EECS 16ML: Introduction to Practical ML
Huang-Sigg-Wang-Wang

UC Berkeley, Spring 2021
February 26, 2021

Graph Spectral Clustering Quiz

Last Name	First Name	SID

***Rules**:*

- **Unless otherwise stated, all your answers need to be justified and your work must be shown. Answers without sufficient justification will get no credit.**
- You have 30 minutes to complete the quiz.
- Collaboration with others is strictly prohibited.
- You may not reference your notes, the textbook, and any material that can be found through the course website. You may not use Google to search up general knowledge.
- For any clarifications you have, please create a private Piazza post. We will have a Google Doc that shows our official clarifications.

1 True/False [1 point each]

1) The largest eigenvectors of the similarity graph Laplacian matrix should be used to construct the eigenvector matrix that will eventually be used for clustering.

   (a) True

   (b) False

2) When performing k-means in in the graph spectral clustering algorithm, k-means should be performed on the columns of the eigenvector matrix of the Laplacian matrix.

   (a) True

   (b) False

3) Graph spectral clustering builds on a centroid-based clustering algorithm to achieve more complex decision boundaries between clusters.

   (a) True

   (b) False

4) All entries in the similarity matrix used in graph spectral clustering are nonnegative.

   (a) True

   (b) False

5) The goal of spectral graph clustering is to separate points into clusters such that intra-cluster (within each cluster) similarity is minimized and inter-cluster (between different clusters) similarity is maximized.

   (a) True

   (b) False

2 Multiple Choice [2 points each]

1) Which matrix has diagonal entries that correspond to the sum of the weights of edges adjacent to each vertex?

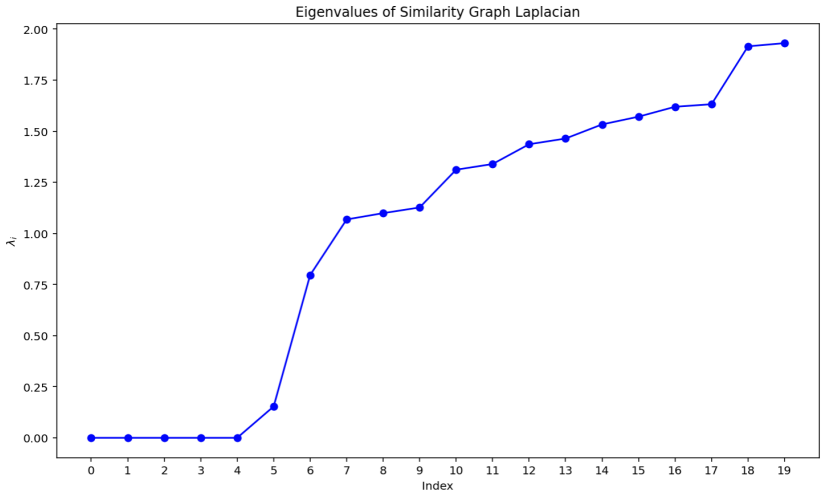
   (a) Degree matrix

   (b) Adjacency matrix

   (c) Laplacian matrix

   (d) None of the above

2) Looking at the plot below for eigenvalue growth rate, which of the following values of k would be best to choose?



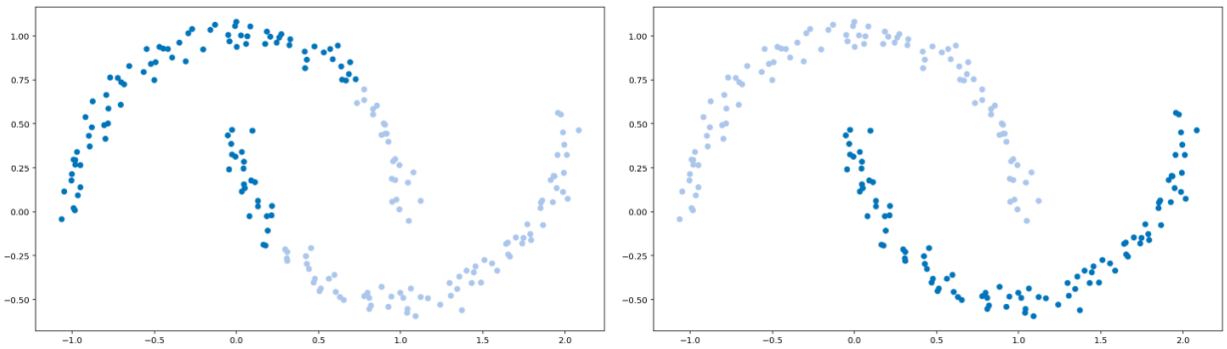
(a) 3

(b) 5

(c) 7

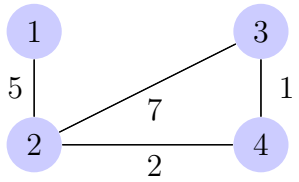
(d) 9

3) For the two plots shown below, one is the result of spectral clustering with a Gaussian similarity function with $\sigma = 0.1$, and another is the result of spectral clustering with a Gaussian similarity function with $\sigma = 1$. Label the plots with the corresponding σ value of the Gaussian similarity function used.



3 Short Answers [3 points each]

1) Find the Laplacian matrix for this graph:



2) What is the role of a similarity function in graph spectral clustering? List 2 other examples of valid similarity functions.

3) Explain why it is always possible to compute the first k eigenvectors of the Laplacian matrix in spectral clustering.

4) What effect does increasing σ have on the RBF kernel?

5) Why do we prefer using k-means++ over k-means in general? Give two reasons.

6) Fill in the blanks to complete the pseudocode of the normalized graph spectral clustering algorithm.

   Return clusters $A_1, \cdots, A_k$
   Given data $X \in \mathbb{R}^{n \times d}$, number of desired clusters k

   (a) Construct a _____ $S \in \mathbb{R}^{n \times n}$ according to some well defined metric, such as the Gaussian kernel or Euclidean distance.

   (b) Compute $L_n =$ _____, the symmetric normalized graph Laplacian of S.

   (c) Compute the first k _____ of L, where the first k _____ correspond to _____.

   (d) Construct a matrix H for L with _____, making sure to normalize the _____ to have norm 1.

   (e) Run _____ on _____ to cluster the data.