ECON 355: Value Function Iteration Flipped Classroom
Spring 2021
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This assignment will be started in class during a flipped class session. This will count as your third homework assignment and be due at the start of class Monday, March 22. When asked to plot something, be sure to include the figure in your results. You will be expected to hand in a pdf for the homework with all of your answers as well as your MATLAB code.

A Robinson Crusoe model can be defined by the following utility function

$$\sum_{i=0}^{\infty} \beta u\left(c_{t+i}\right)$$

with the budget constraints

$$k_{t+1} = (1-\delta)k_t + i_t$$
$$f\left(k_t\right) = k_t^{\theta} \geq c_t + i_t.$$

1. (10 points) Suppose the instantaneous utility function is defined as

$$u\left(c_t\right) = \ln c_t$$

a. Assume the following calibration of $\beta = 0.98$, $\delta = 0.1$ and $\theta = 0.4$ Numerically solve for the value function using MATLAB. To do so create a grid going from fifty percent of the non-zero steady state value of capital to 1.5 times the steady state value of capital. Have 100 points in your grid. Plot the value function.

b. Using the results from part b, plot the policy function, $k_{t+1} = g\left(k_t\right)$. Include the 45° line in your figure. Is the non-zero steady state stable?

2. (35 points) Suppose the instantaneous utility function is defined as

$$u\left(c_t\right) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}.$$

a. Set up Robinson's problem as a dynamic programming problem. What is the Bellman equation? What is the first order condition? What is the non-zero steady state of capital? Show your work.

b. Set up Robinson's problem as a Lagrangian with consumption and capital as the only two economic variables (plug in for investment). What are the first order conditions? What are the non-zero steady state of consumption? Show your work.

c. Assume the following calibration of $\beta = 0.95$, $\delta = 0.025$, $\theta = 0.3$ and $\sigma = 1.5$. Numerically solve for the value function using MATLAB. To do so create a grid going from fifty percent of the steady state value of capital to 1.5 times the steady state value of capital. Have 100 points in your grid. Plot the value function.

d. Using the results from part b, plot the policy function, $k_{t+1} = g\left(k_t\right)$. Include the 45° line in your figure. Is the non-zero steady state stable?

e. Repeat parts c and d but assume that the budge constraint is modified so there is an endowment that falls from the sky every period. The feasibility condition is now given by

$$k_t^{\theta} + e \geq c_t + i_t.$$

Where $e = 20$ and the rest of the calibration remains the same as from part c. Include these figures. Do you notice anything different between these figures and those from part c and d?