A) $\mathcal{L} = \max_{P_t(j)} E_0 \beta^t \frac{1}{C_t}\left[\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon_t} Y_t\left(\frac{1}{P_t}\right) - W_t N_t(j) - \frac{\phi}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)^2 Y_t\right]$

$$\frac{\partial \mathcal{L}}{\partial P_t(j)} = 0 = (-\varepsilon \varepsilon_t)\frac{1}{C_t}\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon\varepsilon_t - 1} Y_t\left(\frac{1}{P_t}\right) - \frac{\phi}{C_t}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)\frac{Y_t}{\pi P_{t-1}(j)} +$$
$$\beta\phi E_t\left[\frac{1}{C_{t+1}}\left(\frac{P_{t+1}(j)}{P_t(j)} - 1\right) Y_{t+1}\frac{P_{t+1}(j)}{\pi P_t(j)^2}\right] \quad (3)$$

B)

Symmetric equilibrium

$P_t(j)$ is the same for all firms

So $P_t = P_t(j)$

$$= (-\varepsilon\varepsilon_t)\frac{1}{C_t}\left(\frac{P_t}{P_t}\right)^{-\varepsilon\varepsilon_t - 1} Y_t\left(\frac{1}{P_t}\right) - \frac{\phi}{C_t}\left(\frac{P_t}{\pi P_{t-1}} - 1\right)\frac{Y_t}{\pi P_{t-1}}$$
$$\beta\phi E_t\left[\frac{1}{C_{t+1}}\left(\frac{P_{t+1}}{P_t} - 1\right) Y_{t+1}\frac{P_{t+1}}{\pi P_t^2}\right] \quad (4)$$

devide by $\frac{Y_t}{P_t C_t}$

$$(\varepsilon\varepsilon_t) = -\phi\left(\frac{\pi_t}{\pi} - 1\right)\left(\frac{\pi_t}{\pi}\right) + \beta\phi E_t\left(\frac{C_t}{C_{t+1}}\right)\left(\frac{\pi_{t+1}}{\pi} - 1\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{\pi_{t+1}}{\pi}\right)$$

from HH FOC (1) $w_t = \psi H_t^{\eta}\frac{1}{C_t}$

from production fn $H_t = Y_t$

plug in from above foc w/ $C_t = \frac{\psi Y_t^{\eta}}{w_t}$

$$(\varepsilon\varepsilon_t) = -\phi\left(\frac{\pi_t}{\pi} - 1\right)\left(\frac{\pi_t}{\pi}\right) + \beta\phi E_t\left(\frac{\psi Y_t^{\eta} w_{t+1}}{w_t \psi Y_{t+1}^{\eta}}\right)\left(\frac{\pi_{t+1}}{\pi} - 1\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(\frac{\pi_{t+1}}{\pi}\right)$$

LHS $\varepsilon(\ln(\varepsilon_{t+1}) + \pi v_t)$

$$\text{RHS} = -\phi\left(\frac{\bar{\pi}}{\bar{\pi}}e^{\tilde{\pi}_t}-1\right)\left(\frac{\bar{\pi}}{\bar{\pi}}c^{\tilde{\pi}_t}\right)+\beta\phi E_t\left[\frac{\left(\frac{\bar{y}}{\bar{y}}c^{\tilde{y}_t}\right)^{\eta}\left(\frac{\bar{w}}{\bar{w}}c^{\tilde{w}_{t+1}}\right)}{\left(\frac{\bar{w}}{\bar{w}}e^{\tilde{w}_t}\right)\left(\frac{\bar{y}}{\bar{y}}e^{\tilde{y}_{t+1}}\right)^{\eta}}\right]\left(\frac{\bar{\pi}}{\bar{\pi}}c^{\tilde{\pi}_{t+1}}-1\right)\left(\frac{\bar{\pi}}{\bar{\pi}}e^{\tilde{\pi}_{t+1}}\right)\left(\frac{\bar{y}}{\bar{y}}e^{\tilde{y}_{t+1}}\right)\left(\frac{\bar{\pi}}{\bar{\pi}}e^{-\tilde{\pi}_{t+1}}\right)$$

$$-\phi\left(c^{\pi_t}-1\right)\left(c^{\pi_t}\right)+\beta\phi E_t\left[\frac{\left(e^{\tilde{y}_t}\right)^{\eta}e^{\tilde{w}_{t+1}}}{e^{\tilde{w}_t}\left(e^{\tilde{y}_{t+1}}\right)^{\eta}}\right]\left(e^{\tilde{\pi}_{t+1}}-1\right)\left(c^{\pi_{t+1}}\right)\left(e^{\tilde{y}_{t+1}}-\tilde{y}_t\right)\left(e^{\pi_{t+1}}\right)$$

$$-\phi\left[e^{\pi_t}-e^{\pi_t}\right]+\beta\phi E_t\left[e^{\tilde{y}_t(i)+\tilde{w}_{t+1}-\tilde{w}_t-\tilde{y}_{t+1}(i)}\right]\left(e^{\tilde{\pi}_{t+1}}-1\right)\left(e^{\tilde{\pi}_{t+1}}\right)\left(c^{\tilde{y}_{t+1}-\tilde{y}_t}\right)\left(e^{\pi_{t+1}}\right)$$