Sierra College – Math 30 – **Exam #1**

NAME: _____score: /75

You must have read, signed and submitted the exam contract before taking this test. As a reminder, be certain to show all of your work for each question in order, in a neat and organized manner with explanations, using proper mathematical notation and presenting a logical argument that justifies your conclusions. Do not share these questions in any way with anyone. The only resources allowed are the textbook, your handwritten notes, and a basic calculator. You many not use any type of cell phone or computer (online or offline). You do not need to write any answers on this page, just include your work on your own paper.

(5 points each)

1. Evaluate the limit, if it exists

a. $\lim_{x \to -1^+} \left(\frac{6x^4 - 3x^3 + x^2 - 5x - 15}{12x^4 - 28x^3 - x^2 + 30x - 9} \right)$ 2.3 #11-32 Review #3-20

b.

c. $\lim_{x \to \frac{3}{2}^-} \left(\frac{6x^4 - 3x^3 + x^2 - 5x - 15}{12x^4 - 28x^3 - x^2 + 30x - 9} \right)$ 2.2 #31-44

d. $\lim_{x \to 0^+} \left[\sin^{-1}\left(\frac{e^x - 1}{e^{2x} - 1} \right) \right]$ 2.3 #11-32 Review 3-20

e. $\lim_{x \to \frac{\pi}{2}^+} \left(1 + \frac{\sin x \sqrt{1 - \sin^2 x}}{\sin(2x)} \right)$ 2.3 #41-46

f. $\lim_{x \to \infty} \left(\frac{5x^2 + \cos(2x) - \sin^2(3x)}{x^2 + 10} \right)$ 2.6 #32,39,40

g. $\lim_{x \to -\infty} \left[\csc(\tan^{-1} x) \right]$ 2.6 #35,40

(5 points each)

2. For what value of c will the function $f(x) = \begin{cases} 2x + \ln(x + c) & \text{if } x \geq 1 \\ \ln(cx + 1)^2 & \text{if } x < 1 \end{cases}$ be continuous at $x = 1$? 2.5 #45,46

3. For the function $f(x) = (x - 3)^2$ there are two different tangent lines that pass through the point $(0, -7)$. Find the equation of these two tangent lines. 2.7 #20-30

4. Find the equation of a rational function that satisfies all of the conditions given below. 2.6 #57-59 Review #2,49 2.2 #15-18 2.6 #5-10

$$\lim_{x \to 1^+} f(x) = \infty, \quad \lim_{x \to 1^-} f(x) = -\infty, \quad \lim_{x \to -\infty} f(x) = 3, \quad \lim_{x \to -2} f(x) = 2$$

$$f(0) = 0 \text{ but } f'(-2) \text{ does not exist (DNE)}$$

5. Prove that the equation $\frac{\tan^{-1}(x)}{2x - 1} = 1 + x$ has at least one real solution. 2.5 #53-62, 50-52

6. Evaluate $\lim_{h \to 0} \frac{\left[2(3 + h)^2 - 12(3 + h) + 23 \right] - [5]}{h}$ ***NOT*** ***DIRECTLY*** but by interpreting the limit as the derivative of a function at a specific point on a specific curve. 2.7 #37-42 Review #40

(5 points each = 10 points)

7. Given the statement $\lim_{x \to 1} \frac{2x^3 + 4x^2 - 6x}{x^3 - x^2} = 8$

   a. Prove the statement using the formal definition of a limit. 2.4 #19-32
   b. Explain what this means (by using your own specific example as was discussed in lecture).

(10 points)

8. Given $f(x) = \frac{x}{1 + \sqrt{2x}}$. Find $f'(x)$ using the definition of the derivative. 2.8 #21-31