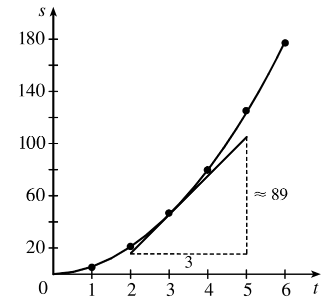
(b) Using the points $(2, 16)$ and $(5, 105)$ from the approximate tangent line, the instantaneous velocity at $t = 3$ is about $\frac{105 - 16}{5 - 2} = \frac{89}{3} \approx 29.7$ ft/s.



8. (a) (i) $s = s(t) = 2\sin \pi t + 3\cos \pi t$. On the interval $[1, 2]$, $v_{\text{ave}} = \frac{s(2) - s(1)}{2 - 1} = \frac{3 - (-3)}{1} = 6$ cm/s.

(ii) On the interval $[1, 1.1]$, $v_{\text{ave}} = \frac{s(1.1) - s(1)}{1.1 - 1} \approx \frac{-3.471 - (-3)}{0.1} = -4.71$ cm/s.

(iii) On the interval $[1, 1.01]$, $v_{\text{ave}} = \frac{s(1.01) - s(1)}{1.01 - 1} \approx \frac{-3.0613 - (-3)}{0.01} = -6.13$ cm/s.

(iv) On the interval $[1, 1.001]$, $v_{\text{ave}} = \frac{s(1.001) - s(1)}{1.001 - 1} \approx \frac{-3.00627 - (-3)}{0.001} = -6.27$ cm/s.

(b) The instantaneous velocity of the particle when $t = 1$ appears to be about -6.3 cm/s.

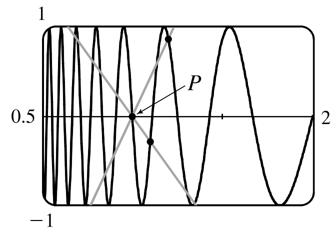
9. (a) For the curve $y = \sin(10\pi/x)$ and the point $P(1, 0)$:

x	Q	m_{PQ}
2	$(2, 0)$	0
1.5	$(1.5, 0.8660)$	1.7321
1.4	$(1.4, -0.4339)$	-1.0847
1.3	$(1.3, -0.8230)$	-2.7433
1.2	$(1.2, 0.8660)$	4.3301
1.1	$(1.1, -0.2817)$	-2.8173

x	Q	m_{PQ}
0.5	$(0.5, 0)$	0
0.6	$(0.6, 0.8660)$	-2.1651
0.7	$(0.7, 0.7818)$	-2.6061
0.8	$(0.8, 1)$	-5
0.9	$(0.9, -0.3420)$	3.4202

As x approaches 1, the slopes do not appear to be approaching any particular value.

(b)



We see that problems with estimation are caused by the frequent oscillations of the graph. The tangent is so steep at P that we need to take x-values much closer to 1 in order to get accurate estimates of its slope.

(c) If we choose $x = 1.001$, then the point Q is $(1.001, -0.0314)$ and $m_{PQ} \approx -31.3794$. If $x = 0.999$, then Q is $(0.999, 0.0314)$ and $m_{PQ} = -31.4422$. The average of these slopes is -31.4108. So we estimate that the slope of the tangent line at P is about -31.4.

2.2 The Limit of a Function

1. As x approaches 2, $f(x)$ approaches 5. [Or, the values of $f(x)$ can be made as close to 5 as we like by taking x sufficiently close to 2 (but $x \neq 2$).] Yes, the graph could have a hole at $(2, 5)$ and be defined such that $f(2) = 3$.

2. As x approaches 1 from the left, $f(x)$ approaches 3; and as x approaches 1 from the right, $f(x)$ approaches 7. No, the limit does not exist because the left- and right-hand limits are different.

3. (a) $\lim\limits_{x \to -3} f(x) = \infty$ means that the values of $f(x)$ can be made arbitrarily large (as large as we please) by taking x sufficiently close to -3 (but not equal to -3).

(b) $\lim\limits_{x \to 4^+} f(x) = -\infty$ means that the values of $f(x)$ can be made arbitrarily large negative by taking x sufficiently close to 4 through values larger than 4.

4. (a) As x approaches 2 from the left, the values of $f(x)$ approach 3, so $\lim\limits_{x \to 2^-} f(x) = 3$.

(b) As x approaches 2 from the right, the values of $f(x)$ approach 1, so $\lim\limits_{x \to 2^+} f(x) = 1$.

(c) $\lim\limits_{x \to 2} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit.

(d) When $x = 2$, $y = 3$, so $f(2) = 3$.

(e) As x approaches 4, the values of $f(x)$ approach 4, so $\lim\limits_{x \to 4} f(x) = 4$.

(f) There is no value of $f(x)$ when $x = 4$, so $f(4)$ does not exist.

5. (a) As x approaches 1, the values of $f(x)$ approach 2, so $\lim\limits_{x \to 1} f(x) = 2$.

(b) As x approaches 3 from the left, the values of $f(x)$ approach 1, so $\lim\limits_{x \to 3^-} f(x) = 1$.

(c) As x approaches 3 from the right, the values of $f(x)$ approach 4, so $\lim\limits_{x \to 3^+} f(x) = 4$.

(d) $\lim\limits_{x \to 3} f(x)$ does not exist since the left-hand limit does not equal the right-hand limit.

(e) When $x = 3$, $y = 3$, so $f(3) = 3$.

6. (a) $h(x)$ approaches 4 as x approaches -3 from the left, so $\lim\limits_{x \to -3^-} h(x) = 4$.

(b) $h(x)$ approaches 4 as x approaches -3 from the right, so $\lim\limits_{x \to -3^+} h(x) = 4$.

(c) $\lim\limits_{x \to -3} h(x) = 4$ because the limits in part (a) and part (b) are equal.

(d) $h(-3)$ is not defined, so it doesn't exist.

(e) $h(x)$ approaches 1 as x approaches 0 from the left, so $\lim\limits_{x \to 0^-} h(x) = 1$.

(f) $h(x)$ approaches -1 as x approaches 0 from the right, so $\lim\limits_{x \to 0^+} h(x) = -1$.

(g) $\lim\limits_{x \to 0} h(x)$ does not exist because the limits in part (e) and part (f) are not equal.

(h) $h(0) = 1$ since the point $(0, 1)$ is on the graph of h.

(i) Since $\lim\limits_{x \to 2^-} h(x) = 2$ and $\lim\limits_{x \to 2^+} h(x) = 2$, we have $\lim\limits_{x \to 2} h(x) = 2$.

(j) $h(2)$ is not defined, so it doesn't exist.

(k) $h(x)$ approaches 3 as x approaches 5 from the right, so $\lim\limits_{x\to 5^+} h(x) = 3$.

(l) $h(x)$ does not approach any one number as x approaches 5 from the left, so $\lim\limits_{x\to 5^-} h(x)$ does not exist.

7. (a) $\lim\limits_{t\to 0^-} g(t) = -1$ (b) $\lim\limits_{t\to 0^+} g(t) = -2$

(c) $\lim\limits_{t\to 0} g(t)$ does not exist because the limits in part (a) and part (b) are not equal.

(d) $\lim\limits_{t\to 2^-} g(t) = 2$ (e) $\lim\limits_{t\to 2^+} g(t) = 0$

(f) $\lim\limits_{t\to 2} g(t)$ does not exist because the limits in part (d) and part (e) are not equal.

(g) $g(2) = 1$ (h) $\lim\limits_{t\to 4} g(t) = 3$

8. (a) $\lim\limits_{x\to -3} A(x) = \infty$ (b) $\lim\limits_{x\to 2^-} A(x) = -\infty$

(c) $\lim\limits_{x\to 2^+} A(x) = \infty$ (d) $\lim\limits_{x\to -1} A(x) = -\infty$

(e) The equations of the vertical asymptotes are $x = -3$, $x = -1$ and $x = 2$.

9. (a) $\lim\limits_{x\to -7} f(x) = -\infty$ (b) $\lim\limits_{x\to -3} f(x) = \infty$ (c) $\lim\limits_{x\to 0} f(x) = \infty$

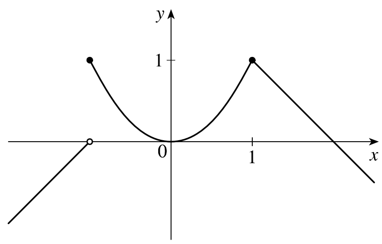
(d) $\lim\limits_{x\to 6^-} f(x) = -\infty$ (e) $\lim\limits_{x\to 6^+} f(x) = \infty$

(f) The equations of the vertical asymptotes are $x = -7$, $x = -3$, $x = 0$, and $x = 6$.

10. $\lim\limits_{t\to 12^-} f(t) = 150$ mg and $\lim\limits_{t\to 12^+} f(t) = 300$ mg. These limits show that there is an abrupt change in the amount of drug in the patient's bloodstream at $t = 12$ h. The left-hand limit represents the amount of the drug just before the fourth injection. The right-hand limit represents the amount of the drug just after the fourth injection.

11. From the graph of

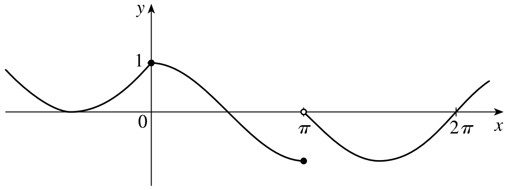
$$f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1\,, \\ 2 - x & \text{if } x \ge 1 \end{cases}$$



we see that $\lim\limits_{x\to a} f(x)$ exists for all a except $a = -1$. Notice that the right and left limits are different at $a = -1$.

12. From the graph of

$$f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \le x \le \pi\,, \\ \sin x & \text{if } x > \pi \end{cases}$$

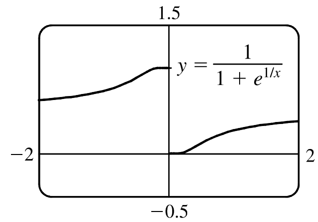


we see that $\lim\limits_{x\to a} f(x)$ exists for all a except $a = \pi$. Notice that the right and left limits are different at $a = \pi$.

13. (a) $\lim\limits_{x\to 0^-} f(x) = 1$

(b) $\lim\limits_{x\to 0^+} f(x) = 0$

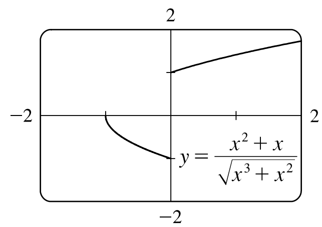
(c) $\lim\limits_{x\to 0} f(x)$ does not exist because the limits in part (a) and part (b) are not equal.



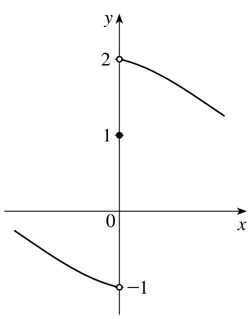
14. (a) $\lim\limits_{x\to 0^-} f(x) = -1$

(b) $\lim\limits_{x\to 0^+} f(x) = 1$

(c) $\lim\limits_{x\to 0} f(x)$ does not exist because the limits in part (a) and part (b) are not equal.

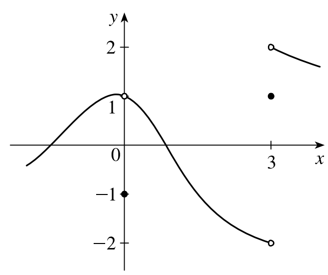


15. $\lim\limits_{x\to 0^-} f(x) = -1,\quad \lim\limits_{x\to 0^+} f(x) = 2,\quad f(0) = 1$



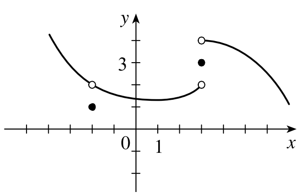
16. $\lim\limits_{x\to 0} f(x) = 1,\ \lim\limits_{x\to 3^-} f(x) = -2,\ \lim\limits_{x\to 3^+} f(x) = 2,$

$f(0) = -1,\ f(3) = 1$



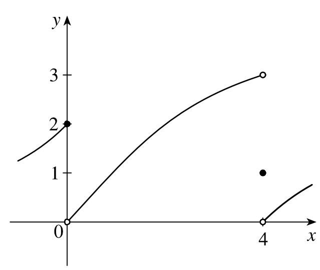
17. $\lim\limits_{x\to 3^+} f(x) = 4,\quad \lim\limits_{x\to 3^-} f(x) = 2,\ \lim\limits_{x\to -2} f(x) = 2,$

$f(3) = 3,\quad f(-2) = 1$



18. $\lim\limits_{x\to 0^-} f(x) = 2,\ \lim\limits_{x\to 0^+} f(x) = 0,\ \lim\limits_{x\to 4^-} f(x) = 3,$

$\lim\limits_{x\to 4^+} f(x) = 0,\ f(0) = 2,\ f(4) = 1$



19. For $f(x) = \dfrac{x^2 - 3x}{x^2 - 9}$:

x	$f(x)$
3.1	0.508 197
3.05	0.504 132
3.01	0.500 832
3.001	0.500 083
3.0001	0.500 008

x	$f(x)$
2.9	0.491 525
2.95	0.495 798
2.99	0.499 165
2.999	0.499 917
2.9999	0.499 992

It appears that $\lim\limits_{x\to 3} \dfrac{x^2 - 3x}{x^2 - 9} = \dfrac{1}{2}$.

20. For $f(x) = \dfrac{x^2 - 3x}{x^2 - 9}$:

x	$f(x)$
-2.5	-5
-2.9	-29
-2.95	-59
-2.99	-299
-2.999	-2999
-2.9999	$-29{,}999$

x	$f(x)$
-3.5	7
-3.1	31
-3.05	61
-3.01	301
-3.001	3001
-3.0001	$30{,}001$

It appears that $\lim\limits_{x\to-3^+} f(x) = -\infty$ and that $\lim\limits_{x\to-3^-} f(x) = \infty$, so $\lim\limits_{x\to-3} \dfrac{x^2 - 3x}{x^2 - 9}$ does not exist.

21. For $f(t) = \dfrac{e^{5t} - 1}{t}$:

t	$f(t)$
0.5	22.364 988
0.1	6.487 213
0.01	5.127 110
0.001	5.012 521
0.0001	5.001 250

t	$f(t)$
-0.5	1.835 830
-0.1	3.934 693
-0.01	4.877 058
-0.001	4.987 521
-0.0001	4.998 750

It appears that $\lim\limits_{t\to0} \dfrac{e^{5t} - 1}{t} = 5$.

22. For $f(h) = \dfrac{(2+h)^5 - 32}{h}$:

h	$f(h)$
0.5	131.312 500
0.1	88.410 100
0.01	80.804 010
0.001	80.080 040
0.0001	80.008 000

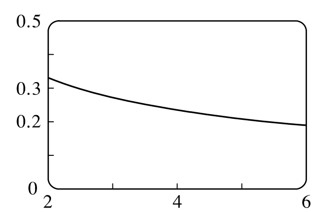
h	$f(h)$
-0.5	48.812 500
-0.1	72.390 100
-0.01	79.203 990
-0.001	79.920 040
-0.0001	79.992 000

It appears that $\lim\limits_{h\to0} \dfrac{(2+h)^5 - 32}{h} = 80$.

23. For $f(x) = \dfrac{\ln x - \ln 4}{x - 4}$:

x	$f(x)$
3.9	0.253 178
3.99	0.250 313
3.999	0.250 031
3.9999	0.250 003

x	$f(x)$
4.1	0.246 926
4.01	0.249 688
4.001	0.249 969
4.0001	0.249 997

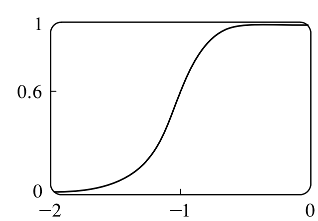


It appears that $\lim\limits_{x\to4} f(x) = 0.25$. The graph confirms that result.

24. For $f(p) = \dfrac{1 + p^9}{1 + p^{15}}$:

p	$f(p)$
-1.1	0.427 397
-1.01	0.582 008
-1.001	0.598 200
-1.0001	0.599 820

p	$f(p)$
-0.9	0.771 405
-0.99	0.617 992
-0.999	0.601 800
-0.9999	0.600 180



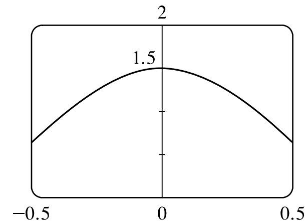
It appears that $\lim\limits_{p\to-1} f(p) = 0.6$. The graph confirms that result.

25. For $f(\theta) = \dfrac{\sin 3\theta}{\tan 2\theta}$:

θ	$f(\theta)$
± 0.1	1.457 847
± 0.01	1.499 575
± 0.001	1.499 996
± 0.0001	1.500 000

It appears that $\lim\limits_{\theta \to 0} \dfrac{\sin 3\theta}{\tan 2\theta} = 1.5$.

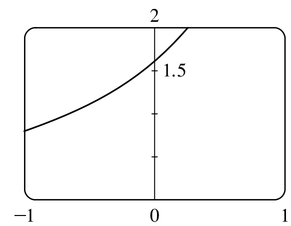
The graph confirms that result.



26. For $f(t) = \dfrac{5^t - 1}{t}$:

t	$f(t)$
0.1	1.746 189
0.01	1.622 459
0.001	1.610 734
0.0001	1.609 567

t	$f(t)$
-0.1	1.486 601
-0.01	1.596 556
-0.001	1.608 143
-0.0001	1.609 308



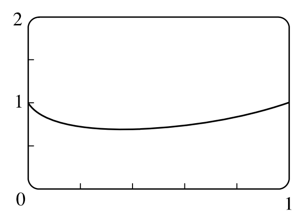
It appears that $\lim\limits_{t \to 0} f(t) \approx 1.6094$. The graph confirms that result.

27. For $f(x) = x^x$:

x	$f(x)$
0.1	0.794 328
0.01	0.954 993
0.001	0.993 116
0.0001	0.999 079

It appears that $\lim\limits_{x \to 0^+} f(x) = 1$.

The graph confirms that result.

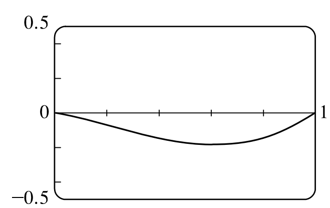


28. For $f(x) = x^2 \ln x$:

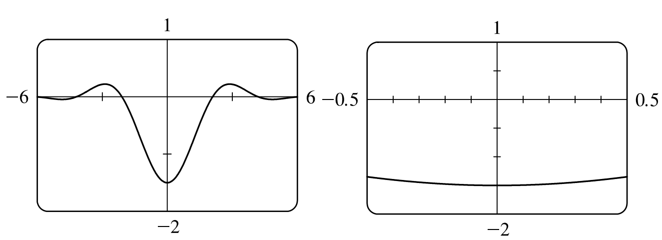
x	$f(x)$
0.1	$-0.023\,026$
0.01	$-0.000\,461$
0.001	$-0.000\,007$
0.0001	$-0.000\,000$

It appears that $\lim\limits_{x \to 0^+} f(x) = 0$.

The graph confirms that result.



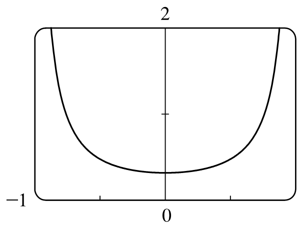
29. (a) From the graphs, it seems that $\lim\limits_{x \to 0} \dfrac{\cos 2x - \cos x}{x^2} = -1.5$.

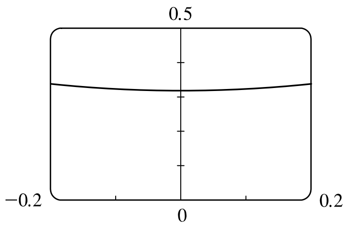


(b)

x	$f(x)$
± 0.1	$-1.493\,759$
± 0.01	$-1.499\,938$
± 0.001	$-1.499\,999$
± 0.0001	$-1.500\,000$

30. (a) From the graphs, it seems that $\lim\limits_{x\to 0} \dfrac{\sin x}{\sin \pi x} = 0.32$.





(b)

x	$f(x)$
± 0.1	0.323 068
± 0.01	0.318 357
± 0.001	0.318 310
± 0.0001	0.318 310

Later we will be able to show that the exact value is $\dfrac{1}{\pi}$.

31. $\lim\limits_{x\to 5^+} \dfrac{x+1}{x-5} = \infty$ since the numerator is positive and the denominator approaches 0 from the positive side as $x \to 5^+$.

32. $\lim\limits_{x\to 5^-} \dfrac{x+1}{x-5} = -\infty$ since the numerator is positive and the denominator approaches 0 from the negative side as $x \to 5^-$.

33. $\lim\limits_{x\to 1} \dfrac{2-x}{(x-1)^2} = \infty$ since the numerator is positive and the denominator approaches 0 through positive values as $x \to 1$.

34. $\lim\limits_{x\to 3^-} \dfrac{\sqrt{x}}{(x-3)^5} = -\infty$ since the numerator is positive and the denominator approaches 0 from the negative side as $x \to 3^-$.

35. Let $t = x^2 - 9$. Then as $x \to 3^+$, $t \to 0^+$, and $\lim\limits_{x\to 3^+} \ln(x^2-9) = \lim\limits_{t\to 0^+} \ln t = -\infty$ by (5).

36. $\lim\limits_{x\to 0^+} \ln(\sin x) = -\infty$ since $\sin x \to 0^+$ as $x \to 0^+$.

37. $\lim\limits_{x\to (\pi/2)^+} \dfrac{1}{x} \sec x = -\infty$ since $\dfrac{1}{x}$ is positive and $\sec x \to -\infty$ as $x \to (\pi/2)^+$.

38. $\lim\limits_{x\to \pi^-} \cot x = \lim\limits_{x\to \pi^-} \dfrac{\cos x}{\sin x} = -\infty$ since the numerator is negative and the denominator approaches 0 through positive values as $x \to \pi^-$.

39. $\lim\limits_{x\to 2\pi^-} x \csc x = \lim\limits_{x\to 2\pi^-} \dfrac{x}{\sin x} = -\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x \to 2\pi^-$.

40. $\lim\limits_{x\to 2^-} \dfrac{x^2-2x}{x^2-4x+4} = \lim\limits_{x\to 2^-} \dfrac{x(x-2)}{(x-2)^2} = \lim\limits_{x\to 2^-} \dfrac{x}{x-2} = -\infty$ since the numerator is positive and the denominator approaches 0 through negative values as $x \to 2^-$.

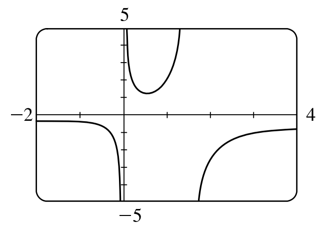
41. $\lim\limits_{x\to 2^+} \dfrac{x^2-2x-8}{x^2-5x+6} = \lim\limits_{x\to 2^+} \dfrac{(x-4)(x+2)}{(x-3)(x-2)} = \infty$ since the numerator is negative and the denominator approaches 0 through negative values as $x \to 2^+$.

42. $\lim\limits_{x\to 0^+} \left(\dfrac{1}{x} - \ln x\right) = \infty$ since $\dfrac{1}{x} \to \infty$ and $\ln x \to -\infty$ as $x \to 0^+$.

43. $\lim\limits_{x\to 0} (\ln x^2 - x^{-2}) = -\infty$ since $\ln x^2 \to -\infty$ and $x^{-2} \to \infty$ as $x \to 0$.

44. (a) The denominator of $y = \dfrac{x^2+1}{3x-2x^2} = \dfrac{x^2+1}{x(3-2x)}$ is equal to zero when $x = 0$ and $x = \frac{3}{2}$ (and the numerator is not), so $x = 0$ and $x = 1.5$ are vertical asymptotes of the function.

(b)



45. (a) $f(x) = \dfrac{1}{x^3-1}$.

From these calculations, it seems that

$\lim\limits_{x\to 1^-} f(x) = -\infty$ and $\lim\limits_{x\to 1^+} f(x) = \infty$.

x	$f(x)$
0.5	−1.14
0.9	−3.69
0.99	−33.7
0.999	−333.7
0.9999	−3333.7
0.99999	−33,333.7

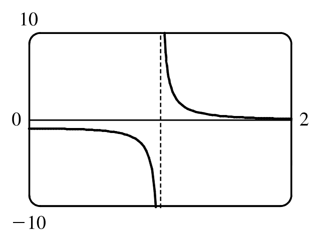
x	$f(x)$
1.5	0.42
1.1	3.02
1.01	33.0
1.001	333.0
1.0001	3333.0
1.00001	33,333.3

(b) If x is slightly smaller than 1, then $x^3 - 1$ will be a negative number close to 0, and the reciprocal of $x^3 - 1$, that is, $f(x)$, will be a negative number with large absolute value. So $\lim\limits_{x\to 1^-} f(x) = -\infty$.

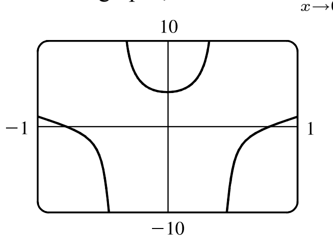
If x is slightly larger than 1, then $x^3 - 1$ will be a small positive number, and its reciprocal, $f(x)$, will be a large positive number. So $\lim\limits_{x\to 1^+} f(x) = \infty$.

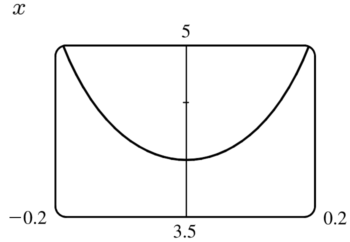
(c) It appears from the graph of f that

$\lim\limits_{x\to 1^-} f(x) = -\infty$ and $\lim\limits_{x\to 1^+} f(x) = \infty$.



46. (a) From the graphs, it seems that $\lim\limits_{x\to 0} \dfrac{\tan 4x}{x} = 4$.





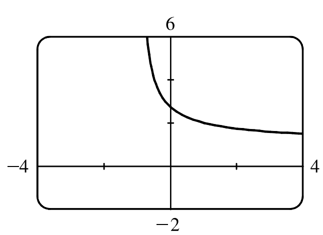
(b)

x	$f(x)$
±0.1	4.227 932
±0.01	4.002 135
±0.001	4.000 021
±0.0001	4.000 000

47. (a) Let $h(x) = (1+x)^{1/x}$.

x	$h(x)$
−0.001	2.71964
−0.0001	2.71842
−0.00001	2.71830
−0.000001	2.71828
0.000001	2.71828
0.00001	2.71827
0.0001	2.71815
0.001	2.71692

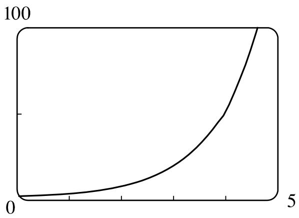
(b)

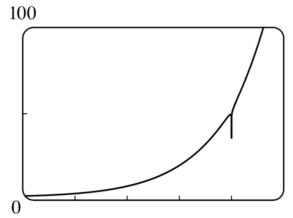


It appears that $\lim\limits_{x\to 0} (1+x)^{1/x} \approx 2.71828$, which is approximately e.

In Section 3.6 we will see that the value of the limit is exactly e.

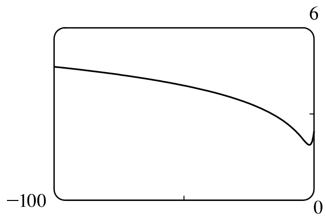
48. (a)

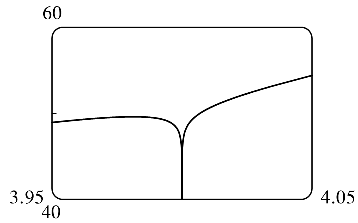




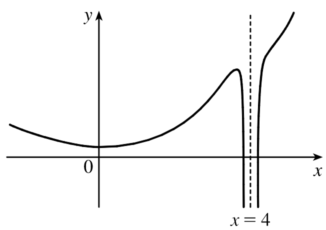
No, because the calculator-produced graph of $f(x) = e^x + \ln|x - 4|$ looks like an exponential function, but the graph of f has an infinite discontinuity at $x = 4$. A second graph, obtained by increasing the `numpoints` option in Maple, begins to reveal the discontinuity at $x = 4$.

(b) There isn't a single graph that shows all the features of f. Several graphs are needed since f looks like $\ln|x - 4|$ for large negative values of x and like e^x for $x > 5$, but yet has the infinite discontiuity at $x = 4$.





A hand-drawn graph, though distorted, might be better at revealing the main features of this function.



49. For $f(x) = x^2 - (2^x/1000)$:

(a)

x	$f(x)$
1	0.998 000
0.8	0.638 259
0.6	0.358 484
0.4	0.158 680
0.2	0.038 851
0.1	0.008 928
0.05	0.001 465

It appears that $\lim_{x \to 0} f(x) = 0$.

(b)

x	$f(x)$
0.04	0.000 572
0.02	−0.000 614
0.01	−0.000 907
0.005	−0.000 978
0.003	−0.000 993
0.001	−0.001 000

It appears that $\lim_{x \to 0} f(x) = -0.001$.

50. For $h(x) = \dfrac{\tan x - x}{x^3}$:

(a)

x	$h(x)$
1.0	0.557 407 73
0.5	0.370 419 92
0.1	0.334 672 09
0.05	0.333 667 00
0.01	0.333 346 67
0.005	0.333 336 67

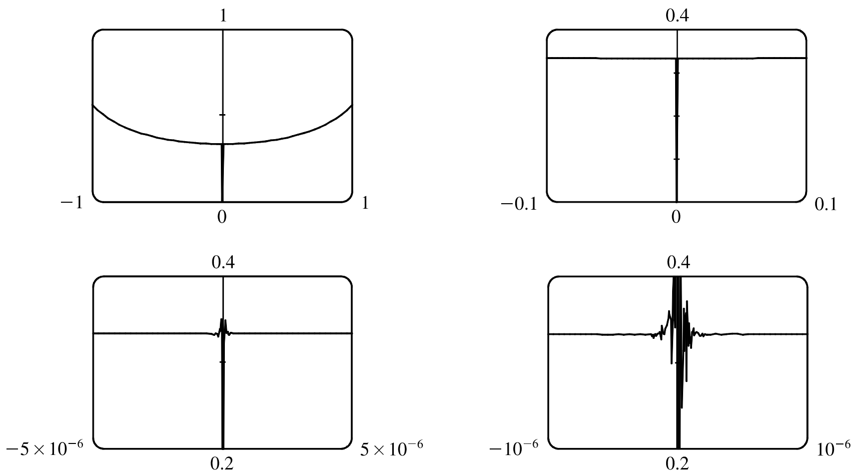
(b) It seems that $\lim_{x \to 0} h(x) = \frac{1}{3}$.

(c)

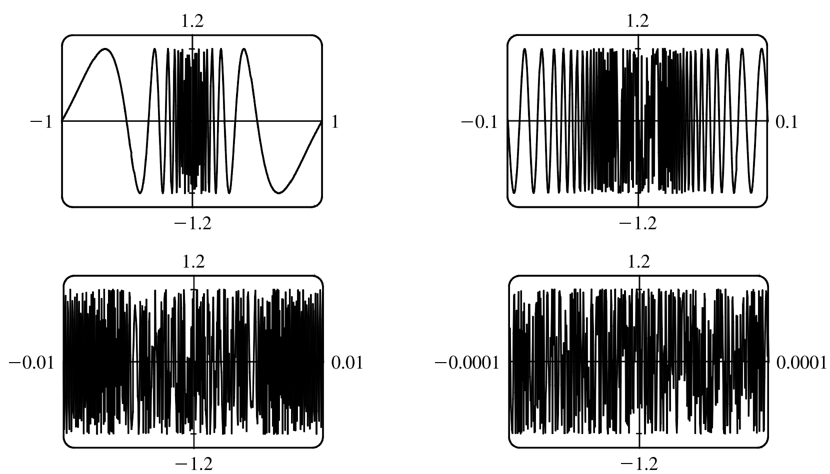
x	$h(x)$
0.001	0.333 333 50
0.0005	0.333 333 44
0.0001	0.333 330 00
0.00005	0.333 336 00
0.00001	0.333 000 00
0.000001	0.000 000 00

Here the values will vary from one calculator to another. Every calculator will eventually give *false values*.

(d) As in part (c), when we take a small enough viewing rectangle we get incorrect output.



51. No matter how many times we zoom in toward the origin, the graphs of $f(x) = \sin(\pi/x)$ appear to consist of almost-vertical lines. This indicates more and more frequent oscillations as $x \to 0$.



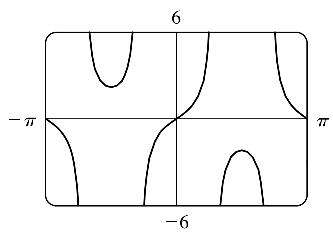
52. (a) For any positive integer n, if $x = \dfrac{1}{n\pi}$, then $f(x) = \tan \dfrac{1}{x} = \tan(n\pi) = 0$. (Remember that the tangent function has period π.)

(b) For any nonnegative number n, if $x = \dfrac{4}{(4n+1)\pi}$, then

$$f(x) = \tan\frac{1}{x} = \tan\frac{(4n+1)\pi}{4} = \tan\left(\frac{4n\pi}{4} + \frac{\pi}{4}\right) = \tan\left(n\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1$$

(c) From part (a), $f(x) = 0$ infinitely often as $x \to 0$. From part (b), $f(x) = 1$ infinitely often as $x \to 0$. Thus, $\lim\limits_{x\to 0} \tan\dfrac{1}{x}$ does not exist since $f(x)$ does not get close to a fixed number as $x \to 0$.

53.



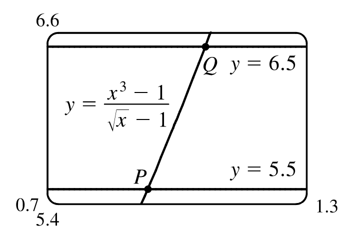
There appear to be vertical asymptotes of the curve $y = \tan(2\sin x)$ at $x \approx \pm 0.90$ and $x \approx \pm 2.24$. To find the exact equations of these asymptotes, we note that the graph of the tangent function has vertical asymptotes at $x = \frac{\pi}{2} + \pi n$. Thus, we must have $2\sin x = \frac{\pi}{2} + \pi n$, or equivalently, $\sin x = \frac{\pi}{4} + \frac{\pi}{2}n$. Since $-1 \le \sin x \le 1$, we must have $\sin x = \pm\frac{\pi}{4}$ and so $x = \pm\sin^{-1}\frac{\pi}{4}$ (corresponding to $x \approx \pm 0.90$). Just as 150° is the reference angle for 30°, $\pi - \sin^{-1}\frac{\pi}{4}$ is the reference angle for $\sin^{-1}\frac{\pi}{4}$. So $x = \pm\left(\pi - \sin^{-1}\frac{\pi}{4}\right)$ are also equations of vertical asymptotes (corresponding to $x \approx \pm 2.24$).

54. $\lim\limits_{v\to c^-} m = \lim\limits_{v\to c^-} \dfrac{m_0}{\sqrt{1 - v^2/c^2}}$. As $v \to c^-$, $\sqrt{1 - v^2/c^2} \to 0^+$, and $m \to \infty$.

55. (a) Let $y = \dfrac{x^3 - 1}{\sqrt{x} - 1}$.

From the table and the graph, we guess that the limit of y as x approaches 1 is 6.

x	y
0.99	5.925 31
0.999	5.992 50
0.9999	5.999 25
1.01	6.075 31
1.001	6.007 50
1.0001	6.000 75



(b) We need to have $5.5 < \dfrac{x^3 - 1}{\sqrt{x} - 1} < 6.5$. From the graph we obtain the approximate points of intersection $P(0.9314, 5.5)$ and $Q(1.0649, 6.5)$. Now $1 - 0.9314 = 0.0686$ and $1.0649 - 1 = 0.0649$, so by requiring that x be within 0.0649 of 1, we ensure that y is within 0.5 of 6.