ECON 355: Homework 2
Spring 2021
C. Richard Higgins

This homework is due February 26 at noon. Please upload a single pdf file with your answers to our shared folder on Google Drive. You should also upload your MATLAB code files separately. Be sure to show your work.

1. Assume a standard overlapping generations (OLG) model where members of generation t work only in period t for this problem. The economy features Cobb-Douglas production defined as $Y(t) = K(t)^{\theta} H(t)^{1-\theta}$. A member h of generation t has the utility function

$$u\left(c_t^h(t), c_t^h(t+1)\right) = \frac{c_t^h(t)^{1-\sigma}}{1-\sigma} + \beta \frac{c_t^h(t+1)^{1-\sigma}}{1-\sigma}.$$

a. What will aggregate consumption for generation t be in period t. Leave this as a function in terms of $R(t)$ and $Y(t)$. Be sure to plug in for $w(t)$ and simplify.

b. Take the partial derivative for your answer from part a with respect to R(t). Assume $R(t) = 1.05$, $\beta = 0.99$, $Y(t) = 2$, $\theta = 0.5$ and $\sigma = 0.5$. What is the partial derivative at these values? What is the partial derivative if all of those values are the same except $\sigma = 2$?

c. What happens to the function from part a if you set $\sigma = 1$. You do not need to repeat all of the steps from part a, you can simply take your answer from a and set $\sigma = 1$. What do you notice about the function?

d. How does your answer from part a compare to the answer we found in class when we assume log utility? How does the behavior of the agents in the model differ with this utility specification, assuming $\sigma \neq 1$?

2. Assume a standard OLG model with a pay-as-you-go social security system. The economy features Cobb-Douglas production defined as $Y(t) = K(t)^{\theta} H(t)^{1-\theta}$. Members of generation t work only in period t. A member h of generation t has the utility function $u\left(c_t^h(t), c_t^h(t+1)\right) = \ln c_t^h(t) + \beta \ln c_t^h(t+1)$. Young individuals pay a lump-sum tax T that is transferred to old individuals who receive payments of $(1+n)T$.

a. What will aggregate consumption for generation t be in period t. You should plug in for $R(t)$ and $w(t)$ then simplify as much as possible.

b. Under what conditions will aggregate consumption of the young be higher with pay-as-you-go social security than without?

c. Suppose the government instead operates a fully funded social security system. Individuals pay the lump sum tax T when young and receive $R(t)T$ back when they are old. What are the budget constraints for individual h of generation t when young and old? What is this individual's lifetime budget constraint? Will consumption by the young be different in a fully funded social security system than in a system without a social security system? Briefly explain (this can be very brief).

3. Assume a standard OLG model that features Cobb-Douglas production defined as $Y(t) = K(t)^{\theta} H(t)^{1-\theta}$. Members of generation t work only in period t. A member h of generation t has the utility function

$$u\left(c_t^h(t), c_t^h(t+1)\right) = \ln c_t^h(t) + \beta \ln c_t^h(t+1).$$

The parameters of the model are defined as $\theta = 0.33$, $\beta = 0.98$, and $n = 0.01$.

a. What is the function for the evolution of capital per unit of labor, $k(t+1) = g(k(t))$. What is the non-zero steady state of capital per unit of labor?

b. Use MATLAB to determine if the steady state is stable. Create a grid of values from 0 to 0.4 stepping by 0.001 for $k(t)$ and solve for $g(k(t))$ for these values. Plot this function with $k(t)$ on the x-axis and $k(t+1)$ on the y-axis. Plot the 45° line as well. Include the figure in the pdf. Is the non-zero steady state stable?

c. Assume there are 1,000 people in generation 1 and all individuals in every generation provide 1 unit of labor when they are young (per worker and per unit of labor are now the same thing). There are 5,000 units of capital (not capital per worker) in the first period. Use MATLAB to plot how output per worker and average consumption by the young (young consumption per worker) will evolve over 20 periods (starting with period 1). Include the figure in the pdf. Does the economy converge to the steady state quickly or slowly compared to the examples we looked at in homework 1 for the Solow model? What do you think could explain this difference?

4. Consider a Robinson Crusoe model with instantaneous utility given by $u(c_t) = \ln c_t$. Robinson faces a budget constraint given by $k_{t+1} = (1-\delta)k_t + i_t$ and a feasibility constraint $y_t = f(k_t) = k_t^\alpha \geq c_t + i_t$.

a. What are the first-order conditions for this economy after Robinson maximizes expected lifetime utility by selecting capital, consumption and investment? Be sure to include your budget constraints. You should simplify these so you have three first-order conditions.

b. What are the steady states of capital, consumption and investment?

c. What is the transversality condition for this economy?