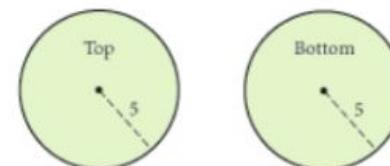
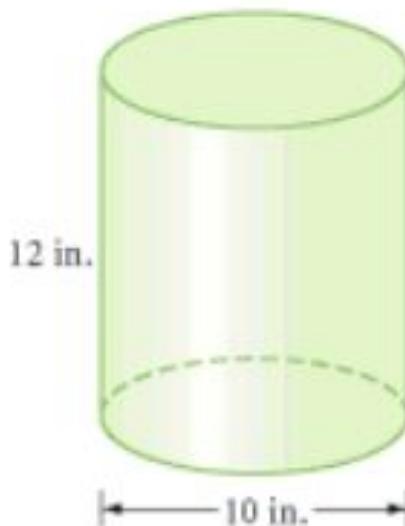


# Surface Area of 3-D Shapes - Prism



$$\text{surface area} = 2(\text{base area}) + (\text{lateral surface area})$$

# Surface Area of 3-D Shapes - Cylinder



Bases

$$b = C = 2\pi r$$

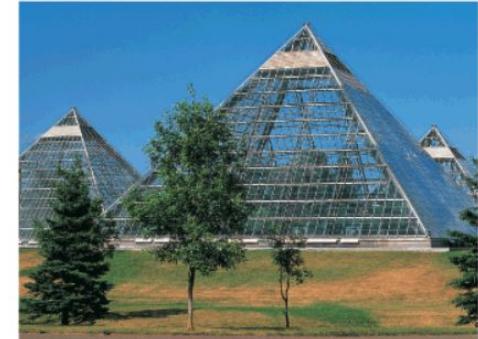


$$\text{Surface Area of Cylinder} = 2\pi r(r + h)$$

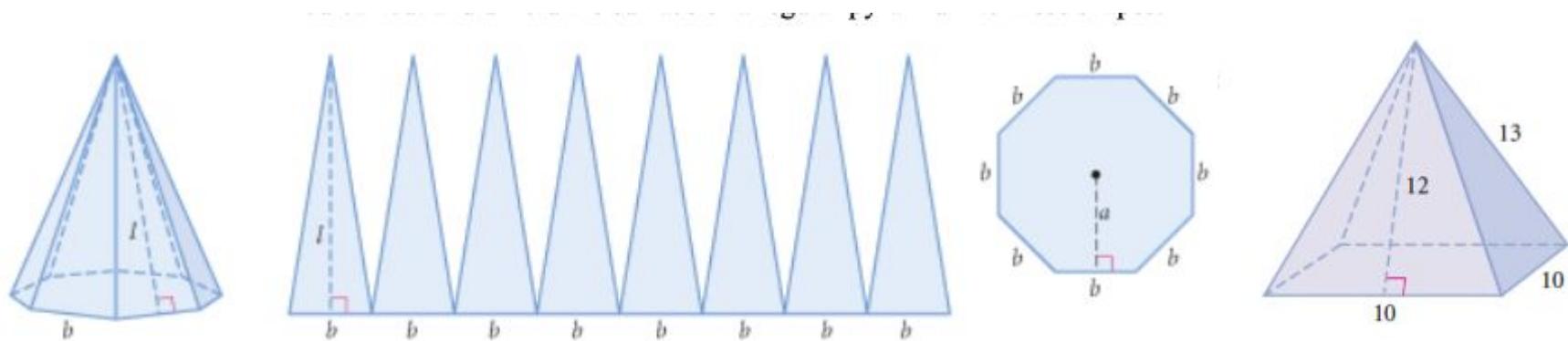


This ice cream plant in Burlington, Vermont, uses cylindrical containers for its milk and cream.

# Surface Area of 3-D Shapes - Pyramid



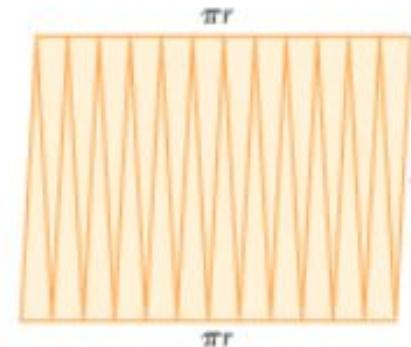
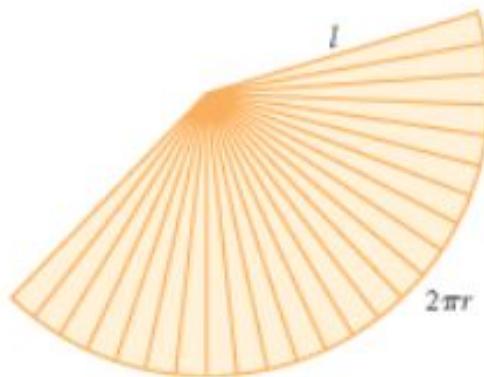
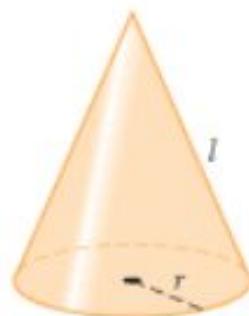
These conservatories in Edmonton, Canada, are glass pyramids.



$$\text{Surface Area of Pyramid} = \frac{1}{2}bn(l + a)$$

$$\text{Surface Area} = 340 \text{ sq-cm}$$

# Surface Area of 3-D Shapes - Cone



$$\text{Surface Area of Cone} = \pi r(r + l)$$

# Do problems from Homework

## Homework assigned 8.7 Surface Area

In Exercises 1–10, find the surface area of each solid. All quadrilaterals are rectangles, and all given measurements are in centimeters. Round your answers to the nearest 0.1 cm<sup>2</sup>.

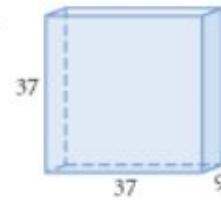


for Exercise 13

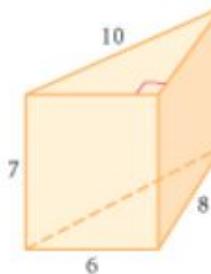
1. <http://www.keymath.com/DG4/chapter8/mpys>



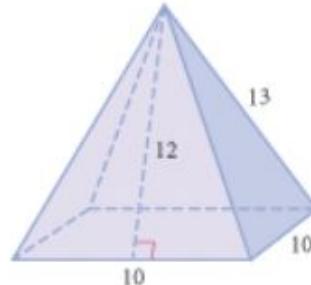
2.



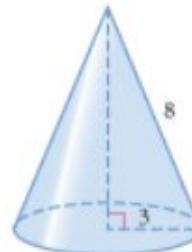
3.



4.



5.

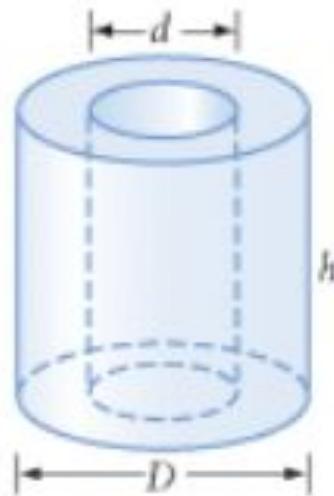


6.



# Do problems from Homework

9.  $D = 8, d = 4, h = 9$  



# Reminders

Submit Quiz Corrections for 100% marks

You need to explain your mistake and how you corrected it in your submission or verbally

Project on Radical Expressions & Special right-triangles due today (this weekend)

Homework assigned 8.7 Surface Area

Conjectures Notebook Review and Reflection Assigned

Start studying for your next quiz! (8.4 - 8.7) (regular polygons, circles, pieces of circles, and surface area)

Best way to do this is to do & VALIDATE your Homework Problems using AK!

Mon Jan 31st

# What's happening today?

## Check-in

[Submitted Quiz Corrections for 100% marks?](#)

[Completed Chap 9.3 Radicals & Equations Project?](#)

[Completed Chap 8.5 and 8.6 Problems on Sectors, Segments & Annulii?](#)

[Completed Chap 8.7 Surface Area Problems?](#)

## References

[AKs for Chapter Homework 8.4, 8.5, 8.6, 8.7, 9.3 posted - Review & Validate!](#)

[What Goes in your Toolbox?](#)

## Today

[Divide into teams of 2](#)

Geogebra 3D!

[Area Problems Practice!](#) Where do you need help?

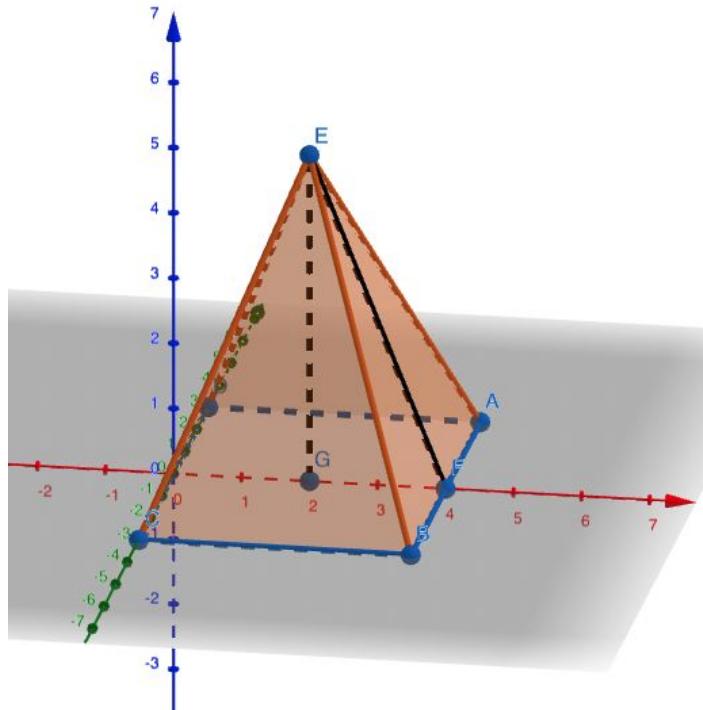
Sectors, Segments, Annulii, Surface Area

Examine Conjectures Notebook (Toolbox) - Conjectures List Posted

## Reminders

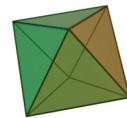
U4Q2 - Quiz on 8.4, 8.5, 8.6, 8.7, 9.3 scheduled for Friday, Feb 4th

# Let's build some 3D Shapes in Geogebra



Can you build a Cone??

Can you build a Octahedron?

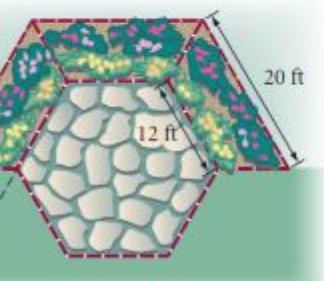


# Problems that make you think

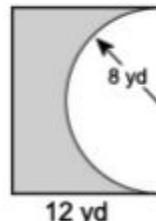
## Try these Area Practice Problems

Start with these ..

2. A landscape architect is designing three trapezoidal flowerbeds to wrap around three sides of a hexagonal flagstone patio, as shown. What is the area of the entire flowerbed? The landscape architect's fee is \$100 plus \$5 per square foot. What will the flowerbed cost?



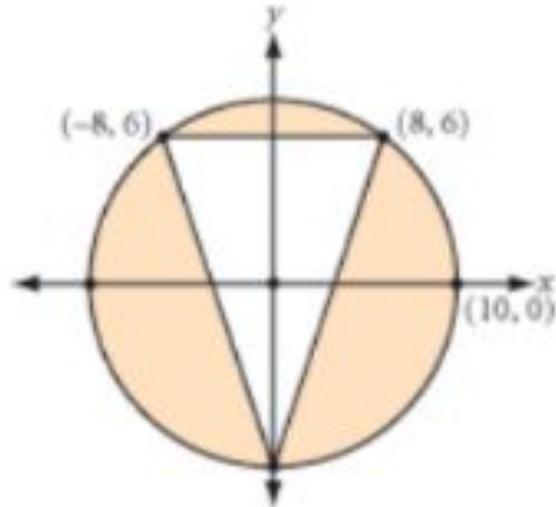
5. Find the area of the shaded region. (Hint: draw a line segment in a clever place)



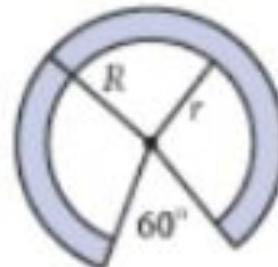
6. Find the area of an equilateral triangle with sides of length 8 cm. Give an exact, reduced answer.

# Problems that make you think ..

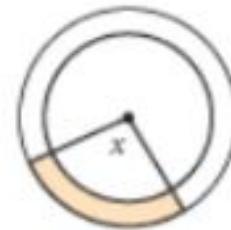
What is the area of the shaded region between the circle and the triangle?

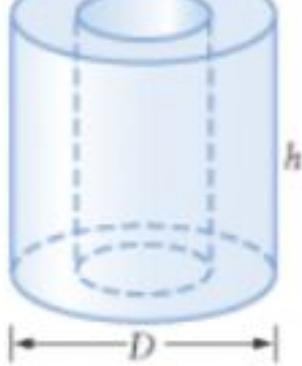


$$R = 12 \text{ cm}$$
$$r = 9 \text{ cm}$$



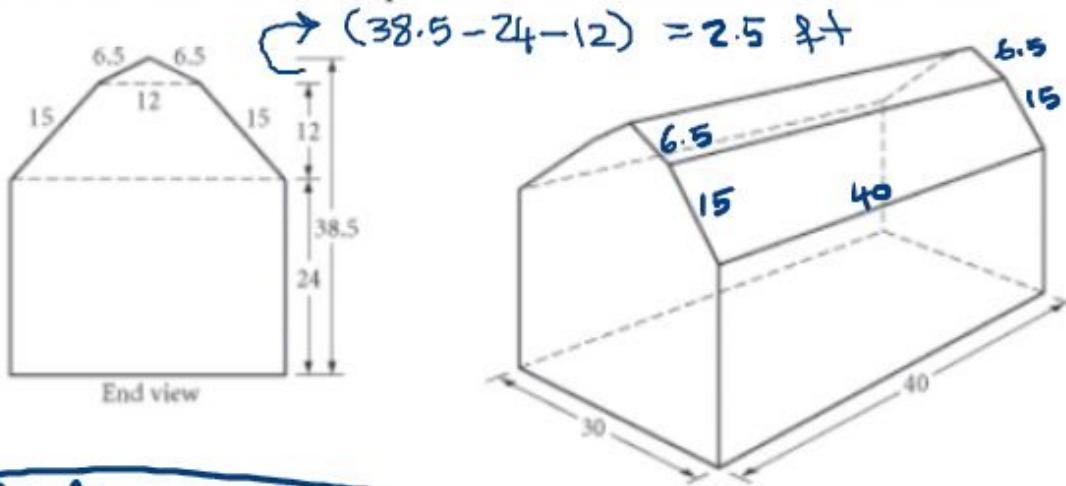
The shaded area is  $10\pi \text{ cm}^2$ .  
The radius of the large circle is 10 cm, and the radius of the small circle is 8 cm. Find  $x$ .





ake you think ..

12. *Application* Claudette and Marie are planning to paint the exterior walls of their country farmhouse (all vertical surfaces) and to put new cedar shingles on the roof. The paint they like best costs \$25 per gallon and covers 250 square feet per gallon. The wood shingles cost \$65 per bundle, and each bundle covers 100 square feet. How much will this home improvement cost? All measurements are in feet.



Wed Feb 2nd

# What's happening today?

Check-in

References - [Conjectures List Posted](#), [Area-Volume Project Posted](#)

Today

Solids - Chapter 10 Volume!

Assign Project: Partners -

```
[ ' Luli', ' Anusha']
[ ' Lincoln', 'Lulu']
[ ' Hedia', ' Charlotte']
[ ' Luc', ' Bella']
[ ' Declan', ' Cophine']
[ ' Cam', ' Kenzo']
```

Examine Conjectures Notebook (Toolbox)

Reminders

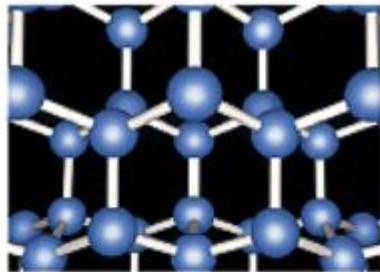
Complete your Conjectures Notebook - [Conjectures List Posted](#)

[Work on Volume Problems Assigned](#)

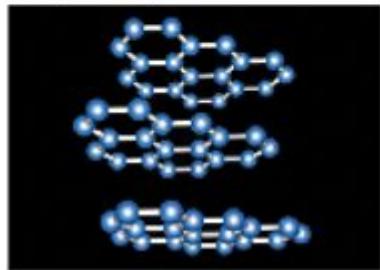
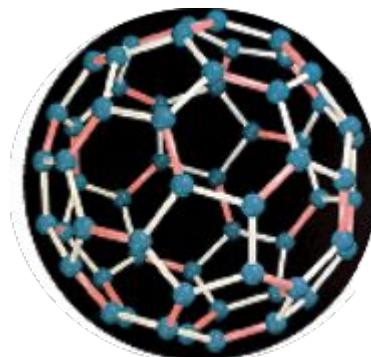
Work on [Area-Volumes Project](#)

U4Q2 - Quiz on 8.4-7, 9.3 scheduled for Friday, Feb 4th

# Solids - Volume Vocabulary

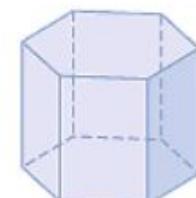
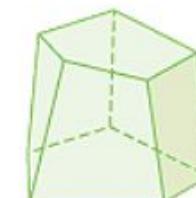
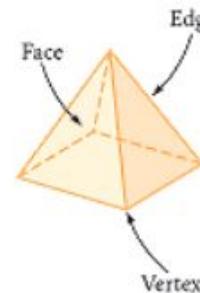
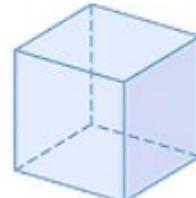


The geometry of diamonds

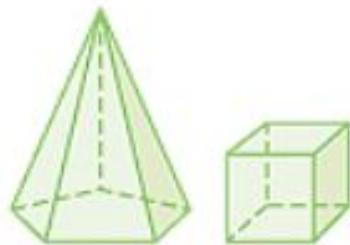


The geometry of graphite

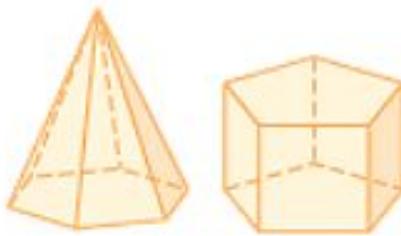
Polyhedron  
Face  
Edge  
Vertex



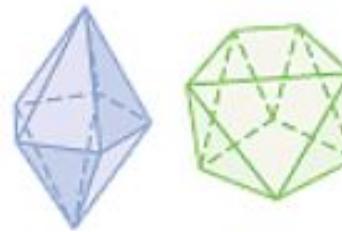
# Polyhedrons



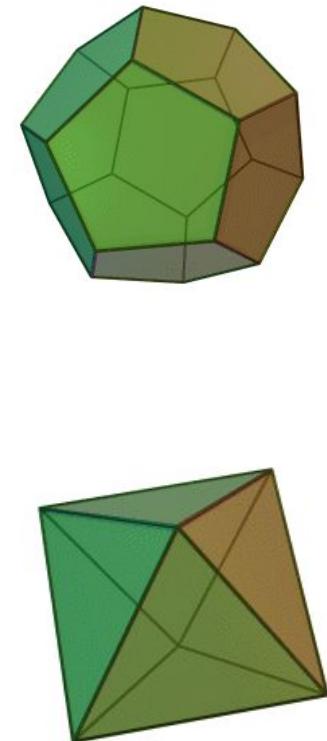
Hexahedrons



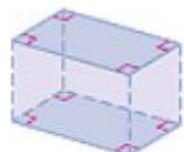
Heptahedrons



Decahedrons



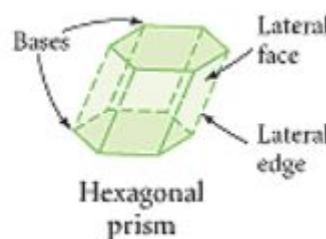
# Pyramids Prisms Cylinders Cones Spheres- Oblique & Right



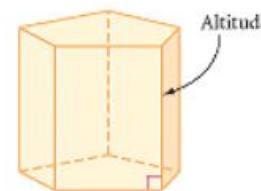
Rectangular prism



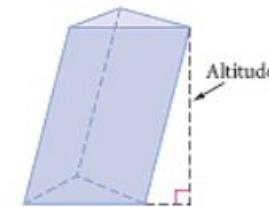
Triangular prism



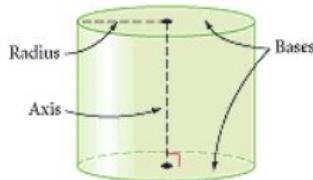
Hexagonal prism



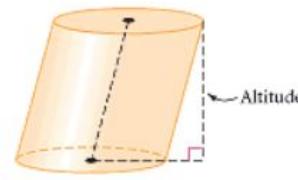
Right pentagonal prism



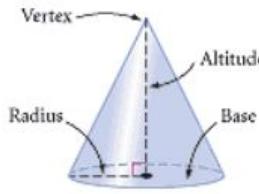
Oblique triangular prism



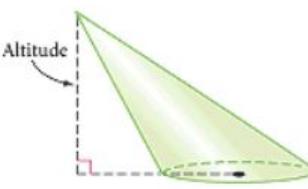
Right cylinder



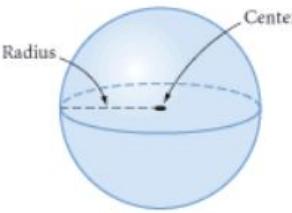
Oblique cylinder



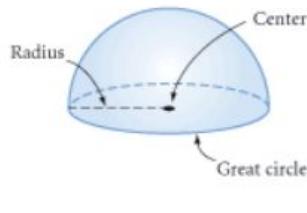
Right cone



Oblique cone

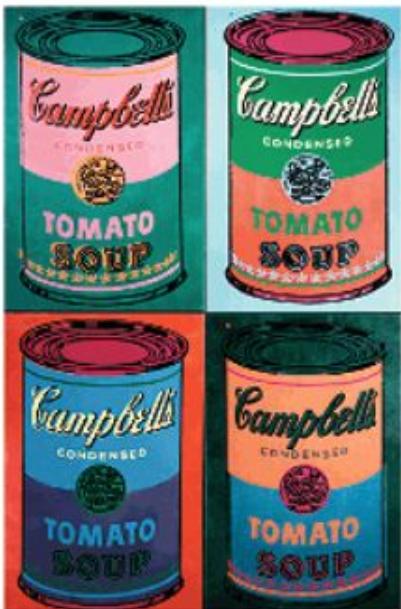


Sphere



Hemisphere

# Volume



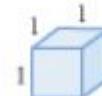
100 Cans (1962 oil on canvas), by pop art artist Andy Warhol (1925–1987), repeatedly uses the cylindrical shape of a soup can to make an artistic statement with a popular image.

**Volume** is the measure of the amount of space contained in a solid. You use cubic units to measure volume: cubic inches ( $\text{in}^3$ ), cubic feet ( $\text{ft}^3$ ), cubic yards ( $\text{yd}^3$ ), cubic centimeters ( $\text{cm}^3$ ), cubic meters ( $\text{m}^3$ ), and so on .

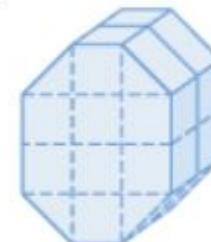
The volume of an object is the number of unit cubes that completely fill the space within the object.

1

Length: 1 unit

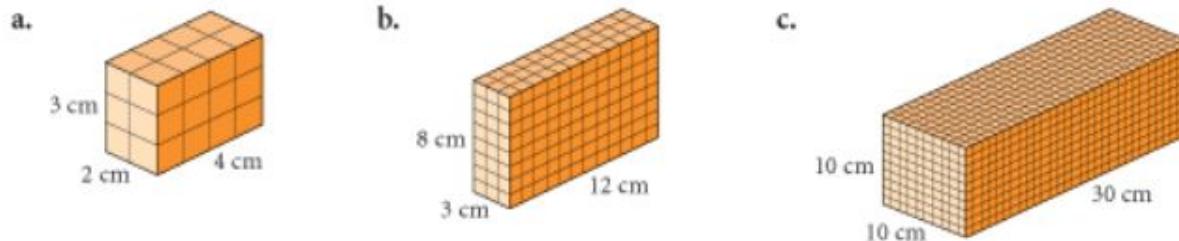


Volume: 1 cubic unit



Volume: 20 cubic units

# Volume Conjectures



## Rectangular Prism Volume Conjecture

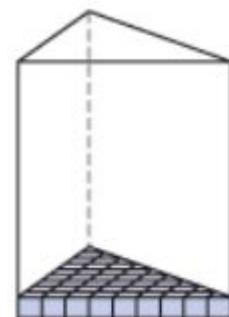
C-86a

If  $B$  is the area of the base of a right rectangular prism and  $H$  is the height of the solid, then the formula for the volume is  $V = \underline{\hspace{2cm}} \cdot BH$ .

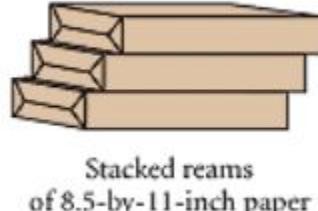
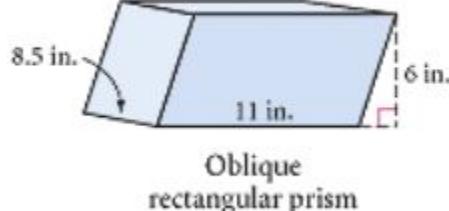
## Right Prism-Cylinder Volume Conjecture

C-86b

If  $B$  is the area of the base of a right prism (or cylinder) and  $H$  is the height of the solid, then the formula for the volume is  $V = \underline{\hspace{2cm}} \cdot BH$ .



# Right or Oblique Volume - is it different?



## Prism-Cylinder Volume Conjecture

C-86

The volume of a prism or a cylinder is the    multiplied by the   .

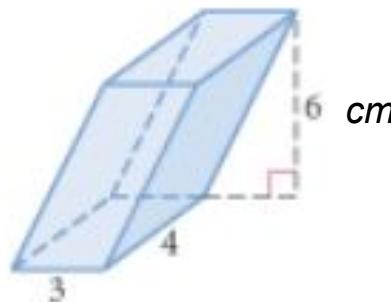
Base Area

Height  
perpendicular  
to Base

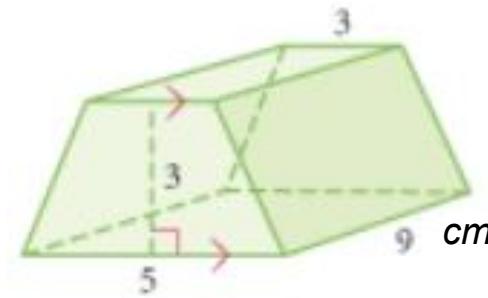


# Volume Problems

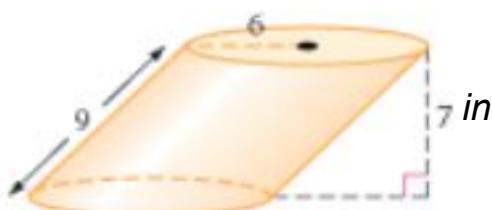
Oblique rectangular prism



Right trapezoidal prism



Oblique Cylinder

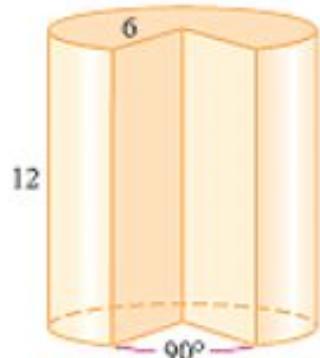


$$252\pi \text{ in}^3$$

Right cylinder with a  
90° slice removed 

$$72 \text{ cm}^3$$

$$108 \text{ cm}^3$$



$$324\pi \text{ cm}^3$$

Fri, Feb 4th

# What's happening today?

Check-in

References

Today

Work on Volume Problems (Chap 10.1 10.2)

Work on Volumes Project

U4Q2 - Areas Quiz

Reminders

Work on Volumes Project

Tue, Feb 8th

# What's happening today?

## Check-in

U2Q2 Quizzes Returned

Submit Quiz Corrections!

## References

## Today

Review Prisms & Cylinders Volume

Do Volume Exercise with water

Chapter 10.3 Volume of Cones and Pyramids

Do some problems - [Homework assigned](#)

## Reminders

Work on Volumes Project

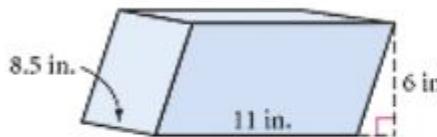
Construct a Platonic Solid!

Do Volume of a Sphere Worksheet

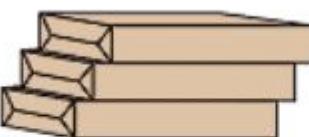
Work on U4Q2 Quiz Corrections

# Review Volume of Right/Oblique Prisms/Cylinders

GOALS FOR THIS LESSON



Oblique  
rectangular prism



Stacked reams  
of 8.5-by-11-inch paper



Stacked sheets  
of paper



Sheets of paper  
stacked straight

## Prism-Cylinder Volume Conjecture

C-86

The volume of a prism or a cylinder is the    multiplied by the   .

Base Area

Height  
perpendicular  
to Base



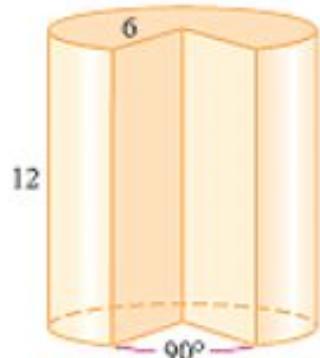
# Review Problems

Oblique Cylinder

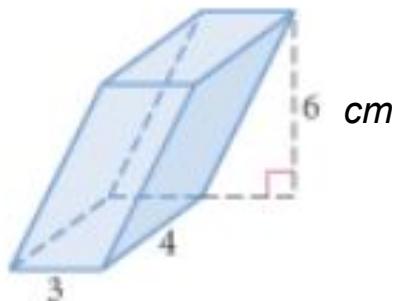


$$252\pi \text{ in}^3$$

Right cylinder with a  
90° slice removed

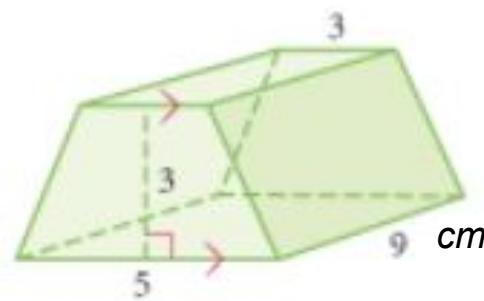


Oblique rectangular prism



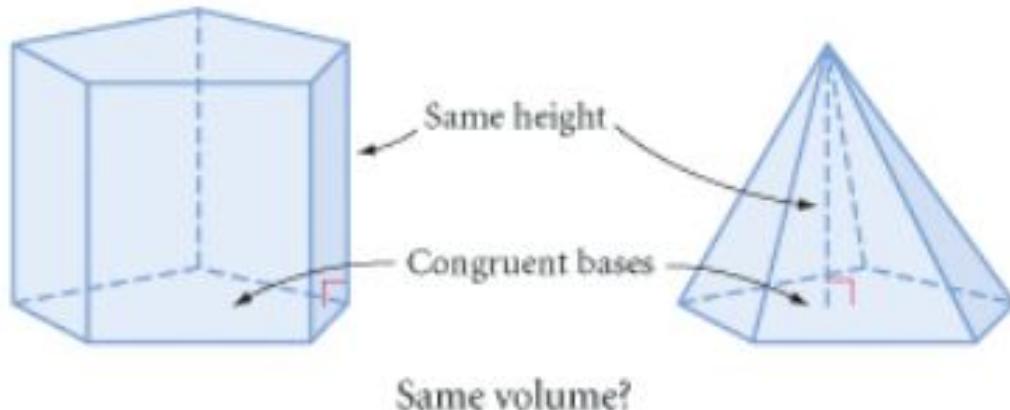
$$72 \text{ cm}^3$$

Right trapezoidal prism



$$108 \text{ cm}^3$$

# Volume of a Pyramid



It's actually 1/3rd!    Pyramid/Cone Volume =  $(\frac{1}{3}) * BH$

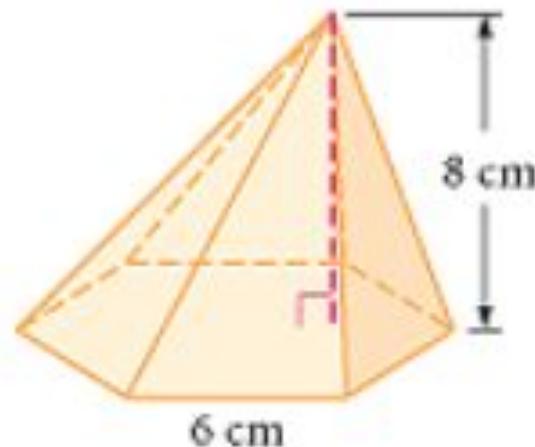
## Pyramid-Cone Volume Conjecture

C-87

If  $B$  is the area of the base of a pyramid or a cone and  $H$  is the height of the solid, then the formula for the volume is  $V = \underline{\hspace{2cm}}$ .

# Example Problem

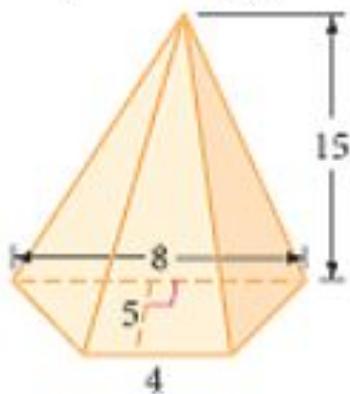
Find the volume of a regular hexagonal pyramid with a height of 8 cm. Each side of its base is 6 cm.



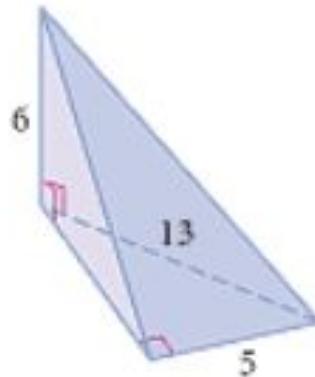
# Problems

3. Trapezoidal pyramid

(h)

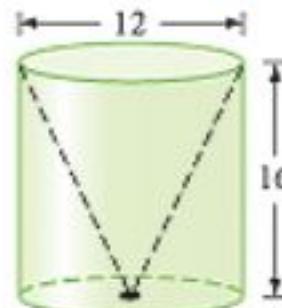


4. Triangular pyramid



6. Cylinder with cone removed

(h)



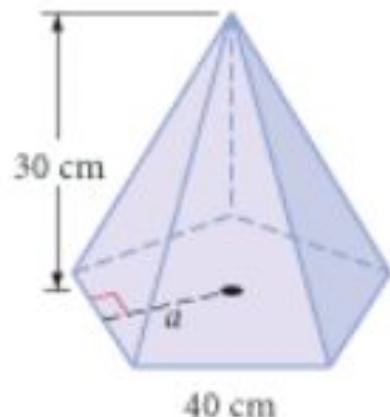
Once you solve this, [See Published Answer Key](#) for solution

# Concept of Mass & Density

$$\text{Mass (gm)} = \text{Volume (cc)} * \text{Density (g/cc)}$$

14. Bretislav has designed a crystal glass sculpture. Part of the piece is in the shape of a large regular pentagonal pyramid, shown at right. The apothem of the base measures 27.5 cm. How much will this part weigh if the glass he plans to use weighs 2.85 grams per cubic centimeter?

Once you solve this, [See Published Answer Key](#) for solution



# Word Problem

Your garden is shaped like a rectangle with dimensions 50 feet by 20 feet.

You want to buy compost, covering the entire garden at a depth of 2 inches.

How many cubic yards of compost should you buy?

# Reminders

Work on Volumes Project

Do Volume of a Sphere Worksheet

Submit U4Q2 Quiz Corrections by Thursday!

Complete your Volumes of Prism & Cylinder Homework

Complete your Volumes of Pyramid & Cone Homework

**Consolidated Unit 4 Test is on Friday, Feb 18th**

**Includes Chapters 8.1, 8.2, 8.4, 8.5, 8.6, 8.7, 9.3, 10.1, 10.2, 10.3, 10.6, 10.7**

Study for it by:

Completing your Homework Assignments & Projects!

Verifying your Answers with the AKs

Submitting your Quiz Corrections

Conference with me - ALL OF YOU!

Thu, Feb 10th

# What's happening today?

Check-in

Review

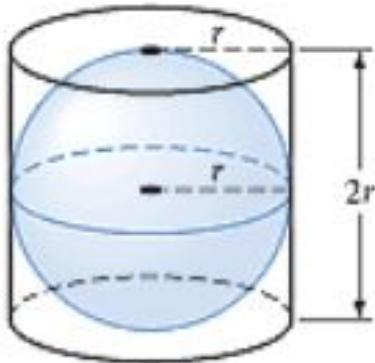
- Volumes of Cylinder & Prism
- Volumes of Cone & Pyramid

Today

- Chap 10.6: Derive Volume of Sphere
- Chap 10.7: Derive Surface Area of Sphere
- Do Sphere Volume and SA problems
- Project Work
- Previous Homework

Reminders

# Volume of Sphere



## Sphere Volume Conjecture

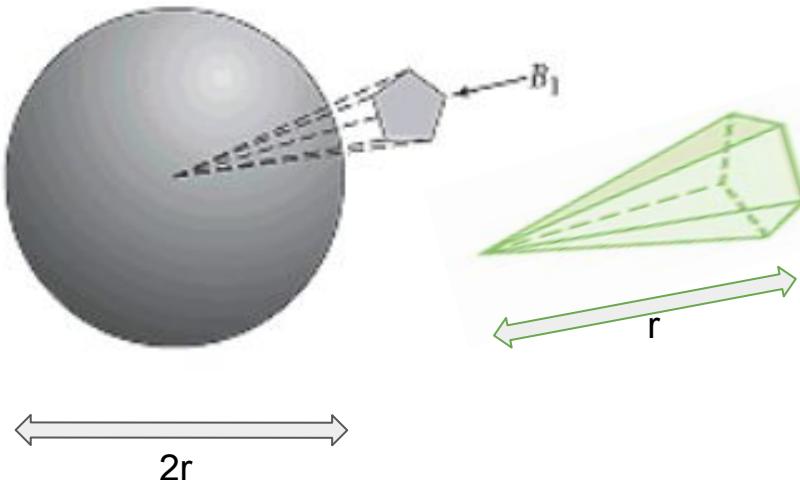
C-88

The volume of a sphere with radius  $r$  is given by the formula  $\frac{4}{3} \pi r^3$ .

$$\frac{4}{3} \pi r^3$$

# Surface Area of Sphere

[Link to Sphere SA Conj Derivation](#)



## Sphere Surface Area Conjecture

C-89

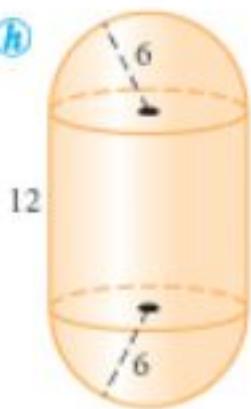
The surface area,  $S$ , of a sphere with radius  $r$  is given by the formula ?.

$$4\pi r^2$$

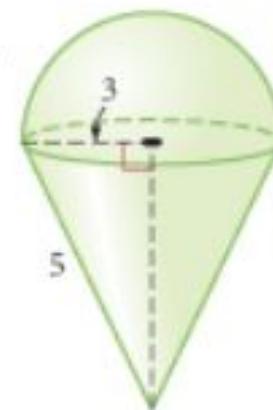
# Do some Sphere Volume Problems

10.6 E&P 2: 4, 6, 8, 13, 16

4.



5.



Once you solve this, [See Published Answer Key](#) for solution

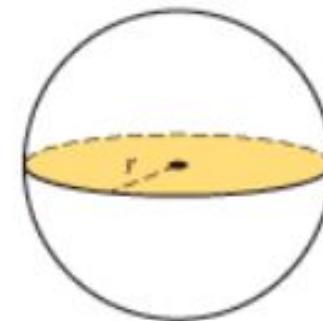
# Do some Sphere Surface Area Problems

10.7 E&P 1: 2, 5, 6, 10, 11

4. The shaded circle at right has area  $40\pi\text{cm}^2$ . Find the surface area of the sphere. [\(h\)](#)

5. Find the volume of a sphere whose surface area is  $64\pi\text{cm}^2$ .

6. Find the surface area of a sphere whose volume is  $288\pi\text{cm}^3$ .



Once you solve this, [See Published Answer Key](#) for solution

# Reminders

Work on Volumes Project

Do Volume of a Sphere Worksheet

Do Sphere Volume & SA Problems

Submit U4Q2 Quiz Corrections by Thursday!

Complete your Volumes of Prism & Cylinder Homework

Complete your Volumes of Pyramid & Cone Homework

Complete your Volume & SA of Sphere Homework!

**Consolidated Unit 4 Test is on Friday, Feb 18th**

**Includes Chapters 8.1, 8.2, 8.4, 8.5, 8.6, 8.7, 9.3, 10.1, 10.2, 10.3, 10.6, 10.7**

Study for it by:

Completing ALL Homework Assignments & Projects!

Verifying your Answers with the AKs

Submitting your Quiz Corrections

Conference with me - **ALL OF YOU!**

Mon-Wed-Fri,  
Feb 14-16-18th

# What's happening today?

## Check-in

Happy Valentine's day!

## References

[AKs to Homework assignments & Quiz U4Q1 have been published](#)

[AKs to Quiz U4Q2 have been published for those who have already submitted QC](#)

[AKs to Practice Problems have been published](#)

[48-problem Practice slideshow published](#)

## Today

Review Toolbox (Conjectures Notebook Day)

Work Catch-up Day

Project Catch-up Day

Unit 4 Test Prep Day

Construct a Platonic Solid Day

## Reminders

# Details for today

1. Review Toolbox (Conjectures Notebook).
2. Complete & Submit your Quiz Corrections with an explanation of problems. you got incorrectly. [The AK\\_U4Q1 is posted here](#). For AK\_U4Q2 you need to schedule conference & submit a Quiz Correction first for problems you got wrong.
3. Review and Validate your answers to your Homework Problems (most recent first).  
[The AKs for all Homework problems and Practice are posted here](#). If incorrect redo them taking clues from AK.
4. [\(New\) I have linked a slideshow of 48 Problems that you can test yourself on. You are welcome to attempt up to 4 problems from the set](#). Solve problem on paper first with images of sketches as needed. Paste it into the slide along with your name on the Title. It will automatically save it there. This is NOT a graded assignment. It is given as a practice problem set. I will accept in lieu for a Quiz Correction problem if you are missing them.

# Unit 4 Test Details

- 1. Consolidated Unit 4 Test is on Friday, Feb 18th**
  - a. Includes Chapters 8.1, 8.2, 8.4, 8.5, 8.6, 8.7, 9.3, 10.1, 10.2, 10.3, 10.6, 10.7**
2. Test expected to be 1-hour long
3. Bring your Conjectures Notebook (Toolbox), Geo Instruments, Calculator.
4. Phones will not be permitted for use as a calculator. Laptops are not allowed,
5. Upto a maximum of 30% can be earned back after submitting a Test Correction with verbal explanation via conference

Tue, Mar 1st

# Tuesday, March 1st

## Check-in

Welcome back! Nice break?

Test Review & Corrections(Please see me in conference)

1. [Access your graded Unit 4 Test & submit Corrections here](#)
2. Good sketches!
3. Trouble with Units for Area/Volume and Units Conversions
4. Issues with 30-60-90 & Isosceles rt-triangles
5. Algebra mistakes
6. Volume & SA Formulas issues

HW - Due Thursday, March 3rd

1. [Watch Ratio, Similarity, and Proportions Video linked here](#) and Portal
2. [Complete Similarity Investigation linked here](#) and on Portal
3. [Submit your Test Corrections here](#)

# Algebra Mistakes

Consider the expression

$$\boxed{a \cdot p + b}$$

what if I want to factor out  $p$ ?

Is it  $(a+b)p$ ?? NO!!

What is

$$\boxed{6\pi + 4} \quad ??$$

- $2^*(3\pi+2)$

# Length Area Volume Units & Conversion

- Units of length in, ft, yd, cm, m, km
- Units of area  $\text{cm}^2$ ,  $\text{m}^2$ ,  $\text{ft}^2$ ,  $\text{in}^2$ ,  $\text{yd}^2$
- Units of volume  $\text{cm}^3$  (cc),  $\text{m}^3$ ,  $\text{in}^3$ ,  $\text{yd}^3$ , gal, litre
- Units of weight kg, gm, lb
- Units of density  $\text{g/cm}^3$ ,  $\text{kg/m}^3$ ,  $\text{lb/in}^3$
- Units of speed miles/hr, km/hr, m/s
- Units of time sec, min, hr
- 12 inches to a foot
- 3-feet per yard
- 2.54 cm to an inch
- 2.2 pounds in a kg
- 100 cm in 1 m
- 8 km in 5 mile
- 1000 g in a kg
- 1000 cc in a litre
- 128 fl-oz in a gal
- 231  $\text{in}^3$  in a gal

How many m are in 1 cm ?  $(1/100) = 0.01 \text{ m}$

How many km are in 1 mile?  $(8/5) = 1.6 \text{ km}$

How many  $\text{m}^2$  are in 200  $\text{cm}^2$  ?  $(1/100)*(1/100)*200 = 0.02 \text{ m}^2$

How many  $\text{yd}^3$  of empty space are in a 5-ft diameter & 18" deep hole with a 5x8" post in it?

$$\begin{aligned} &= \{[\pi*(5/2)^2 \text{ ft}^2] - 5*8 \text{ in}^2 * (1/12)(1/12) \text{ ft}^2/\text{in}^2\} * 18 \text{ in} * (1/12) \text{ ft/in} * (1/27) \text{ yd}^3/\text{ft}^3 \\ &= \{[19.635 - 0.278]*3/2\}*(1/27) = 1.075 \text{ yd}^3 \end{aligned}$$

# Volume conversion problem

How many  $\text{yd}^3$  of empty space are in a 5-ft diameter & 18" deep hole with a 5x8" post in it?

$$\begin{aligned}1 \text{ yd} &= 3 \text{ ft} & 12 \text{ in} &= 1 \text{ ft} \\ \text{Area of Annulus or 'Base Area'} &= \pi \left(\frac{5}{2}\right)^2 \text{ ft}^2 - (5 \times 8) \text{ in}^2 \cdot \frac{1}{144} \frac{\text{ft}^2}{\text{in}^2} \\ &= \frac{25}{4} \pi \text{ ft}^2 - \frac{40}{144} \text{ ft}^2 = \frac{25}{4} \pi - \frac{5}{18} \text{ ft}^2\end{aligned}$$

$$\text{Hole depth} = 18 \text{ in} \times \frac{1}{12} \frac{\text{ft}}{\text{in}} = \frac{18}{12} = \frac{3}{2} \text{ ft}$$

$$\begin{aligned}\text{Volume of hole} &= \left(\frac{25}{4} \pi - \frac{5}{18}\right) \left(\frac{3}{2}\right) \text{ ft}^3 \\ &= \left(\frac{25}{4} \pi - \frac{5}{18}\right) \cancel{\left(\frac{3}{2}\right) \text{ ft}^3} \times \frac{1}{27} \frac{\text{yd}^3}{\text{ft}^3} \\ &= \left(\frac{25}{4} \pi - \frac{5}{18}\right) \frac{1}{18} \text{ yd}^3 \\ &= 1.075 \text{ yd}^3\end{aligned}$$

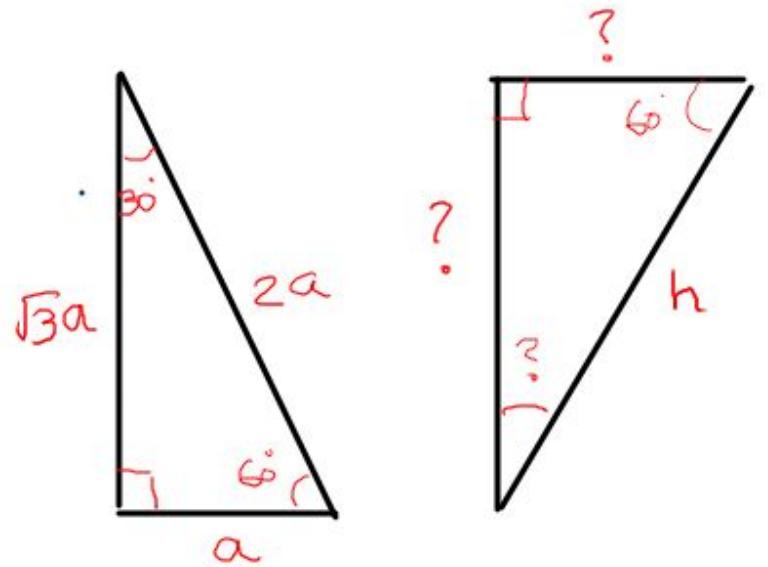
$$\begin{aligned}12 \text{ in} &= 1 \text{ ft} \\ 1 \text{ in} &= \frac{1}{12} \text{ ft} \\ (1 \text{ in})^2 &= \left(\frac{1}{12}\right)^2 \text{ ft}^2 \\ 1^2 \text{ in}^2 &= \frac{1}{144} \text{ ft}^2\end{aligned}$$

$$\begin{aligned}1 \text{ yd} &= 3 \text{ ft} \\ \frac{1}{3} \text{ yd} &= 1 \text{ ft} \\ \frac{1}{27} \text{ yd}^3 &= 1 \text{ ft}^3\end{aligned}$$

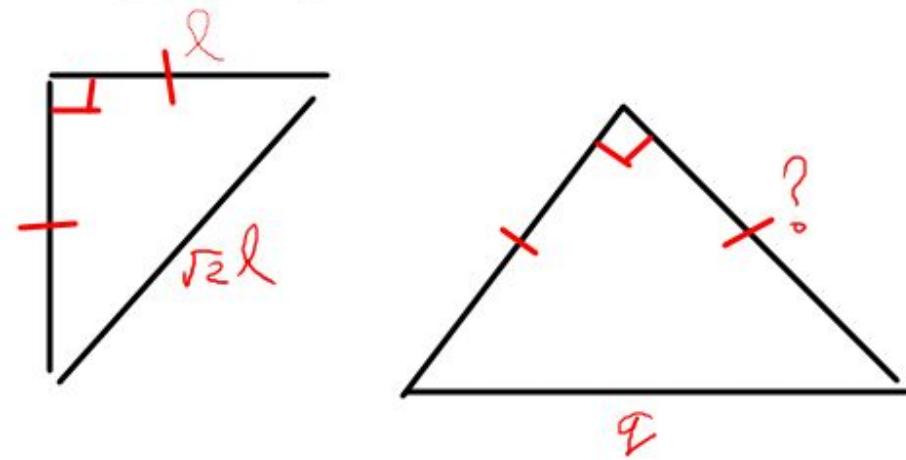
How many  $\text{yd}^3$  of empty space are in a 5-ft diameter & 18" deep hole with a 5x8" post in it?

# 30-60-90 Triangles & Isosceles Rt-Triangles

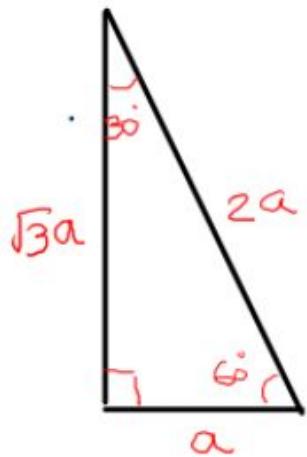
$30^\circ-60^\circ-90^\circ$  triangles



Isosceles Rt-triangles



# 30-60-90 Triangles & Isosceles Rt-Triangles



$$c = \sqrt{(\sqrt{3}a)^2 + a^2} = \sqrt{3a^2 + a^2} = 2a \quad \checkmark$$

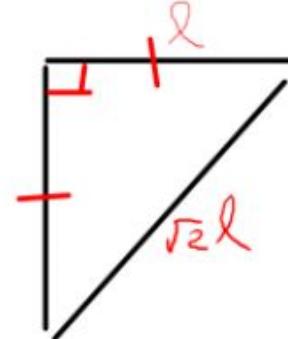
$$\frac{x}{h} = \frac{a}{2a} = \frac{1}{2}$$

$$x = \frac{h}{2}$$

$$h^2 = \left(\frac{h}{2}\right)^2 + \left(\frac{\sqrt{3}h}{2}\right)^2$$

$$= \frac{h^2}{4} + \frac{3h^2}{4}$$

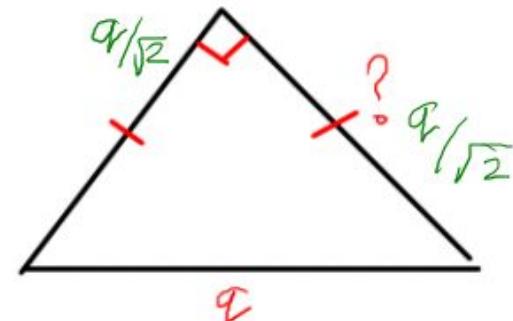
$$= h^2 \quad \checkmark$$



$$c^2 = l^2 + l^2$$

$$= 2l^2$$

$$c = \sqrt{2}l \quad \checkmark$$



$$\frac{q}{x} = \sqrt{2}$$

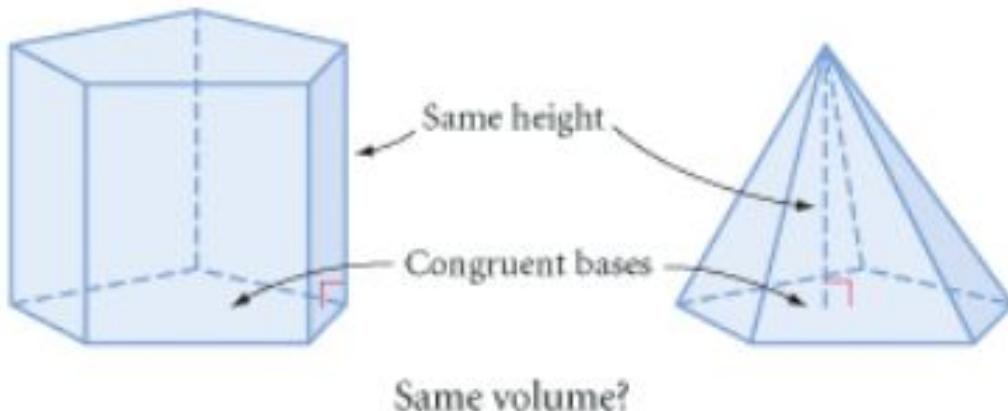
$$x = \frac{q}{\sqrt{2}}$$

$$q^2 = \left(\frac{q}{\sqrt{2}}\right)^2 + \left(\frac{q}{\sqrt{2}}\right)^2$$

$$= \frac{q^2}{2} + \frac{q^2}{2}$$

$$= q^2 \quad \checkmark$$

# Volume of a Pyramid



It's actually 1/3rd!    Pyramid/Cone Volume =  $(\frac{1}{3}) * BH$

## Pyramid-Cone Volume Conjecture

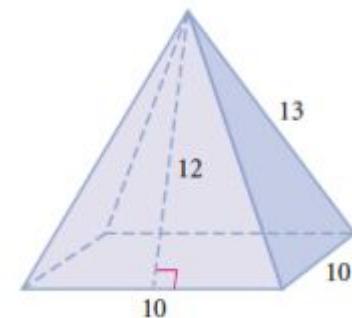
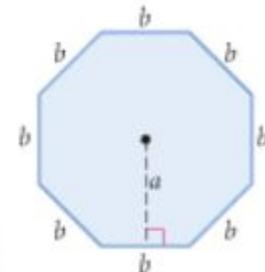
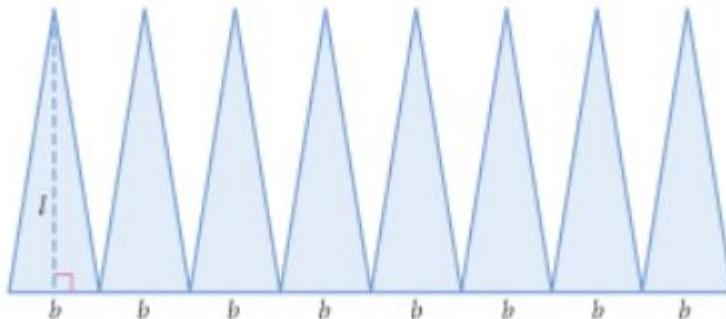
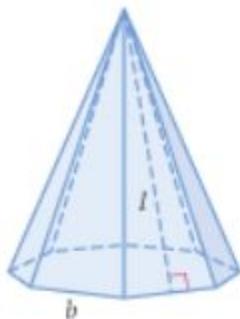
C-87

If  $B$  is the area of the base of a pyramid or a cone and  $H$  is the height of the solid, then the formula for the volume is  $V = \underline{\hspace{2cm}}$ .

# Surface Area of 3-D Shapes - Pyramid



These conservatories in Edmonton, Canada, are glass pyramids.



$$\text{Surface Area of Pyramid} = \frac{1}{2}bn(l + a)$$

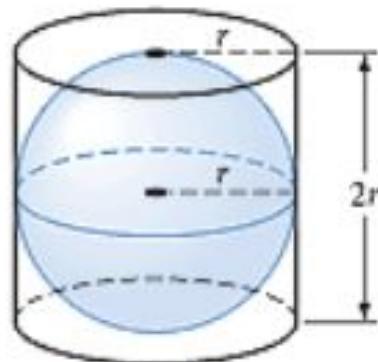
$$\text{Surface Area} = 340 \text{ sq-cm}$$

# Volume of Sphere

By Investigation we discovered that 1 hemisphere of water fills  $\frac{1}{3}$  rd of the cylinder containing it

OR a sphere of water fills  $\frac{2}{3}$  rd of the cylinder containing it

Therefore: Volume of Sphere = ?? in terms of the Cylinder containing it ??



## Sphere Volume Conjecture

C-88

The volume of a sphere with radius  $r$  is given by the formula  $\underline{\quad}$ .

$$\frac{4}{3} \pi r^3$$

Thu, Mar 3rd

# What's happening today?

## Check-in

Did you watch video? Complete 11.1 Investigation?  
Working on Test Correction?

## Today

[Form Random Teams!](#)

Triangle similarity Complete Inv 11.1 & 11.2  
Do problems 11.1: 3, 7, 8, 13, 15

# Units and Proportions practice

1. My car can go 380 miles for a full tank of gasoline
    - a. How many gallons of gas do I need to go to Portland (174 miles)?
    - b. How long will it take at an avg speed of 55 mph?
  2. Chandru takes 9 minutes to run 1 mile
    - a. How long will he take to run a Half-Marathon?
    - b. What is his speed in km/hr?
  3. Tom needs 2" of compost to layer his 300 ft<sup>2</sup> garden.
    - a. How many yd<sup>3</sup> should he buy?
  4. A steel ball has a 1-ft diameter
    - a. Steel has a density of 0.28 lb/in<sup>3</sup>, How heavy is it? Can you lift it?
    - b. Al has a density of 2.7 g/cm<sup>3</sup>, How heavy is an Al ball of the same size?
    - c. What if the Aluminium (or Steel) ball has a 2 ft diameter?
    - d. Can you guess how many times the Volume (or weight) increased?
  5. All sides of a rectangle triple in size
    - a. What proportion does the area increase by?
- 12 inches to a foot
  - 3-feet per yard
  - 2.54 cm to an inch
  - 2.2 pounds in a kg
  - 100 cm in 1 m
  - 1.61 km in 1 mile
  - 1000 g in a kg
  - 1000 cc in a litre
  - Marathon is 26.2 miles
  - Portland is 174 miles from Seattle
  - Chandru's car tank = 15 gallons
  - Mass = Volume x Density in consistent units
  - Volume of a sphere =  $(4/3) \pi r^3$

# Cross-Multiplication Trick for calculating conversions

380 miles => 1 Full tank

380 miles => 15 Gallons

174 miles => ?? Gallons

$174 \times 15 \div 380 \Rightarrow ?? \text{ Gallons} = 6.87 \text{ Gallons}$

1 hour => 55 miles

?? Hours => 174 miles

?? Hours =>  $174 \times 1 \div 55 = 3.16 \text{ Hours or } 3 \text{ hours } 10 \text{ minutes}$

1. 12 inches to a foot
2. 3-feet per yard
3. 2.54 cm to an inch
4. 2.2 pounds in a kg
5. 100 cm in 1 m
6. 1.61 km in 1 mile
7. 1000 g in a kg
8. 1000 cc in a litre
9. Marathon is 26.2 miles
10. Portland is 174 miles from Seattle
11. Chandru's car tank = 15 gallons
12. Mass = Volume x Density in consistent units
13. Volume of a sphere =  $4/3 \pi r^3$

# Cross-multiply - Pb #2

Chandru takes 9 minutes for running 1 Mile How long for  $\frac{1}{2}$  marathon?

$$1 \text{ Mile} \Rightarrow 9 \text{ Minutes}$$

Divide!!

Multiply!

$$\frac{1}{2} * 26.2 \text{ Miles} \Rightarrow \frac{1}{2} * 26.2 * 9 \div 1 = 118 \text{ minutes}$$

$$118 \Rightarrow \text{Minutes} = 1 \text{ hr } 58 \text{ minutes}$$

- 12 inches to a foot
- 3-feet per yard
- 2.54 cm to an inch
- 2.2 pounds in a kg
- 100 cm in 1 m
- 1.61 km in 1 mile
- 1000 g in a kg
- 1000 cc in a litre
- Marathon is 26.2 miles
- Portland is 174 miles from Seattle
- Chandru's car tank = 15 gallons
- Mass = Volume x Density
- Volume of a sphere =  $\frac{4}{3} \pi r^3$

# Trick for calculating conversions - Pb #2 continued

What is Chandru's speed in Km/Hr?

$$\frac{1}{2} * 26.2 \text{ Miles} \Rightarrow 118 \text{ min}$$

$$?? \text{ Miles} \Rightarrow 60 \text{ minutes}$$

$$?? \text{ Miles} \Rightarrow 60 * 13.1 \div 118 = 6.66 \text{ miles in 60 minutes}$$

or 6.6 Miles/Hr

$$1 \text{ Mile} = 1.61 \text{ Km}$$

Divide!!

Multiply!

$$6.6 \text{ M} = 10.56 \text{ Km}$$

or 10.56 Km/hr

- 12 inches to a foot
- 3-feet per yard
- 2.54 cm to an inch
- 2.2 pounds in a kg
- 100 cm in 1 m
- 1.61 km in 1 mile
- 1000 g in a kg
- 1000 cc in a litre
- Marathon is 26.2 miles
- Portland is 174 miles from Seattle
- Chandru's car tank = 15 gallons
- Mass = Volume x Density
- Volume of a sphere =  $\frac{4}{3} \pi r^3$

# Trick for calculating conversions - Pb #3

Tom needs 2" of compost to layer his 300 ft<sup>2</sup> garden. How many yd<sup>3</sup> should he buy?

Volume of Compost = Area of Base X Height

$$12 \text{ in} = 1 \text{ ft}$$

$$2 \text{ in} = 2 * 1 \div 12 \text{ ft} = \frac{1}{6} \text{ ft}$$

$$\text{Volume of compost} = 300 \text{ ft}^2 * \frac{1}{6} \text{ ft} = 50 \text{ ft}^3$$

$$\begin{array}{rcl} 1 \text{ yd} & = & 3 \text{ ft} \\ 1 * 1 * 1 \text{ yd}^3 & = & 3 * 3 * 3 \text{ ft}^3 \end{array}$$

$$\begin{array}{rcl} 1 \text{ yd}^3 & = & 27 \text{ ft}^3 \\ \text{Divide!!} & & \\ \text{Multiply!} & & \\ ??? \text{ yd}^3 & = & 50 \text{ ft}^3 \end{array}$$

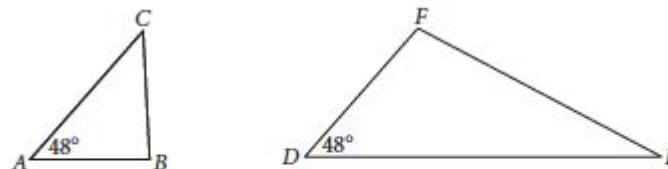
$$??? \text{ yd}^3 = 50 * 1 \div 27 = 1.85 \text{ yd}^3$$

- 12 inches to a foot
- 3-feet per yard
- 2.54 cm to an inch
- 2.2 pounds in a kg
- 100 cm in 1 m
- 1.61 km in 1 mile
- 1000 g in a kg
- 1000 cc in a litre
- Marathon is 26.2 miles
- Portland is 174 miles from Sea
- Chandru's car tank = 15 gallon
- Mass = Volume x Density
- Volume of a sphere =  $\frac{4}{3} \pi r^3$

# Similar Triangles

Similar Quadrilaterals requires corresponding angles to be congruent and corresponding sides to be proportional

Triangles are special - We will look at Similarity Shortcuts (remember Congruency Shortcuts?)



$\angle A \cong \angle D$ , but  $\triangle ABC$  is not similar to  $\triangle DEF$  or to  $\triangle DFE$ .

How about two sets of congruent angles?

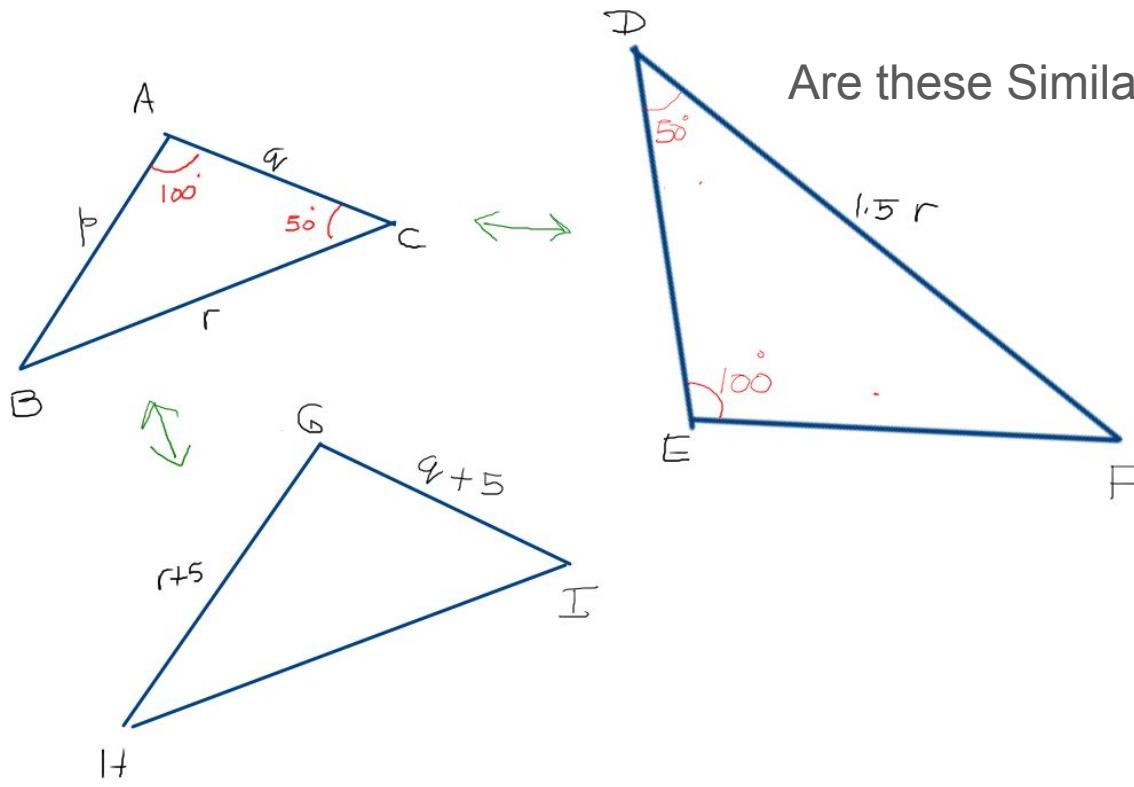
## AA Similarity Conjecture

If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

C-91

$\frac{AB}{DE} \approx \frac{AC}{DF} \approx \frac{BC}{EF}$ ? The ratios should be equal.

# Similar Triangles



# Congruence Shortcuts and Similarity Shortcuts

The Triangle Congruence you studied are: ?

SSS, SAS, ASA, SAA

Can you think how you might write Similarity Shortcuts?

[AA Triangle Similarity Geogebra](#)

[SSS Triangle Similarity Geogebra](#)

[SAS Triangle Similarity Geogebra](#)

# Similar Triangles

Is SSS a similarity Shortcut (Corresponding Sides are Proportional)

Is SAS a similarity Shortcut (Sides Proportional with Included Angle congruent)

What about ASA and SAA ?? (Congruent Angles, Proportional Side)

Lastly SSA - not a Congruency shortcut - is NOT a Similarity shortcut

With this you should be able to do [Similarity Investigation 11-2](#)

Split into teams: [Form Random Teams!](#)

Mon-Wed, Mar 7th & 9th

# What's happening today & next class?

## Check-in/Reminders

Unit 4 Test Correction is due today. AK will be posted early tomorrow and corrections can no longer be accepted.

## Today in class (doing lots of problems in class)

Review Triangle similarity Conjectures

Do Similar Polygons Homework problems 11.1: 3, 7, 8, 13, 15

Do Similar Triangles Homework problems 11.2: 2, 3, 5, 6, 8

Chapter 11.3: Indirect Measurements with Similar Triangles Project

## Announcement

Unit 5 Quiz 1 on 11.1 11.2 11.3 11.4 on Tuesday March 15th

# What does Similar mean?

## Similar Polygons

- Corresponding Angles are Congruent and the Corresponding Sides being Proportional in length
- This definition works for Polygons not so much for Circles and Spheres!
- For Triangles Corresponding Sides can be found by using the “Opposite Side” of the angle idea

# What is Dilation?

Dilation is to Enlarge or Reduce figures by the same Proportion (a.k.a Scale Factor) without distorting the shape

Think enlarging/reducing photographs without making one look fatter or thinner!

For a Rectangle: Scale Factor is the ratio Width / Height

Original 1"x.84"    Proportional 2"x1.68"    Not Proportional



2"x2.84"



Dilation Similarity Conjecture

C-90

the polygons are similar

If one polygon is a dilated image of another polygon, then   .

# Similar Triangles Conjectures

## AA Similarity Conjecture

C-91

If 2 angles of one triangle are congruent to 2 angles of another triangle,  
then **Similar**

## SSS Similarity Conjecture

C-92

If the three sides of one triangle are proportional to the three sides of another  
triangle, then the two triangles are **Similar**

## SAS Similarity Conjecture

C-93

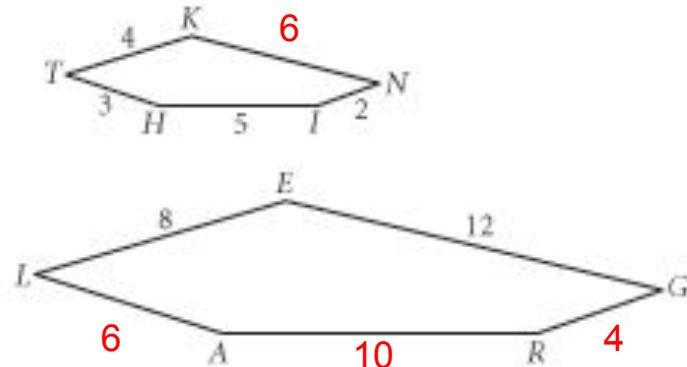
If two sides of one triangle are proportional to two sides of another triangle  
and ?, then the ? **Triangles are similar**  
**included angle is congruent**

# Do some problems

Chapter 11.1: 3, 7, 8, 13, 15 (Pg 587)

7. *THINK ~ LARGE*

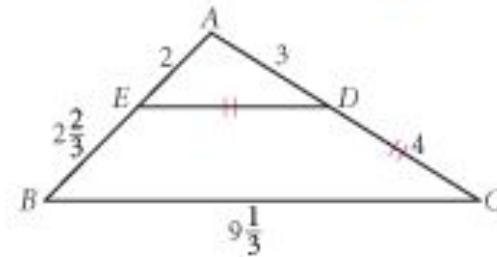
Find  $AL$ ,  $RA$ ,  $RG$ , and  $KN$ .



13.  $\overline{DE} \parallel \overline{BC}$

Are the corresponding angles congruent in  $\triangle AED$  and  $\triangle ABC$ ? Are the corresponding sides proportional?

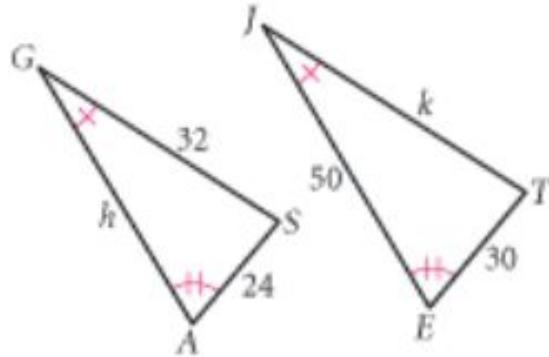
Is  $\triangle AED \sim \triangle ABC$ ? (h)



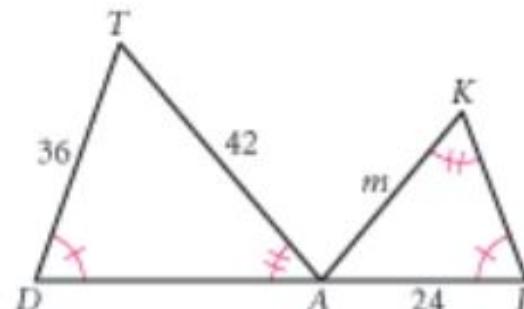
# Do some problems

Chapter 11.2: 2, 3, 5, 6, 8 (Pg 591)

2.  $h = \underline{?}$ ,  $k = \underline{?}$        $h = k = 40 \text{ cm}$



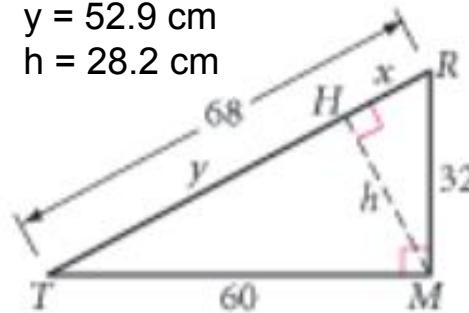
3.  $m = \underline{?}$



8. Why is  $\triangle TMR \sim \triangle THM \sim \triangle MHR$ ?

Find  $x$ ,  $y$ , and  $h$ .

$$\begin{aligned}x &= 15.1 \text{ cm} \\y &= 52.9 \text{ cm} \\h &= 28.2 \text{ cm}\end{aligned}$$



Find the similar triangles first before proceeding. There are 3 of them!

# Manipulating Fractions

Conversions :

Improper to Proper  
or Mixed

Improper Fraction }  $\frac{17}{4}, \frac{3}{2}$     Mixed Fraction :  $2\frac{1}{3}, 1\frac{6}{7}$

$$\begin{aligned} & \text{Mixed to Improper } \left\{ \begin{array}{l} \text{or proper} \\ \text{or simply} \end{array} \right\} : 7\frac{1}{8} = 7 + \frac{1}{8} = \frac{7}{1} + \frac{1}{8} = \frac{56+1}{8} \leftarrow \text{LCD} = \frac{57}{8} \\ & \quad \begin{array}{l} \text{add} \\ \text{mul} \end{array} \quad \begin{array}{l} \text{add} \\ \text{mul} \end{array} \quad \checkmark \end{aligned}$$

Multiplying 3

$$7\frac{2}{3} \times 6\frac{5}{4} = \frac{23}{3} \times \frac{29}{4} = \frac{667}{12}$$

Adding :

$$\frac{3}{7} + \frac{4}{5} = \frac{15+28}{35} \quad \begin{array}{l} \text{find LCD} \\ \checkmark \end{array}$$

Dividing 3

$$\frac{\frac{a}{c}}{\left(\frac{b}{c}\right)} = \frac{ac}{b} \quad \text{and} \quad \frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\begin{aligned} \frac{\left(\frac{1}{3}\right)}{5} &= \frac{7}{15} = \frac{5+2}{15} = \frac{1}{3} + \frac{2}{15} \\ &\downarrow \quad \begin{array}{l} \text{num} \\ \text{reciprocal} \end{array} \quad \begin{array}{l} \text{den} \\ \text{den} \end{array} \\ &= \left(\frac{1}{3}\right) \times \frac{1}{5} = \frac{7}{15} \end{aligned}$$

$$\begin{aligned} \frac{7}{\left(\frac{3}{5}\right)} &= \frac{35}{3} = \frac{33+2}{3} = 11\frac{2}{3} \\ &\downarrow = 7 \times \frac{5}{3} \quad \begin{array}{l} \text{Reciprocal} \\ \text{num} \end{array} \quad \begin{array}{l} \text{den} \\ \text{den} \end{array} \end{aligned}$$

Cross Multiplying

$$\begin{aligned} \text{If } \frac{a}{b} = \frac{c}{d} \quad \text{then } ad = bc \quad \begin{array}{l} \text{Calc X} \\ \frac{1}{4} - \frac{2}{3} \\ \cancel{X} \end{array} \Rightarrow \begin{cases} 7x = 4 \times \frac{8}{3} = \frac{32}{3} \\ x = \frac{32}{21} \end{cases} \end{aligned}$$

Linked PDF here

## Procedure for Solving Similar Polygons/Triangles

- ① Name all vertices
- ② Mark all givens and measures
- ③ Find Congruent angles & Mark them
- ④ Find Congruent Sides & Mark them
- ⑤ Split into suspected Similar triangles using same vertex Names
- ⑥ Transfer all notations to all split triangles
- ⑦ Apply AA, SSS, SAS Similarity Conjectures to determine Similarity
- ⑧ Apply Polygon Similarity Conjecture if not triangle (All angles Congruent)  
(All sides Proportional)
- ⑨ For Similar triangles Mark all angles Congruent (Reverse Similarity Conjecture)
- ⑩ Set up ratio Numerators for each triangle pair Line Segments only
- ⑪ Follow Congruent angle to the similar triangle to find opposite side
- ⑫ Set up ratio Denominators for each triangle pair Line Segments Only
- ⑬ Substitute known and unknown measures for Line Segments
- ⑭ Cross multiply and solve for unknowns!

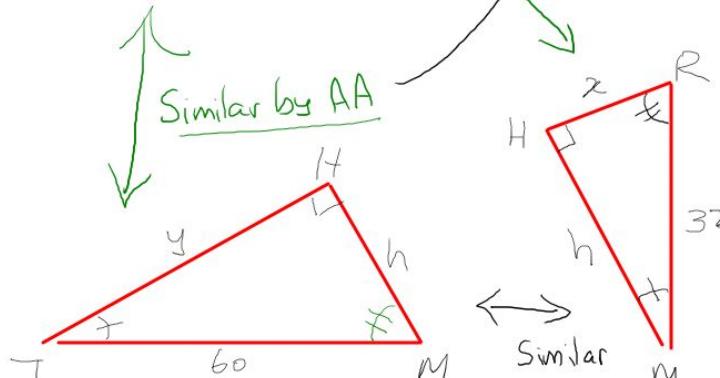
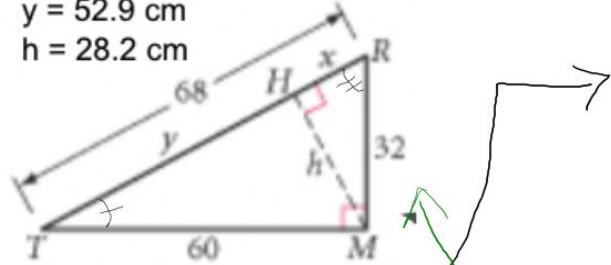
[Linked PDF here](#)

# Problem #8 solution

$$x = 15.1 \text{ cm}$$

$$y = 52.9 \text{ cm}$$

$$h = 28.2 \text{ cm}$$



$\angle TMH \cong \angle TRM !$

Because  $\triangle TMR \sim \triangle THM$

$$\frac{RM}{HM} = \frac{TM}{TH} = \frac{TR}{TM}$$

$$\frac{32}{h} = \frac{60}{y} = \frac{x+y}{60} = \frac{68}{60}$$

$$h = \frac{60 \times 32}{68} = 28.2 \quad y = \frac{60 \times 60}{68} = 52.9$$

$$\frac{TM}{RM} = \frac{MH}{HR} = \frac{TH}{HM}$$

$$\frac{60}{32} = \frac{h}{x} = \frac{y}{h}$$

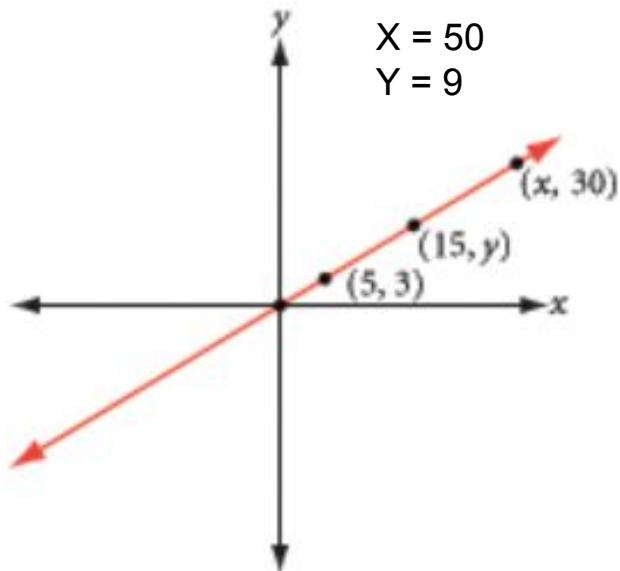
$$\frac{60}{32} = \frac{28.2}{x} = \frac{52.9}{28.2}$$

$$x = \frac{28.2 \times 32}{60} = 15.05$$

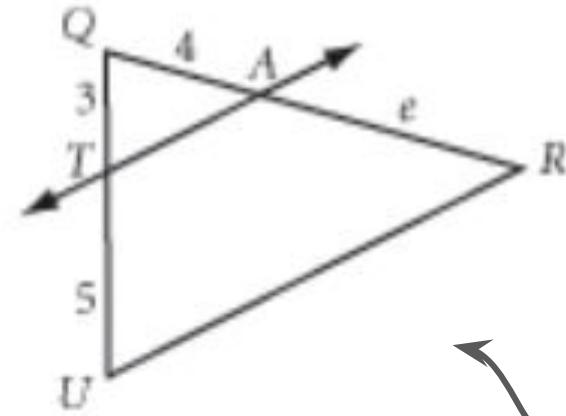
# Do some more problems

Chapter 11.2: 9, 15 (Pg 592)

15. Find  $x$  and  $y$ . (h)



9.  $\overline{TA} \parallel \overline{UR}$   
 $e = ?$  6.666 cm



Find the similar triangles first before proceeding.

# Reminders

Work on Test Correction

Complete [11.1 Homework](#) by Wednesday

Complete [11.2 Homework](#) by Friday

Complete [11.3 Homework](#) by Tuesday, March 15th

Complete [Mirror Mirror Project](#) by Tuesday, March 15th

U5Q1: Quiz on 11.1, 11.2 & 11.3 on March 15th

Fri, Mar 11th

# What's happening today?

## Check-in

Unit 4 Test Correction is closed. Grades returned. If you want to still improve your grade, you can do and submit a bonus problem from that test

Grade sheets sent

## Today in class

Do some Quiz practice problems using procedure discussed in last class

Review Similarity Project (Do during weekend)

Chapter 11.3: Indirect Measurements with Similar Triangles Project

Do Chapter 11.3 Problems & Proofs

## Reminders

Complete 11.1, 11.2, 11.3 Homework & Project. This is the Practice for Quiz!!

Unit 5 Quiz 1 on 11.1 11.2 11.3 11.4 on Tuesday March 15th

11.1 AK published, 11.2 & 11.3 AK will be published over weekend

# Similar Triangles Conjectures

## AA Similarity Conjecture

C-91

If 2 angles of one triangle are congruent to 2 angles of another triangle,  
then **Similar**

## SSS Similarity Conjecture

C-92

If the three sides of one triangle are proportional to the three sides of another  
triangle, then the two triangles are **Similar**

## SAS Similarity Conjecture

C-93

If two sides of one triangle are proportional to two sides of another triangle  
and ?, then the ? **Triangles are similar**  
**included angle is congruent**

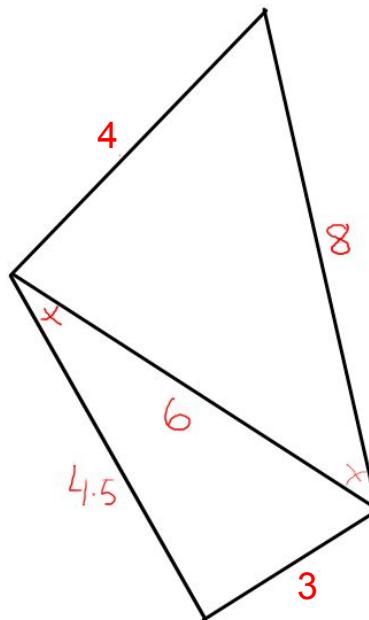
## Procedure for Solving Similar Polygons/Triangles

- ① Name all vertices
- ② Mark all givens and measures
- ③ Find Congruent angles & Mark them
- ④ Find Congruent Sides & Mark them
- ⑤ Split into suspected Similar triangles using same vertex Names
- ⑥ Transfer all notations to all split triangles
- ⑦ Apply AA, SSS, SAS Similarity Conjectures to determine Similarity
- ⑧ Apply Polygon Similarity Conjecture if not triangle (All angles Congruent)  
(All sides Proportional)
- ⑨ For Similar triangles Mark all angles Congruent (Reverse Similarity Conjecture)
- ⑩ Set up ratio Numerators for each triangle pair Line Segments only
- ⑪ Follow Congruent angle to the similar triangle to find opposite side
- ⑫ Set up ratio Denominators for each triangle pair Line Segments Only
- ⑬ Substitute known and unknown measures for Line Segments
- ⑭ Cross multiply and Solve for Unknowns!

[Linked PDF here](#)

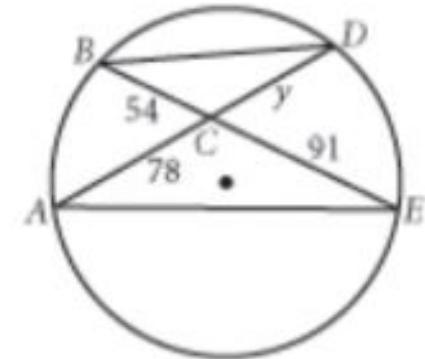
# Quiz Practice - Check if you can figure this out

Strictly follow the steps established in previous class  
Think of these problems as a mini quiz!



Find all missing dimensions

12. Find  $y$ .

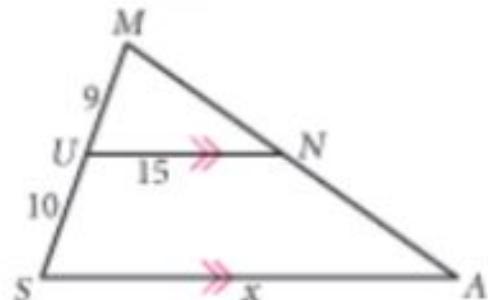


$$y = 63$$

Hint: Use Inscribed Angles Conjecture.  
Look in your notebook for Circles conjectures

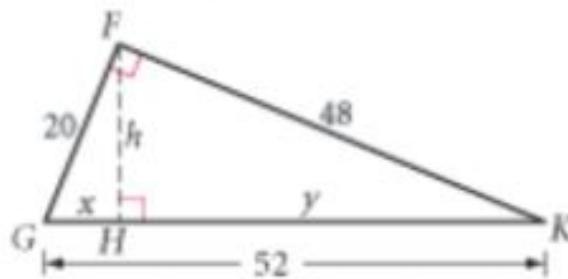
# More Quiz Practice

11. Find  $x$ . 



$$x = 31.666$$

13. Find  $x$ ,  $y$ , and  $h$ . 



$$x = 7.69$$

$$y = 44.3$$

$$h = 18.46$$

# Chapter 11.3: Indirect Measurements

Pg# 598

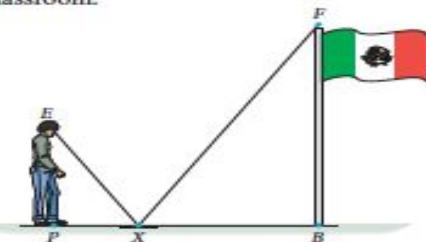


## Investigation Mirror, Mirror

### You will need

- metersticks
- masking tape or a soluble pen
- a mirror

Choose a tall object with a height that would be difficult to measure directly, such as a football goalpost, a basketball hoop, a flagpole, or the height of your classroom.



Mark crosshairs on your mirror. Use tape or a soluble pen. Call the intersection point  $X$ . Place the mirror on the ground several meters from your object.

An observer should move to a point  $P$  in line with the object and the mirror in order to see the reflection of an identifiable point  $F$  at the top of the object at point  $X$  on the mirror. Make a sketch of your setup, like this one.

Measure the distance  $PX$  and the distance from  $X$  to a point  $B$  at the base of the object directly below  $F$ . Measure the distance from  $P$  to the observer's eye level,  $E$ .

Think of  $\overline{FX}$  as a light ray that bounces back to the observer's eye along  $\overline{XE}$ . Why is  $\angle B \cong \angle P$ ? Name two similar triangles. Tell why they are similar.

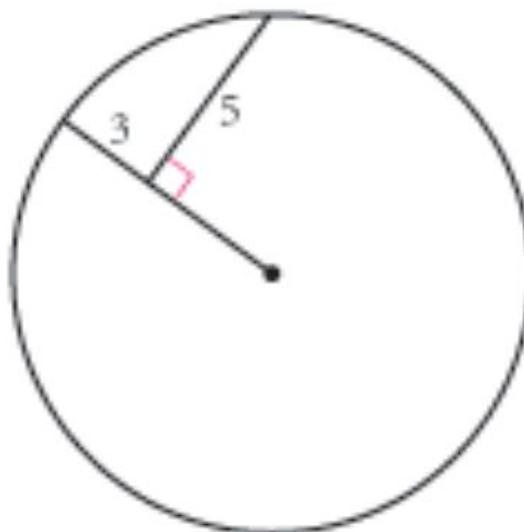
Set up a proportion using corresponding sides of similar triangles. Use it to calculate  $FB$ , the approximate height of the tall object.

Write a summary of what you and your group did in this investigation. Discuss possible causes for error.

PROJECT: Mirror Mirror Investigation Assigned

## 11.3 - Do some problems

15. Find the radius of the circle.



Hint: Nothing to do with  
similar triangles!

$$\begin{aligned}\text{Radius} &= 5 \frac{2}{3} \\ &= 5.666\end{aligned}$$

# Reminders

Work on Test Correction

Complete [11.1 Homework](#) by Wednesday

Complete [11.2 Homework](#) by Friday

Complete [11.3 Homework](#) by Tuesday, March 15th

Complete [Mirror Mirror Project](#) by Tuesday, March 15th

U5Q1: Quiz on 11.1, 11.2 & 11.3 on March 15th

Tue, Mar 15th

# What's happening today?

Check-in

[AK for 11\\_2, 11\\_3 Published](#)

Today in class

Review

Project completed?

Quiz U5Q1

Reminders

Tue/Thu, Mar 22/24

# What's happening today?

Check-in

Welcome back!

Thought question - Are all circles similar?

Today in class - Area, Surface Area and Volume Ratios of similar shapes

Open grades week

Review Quiz, Project

Skipping Chapter 11.4 ...

Proportions with Areas & Volumes (Chapter 11.5 & 11.6)

Do problems from 11.5:1-4 & 11.6:1-4

Skipping Chapter 11.7 ...

Reminders

Finish these as Homework

Compare/Review your answers with AK posted here!

# Proportions with Areas

## Proportional Areas Conjecture

C-96

If corresponding side lengths of two similar polygons or the radii of two circles compare in the ratio  $\frac{m}{n}$ , then their areas compare in the ratio  $\frac{?}{\frac{m^2}{n^2}}$ .

This conjecture is also true for Circles!

This conjecture is also true for Surface Areas of solids!

# How do you construct proportional 2D shapes

Let's examine formulas for Areas & Surface Areas: Slides Chap 8 slides 54-60, 94, 110

Let's examine formulas for Volumes: Slides 76, 77, 83, 85, 93 etc

# Let's do some problems

## **EXAMPLE**

If you need 3 oz of shredded cheese to cover a medium 12 in. diameter pizza, how much shredded cheese would you need to cover a large 16 in. diameter pizza?

$$5\frac{1}{3} \text{ oz}$$

# Let's do some problems

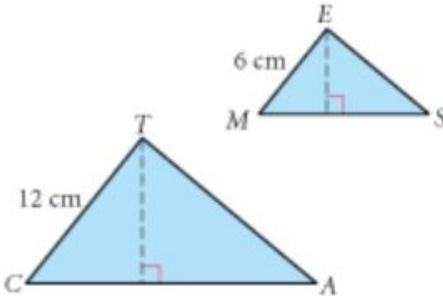
Chap  
11.5:1-4

1.  $\triangle CAT \sim \triangle MSE$

area of  $\triangle CAT = 72 \text{ cm}^2$

area of  $\triangle MSE = ?$  (h)

$18 \text{ cm}^2$



2.  $RECT \sim ANGL$

$\frac{\text{area of } RECT}{\text{area of } ANGL} = \frac{9}{16}$

$TR = ? \quad 18$

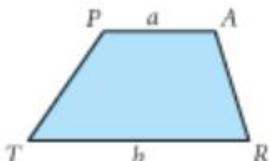
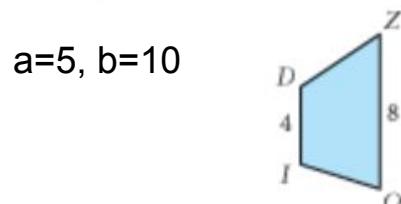


3.  $TRAP \sim ZOID$  (h)

$\frac{\text{area of } ZOID}{\text{area of } TRAP} = \frac{16}{25}$

$a = ?$ ,  $b = ?$

$a=5$ ,  $b=10$

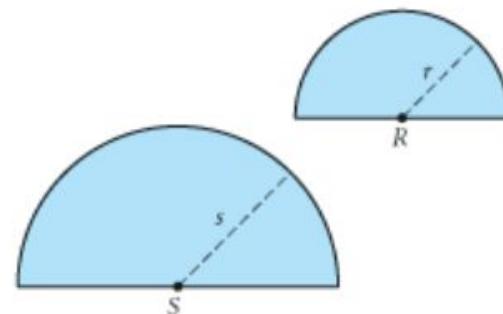


4. semicircle  $R \sim$  semicircle  $S$

$\frac{r}{s} = \frac{3}{5}$

area of semicircle  $S = 75\pi \text{ cm}^2$

area of semicircle  $R = ? \quad 27\pi \text{ cm}^2$



# Proportions with Volumes

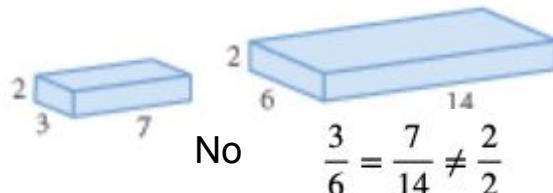
## Proportional Volumes Conjecture

C-97

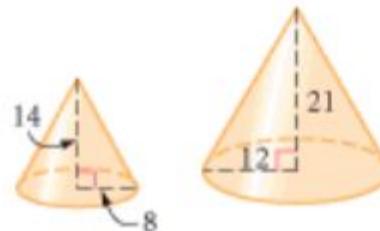
If corresponding edge lengths (or radii, or heights) of two similar solids compare in the ratio  $\frac{m}{n}$ , then their volumes compare in the ratio  $\frac{m^3}{n^3}$ .

This conjecture is true for Prisms & Pyramids, Cylinders & Cones

Are these right rectangular prisms similar?



Are these right circular cones similar?



Yes

$$\frac{21}{14} = \frac{12}{8} = \sqrt[3]{\frac{12^2 * 21}{8^2 * 14}} = 1.5$$

# Let's do some real-world problems

The diameter of a soccer ball is about 8.75 in. and the diameter of a tennis ball is about 2.5 in. How many times more surface material is needed to make the outside of a soccer ball than a tennis ball? How many times more air does a soccer ball hold than a tennis ball?

$$12 \frac{1}{4} \quad 42 \frac{7}{8}$$

# Let's do some Volume ratios problems

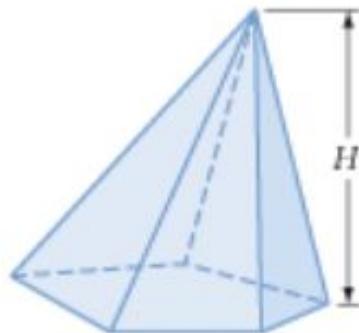
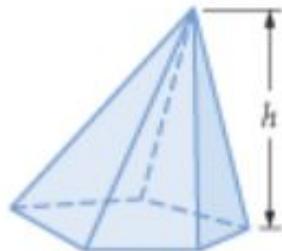
Chap 11.6:1-4

1. The pentagonal pyramids are similar.

$$\frac{h}{H} = \frac{4}{7}$$

volume of large pyramid = ?  $1715 \text{ cm}^3$

volume of small pyramid =  $320 \text{ cm}^3$  



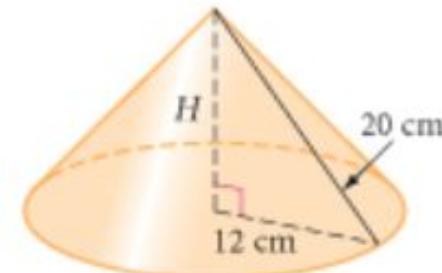
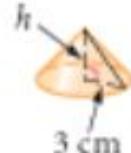
2. These right cones are similar.

$$H = \underline{?}, h = \underline{?} \quad H=16, h=4$$

volume of large cone = ?  $768\pi \text{ cm}^3$

volume of small cone = ?  $12\pi \text{ cm}^3$

$$\frac{\text{volume of large cone}}{\text{volume of small cone}} = \underline{\frac{?}{?}} \quad \frac{64}{1}$$



**Construction tools**  
for Exercises 19 and 20

# Let's do some problems

3. These right trapezoidal prisms are similar.

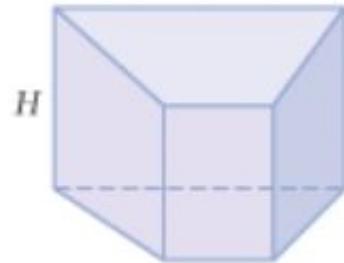
$$\text{volume of small prism} = 324 \text{ cm}^3$$

$$\frac{\text{area of base of small prism}}{\text{area of base of large prism}} = \frac{9}{25}$$

$$\frac{h}{H} = \frac{?}{5}$$

$$\frac{\text{volume of large prism}}{\text{volume of small prism}} = \frac{?}{27}$$

$$\text{volume of large prism} = ? \text{ } 1500 \text{ cm}^3$$



Chap 11.6:1-4

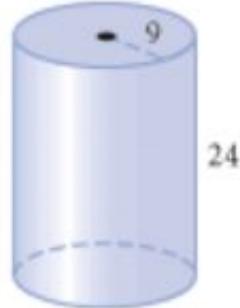
4. These right cylinders are similar.

$$\text{volume of large cylinder} = 4608\pi \text{ ft}^3$$

$$\text{volume of small cylinder} = ? \text{ } 1944 \pi \text{ ft}^3$$

$$\frac{\text{volume of large cylinder}}{\text{volume of small cylinder}} = ? \text{ } 2.37$$

$$H = ? \text{ } 32 \text{ ft}$$



7. The ratio of the weights of two spherical steel balls is  $\frac{8}{27}$ . What is the ratio of their diameters?

$$\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$$

8. **Application** The energy (and cost) needed to operate an air conditioner is proportional to the volume of the space that is being cooled. It costs ZAP Electronics about \$125 per day to run an air conditioner in their small rectangular warehouse. The company's large warehouse, a few blocks away, is 2.5 times as long, wide, and high as the small warehouse. Estimate the daily cost of cooling the large warehouse with the same model of air conditioner.

(h)  $2.5^3 * 125 = \$1953$

9. **Application** A sculptor creates a small bronze statue that weighs 38 lb. She plans to make a version that will be four times as large in each dimension. How much will this larger statue weigh if it is also bronze?  $4^3 * 38 = 2432$  lb



This bronze sculpture by Camille Claudel (1864–1943) is titled *La Petite Chatelaine*. Claudel was a notable French artist and student of Auguste Rodin, whose famous sculptures include *The Thinker*.

10. A tabloid magazine at a supermarket checkout exclaims, “Scientists Breed 4-Foot-Tall Chicken.” A photo shows a giant chicken that supposedly weighs 74 pounds and will solve the world’s hunger problem. What do you think about this headline?

Assuming an average chicken stands 14 inches tall and weighs 7 pounds, would a 4-foot chicken weigh 74 pounds? Is it possible for a chicken to be 4 feet tall? Explain your reasoning.

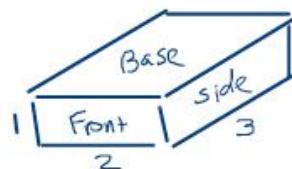
NO!

$$\left(\frac{4 * 12}{14}\right)^3 * 7 = 282 \text{ lb}$$

4-ft chicken would Weigh 282 lb



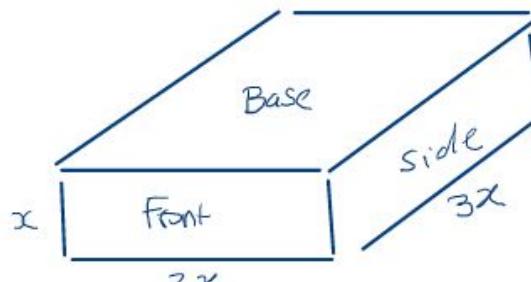
14. Imagine stretching all three dimensions of a 2-cm-by-3-cm-by-1-cm rectangular prism by multiplying them by scale factor  $x$ . Make a table of the surface area and volume of the prism for values of  $x$  from 1 to 5 and plot both sets of points on a graph. Write function  $S(x)$  that gives the surface area of the prism as a function of  $x$  and function  $V(x)$  that gives the volume of the prism as a function of  $x$ . How do their equations and graphs differ?



$$\begin{aligned} SA &= 2 \times \text{Area of Base} \\ &\quad + 2 \times \text{Area of front} \\ &\quad + 2 \times \text{Area of side} \end{aligned}$$

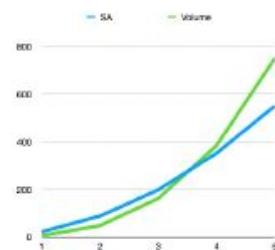
$$\begin{aligned} &= 2 \cdot 2x \cdot 3x + \\ &\quad 2 \cdot 2x \cdot x + \\ &\quad 2 \cdot x \cdot 3x = \underline{\underline{22x^2}} \end{aligned}$$

$$\begin{aligned} Vol &= \frac{\text{Area of Base} \times \text{height}}{x} = 2x \cdot 3x \cdot x = \underline{\underline{6x^3}} \end{aligned}$$



$x$	$22x^2$	$6x^3$
$x$	Surf Area	Volume
1	$22 \text{ cm}^2$	$6 \text{ cm}^3$
2	$88 \text{ cm}^2$	$48 \text{ cm}^3$
3	$198 \text{ cm}^2$	$162 \text{ cm}^3$
4	$352 \text{ cm}^2$	$384 \text{ cm}^3$
5	$550 \text{ cm}^2$	$750 \text{ cm}^3$

$$\begin{aligned} S(x) &= 22x^2 \\ V(x) &= 6x^3 \end{aligned}$$



$x$	$22x^2$	$6x^3$
$x$	SA	Volume
1	22	6
2	88	48
3	198	162
4	352	384
5	550	750

# Let's answer some real-world questions!

## Body Temperature

Every living thing processes food for energy. An animal's body produces energy in proportion to its volume. This energy creates heat that radiates from the animal's surface.



- Step 1      Imagine two similar animals, one with dimensions three times as large as those of the other.
- Step 2      How would the surface areas of these two animals compare? How much more heat could the larger animal radiate through its surface?
- Step 3      How would the volumes of these two animals compare? How many times as much heat would the larger animal produce?
- Step 4      Review your answers from Steps 2 and 3. How many times as much heat must each square centimeter of the larger animal radiate? Would this be good or bad?
- Step 5      Use what you have concluded to answer these questions. Consider size, surface area, and volume.
  - a. Why do large objects cool more slowly than similar small objects?
  - b. Why is a beached whale more likely than a beached dolphin to experience overheating?
  - c. Why are larger mammals found closer to the poles than the equator?
  - d. If a woman and a small child fall into a cold lake, why is the child in greater danger of hypothermia?

AK for 11.5 & 11.6  
plus the additional  
problems in this  
chapter!

Verify your answers to the AK for all problems in this section!

AK for 11.5 & 11.6 plus the additional problems in  
this chapter!

Mon/Wed, Mar 28/29th

# What's happening today?

Check-in

Welcome back!

Reminders

Submitted your U5Q1 Quiz correction?

- REQUIRED TO DO BONUS PROBLEM for 100% MARKS

Completed 11.5 & 11.6 problems?

[Compare/Review your answers with AK posted here!](#)

Today in class

Review Chapter 11 Problems

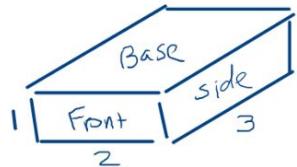
Chapter 12 - Trigonometry!

Reminders

U5Q2 TEST covering Chapter 11 & 12 on Thursday April 7th

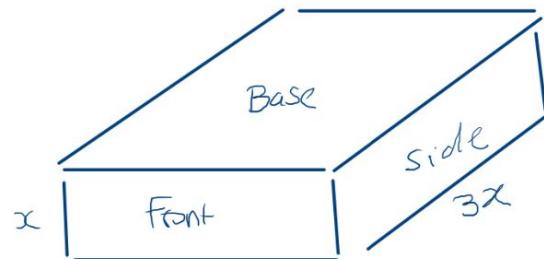
# Review Chapter 11.6 problem about Functions

14. Imagine stretching all three dimensions of a 2-cm-by-3-cm-by-1-cm rectangular prism by multiplying them by scale factor  $x$ . Make a table of the surface area and volume of the prism for values of  $x$  from 1 to 5 and plot both sets of points on a graph. Write function  $S(x)$  that gives the surface area of the prism as a function of  $x$  and function  $V(x)$  that gives the volume of the prism as a function of  $x$ . How do their equations and graphs differ?

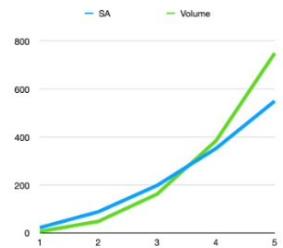


$$\begin{aligned} SA &= 2 \times \text{Area of Base} \\ &\quad + 2 \times \text{Area of front} \\ &\quad + 2 \times \text{Area of side} \end{aligned}$$

$$\begin{aligned} &= 2 \cdot 2x \cdot 3x + 2 \cdot 2x \cdot x + 2 \cdot x \cdot 3x \\ &= 12x^2 + 4x^2 + 6x^2 = \underline{\underline{22x^2}} \end{aligned}$$



$x$	$22x^2$	$6x^3$
	Surf Area	Volume
1	$22 \text{ cm}^2$	$6 \text{ cm}^3$
2	$88 \text{ cm}^2$	$48 \text{ cm}^3$
3	$198 \text{ cm}^2$	$162 \text{ cm}^3$
4	$352 \text{ cm}^2$	$384 \text{ cm}^3$
5	$550 \text{ cm}^2$	$750 \text{ cm}^3$



$x$	$22 \times x \times x$	$6 \times x \times x \times x$
	SA	Volume
1	22	6
2	88	48
3	198	162
4	352	384
5	550	750

# Trigonometry! Study of ratios of Sides of Right Triangles

**T**rigonometry is the study of the relationships between the sides and the angles of triangles. In this lesson you will discover some of these relationships for right triangles.

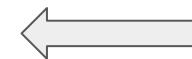
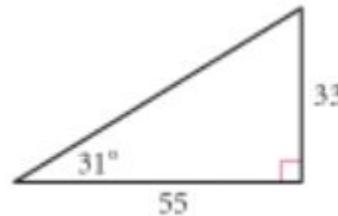
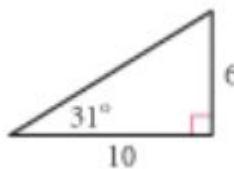
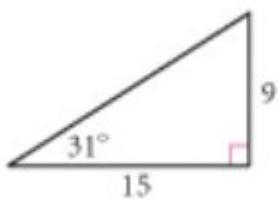
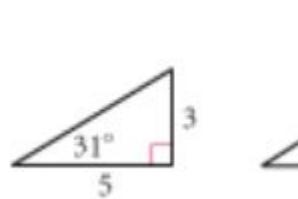
## Astronomy

### CONNECTION

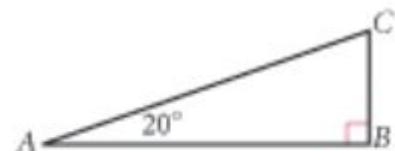
Trigonometry has origins in astronomy. The Greek astronomer Claudius Ptolemy (100–170 C.E.) used tables of chord ratios in his book known as *Almagest*. These chord ratios and their related angles were used to describe the motion of planets in what were thought to be circular orbits. This woodcut shows Ptolemy using astronomy tools.



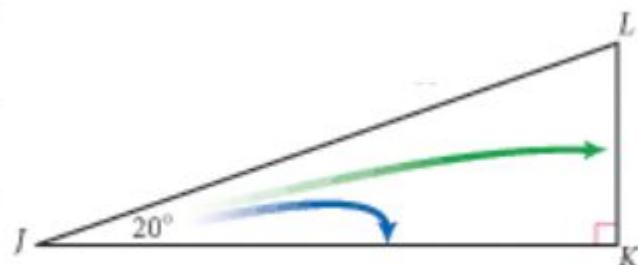
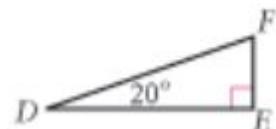
# Angles and Sides of Right-Triangles



Are these  
similar  
Triangles?



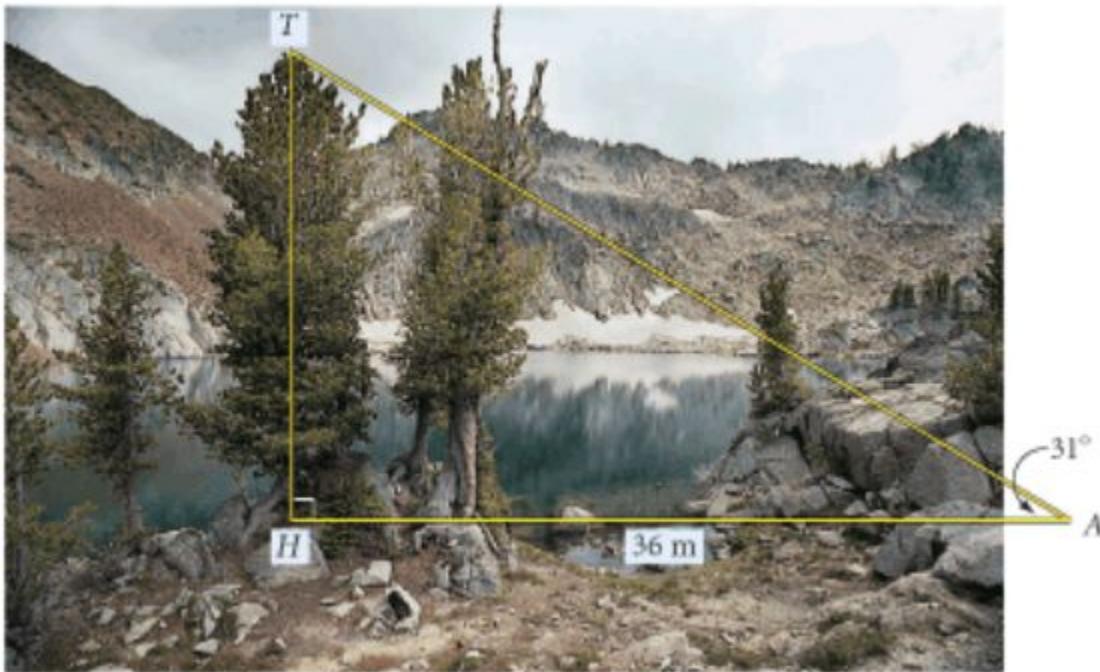
$$\frac{CB}{FE} = \frac{AB}{DE} \Rightarrow CB * DE = FE * AB \Rightarrow \frac{CB * DE}{AB} = FE \Rightarrow \frac{CB}{FE} = \frac{AB}{DE}$$



This side is called  
the **opposite leg**  
because it is across  
from the 20° angle.

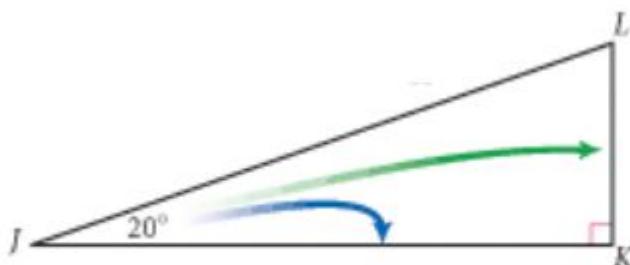
This side is called the **adjacent leg**  
because it is next to the 20° angle.

# Measurements & Trigonometric Tables



Deg.	Sin	Cos	Tan
12.0	0.2079	0.9781	0.2126
.1	.2095	.9778	.2141
.2	.2113	.9774	.2162
.3	.2130	.9770	.2180
.4	.2147	.9767	.2199
.5	.2164	.9763	.2217
.6	.2181	.9759	.2235
.7	.2198	.9755	.2254
.8	.2215	.9751	.2272
.9	.2233	.9748	.2290
13.0	0.2250	0.9744	0.2309
.1	.2267	.9740	.2327
.2	.2284	.9736	.2345
.3	.2300	.9732	.2364
.4	.2317	.9728	.2382
.5	.2334	.9724	.2401
.6	.2351	.9720	.2419
.7	.2368	.9715	.2438
.8	.2385	.9711	.2456
.9	.2402	.9707	.2475
14.0	0.2419	0.9703	0.2493
.1	.2436	.9699	.2512
.2	.2453	.9694	.2530

# Sine, Cosine, Tangent - These are functions!



This side is called the **adjacent leg** because it is next to the  $20^\circ$  angle.

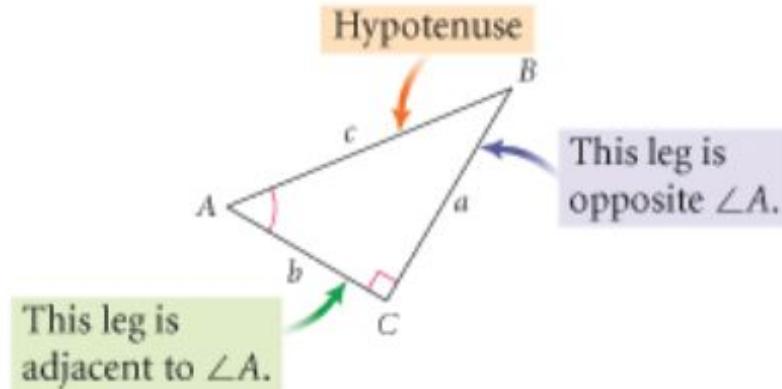
This side is called the **opposite leg** because it is across from the  $20^\circ$  angle.

**Sine**, abbreviated sin, is the ratio of the length of the opposite leg to the length of the hypotenuse.

**Cosine**, abbreviated cos, is the ratio of the length of the adjacent leg to the length of the hypotenuse.

**Tangent**, abbreviated tan, is the ratio of the length of the opposite leg to the length of the adjacent leg.

# Trigonometric Ratios



For any acute angle  $A$  in a right triangle:

$$\text{sine of } \angle A = \frac{\text{length of opposite leg}}{\text{length of hypotenuse}}$$

$$\text{cosine of } \angle A = \frac{\text{length of adjacent leg}}{\text{length of hypotenuse}}$$

$$\text{tangent of } \angle A = \frac{\text{length of opposite leg}}{\text{length of adjacent leg}}$$

In  $\triangle ABC$  above:

$$\sin A = \frac{a}{c}$$

$$\cos A = \frac{b}{c}$$

$$\tan A = \frac{a}{b}$$

# Build a short Trigonometric Table! Verify with your calculator!

In this investigation you will make a small table of trigonometric ratios for angles measuring  $20^\circ$  and  $70^\circ$ .

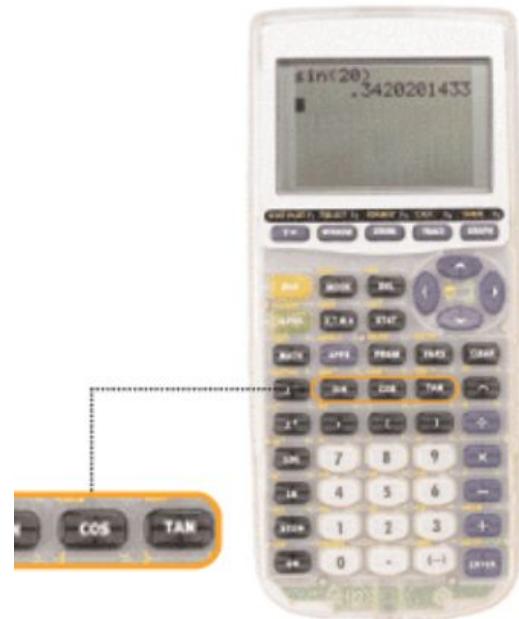
- Step 1 Use your protractor to make a large right triangle  $ABC$  with  $m\angle A = 20^\circ$ ,  $m\angle B = 90^\circ$ , and  $m\angle C = 70^\circ$ .
- Step 2 Measure  $AB$ ,  $AC$ , and  $BC$  to the nearest millimeter.
- Step 3 Use your side lengths and the definitions of sine, cosine, and tangent to complete a table like this. Round your calculations to the nearest thousandth.

$m\angle A$	$\sin A$	$\cos A$	$\tan A$
$20^\circ$			

$m\angle C$	$\sin C$	$\cos C$	$\tan C$
$70^\circ$			

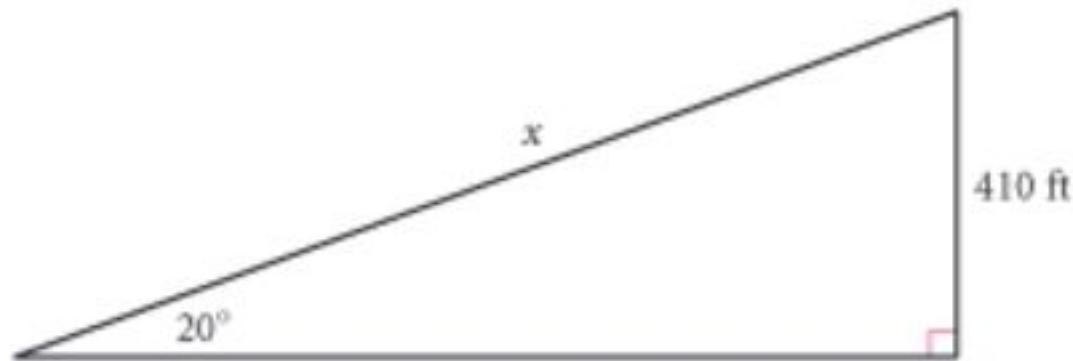
- Step 4 Share your results with your group. Calculate the average of each ratio within your group. Create a new table with your group's average values.

- Step 5 Discuss your results. What observations can you make about the trigonometric ratios you found? What is the relationship between the values for  $20^\circ$  and the values for  $70^\circ$ ? Explain why you think these relationships exist.



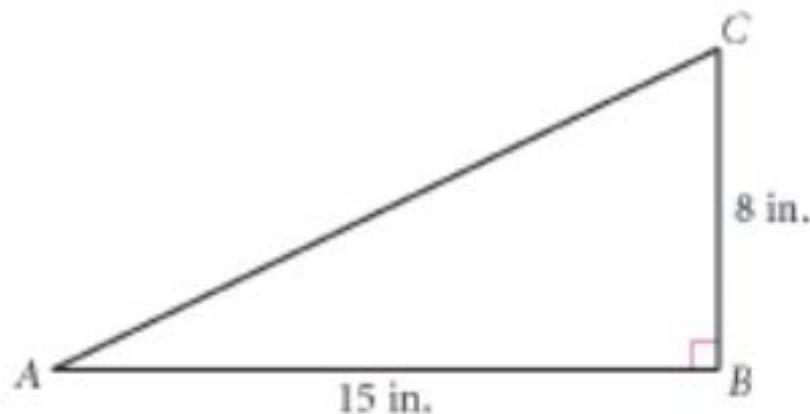
# Do a problem

Find the length of the hypotenuse of a right triangle if an acute angle measures  $20^\circ$  and the leg opposite the angle measures 410 feet.



# Do an (inverse) Problem

A right triangle has legs of length 8 inches and 15 inches. Find the measure of the angle opposite the 8-inch leg.



# Do more problems!

## Chapter 12.1

In Class: 12.1: 4, 5, 6

Homework: 8, 14, 17, 20, 21, 22

For Exercises 4–6, solve for  $x$ . Express each answer accurate to the nearest hundredth of a unit.

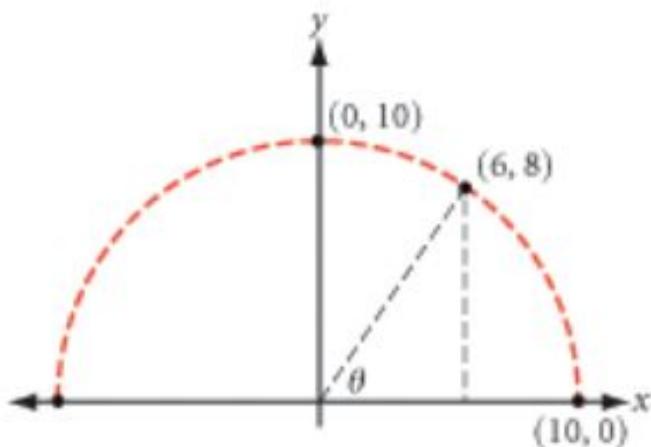
$$4. \sin 40^\circ = \frac{x}{18}$$

$$5. \cos 52^\circ = \frac{19}{x}$$

$$6. \tan 29^\circ = \frac{x}{112}$$

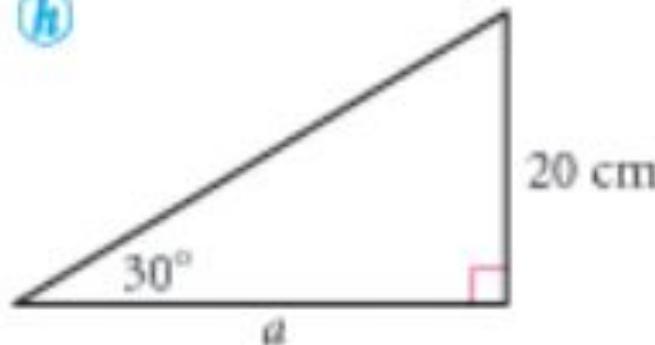
Do more problems!

8.  $\sin \theta = \underline{\hspace{2cm}}$   
 $\cos \theta = \underline{\hspace{2cm}}$   
 $\tan \theta = \underline{\hspace{2cm}}$



14.

(h)

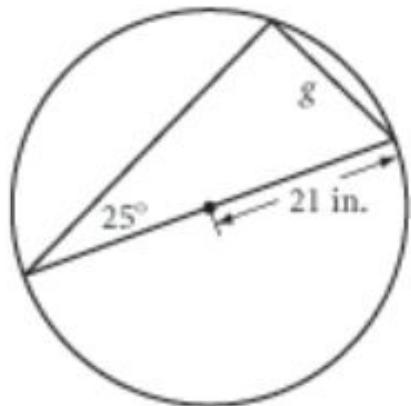


17.

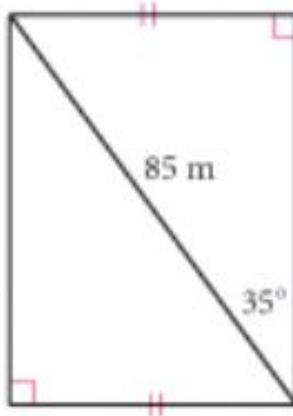


# Do more problems!

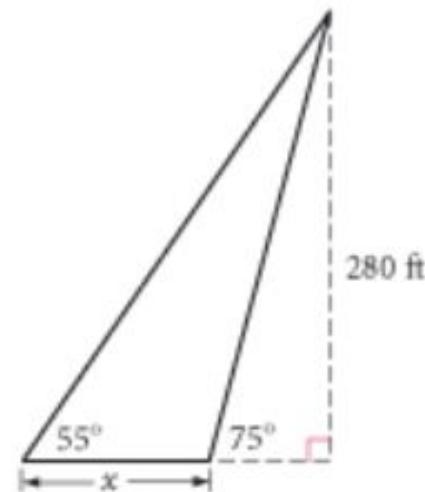
20.



21. Find the perimeter of this quadrilateral. 



22. Find  $x$ .



Fri, Apr 1

# What's happening today?

## Check-in

- Do you know your algebra?
- Do you know your trig functions?

## Reminders

Test on Apr 7th.  $\frac{1}{3}$  rd correction points back. Do it correctly the first time!

You will get 80% of the problem if you:

- Name your vertices

- Setup your trig identities using only line segments and unknowns

- Use proper algebra to cross-multiply

- Use inverse trig functions as needed

Today in class: Do the assigned problems below in [teams of 2](#)

Do problems as though you are taking a test using only a calculator and Conj notebook.

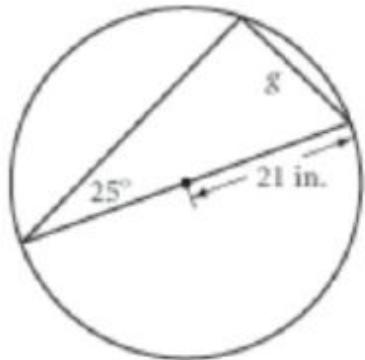
Do not refer to your homework if you have already done these problems.

[Trigonometry Problems 1 assignment #20, #21, #22](#)

[Trigonometry Problems 2 assignment #5, #6, #8, #9](#)

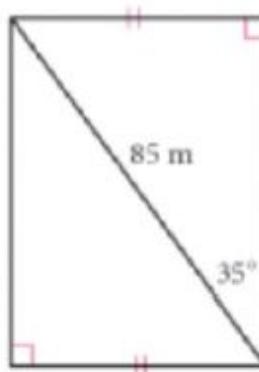
I will come by and check how each of you are doing

20.

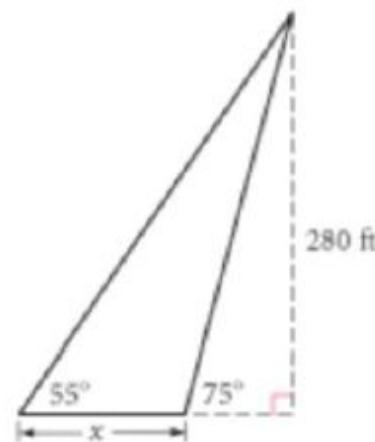


$$g = 17.75 \text{ in}$$

21. Find the perimeter of this quadrilateral.

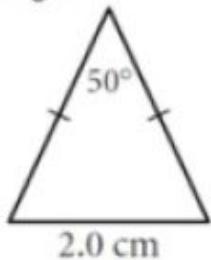


$$236.8 \text{ m}^2$$

22. Find  $x$ .

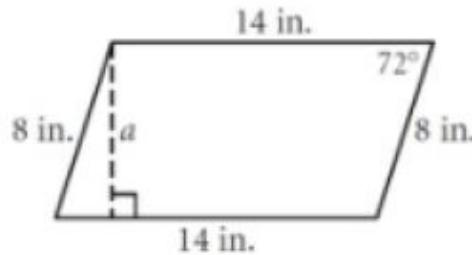
$$x = 121 \text{ ft}$$

5. Find the area of the following triangle.



$$2.14 \text{ cm}^2$$

6. Find the area of the following parallelogram.

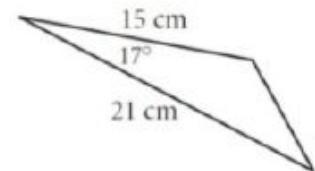


$$106.5 \text{ in}^2$$

8. A ladder 7 meters long stands on level ground and makes a  $73^\circ$  angle with the ground as it rests against a wall.  
How far from the wall is the base of the ladder?

2.05 m

9. Find the area of the triangle at right.



46.04  $\text{cm}^2$

Tue, Apr 5

# What's happening today?

Check-in

Test Details

Preparing for Test on Apr 7th?

Covers all of Unit 5 - Chapters 11, and 12: 11.1, 11.2, 11.3, 11.5, 11.6, 12.1, 12.2, 12.5

Slides since March 3rd (slides #112 to #180)

Test.  $\frac{1}{3}$  rd correction points back. Do it correctly the first time!

You will get 80% of the problem if you:

Follow processes established for Similar Triangle problems & Trigonometry Problems

Use Line Segments and variables/measures before substituting numbers!

Today in class: Review Answers!!

Trigonometry Problems 1

Trigonometry Problems 2

Reminders:

AK for both Problem Trigonometry sets above will be posted Tonight!

Test on Thu Apr 7th. Only use conjectures notebook and a Calculator!

DO THE BONUS PROBLEM!

Tue, Apr 19 & 21

# What's happening today?

## Check-in

Did you have nice break?

Test is graded - corrections due

Grade sheet distributed with missing projects, assignments noted

## Today in class:

New Unit - New Chapter 13: Mathematical System - Proofs (Last Chapter in book!)

## Reminders:

Do Test Corrections - Read Instructions

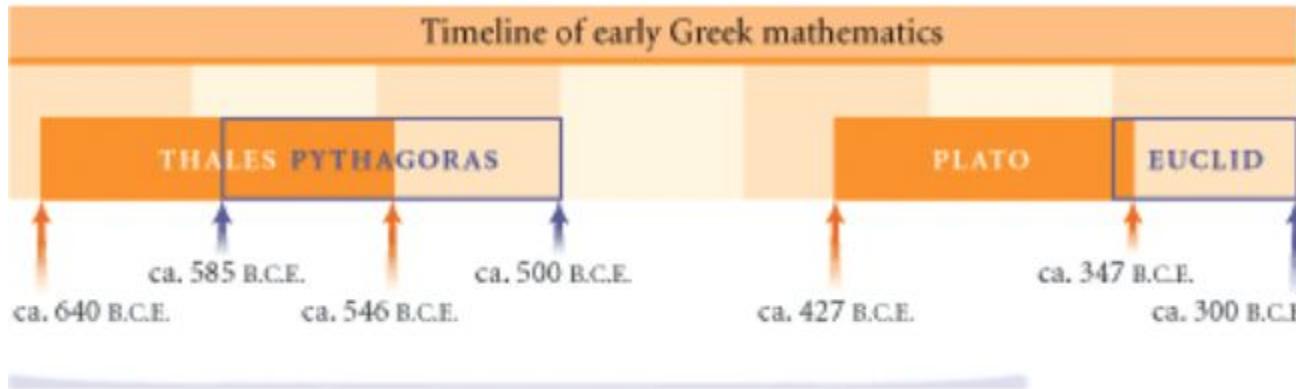
Schedule conferences!

Complete missing Projects & Assignments!

# Semester Calendar

11-Apr	CLOSED (Spring Break)					15-Apr	
18-Apr	Faculty Inservice	DEF - Day 2	CAB - Day 3	FDE - Day 4	BCA - Day 5	22-Apr	
25-Apr	EFD - Day 6	ABC - Day 1	DEF - Day 2	CAB - Day 3	FDE - Day 4	29-Apr	
2-May	BCA - Day 5	EFD - Day 6	ABC - Day 1	DEF - Day 2	CAB - Day 3	6-May	
9-May	FDE - Day 4	RD/Faculty Workday		Final Assessments		13-May	
16-May	Cascades**					20-May	
23-May	Cascades**					27-May	
30-May	CLOSED (Memorial Day)	Cascades					3-Jun
6-Jun	Senior Project/Cascades Presentations/Grad Prep			End of Year Picnic/Closing	Graduation	10-Jun	
13-Jun	Closing Meetings					17-Jun	

# Euclid's Elements



You have learned that Euclid used geometric constructions to study properties of lines and shapes. Euclid also created a **deductive system**—a set of **premises**, or accepted facts, and a set of logical rules—to organize geometry properties. He started from a collection of simple and useful statements he called **postulates**. He then systematically demonstrated how each geometry discovery followed logically from his postulates and his previously proved conjectures, or **theorems**.

# Geometry Premises

## Premises for Logical Arguments in Geometry

1. Definitions and undefined terms
2. Properties of arithmetic, equality, and congruence
3. Postulates of geometry
4. Previously proved geometry conjectures (theorems)

## Geometry Terms & Definitions

- What is Geometry?
- Point and Lines
- Angles
  - Parts of an Angle
- Triangles
  - Triangles Based on Sides
  - Triangles Based on Angles
- Circle
- 2 – Dimensional Shapes
- 3 – Dimensional Shapes
- Prism
- Pyramid
- Platonic Solids

## Properties of Arithmetic

For any numbers  $a$ ,  $b$ , and  $c$ :

### Commutative property of addition

$$a + b = b + a$$

### Commutative property of multiplication

$$ab = ba$$

### Associative property of addition

$$(a + b) + c = a + (b + c)$$

### Associative property of multiplication

$$(ab)c = a(bc)$$

### Distributive property

$$a(b + c) = ab + ac$$

## Coordinate Midpoint Property

If  $(x_1, y_1)$  and  $(x_2, y_2)$  are the coordinates of the endpoints of a segment, then the coordinates of the midpoint are  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ .

## Parallel Slope Property

In a coordinate plane, two distinct lines are parallel if and only if their slopes are equal.

## Perpendicular Slope Property

In a coordinate plane, two nonvertical lines are perpendicular if and only if their slopes are opposite reciprocals of each other.

For coordinate proofs, you also use the coordinate version of the Pythagorean Theorem, the distance formula.

## Distance Formula

The distance between points  $A(x_1, y_1)$  and  $B(x_2, y_2)$  is given by

$$AB^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \text{ or } AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

## Properties of Equality

For any numbers  $a$ ,  $b$ ,  $c$ , and  $d$ :

### Reflexive property

$a = a$  (Any number is equal to itself.)

### Transitive property

If  $a = b$  and  $b = c$ , then  $a = c$ . (This property often takes the form of the **substitution property**, which says that if  $b = c$ , you can substitute  $c$  for  $b$ .)

### Symmetric property

If  $a = b$ , then  $b = a$ .

### Addition property

If  $a = b$ , then  $a + c = b + c$ .

(Also, if  $a = b$  and  $c = d$ , then  $a + c = b + d$ .)

### Subtraction property

If  $a = b$ , then  $a - c = b - c$ .

(Also, if  $a = b$  and  $c = d$ , then  $a - c = b - d$ .)

### Multiplication property

If  $a = b$ , then  $ac = bc$ .

(Also, if  $a = b$  and  $c = d$ , then  $ac = bd$ .)

### Division property

If  $a = b$ , then  $\frac{a}{c} = \frac{b}{c}$  provided  $c \neq 0$ .

(Also, if  $a = b$  and  $c = d$ , then  $\frac{a}{c} = \frac{b}{d}$  provided that  $c \neq 0$  and  $d \neq 0$ .)

### Square root property

If  $a^2 = b$ , then  $a = \pm\sqrt{b}$ .

### Zero product property

If  $ab = 0$ , then  $a = 0$  or  $b = 0$  or both  $a$  and  $b = 0$ .

## Postulates of Geometry

**Line Postulate** You can construct exactly one line through any two points. In other words, two points determine a line.



**Line Intersection Postulate** The intersection of two distinct lines is exactly one point.



**Segment Duplication Postulate** You can construct a segment congruent to another segment.



**Angle Duplication Postulate** You can construct an angle congruent to another angle.



**Midpoint Postulate** You can construct exactly one midpoint on any line segment.



**Angle Bisector Postulate** You can construct exactly one angle bisector in any angle.



**Parallel Postulate** Through a point not on a given line, you can construct exactly one line parallel to the given line.



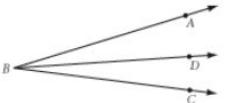
**Perpendicular Postulate** Through a point not on a given line, you can construct exactly one line perpendicular to the given line.



**Segment Addition Postulate** If point  $B$  is on  $\overline{AC}$  and between points  $A$  and  $C$ , then  $AB + BC = AC$ .



**Angle Addition Postulate** If point  $D$  lies in the interior of  $\angle ABC$ , then  $m\angle ABD + m\angle DBC = m\angle ABC$ .

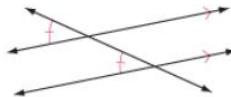


**Linear Pair Postulate** If two angles are a linear pair, then they are supplementary.



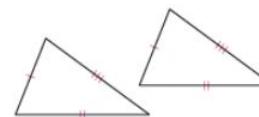
## Corresponding Angles Postulate (CA Postulate)

If two parallel lines are cut by a transversal, then the corresponding angles are congruent. Conversely, if two coplanar lines are cut by a transversal forming congruent corresponding angles, then the lines are parallel.

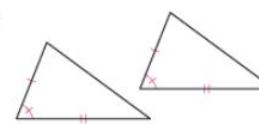


## SSS Congruence Postulate

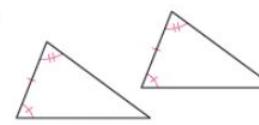
If the three sides of one triangle are congruent to three sides of another triangle, then the two triangles are congruent.



**SAS Congruence Postulate** If two sides and the included angle in one triangle are congruent to two sides and the included angle in another triangle, then the two triangles are congruent.

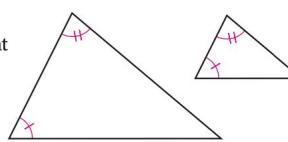


**ASA Congruence Postulate** If two angles and the included side in one triangle are congruent to two angles and the included side in another triangle, then the two triangles are congruent.



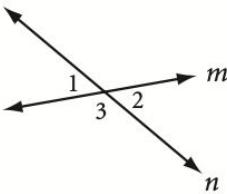
## AA Similarity Postulate

If two angles of one triangle are congruent to two angles of another triangle, then the two triangles are similar.



## Writing a Proof

- Task 1** From the conditional statement, identify what is given and what you must show.
- Task 2** Draw and label a diagram to illustrate the given information.
- Task 3** Restate what is given and what you must show in terms of your diagram.
- Task 4** Plan a proof using your reasoning strategies. Organize your reasoning mentally or on paper.
- Task 5** From your plan, write a proof.



Given  
Lines  $m$  and  $n$   
intersect to form  
vertical angles 1 and 2

►  $\angle 1$  and  $\angle 3$  are  
supplementary  
 $\angle 3$  and  $\angle 2$  are  
supplementary

Linear Pair Postulate

►  $m\angle 1 + m\angle 3 = 180^\circ$   
 $m\angle 3 + m\angle 2 = 180^\circ$

Definition of  
supplementary

►  $m\angle 1 + m\angle 3 =$   
 $m\angle 3 + m\angle 2$

Transitive property  
of equality

$$m\angle 1 = m\angle 2$$

Subtraction property  
of equality

$$\angle 1 \cong \angle 2$$

Definition of  
congruence

Start a theorem list separate  
from your conjecture list.

So the Vertical Angles Conjecture becomes the Vertical Angles (VA) Theorem. It is important when building your mathematical system that you use only the premises of geometry. These include theorems, but not unproved conjectures. You can use all the theorems on your theorem list as premises for proving other theorems. For instance, in Example A you can use the VA Theorem to prove another theorem.

**Given:** Parallel lines  $\ell_1$  and  $\ell_2$  cut by transversal  $\ell_3$  to form alternate interior angles  $\angle 1$  and  $\angle 2$

**Show:**  $\angle 1 \cong \angle 2$

For Task 4, plan a proof. Organize your reasoning mentally or on paper.

Plan:

I need to show that  $\angle 1 \cong \angle 2$ .

Looking over the postulates and theorems, the ones that look useful are the CA Postulate and the VA Theorem.

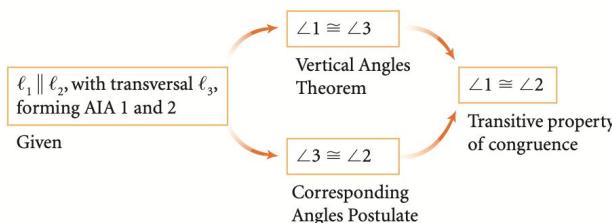
From the CA Postulate, I know that  $\angle 2 \cong \angle 3$  and from the VA Theorem,  $\angle 1 \cong \angle 3$ .

If  $\angle 2 \cong \angle 3$  and  $\angle 1 \cong \angle 3$ , then by substitution  $\angle 1 \cong \angle 2$ .



For Task 5, create a proof from your plan.

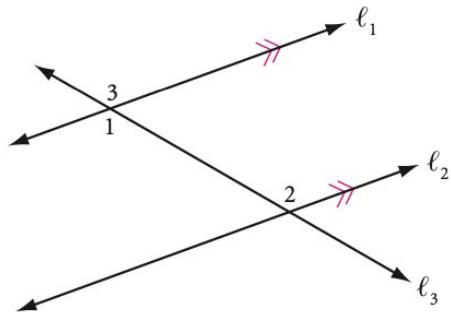
#### Flowchart Proof



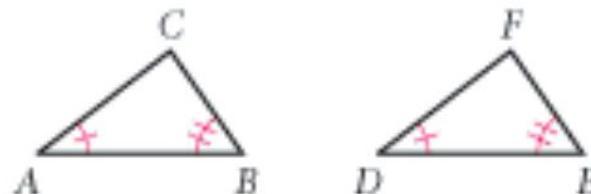
**Given:** Two parallel lines are cut by a transversal

**Show:** Alternate interior angles formed by the lines are congruent

For Task 2, draw and label a diagram.



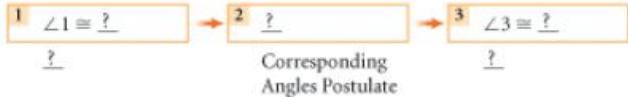
**Third Angle Conjecture:** If two angles of one triangle are congruent to two angles of a second triangle, then the third pair of angles are congruent.



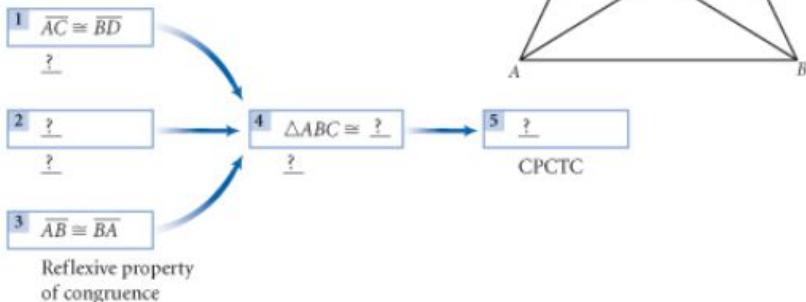
# Proving the Isosceles Triangle Theorem!

## Flow-Chart Proofs

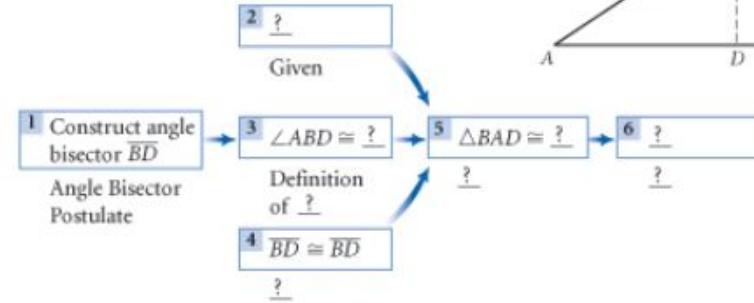
20. Given:  $\angle 1 \cong \angle 2$   
 Show:  $\angle 3 \cong \angle 4$



21. Given:  $\overline{AC} \cong \overline{BD}$ ,  $\overline{AD} \cong \overline{BC}$   
 Show:  $\angle D \cong \angle C$

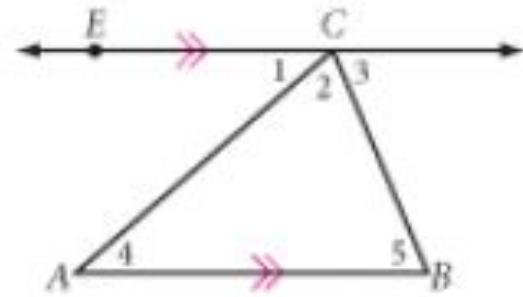


22. Given: Isosceles triangle ABC with  $\overline{AB} \cong \overline{BC}$   
 Show:  $\angle A \cong \angle C$



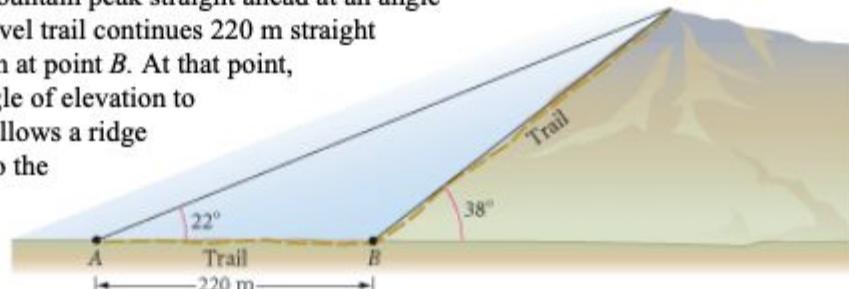
# Statement Proofs

Triangle Sum Conjecture - Can you write a statement proof?



# Solve this problem using the Postulates and Trigonometric Identities

26. Shannon and Erin are hiking up a mountain. Of course, they are packing the clinometer they made in geometry class. At point  $A$  along a flat portion of the trail, Erin sights the mountain peak straight ahead at an angle of elevation of  $22^\circ$ . The level trail continues  $220$  m straight to the base of the mountain at point  $B$ . At that point, Shannon measures the angle of elevation to be  $38^\circ$ . From  $B$  the trail follows a ridge straight up the mountain to the peak. At point  $B$ , how far are they from the mountain peak?



# Reminders

Reminders:

- Do Test Corrections - Read Instructions
- Schedule conferences!
- Complete missing Projects & Assignments!

Mon-Wed-Fri  
Apr 25-27-29

# What's happening this week?

Check-in

Finals Details

**Cumulative Finals Includes Chapters:**

**AREAS: 8.1, 8.2, 8.4, 8.5, 8.6, 8.7**

**PYTHAGOREAN THEOREM: 9.1, 9.2, 9.3**

**VOLUMES & SURFACE AREAS: 10.1, 10.2, 10.3, 10.6, 10.7**

**SIMILARITY & PROPORTIONS: 11.1, 11.2, 11.3, 11.5, 11.6**

**TRIGONOMETRY: 12.1, 12.2, 12.5**

**PROOFS: 13.1, 13.2, 13.3, 13.4, 13.6, 13.7**

**No Test Corrections for Finals. Do it correctly the first time!**

You will get 80% for the problems if you:

Follow orderly processes established in class

Identify Definitions, Postulates, Theorems used

Bring Conjectures Notebook, Calculator, Geo Instruments

Illustrate with Sketches and DO BONUS PROBLEMS

This week in class: **Chapters 13.3-4-6-7**

# Reminders

FINALS on FRI, MAY 13th. REVIEW PROBLEMS ADDED TO CLASSROOM

COMPLETE BONUS PROBLEMS FROM TEST, PROJECTS AND MISSING ASSIGNMENTS IN THAT ORDER!

## HOW TO STUDY FOR FINALS

COMPLETE ALL REMAINING HOMEWORK AND PROJECTS!

STUDY ANSWER KEYS CAREFULLY AND COMPARE TO YOUR HOMEWORK!

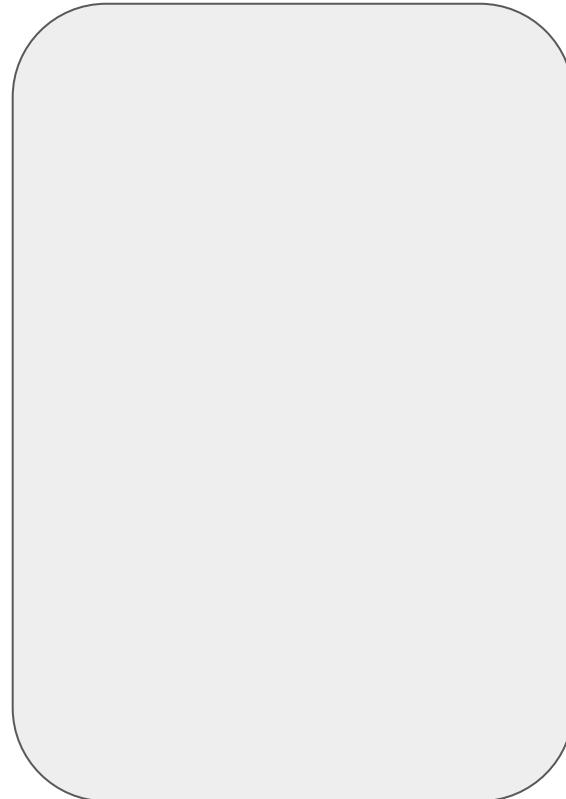
STUDY AND PRACTICE SELECTED HOMEWORK PROBLEMS BY DOING THEM FROM SCRATCH

MAKE SURE THAT YOUR CONJECTURES NOTEBOOK (TOOLBOX) IS ORGANIZED AND COMPLETE

DO SOME REVIEW PROBLEMS POSTED

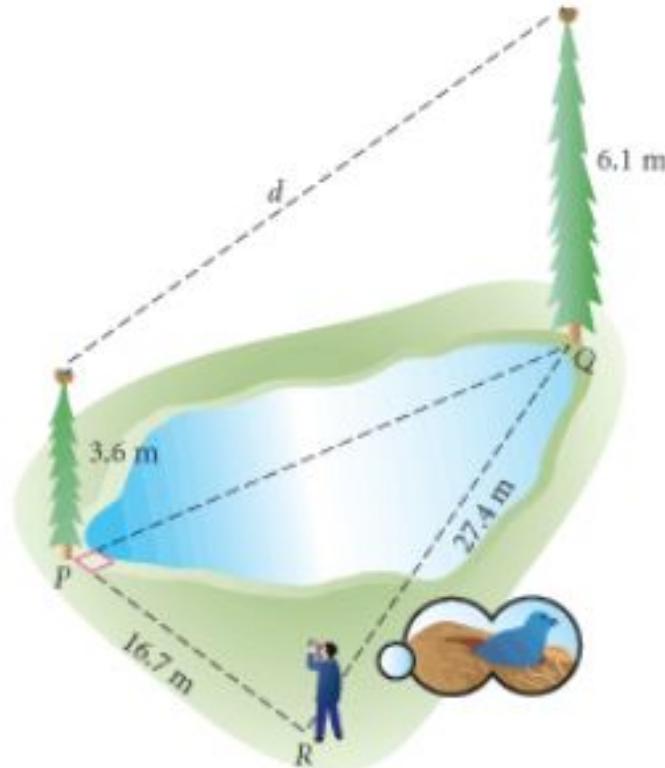
## 13.3: Triangle Proofs

1. If a point is on the perpendicular bisector of a segment, then it is equally distant from the endpoints of the segment. (Perpendicular Bisector Theorem)
2. If a point is equally distant from the endpoints of a segment, then it is on the perpendicular bisector of the segment. (Converse of the Perpendicular Bisector Theorem) 
5. If a point is equally distant from the sides of an angle, then it is on the bisector of the angle. (Converse of the Angle Bisector Theorem) 
8. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles. (Triangle Exterior Angle Theorem)



# Triangle Problem

15. Two bird nests, 3.6 m and 6.1 m high, are on trees across a pond from each other, at points  $P$  and  $Q$ . The distance between the nests is too wide to measure directly (and there is a pond between the trees). A birdwatcher at point  $R$  can sight each nest along a dry path.  $RP = 16.7$  m and  $RQ = 27.4$  m.  $\angle QPR$  is a right angle. What is the distance  $d$  between the nests?

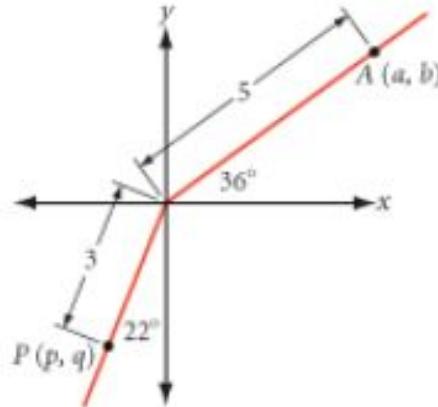


# 13.6: Circle Proofs

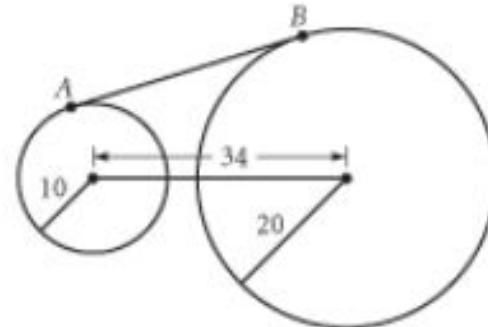
2. The opposite angles of an inscribed quadrilateral are supplementary. (Cyclic Quadrilateral Theorem)
5. Tangent segments from a point to a circle are congruent. (Tangent Segments Theorem)
8. Prove the Angles Inscribed in a Semicircle Conjecture: An angle inscribed in a semicircle is a right angle. Explain why this is a corollary of the Inscribed Angle Theorem.

# Circle Problems

11. Find the coordinates of  $A$  and  $P$  to the nearest tenth.



14.  $\overline{AB}$  is a common external tangent. Find the length of  $\overline{AB}$  (to a tenth of a unit).



## 13.5: Indirect Proofs

In the proofs you have written so far, you have shown *directly*, through a sequence of statements and reasons, that a given conjecture is true. In this lesson you will write a different type of proof, called an indirect proof. In an **indirect proof**, you show something is true by eliminating all the other possibilities. You have probably used this type of reasoning when taking multiple-choice tests. If you are unsure of an answer, you can try to eliminate choices until you are left with only one possibility.

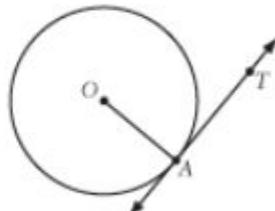
### Proving the Tangent Conjecture

Copy the information and diagram below, then work with your group to complete an indirect proof of the Tangent Conjecture.

**Conjecture:** A tangent is perpendicular to the radius drawn to the point of tangency.

**Given:** Circle  $O$  with tangent  $\overrightarrow{AT}$  and radius  $\overrightarrow{AO}$

**Show:**  $\overrightarrow{AO} \perp \overrightarrow{AT}$



## 13.5: Indirect Proof Problems

5. Fill in the blanks in the indirect proof below.

**Conjecture:** No triangle has two right angles.

**Given:**  $\triangle ABC$

**Show:** No two angles are right angles

### Two-Column Proof

#### Statement

1. Assume  $\triangle ABC$  has two right angles  
(Assume  $m\angle A = 90^\circ$  and  $m\angle B = 90^\circ$   
and  $0^\circ < m\angle C < 180^\circ$ .)
2.  $m\angle A + m\angle B + m\angle C = 180^\circ$
3.  $90^\circ + 90^\circ + m\angle C = 180^\circ$
4.  $m\angle C = ?$

#### Reason

1. ?
2. ?
3. ?
4. ?



But if  $m\angle C = 0$ , then the two sides  $\overline{AC}$  and  $\overline{BC}$  coincide, and thus there is no angle at C. This contradicts the given information. So the assumption is false. Therefore, no triangle has two right angles.

## 13.5: Indirect Proof Problems

7. Write an indirect proof of the conjecture below.

**Conjecture:** In a scalene triangle, the median cannot be the altitude.

**Given:** Scalene triangle  $ABC$  with median  $\overline{CD}$

**Show:** Median  $\overline{CD}$  is not the altitude to  $\overline{AB}$

