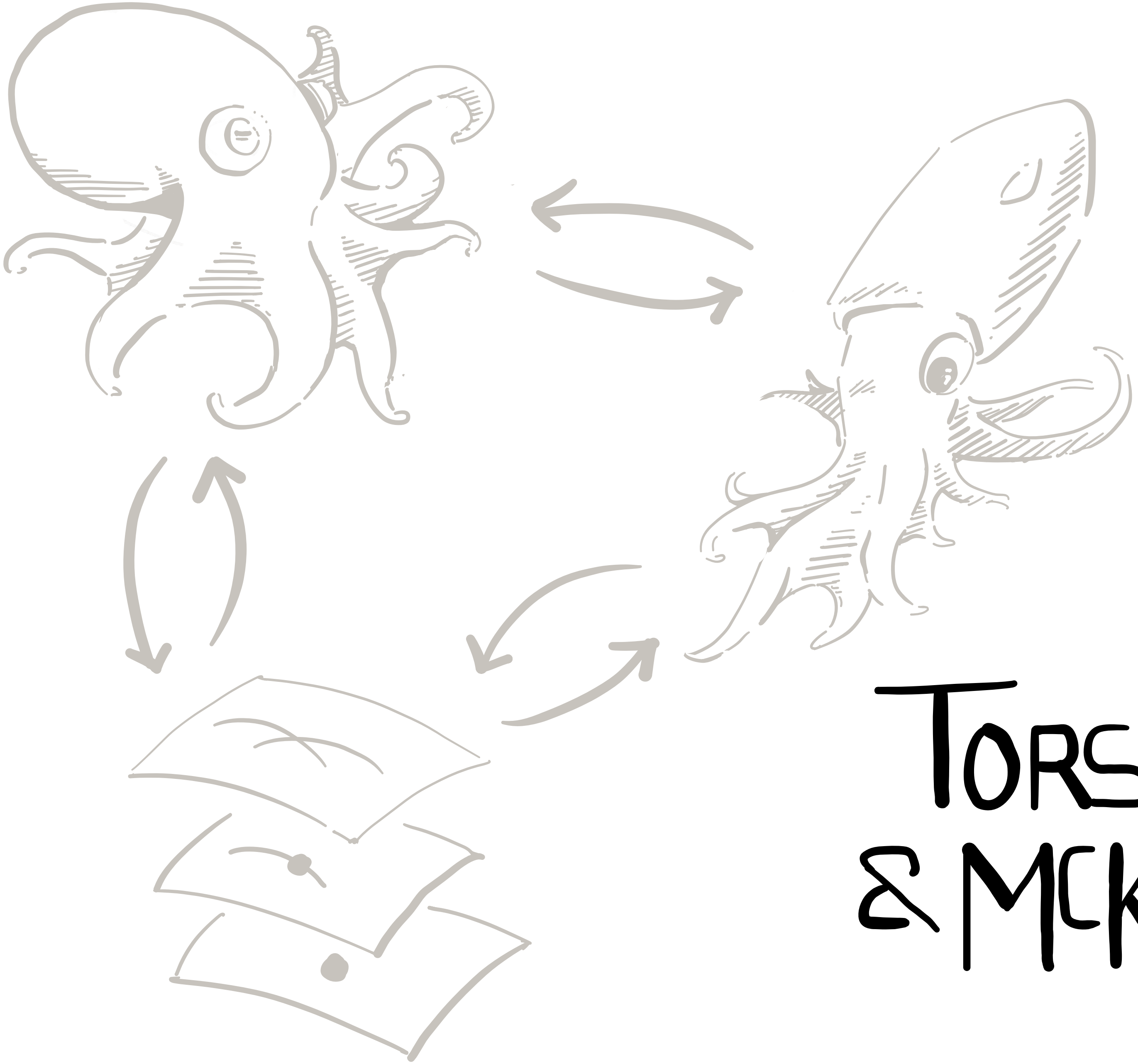
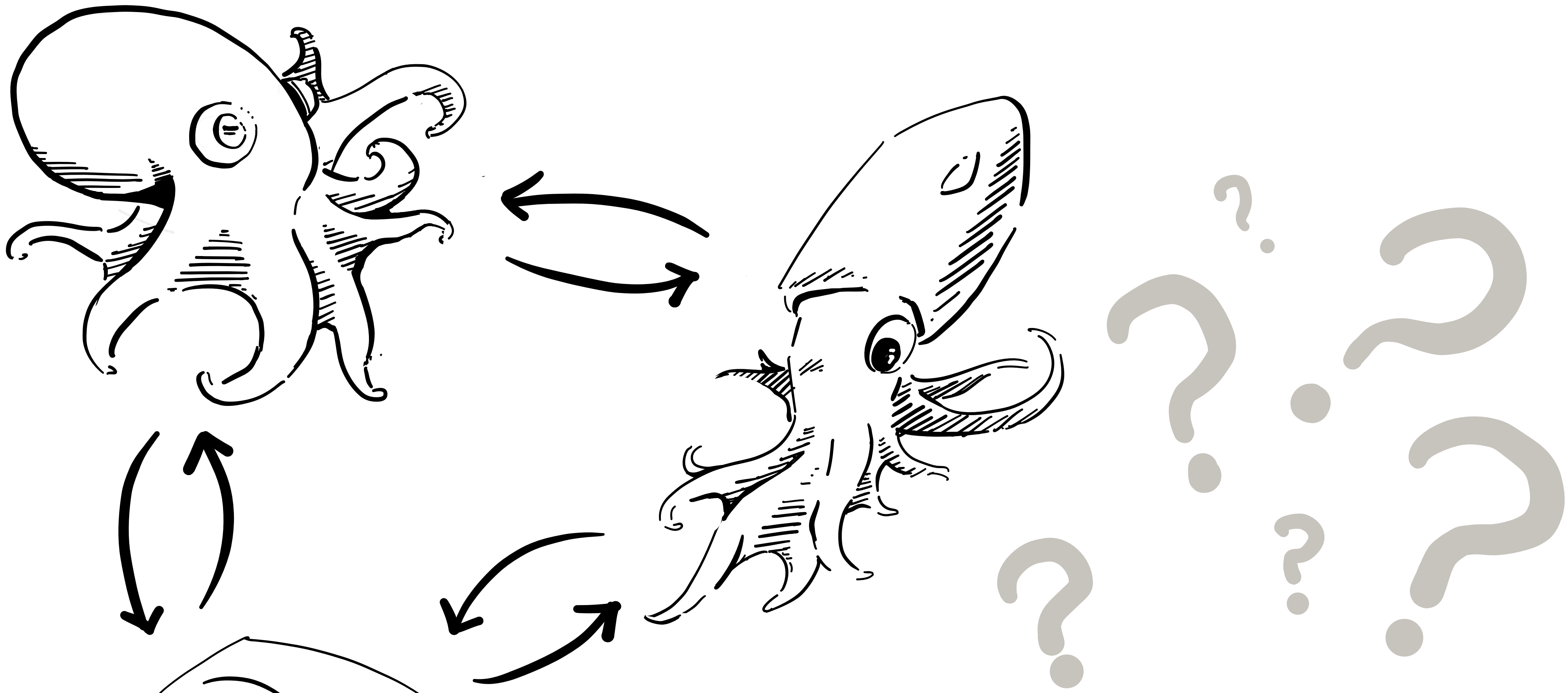


TORSION PAIRS & MCKAY QUIVERS

Parth Shimpi

parth.shimpi@glasgow.ac.uk



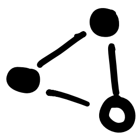
タコ、イカ、$\widetilde{A}_2$-McKay の類似点は何ですか

Let Λ be a (contracted) affine preprojective algebra.

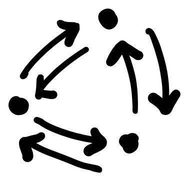
Let Λ be a (contracted) **affine preprojective algebra.**

$\Delta =$  (ADE)

extend

$\underline{\Delta} =$ 

double

$Q =$ 

path algebra

$\mathbb{C}Q = \mathbb{C}\langle \text{paths in } Q \rangle$

add relations

$$\Lambda = \mathbb{C}Q \big/ \left([a, a^*] \mid a \in Q_1 \right)$$

Let Λ be a (contracted) affine preprojective algebra.

$$\Lambda = \Big(\sum_{j\notin J} e_j\Big)\cdot\Lambda'\cdot\Big(\sum_{j\notin J} e_j\Big)$$

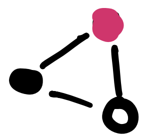
contract

$$\Lambda' = \mathbb{C}Q \big/ \big([a,a^*] \mid a \in Q_1 \big)$$

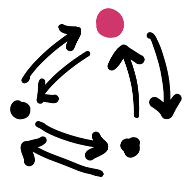
$J \subset \Delta =$ 

(ADE)

extend

$\underline{\Delta} =$ 

double

$Q =$ 

path algebra

$$\mathbb{C}Q = \mathbb{C}\langle \text{paths in } Q \rangle$$

add relations

Let Λ be a (contracted) affine preprojective algebra.

! Many have studied this

Derived autoequivalences. [Crawley-Boevey '00]

[Ishii–Uehara '05]

[Hirano–Wemyss '23]

Tilting theory. [Buan-Iyama-Reiten-Scott '09]

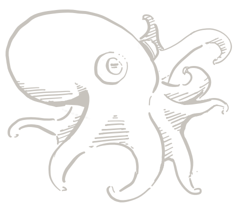
[Amiot–Iyama–Reiten '15]

Stability. [Bridgeland '09]

[Ishii–Ueda–Uehara '10]

[Hirano–Wemyss '18]

[Asai–Iyama '24]



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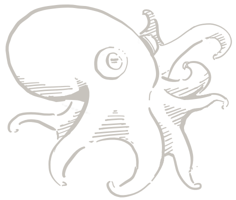
[Hirano–Wemyss '18]
[Asai–Iyama '24]

... so I also tried.



Hearts of t-structures. [S. '25]

Let Λ be a (contracted) affine preprojective algebra.

Question. Is it possible to classify hearts in $D^b \operatorname{mod} \Lambda$?

If $\mathcal{T}$ is a triangulated category, we say $\mathcal{K} \subseteq \mathcal{T}$ is a (bounded) heart if $\operatorname{Hom}(\mathcal{K}, \mathcal{K}[<0]) = 0$ and $\mathcal{T} = \langle \mathcal{K}[n] \mid n \in \mathbb{Z} \rangle$

Hearts of t-structures. [S. '25]

Let Λ be a (contracted) affine preprojective algebra.

Question. Is it possible to classify hearts in $\mathrm{D}^b \operatorname{mod} \Lambda$?

Theorem [S]. If $\mathcal{K}$ is a bounded heart in $\mathrm{D}^b \operatorname{mod} \Lambda$ and $\mathcal{K} \subseteq \operatorname{fl} \operatorname{mod} \Lambda [-1,0]$ then $\mathcal{K}$ is a member of a short, explicit list.

Let Λ be a (contracted) affine preprojective algebra.

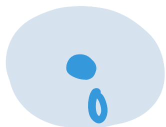
Question. Is it possible to classify hearts in $D^0 \operatorname{mod} \Lambda$?

Theorem [S]. If K is a bounded heart in $D^0 \operatorname{mod} \Lambda$ and $K \subseteq \operatorname{fl} \operatorname{mod} \Lambda[-1,0]$ then K is a member of a short, explicit list.

Full subcategory of objects with finite-length cohomology.

Equivalently, viewing Λ as a sheaf of nc-algebras over $Z = \operatorname{Spec}(Z\Lambda)$, we have

$$D^0\Lambda := D^0 \operatorname{mod} \Lambda = \{ M \in \operatorname{Coh}(Z, \Lambda) \mid \operatorname{Supp} M \subseteq \{0\} \text{ in } Z \}.$$



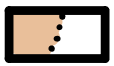
$Z = \operatorname{Spec} \dfrac{\mathbb{C}[x,y]}{(xy = z^3)}$

Let Λ be a (contracted) affine preprojective algebra.

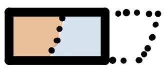
Question. Is it possible to classify hearts in $D^b \operatorname{mod} \Lambda$?

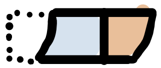
Theorem [S]. If K is a bounded heart in $D^b \operatorname{mod} \Lambda$ and $K \subseteq \operatorname{fl} \operatorname{mod} \Lambda[-1,0]$ then K is a member of a short, explicit list.

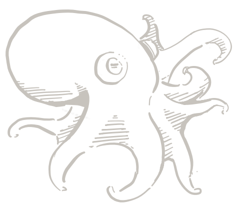
K is a Happel-Reiten-Smalø tilt of $H = \operatorname{fl} \operatorname{mod} \Lambda$

$\therefore$  $T = H \cap K[1]$ is a torsion class in H

 $F = H \cap K$ is a torsion-free class in H

 $H = T * F$

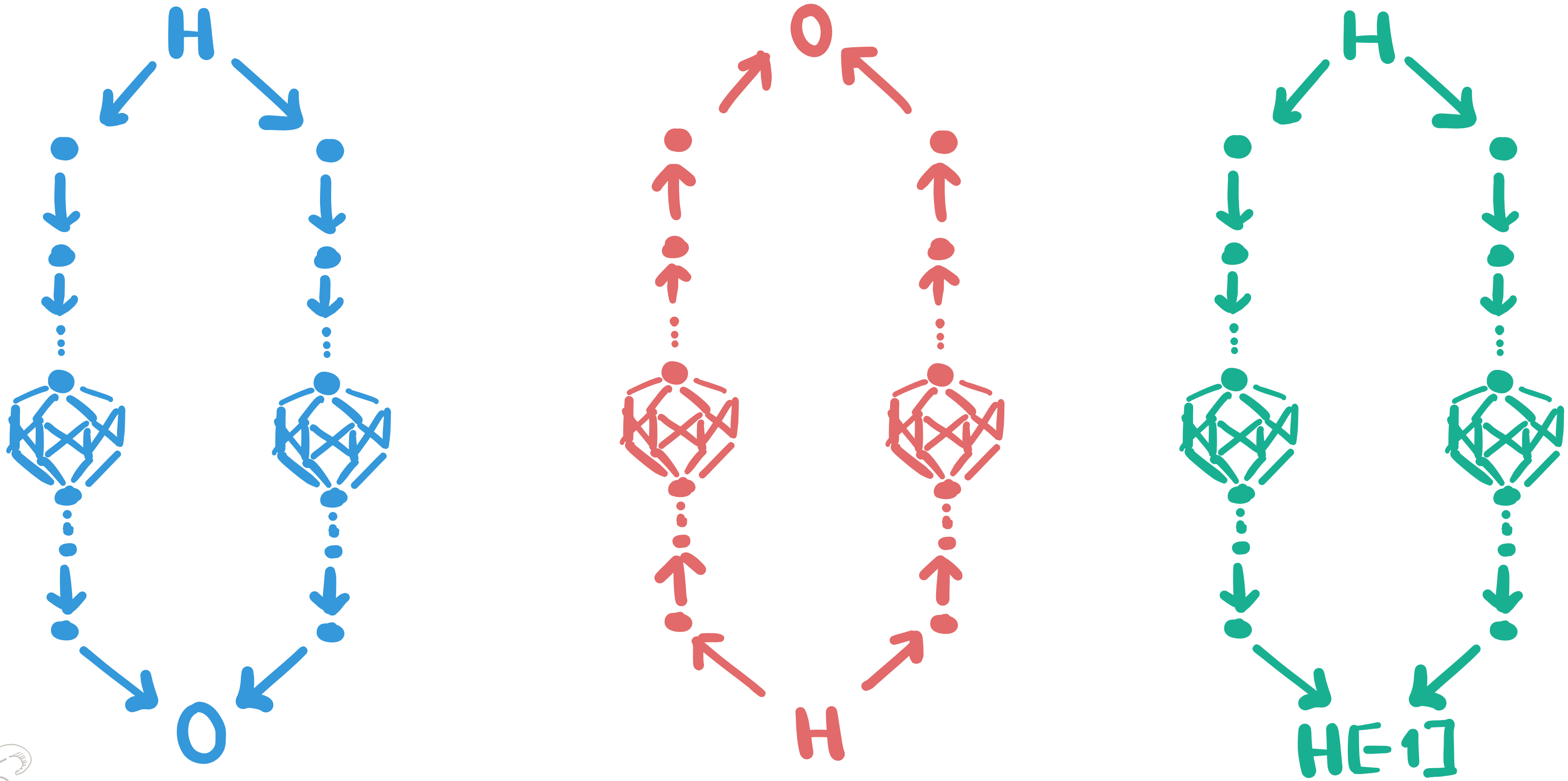
 $K = F * T[-1]$



 $Z = \operatorname{Spec} \dfrac{\mathbb{C}[x,y]}{(xy = z^3)}$

Further, $\mathrm{torf}(\mathcal{H}) \cong \mathrm{tors}(\mathcal{H})^{op} \cong \mathrm{tilt}(\mathcal{H})$ is a complete lattice

poset that admits arbitrary suprema (least upper bounds) and infima (gr. lower bounds)

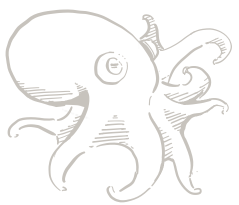


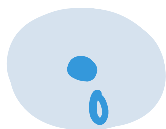
torf(H) tors(H) tilt(H)

What hearts are already known?

① $\mathrm{coh}(Z,\Lambda) \cong \mathrm{fl}\,\mathrm{mod}\,\Lambda$ ... "natural" heart

$$= \{ M \in \mathrm{Coh}\,\Lambda \mid \mathrm{Supp}\,M \subseteq \{0\} \}$$

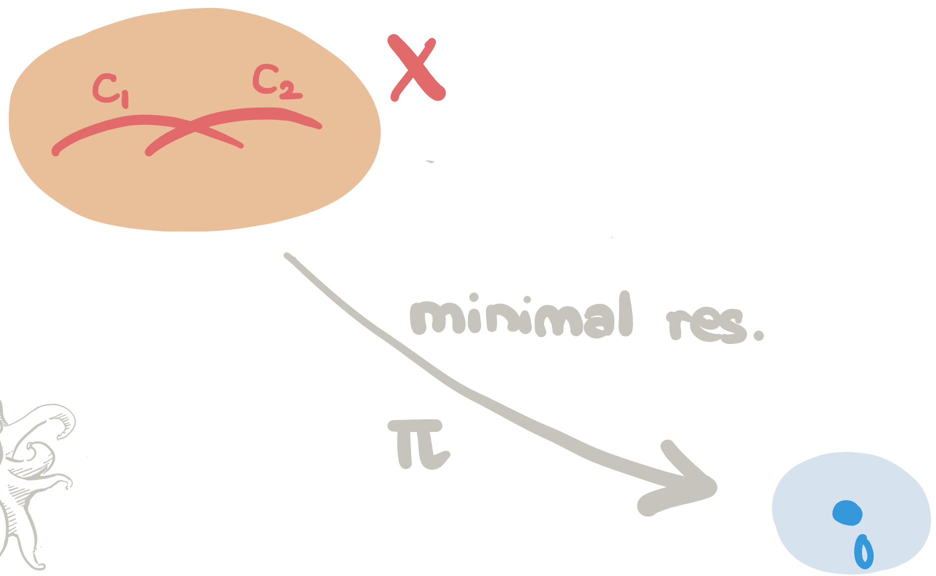




$Z = \mathrm{Spec}\,\dfrac{\mathbb{C}[x,y]}{(xy=z^3)}$

Theorem [Kapranov-Vasserot, Van den Bergh]. There is a derived equivalence between $(X, \mathcal{O}_X)$ and (Z, Λ) that identifies $\operatorname{mod}\Lambda$ with the category of π-perverse sheaves on X.

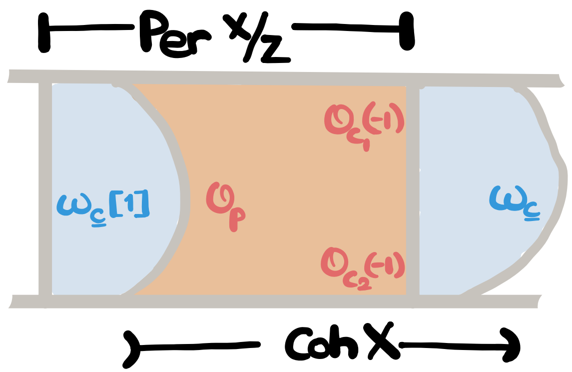
$$\begin{tikzcd}
D^b\operatorname{mod}\Lambda \arrow[r, "\sim"] & D^b\operatorname{Coh}X \\
\operatorname{mod}\Lambda \arrow[r, "\sim"] \arrow[u, hook] & \operatorname{Per}(X/Z) \arrow[u, hook]
\end{tikzcd}$$



$$Z = \operatorname{Spec} \frac{\mathbb{C}[x,y]}{(xy = z^3)}$$

Theorem [Kapranov–Vasserot, Van den Bergh]. There is a derived equivalence between $(X,\mathcal{O}_X)$ and (Z,Λ) that identifies $\operatorname{mod}\Lambda$ with the category of π-perverse sheaves on X.

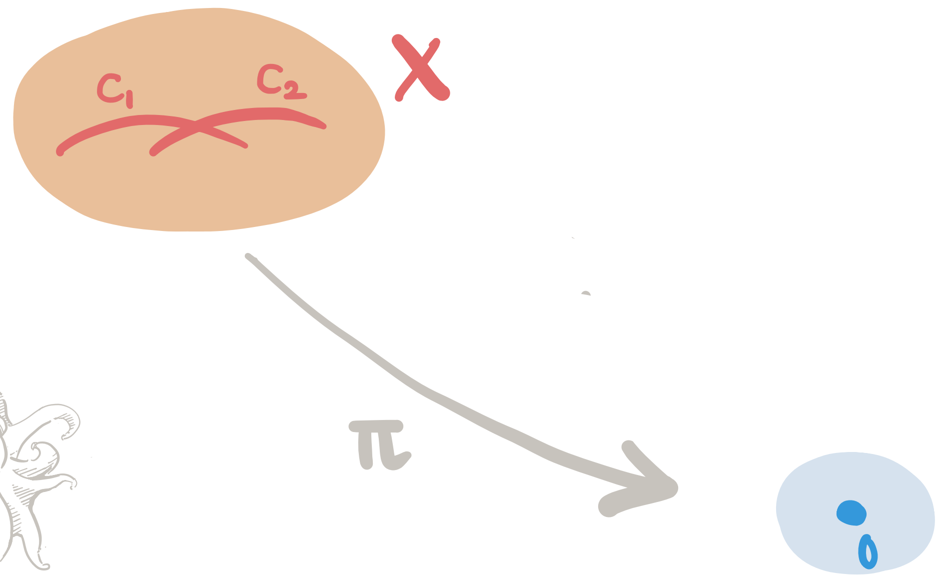
$\mathrm{Per}(X/Z)$ is the (+ve) tilt of $\mathrm{Coh}\,X$ in the torsion class $\ker(R^1\pi_*) \subseteq \mathrm{Coh}\,X$



$$\operatorname{mod}\Lambda \xrightarrow{\sim} \mathrm{Per}(X/Z)$$

$$S_0 \longmapsto \omega_C[1]$$

$$S_i \longmapsto \mathcal{O}_{C_i}(-1)$$

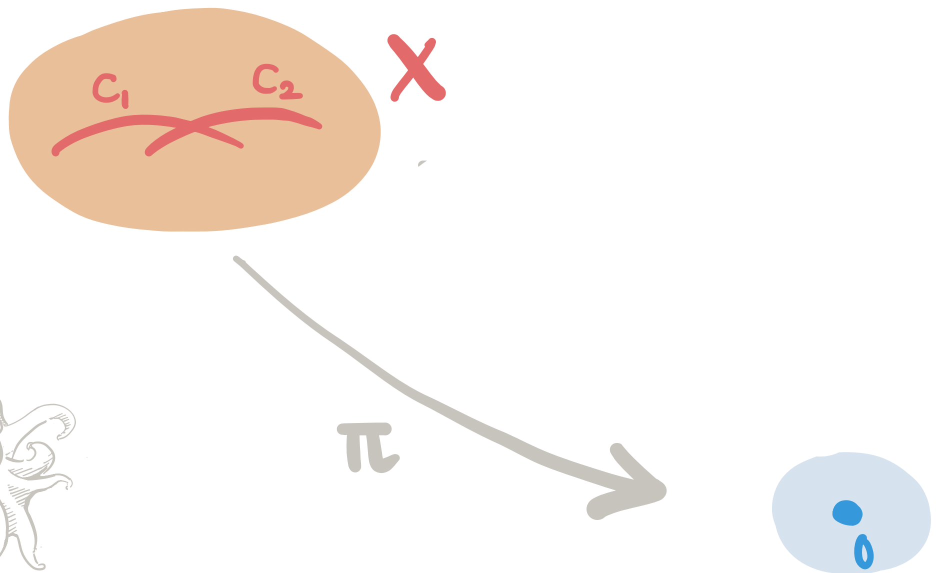


$$Z = \operatorname{Spec} \frac{\mathbb{C}[x,y]}{(xy=z^3)}$$

Theorem [Kapranov-Vasserot, Van den Bergh]. There is a derived equivalence between $(X, \mathcal{O}_X)$ and (Z, Λ) that identifies $\mathrm{mod}\,\Lambda$ with the category of π-perverse sheaves on X.

These are compatible with support-restriction.

$$\begin{tikzcd}
D^b \mathrm{mod}\,\Lambda \arrow[r, "\sim"] & D^b \mathrm{Coh}\, X \\
D^0 \mathrm{mod}\,\Lambda \arrow[u] \arrow[r, "\sim"] & D^0 \mathrm{Coh}\, X \arrow[u] \\
\mathrm{fl}\,\mathrm{mod}\,\Lambda \arrow[u, hook] \arrow[r, "\sim"] & \mathrm{per}(X/Z) \arrow[u, hook]
\end{tikzcd}$$

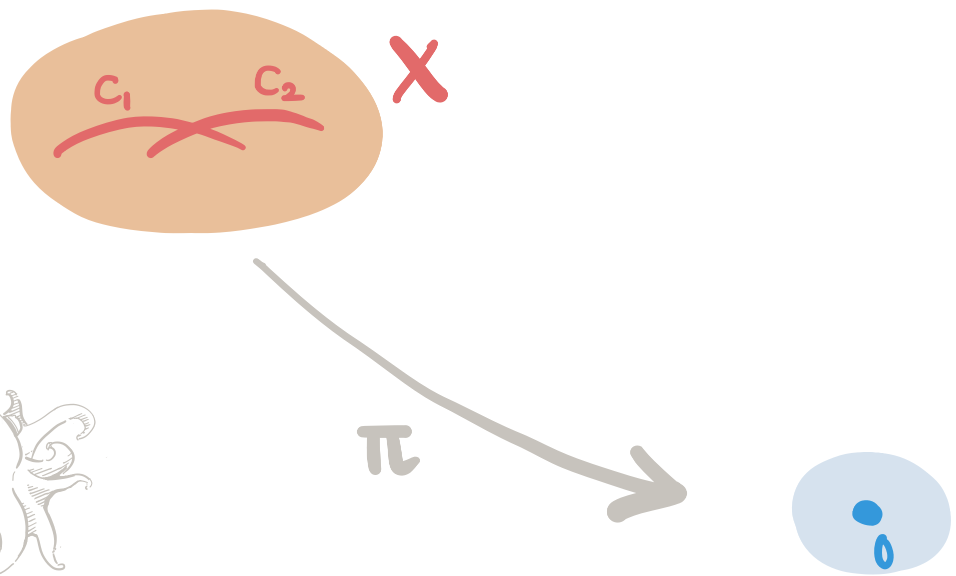


$$Z = \mathrm{Spec}\, \frac{\mathbb{C}[x,y]}{(xy = z^3)}$$

What hearts are already known?

① $\mathrm{coh}(Z, \Lambda) \cong \mathrm{fl}\bmod \Lambda \cong \mathrm{per}\, X/Z$

② $\mathrm{coh}(X, \mathcal{O}_X) = \mathrm{coh}\, X \cong \mathrm{per}\, X/X$



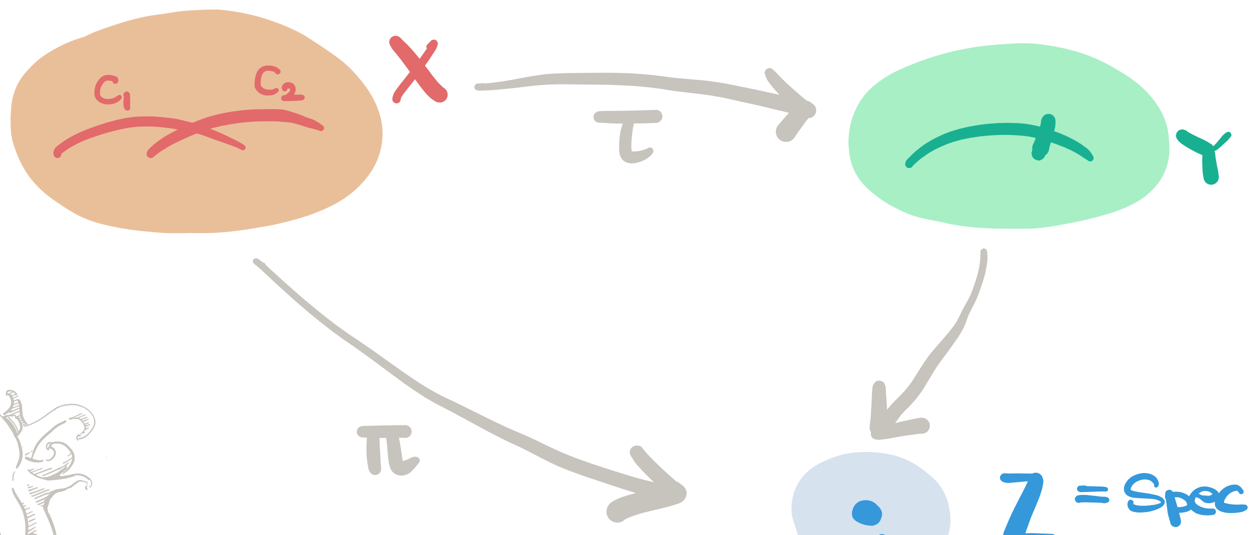
$$Z = \mathrm{Spec}\, \frac{\mathbb{C}[x,y]}{(xy = z^3)}$$

What hearts are already known?

① $\operatorname{coh}(Z,\Lambda) \cong \operatorname{fl} \operatorname{mod} \Lambda \cong \operatorname{per}(X/Z)$

② $\operatorname{coh}(X,\mathcal{O}_X) = \operatorname{coh} X \cong \operatorname{per}(X/X)$

③ $\operatorname{coh}(Y,\mathcal{A}_h) \cong \operatorname{per}(X/Y)$ for each partial contraction.



$$Z = \operatorname{Spec} \frac{\mathbb{C}[x,y]}{(xy=z^3)}$$

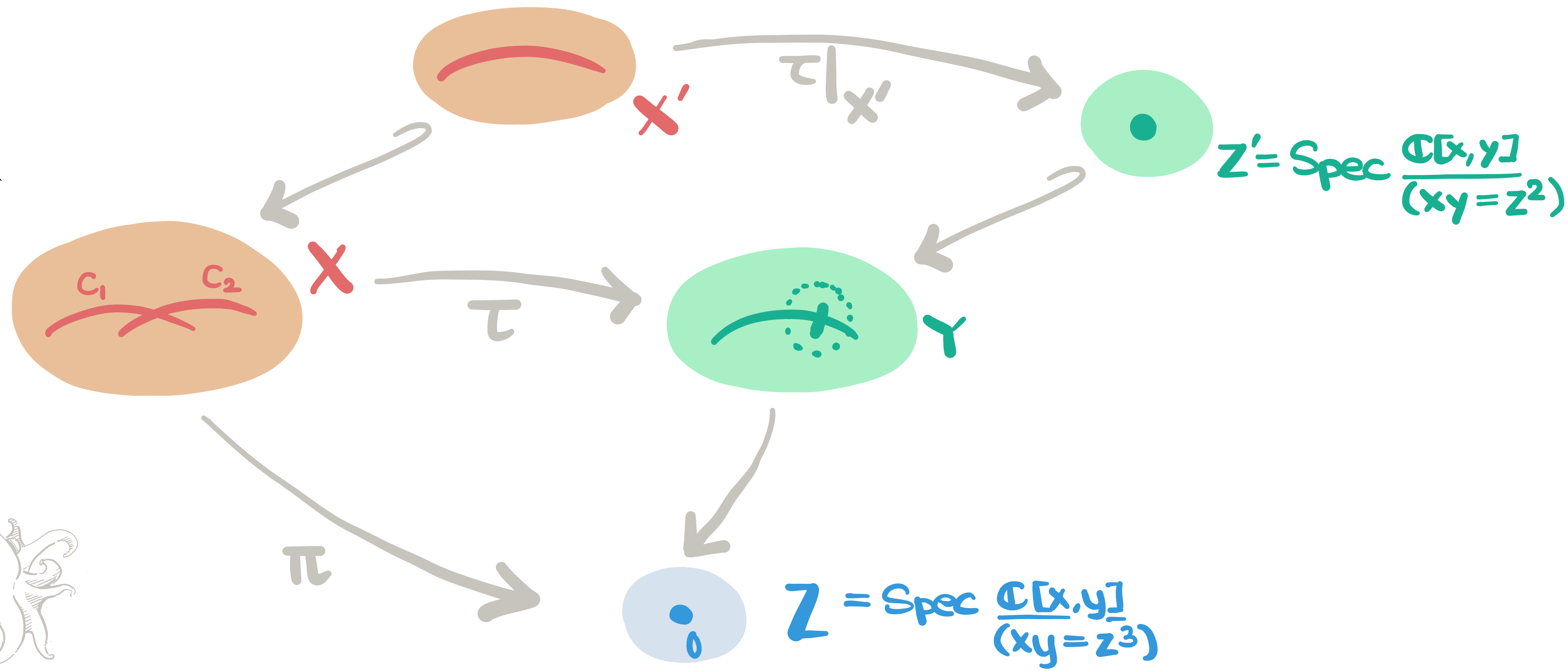
The construction of per from coh works flat-locally.

$\Rightarrow$ Y has a sheaf of nc-algebras Λ such that

$\Lambda \cong \mathcal{O}_Y$ away from contracted locus,

Λ is an NCCR near the contracted locus,

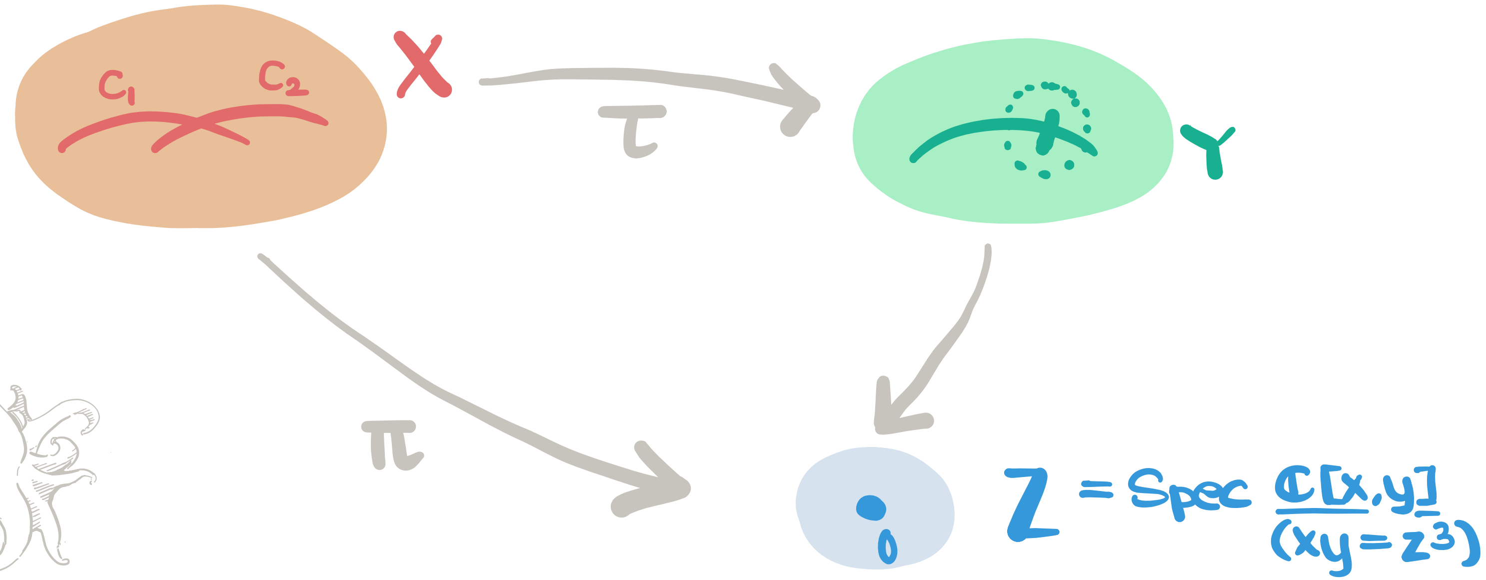
and $(X, \mathcal{O}_X)$ is derived equivalent to (Y, Λ).



Lemma. $$\mathrm{per}(X/Y) = \Big\langle \underbrace{\{\mathcal{F} \in \mathrm{per}(X/Z) \mid \mathrm{Supp}\,\mathcal{F} \text{ contracted by } \tau\}}_{=\ \tau\text{-perverse supported on contracted}},\ \underbrace{\{\mathcal{F} \in \mathrm{per}(X/X) \mid \tau \text{ is an iso. on } \mathrm{Supp}\,\mathcal{F}\}}_{=\ \tau\text{-perverse supported on uncontracted.}} \Big\rangle$$

"="

So semi-geometric hearts are "made up of" coh X and $H = \mathrm{flmod}\,\Lambda$.



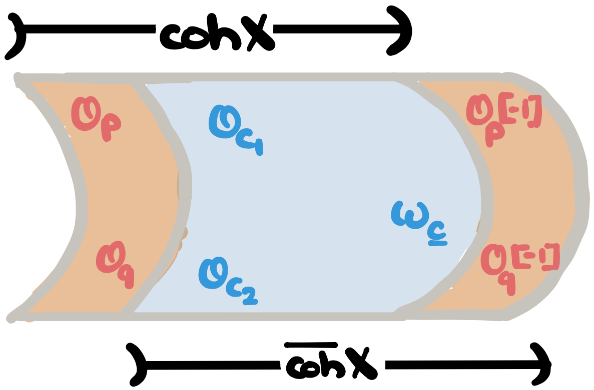
What hearts are already known?

① $\mathrm{coh}(Z,\Lambda) \cong \mathrm{fl\ mod}\,\Lambda \cong \mathrm{per}(X/Z)$

② $\mathrm{coh}(X,\mathcal{O}_X) = \mathrm{coh}\,X \cong \mathrm{per}(X/X)$

③ $\mathrm{coh}(Y,\mathcal{A}) \cong \mathrm{per}(X/Y)$ for each partial contraction.

More hearts come from modifying the above, eg by tilting coh X in (some or all) skyscrapers.



To get a whole interval $[\overline{\mathrm{coh}}\,X, \mathrm{coh}\,X] \cong \mathrm{Bool}(C)$ in $\mathrm{tilt}(H)$.
(Likewise tilt $\mathrm{per}(X/Y)$ in skyscrapers in "geometric locus").

More interestingly, have derived equivalences from tilting modules.

$T \in \operatorname{mod} \Lambda$ is tilting if

$\operatorname{Ext}^i(T,T)=0 \quad \forall i \neq 0,$

and there exist sequences

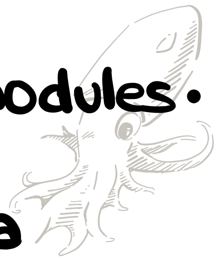
$0 \to P_2 \to P_1 \longrightarrow T \longrightarrow 0 \quad$ w/ $P_1, P_2 \in \operatorname{add} \Lambda$

$0 \to \Lambda \to T_2 \longrightarrow T_1 \longrightarrow 0 \quad$ w/ $T_1, T_2 \in \operatorname{add} T$

Get equivalence $\Psi_T : D^b(\operatorname{End} T) \xrightarrow{\mathbf{R}\operatorname{Hom}(T,-)} D^b \Lambda$ such that

$\Psi_T(\operatorname{mod}(\operatorname{End} T)) \in \operatorname{tilt}(\operatorname{mod} \Lambda)$, equivalently

$\Psi_T^{-1}(\operatorname{mod}(\Lambda)[-1]) \in \operatorname{tilt}(\operatorname{mod}(\operatorname{End} T))$.



More interestingly, have derived equivalences from tilting modules.

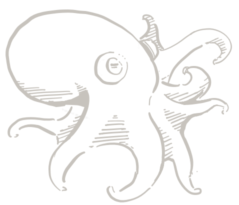
[Buan-Iyama-Reiten-Scott, Iyama-Wemyss] provide a complete understanding of tilting Λ-modules.

① in the uncontracted setting, for all T tilting, $\mathrm{End}(T)$ is canonically isomorphic to Λ.

$$\Psi_T(\mathrm{mod}(\mathrm{End}\,T)) \in \mathrm{tilt}(\mathrm{mod}\,\Lambda)$$

$$\Psi_T^{-1}(\mathrm{mod}(\Lambda)[-1]) \in \mathrm{tilt}(\mathrm{mod}(\mathrm{End}\,T)).$$

$\Rightarrow$ For each tilting module T, get two tilts $\Psi_T(\mathcal{H})$ and $\Psi_T^{-1}(\mathcal{H}[-1]) =: \Phi_T(\mathcal{H}[-1])$.



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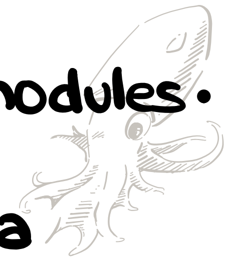
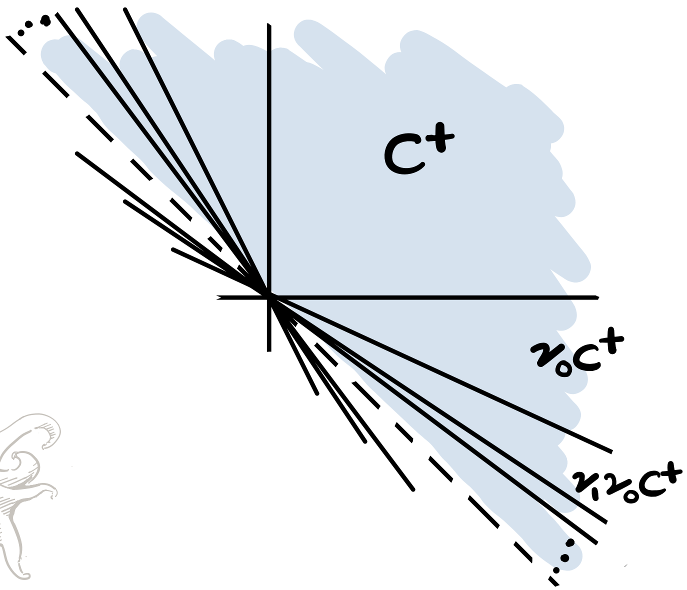


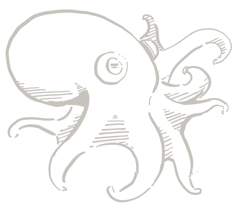
[Buan-Iyama-Reiten-Scott, Iyama-Wemyss] provide a complete understanding of tilting Λ-modules.

① in the uncontracted setting, for all T tilting, $\mathrm{End}(T)$ is canonically isomorphic to Λ.

② Iso-classes of tilting Λ-modules are in bijection with chambers of the $\hat{\Delta}$-Tits cone.



$\Rightarrow$ For each sequence of wall-crossings γ from starting chamber C^+, get a tilting module T_γ with corresponding tilted hearts $\Psi_\gamma(\mathcal{H})$, $\Phi_\gamma(\mathcal{H}[-1])$.

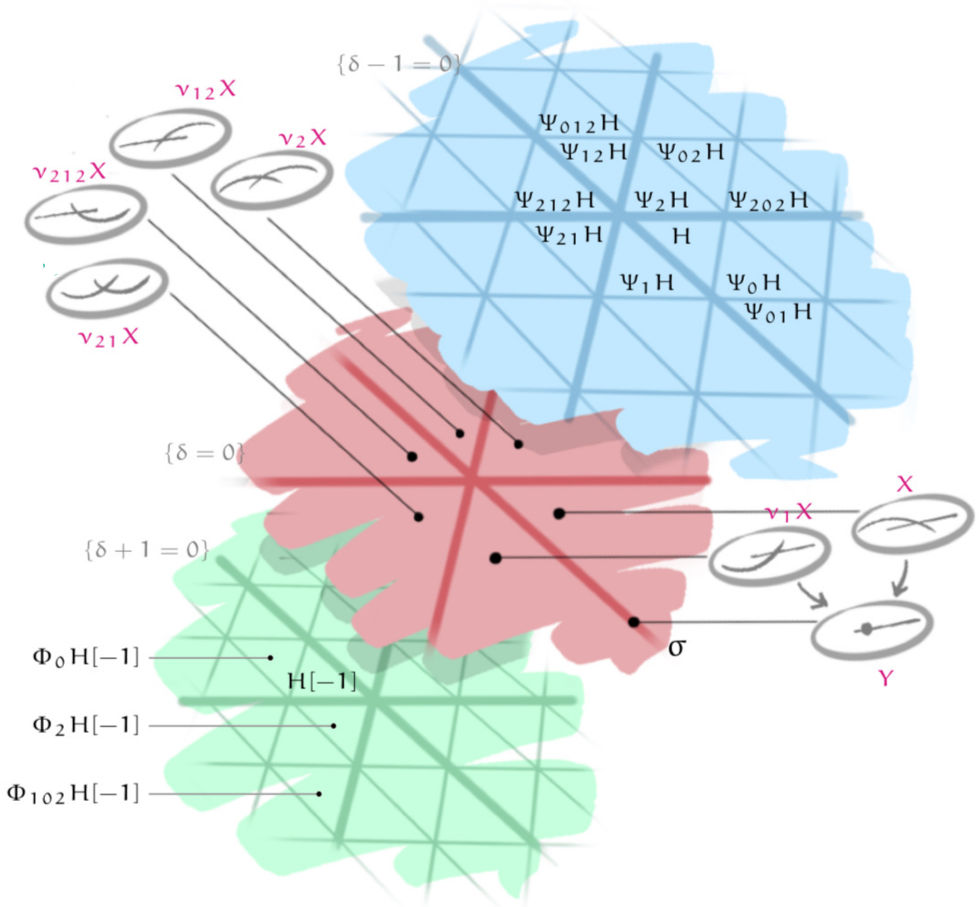


What hearts are already known?

① $H = \mathrm{coh}(Z, \Lambda)$, $\Psi_\nu H$, and $\Phi_\nu H[-1]$ for any ν.

② $K \in [\overline{\mathrm{coh}}\, X, \mathrm{coh}\, X]$, and $\Psi_\nu K$ for ν not containing 0.

③ Appropriate combinations of above, eg $\mathrm{per}(X/Y)$ and $\overline{\mathrm{per}}\, X/Y$.



Let Λ be a (contracted) affine preprojective algebra.

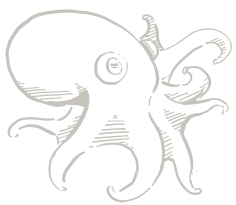
Question. Is it possible to classify hearts in $D^b \mathrm{mod}\,\Lambda$?

Theorem [S]. If $\mathcal{K}$ is a bounded heart in $D^b \mathrm{mod}\,\Lambda$ and $\mathcal{K} \subseteq \mathrm{fl}\,\mathrm{mod}\,\Lambda[-1,0]$ then one of the following holds.

① $\mathcal{K} = \mathcal{H} = \mathrm{coh}(Z,\Lambda)$, $\Psi_\nu \mathcal{H}$, or $\Phi_\nu \mathcal{H}[-1]$ for any ν.

② $\mathcal{K} \in \Psi_\nu^{-1}[\overline{\mathrm{coh}}\, X, \mathrm{coh}\, X]$ for ν not containing 0.

③ $\mathcal{K}$ is an appropriate combination of above.



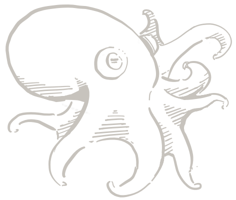
Let Λ be a (contracted) affine preprojective algebra.

Question. Is it possible to classify hearts in $D^b \mathrm{mod}\,\Lambda$?

Theorem [S]. If K is a bounded heart in $D^b \mathrm{mod}\,\Lambda$ and $K \subseteq \mathrm{fl}\,\mathrm{mod}\,\Lambda[-1,0]$ then one of the following holds.

1. $K = H = \mathrm{coh}(\mathbb{Z}, \Lambda)$, $\Psi_\gamma H$, or $\Phi_\gamma H[-1]$ for any γ.
2. $K \in \Psi_\gamma^{-1} [\overline{\mathrm{coh}}\, X, \mathrm{coh}\, X]$ for γ not containing 0.
3. K is an appropriate combination of above.

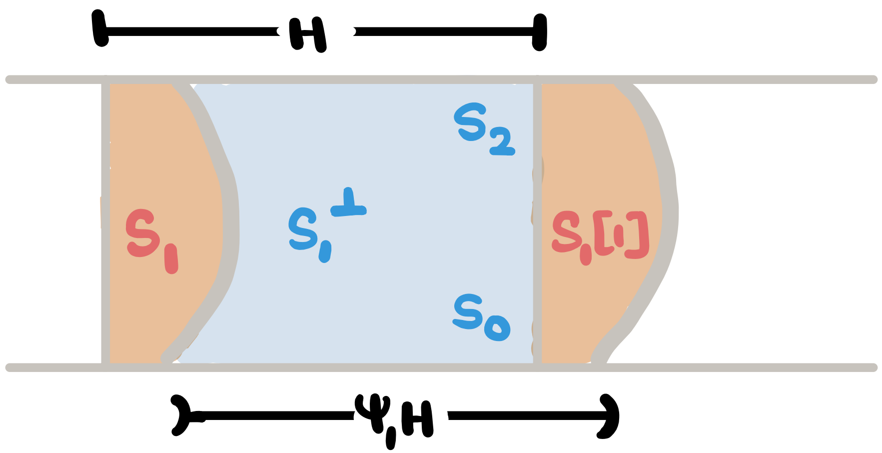
Corollary. If $M \in \mathrm{mod}\,\Lambda$ is a finite length brick, then M is either a simple Λ-module or a skyscraper up to mutation and shifts.



Local understanding of the poset.

$\Psi_i H$ is a <u>simple tilt</u> of H, i.e. the corresponding torsion class is minimal containing S_i.

$\Rightarrow$ the relation $H > \Psi_i H$ is <u>covering</u>, and all covering relations $H > ?$ arise in this way.

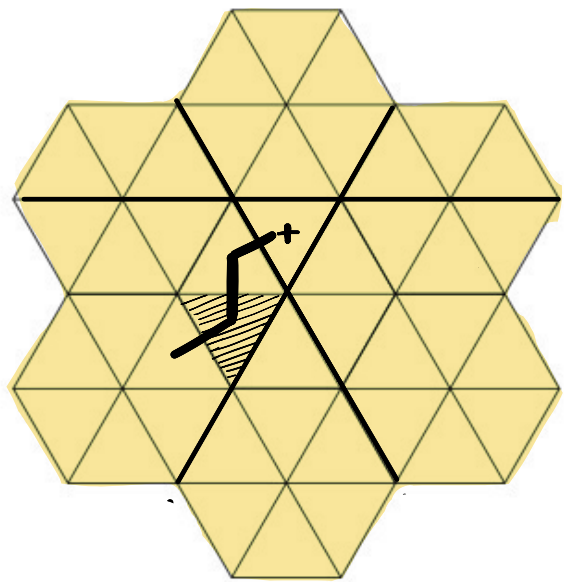


Local understanding of the poset.

$\Psi_i \cdot H$ is a <u>simple tilt</u> of H, i.e. the corresponding torsion class is minimal containing S_i.

$\Rightarrow$ the relation $H > \Psi_i H$ is <u>covering</u>, and all covering relations $H > ?$ arise in this way.

Likewise if $\gamma = \gamma_i \mu$ is a minimal path, then $\Psi_\gamma H$ is a simple tilt of $\Psi_\mu H$ and $\Phi_\mu H$ is a simple tilt of $\Phi_\gamma H[-1]$.



Global, numerical understanding of the poset.

Recap. The $\tilde{\Delta}$-Cartan algebra $\mathfrak{h}$ can be identified with $K_0\Lambda$ in a way that

- $[S_i] \in K_0\Lambda$ correspond to simple roots $\alpha_i \in \mathfrak{h}$

- $[\mathcal{O}_p] \in K_0 X$ correspond to imaginary root $\delta \in \mathfrak{h}$

- $\Theta = \mathrm{Hom}(K_0\Lambda, \mathbb{R})$ is identified with $K^{\mathrm{split}}_{\mathrm{proj}}\Lambda$

- Action of mutation coincides with Weyl group
  i.e. Ψ_i acts by simple reflection in α_i.

Then [BIRS] obtain the bijection

$$\{\text{Tilting } \Lambda\text{-modules}\} \longrightarrow \left\{\begin{array}{l}\text{chambers in Tits cone} \\ \text{in } \Theta\end{array}\right\}$$

by assigning T to its <u>g-vector cone</u>, which records 2-term projective resolutions of the summands of T.

Turns out, it can be obtained differently.

$$\{\text{Tilting } \Lambda\text{-modules}\} \longrightarrow \left\{\begin{array}{l}\text{chambers in Tits cone}\\ \text{in } \Theta\end{array}\right\}$$

$$T \longmapsto \{\theta \in \Theta \mid \theta[k] \geqslant 0 \ \forall k \in \Psi_T H\}$$

Forget tilting altogether?

$$\{\Psi_z H \mid z \text{ a path}\} \longrightarrow \left\{\begin{matrix}\text{chambers in Tits cone} \\ \text{in } \Theta\end{matrix}\right\}$$

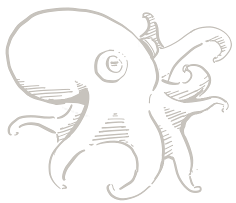
$$K \longmapsto \{\theta \in \Theta \mid \theta[k] \geq 0 \;\; \forall k \in K\}$$

Theorem [Broomhead–Pauksztello–Ploog–Woolf].
If $\mathcal{H}$ is an algebraic abelian category, then assigning each $K \in \operatorname{tilt}(\mathcal{H})$ to its **heart cone**

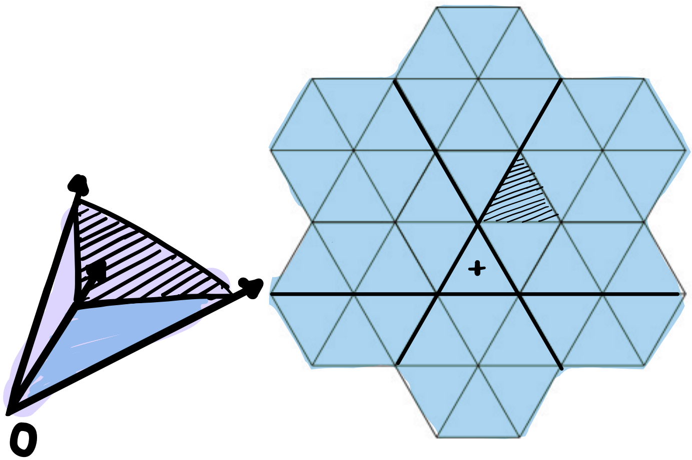
$$C_K = \{\theta \in \operatorname{Hom}(K_0\mathcal{H}, \mathbb{R}) \mid \theta(k) \geq 0 \ \forall k \in K\}$$

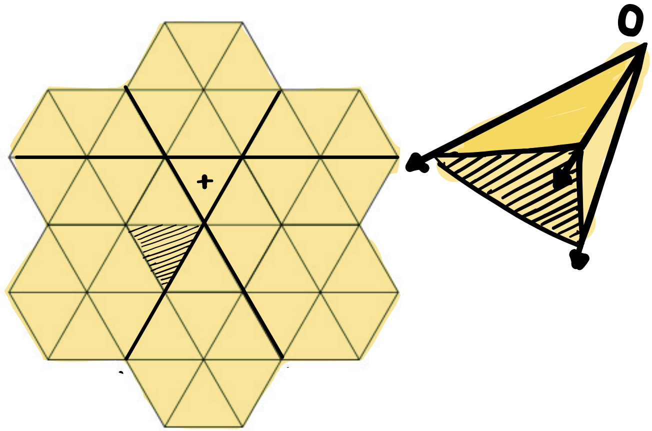
gives a complete simplicial fan, **the heart fan of $\mathcal{H}$**

$$\operatorname{HFan}(\mathcal{H}) = \{C_K \mid K \in \operatorname{tilt}(\mathcal{H})\}.$$



For $H = \mathrm{fl} \bmod \Lambda$, thus fill out "most of" Θ.

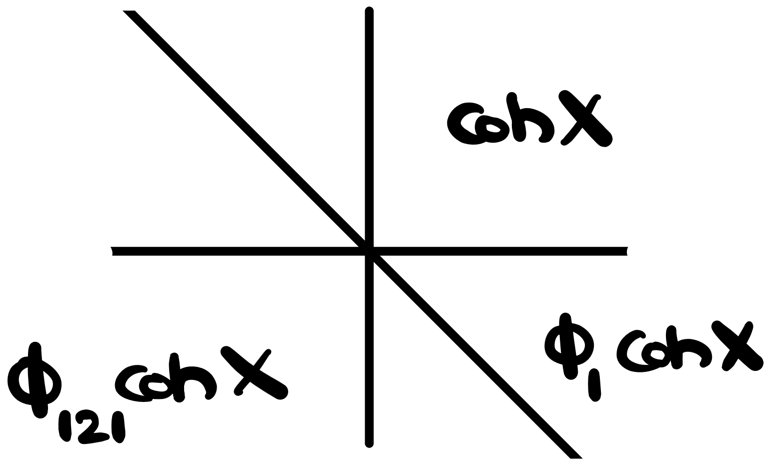




$$C(\Psi_\gamma H) = \gamma \cdot C^+ \qquad C(\Phi_\gamma H[-1]) = -(\gamma \cdot C^+).$$

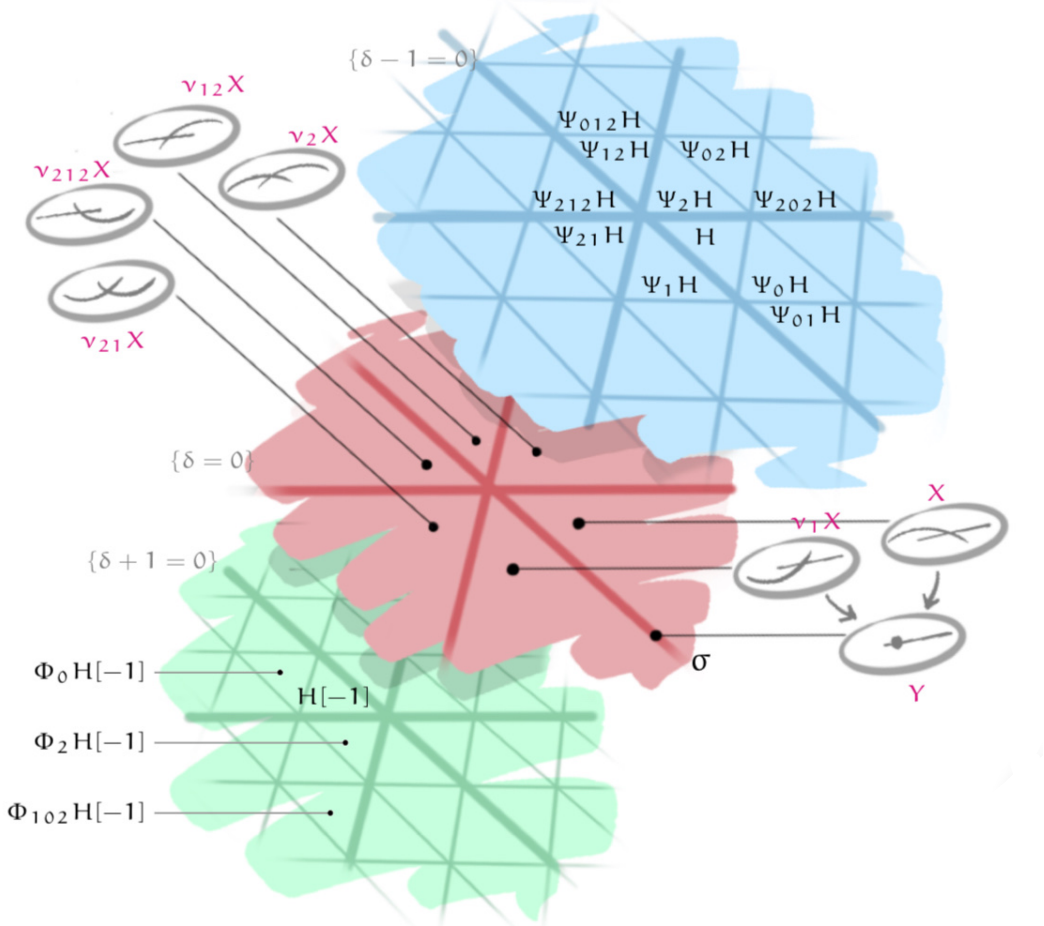
(Draw slices wrt $\delta = \sum_i [S_i]$ normalised at ± 1)

Identifying $\{\delta = 0\} \subset \Theta$ with $\mathbb{R} \otimes \mathrm{Pic}\, X$ shows the chamber $C^0 = \{\theta \mid \theta(\delta) = 0,\ \theta(\alpha_i) \geq 0\ \forall i \neq 0\}$ is the heart cone of $\mathrm{coh}\, X$,



Likewise for semi-geometric hearts.

Thus HFan(flmod Λ) is induced by root hyperplanes in Θ.



Theorem [Asai–Pfeifer]. If $\sigma \in \mathrm{HFan}(H)$ is a heart cone, then the set

$$\{K \in \mathrm{tilt}(H) \mid C_K \sqsupseteq \sigma\}$$

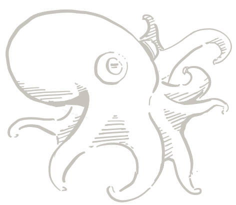
is an <u>interval</u> of the form

$$\{K \mid SS(\theta) * A(\theta) \geq K \geq A(\theta) * SS(\theta)[-1].\}$$

$SS(\theta)$: Abelian subcategory of θ-semistables in H

$$A(\theta) = \{h \in H \mid \theta(s) > 0 \ \forall\, 0 \neq s \hookrightarrow h\} * \{h \in H \mid \theta(f) < 0 \ \forall\, h \twoheadrightarrow f \neq 0\}[-1]$$

for $\theta \in \sigma$ generic.



- Thus full dimensional cones (where $SS(\theta)=\emptyset$) correspond to a unique tilt of H.

- If σ is not a full dimensional cone, then $\{K \mid \sigma \subseteq CK\}$ is in bijection with $\mathrm{tilt}(SS(\theta))$

  (If $SS(\theta)=U*V$ is a torsion pair then consider $K = V * A(\theta) * U[-1]$)

  so get inductive behaviour.

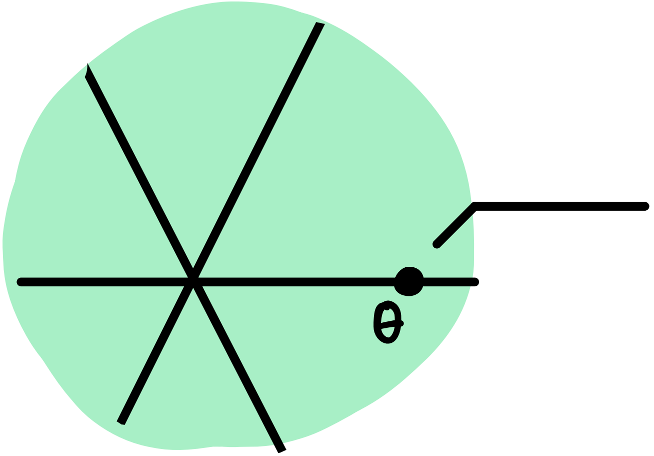
Eg for $\sigma = C_0$, compute

$$SS(\Theta) = \langle \Theta_p \mid p \in C \rangle$$

So must have $\quad \operatorname{coh} X = SS(\Theta) * A(\Theta)$

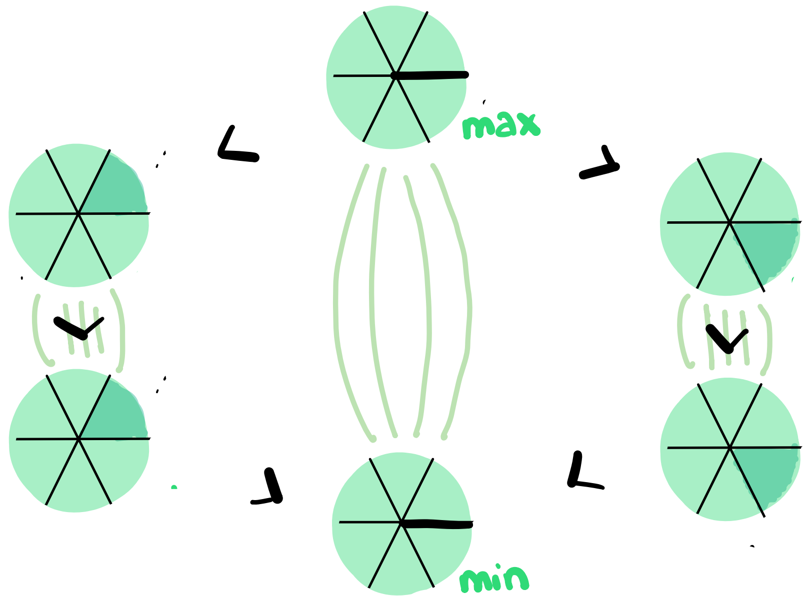
$$\Rightarrow \quad A(\Theta) * SS(\Theta)[-1] = \overline{\operatorname{coh}} X$$

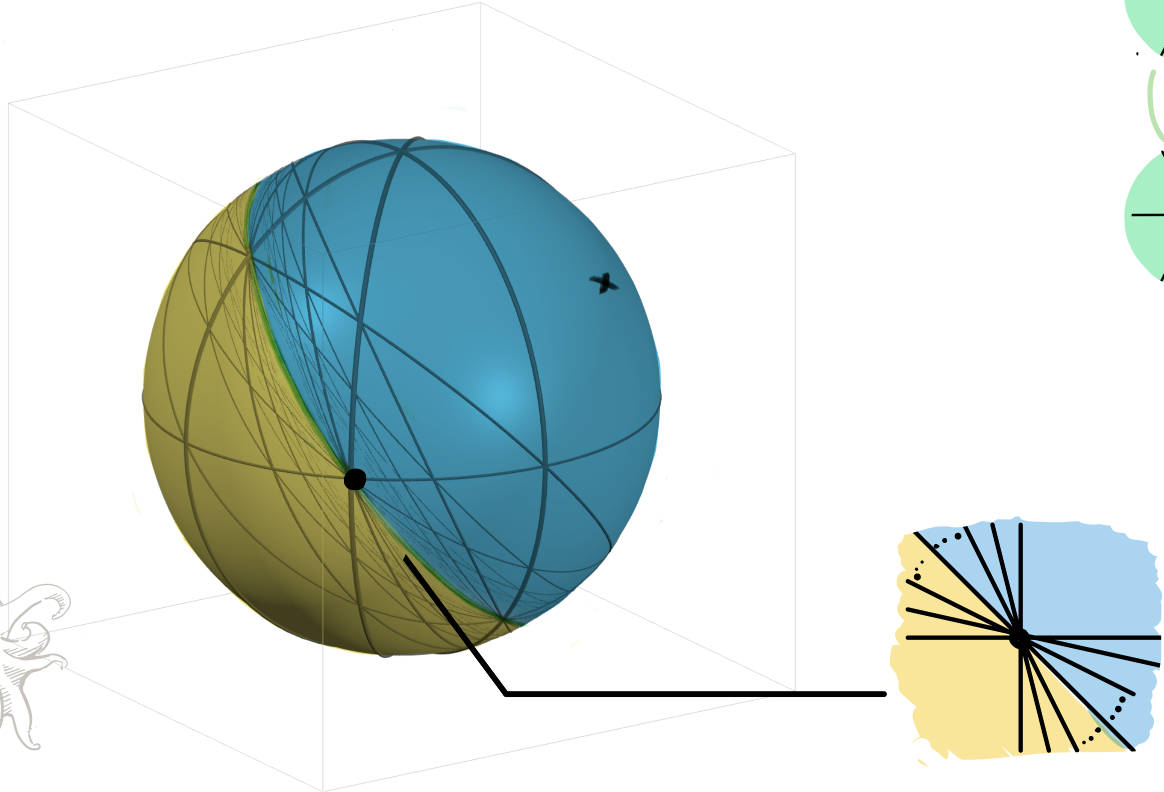
i.e. $K \in \operatorname{tilt}(H)$ has heart cone C_0 iff it is a tilt of $\operatorname{coh} X$ in skyscrapers.



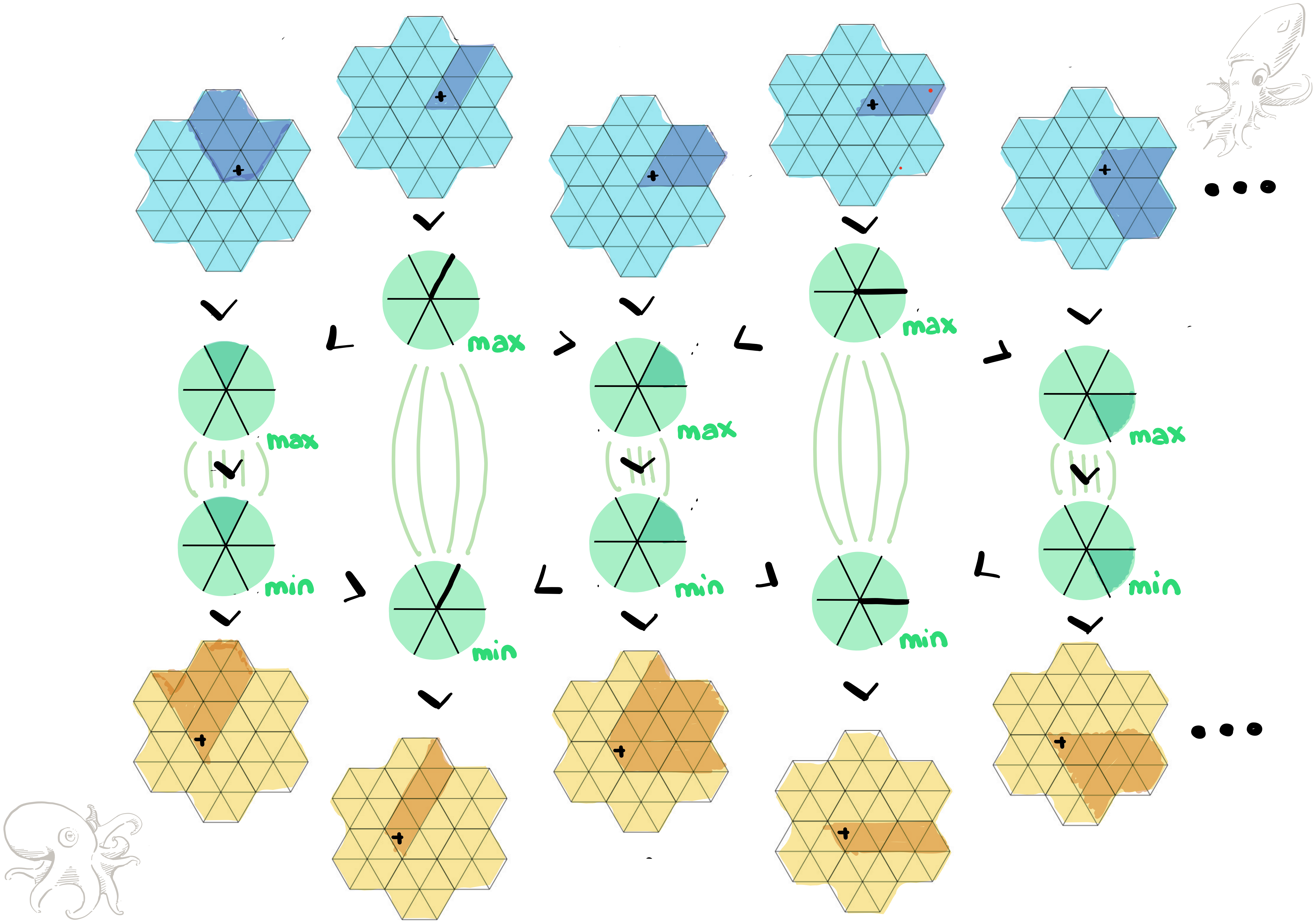
$H^\theta = \mathrm{per}(X/Y)$, $SS(\theta) = \langle$ skyscrapers in geom, all of algebraic $\rangle$

Poset is $\mathrm{int}(\mathrm{flmod}\,\Lambda_{\tilde{A}_1}) \times \mathrm{Bool}(E \setminus E_1)$





$\heartsuit$-fan of $SS(\theta)$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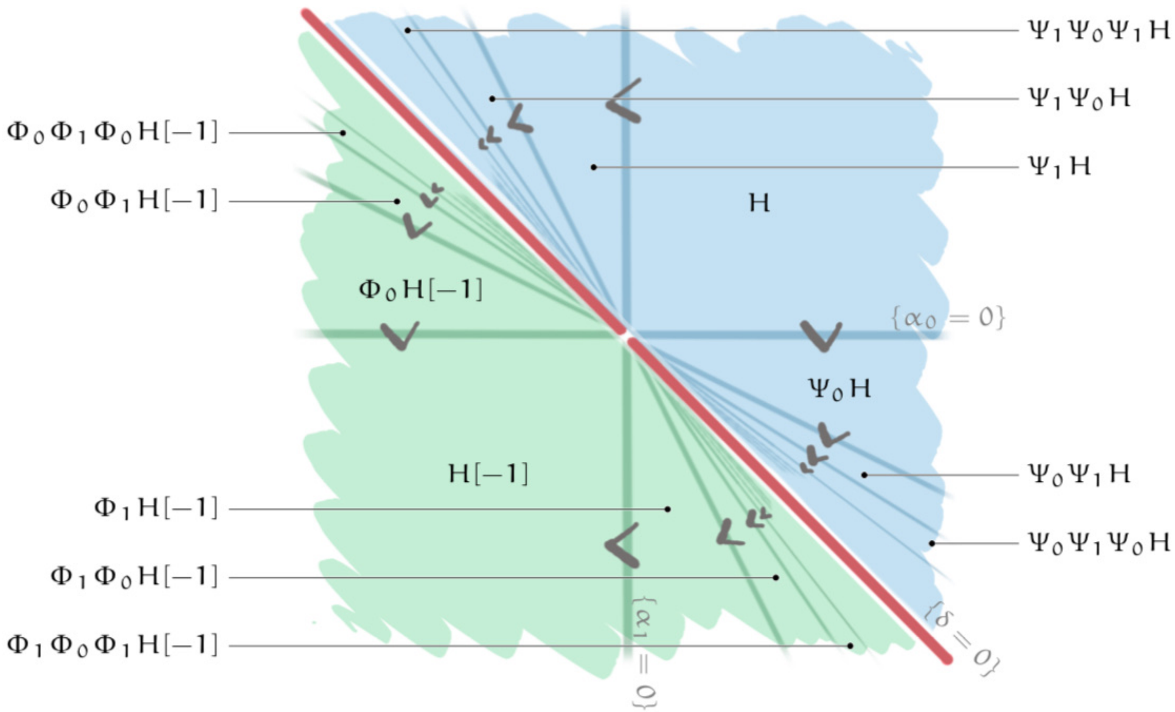
Lastly (and most importantly), 0 is not a heart cone i.e. $\forall K \in \mathrm{tilt}(H)$, $C K \neq 0$.

In single curve case, this is immediate from the fact that relations between algebraic hearts are covering.



($H > \Psi_1 H > \Psi_{01} H > \dots > K$

$\Rightarrow \mathrm{coh}\, X \geq K$

$K < \dots < \Phi_{10} H[-1] < \Phi_0 H[-1]$

$\Rightarrow K \leq \overline{\mathrm{coh}}\, X$)

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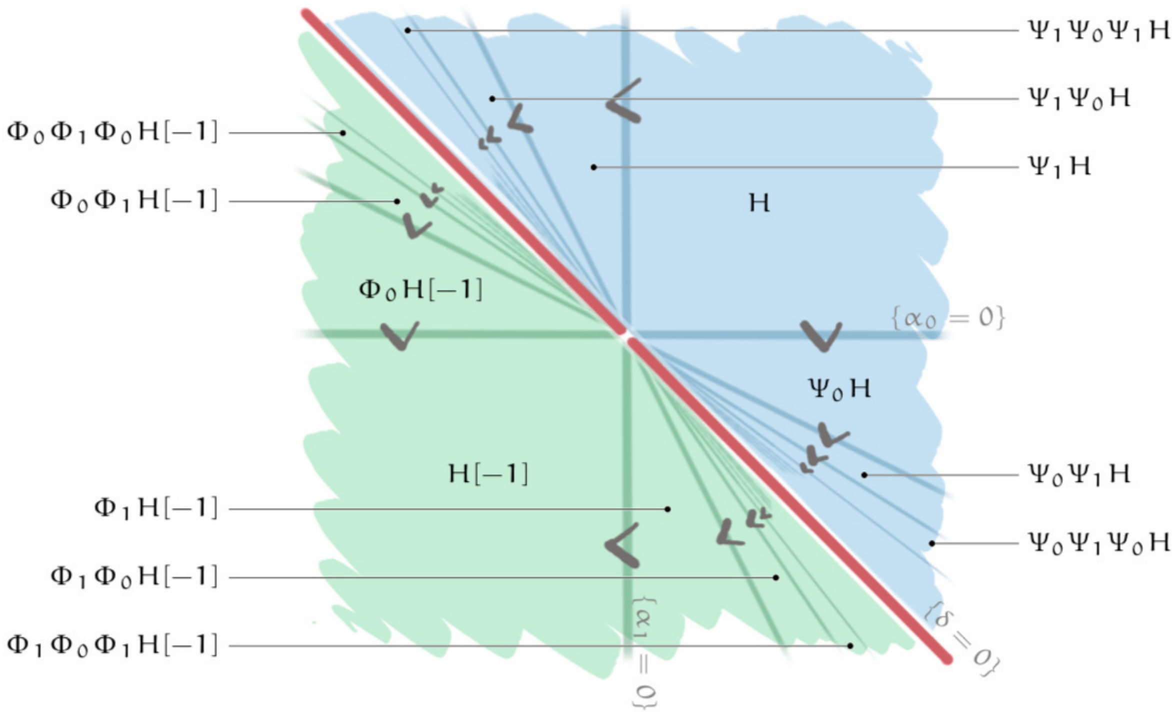

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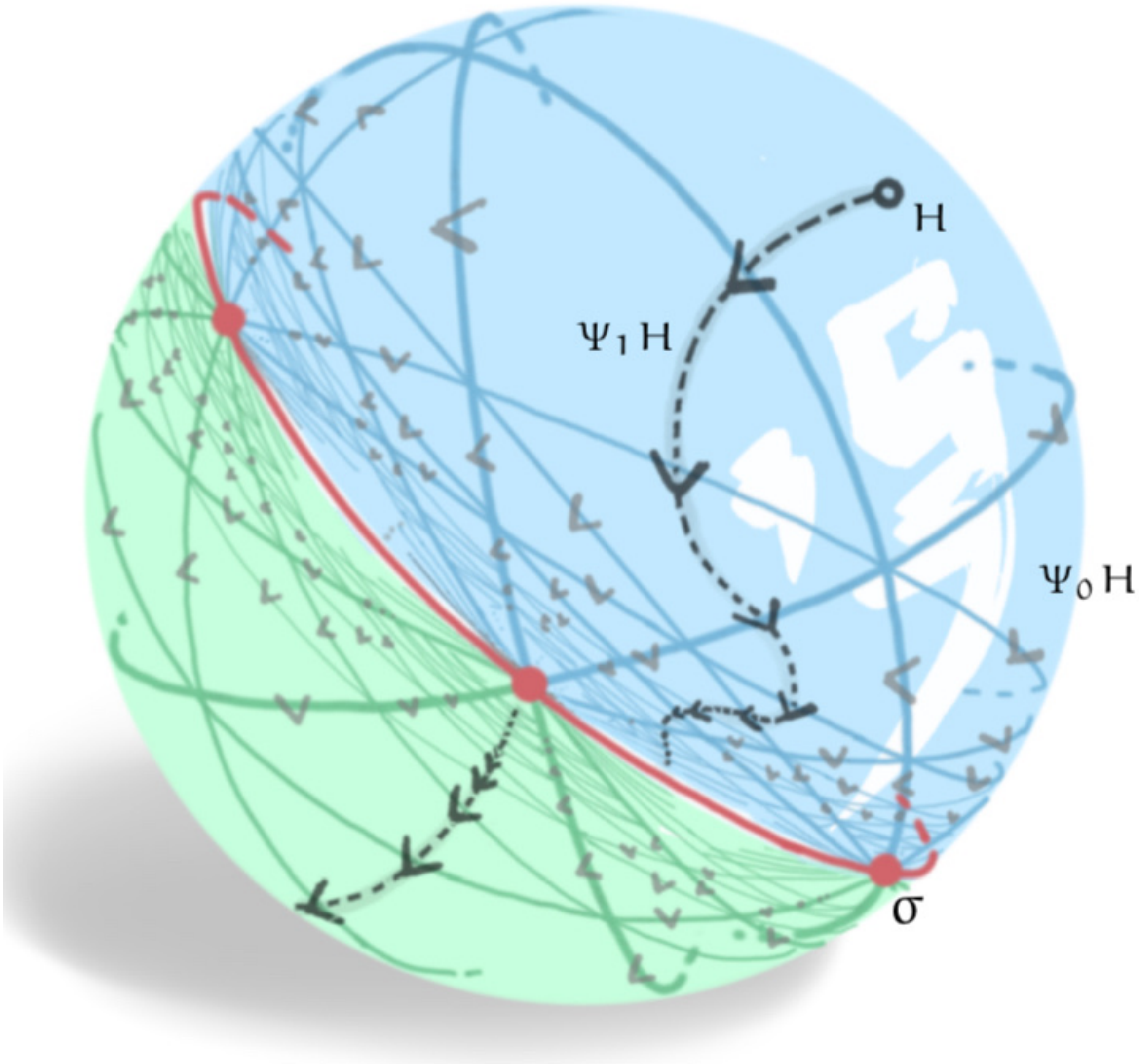
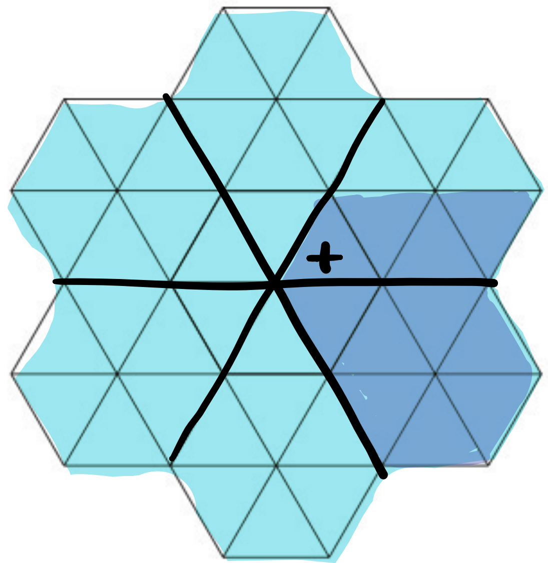
Need more control in dim $\geqslant 3$.



Lemma. For $K \in \mathrm{tilt}(H)$ not algebraic, let ν be the longest path such that ① ν does not contain 0

② $\Psi_2 H > K$.

Then for every $p \in X$, $\Psi_2 \mathcal{O}_p$ lies in K or $K[1]$.



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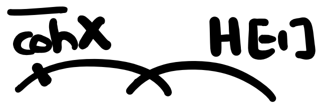
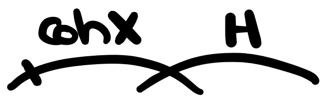
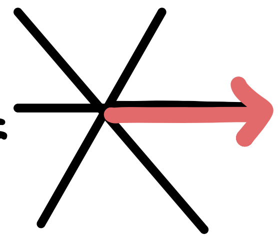
- Replace K by $\Psi_2^{-1} K$ if necessary.

If $p \in C_1$ and $\mathcal{O}_p \in K$, then get $K \geq$ 

$\mathcal{O}_p \in K[1]$, then get 

$\geq K$

so if $\exists\, p, q \in C_1$ with $\mathcal{O}_p \in K$, $\mathcal{O}_q \in K[1]$ then $CK =$ 

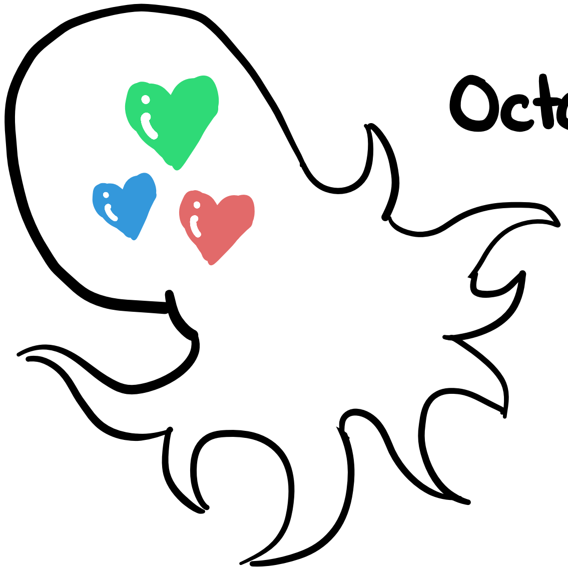
Else have (wlog) $\mathcal{O}_p \in \mathcal{K}$ $\forall p \in C$, so that

$$\mathcal{K} \geqslant \sup\left(\overline{\mathrm{coh}}\,X \quad H[-1] \;,\; H[-1] \quad \overline{\mathrm{coh}}\,X \right) = \overline{\mathrm{coh}}\,X .$$

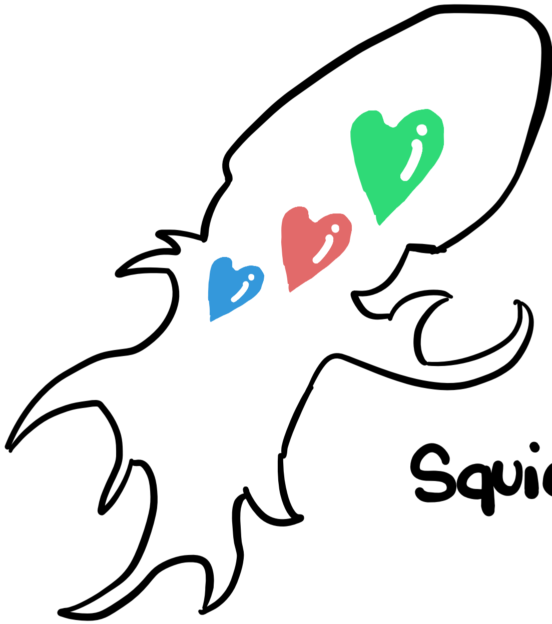
Chasing simple tilts gives $\mathrm{coh}\,X \quad H \geqslant \mathcal{K}$ for some curve, so that

$$\mathrm{coh}\,X \quad H \;\geqslant\; \mathcal{K} \;\geqslant\; \overline{\mathrm{coh}}\,X \;\geqslant\; \overline{\mathrm{coh}}\,X \quad H[-1] .$$

Again, $C\mathcal{K} \neq 0$.

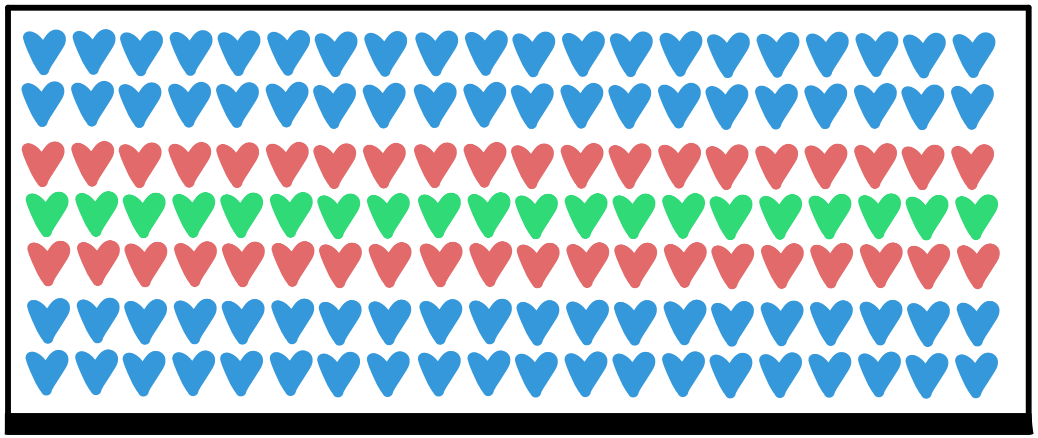


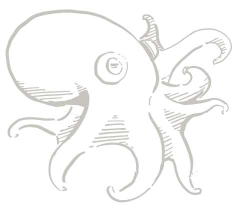
Octopus



Squid

$D^{fl}mod\Lambda$





*Pictures may be anatomically inaccurate.