Answers to questions in

Lab 1: Filtering operations

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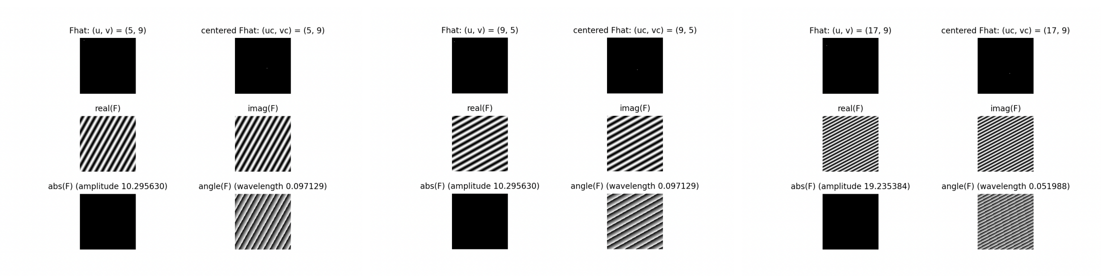
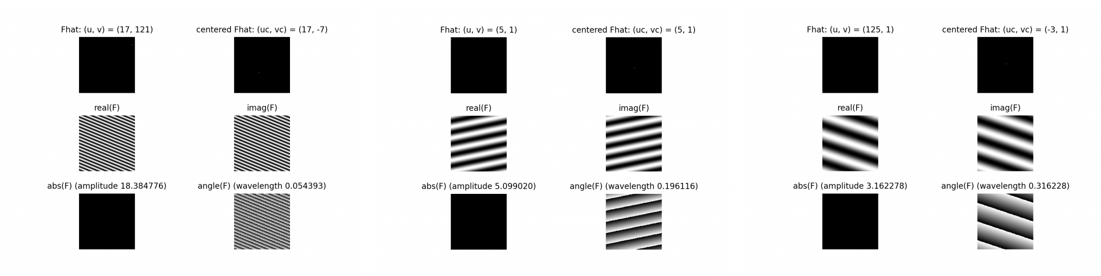
Instructions: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

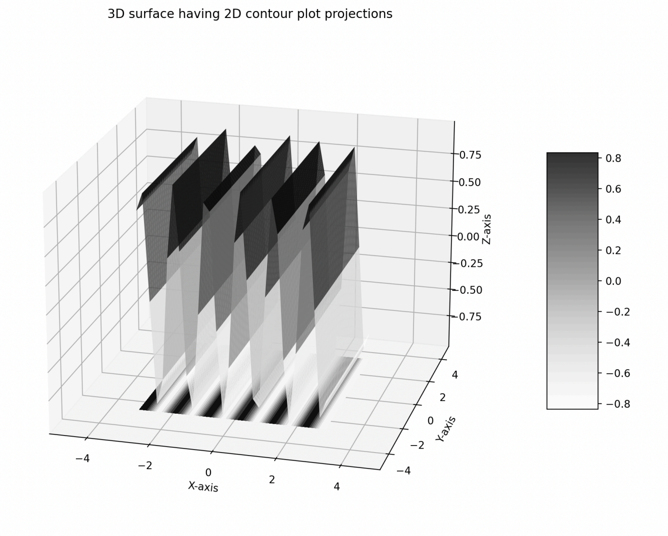
Question 1: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?





Answers:
The x axis corresponds to horizontal frequencies and the y axis represents the vertical frequencies. Combination of x and y result in a combination of both an horizontal and a vertical component and thus produces a diagonal sine wave.

Question 2: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.



Answers:
A single point in the Fourier domain represent one pure sine wave and thus is converted to a single pure sine pattern in the time domain. As can be seen in the image above under the 3d curve, a repeating black and white pattern on the image in the direction dictated by the frequency component of the original pixel in the Fourier domain is created.

Question 3: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:
The amplitude of the signal is given by:

$$x = \sqrt{Re^2[\hat{f}(u,v)] + Im^2[\hat{f}(u,v)]}$$

Question 4: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

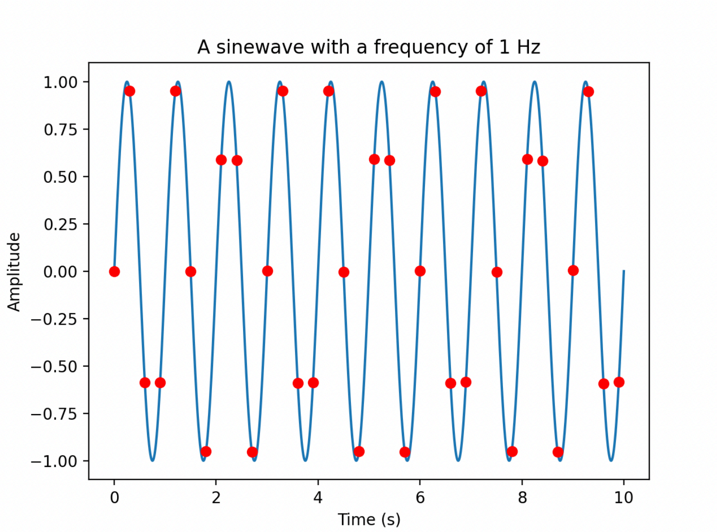
The angle of the signal is given by:

$$\theta = \tan^{-1}(\frac{u}{v})$$

The wavelength of the signal is given by:

$$T = \frac{1}{\sqrt{u^2 + v^2}}$$

Question 5: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!



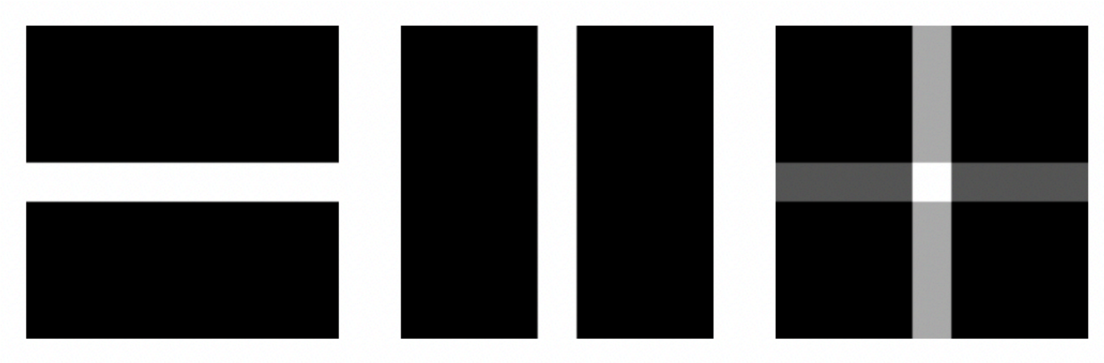
Answers:
Since the pixel represent the sampler of the signal, in order to sample the signal correctly we need to, at least, sample the signal at double it's maximum frequency. Past the halfway point of the FFT domain, all the frequencies are above that threshold, thus they get wrapped back to slower and slower frequencies even though in reality they are supposed to be higher frequencies.

Question 6: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

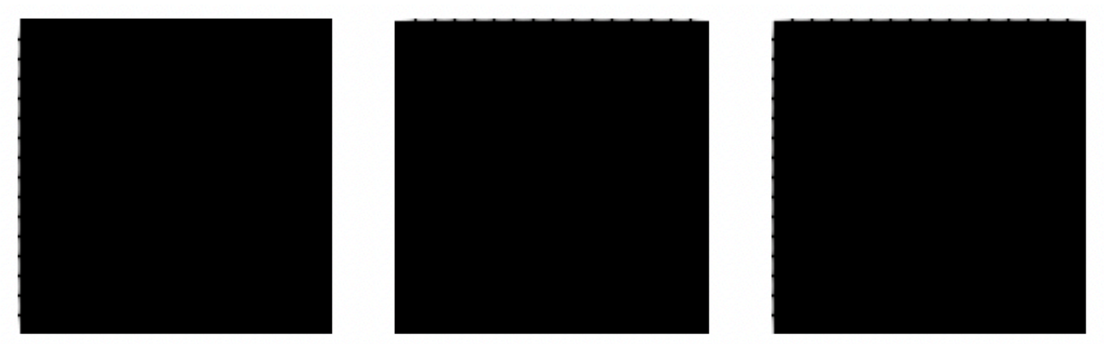
Answers:
It corrects the coordinates we gave for the centered version.

Question 7: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Here are the original image and there linear combination:



Here are the FFT of each of these images:



We see that the last one it is simply a linear combination of the first too.

Answers:
The line are either perfectly vertical or horizontal and thus require no signal going diagonally (represented on the FFT in 4 quadrants excluding the center). Also, they wrap perfectly around the image (since in theory this image tiles the infinite 2D plane). This makes it so there are no extra high frequencies that would be created by an edge at the border of the image if the opposite side was not exactly the same. The line are thus represented by a single line of frequencies in the FFT domain. Combining both images simply combines their FFT together.

Question 8: Why is the logarithm function applied?

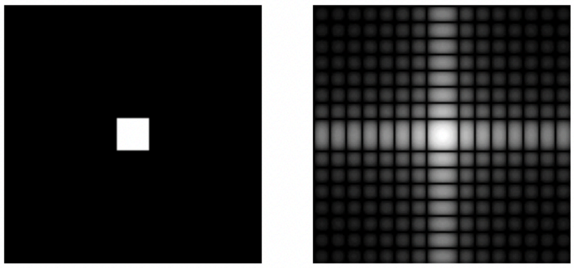
Answers:
Since the value in the center of the FFT are so large compared to the rest of the higher frequency amplitude, we apply a logarithm to bring all the values in the same range (if you have values like 1 000 000 in the center and a bunch of small 10 in the higher frequencies, the small number will be remapped to zero if you don't apply a log and make those 10 become 1 an the 1 000 000 become a 6).

Question 9: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:

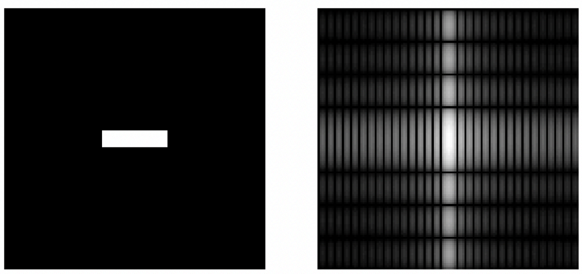
$$\hat{h}(u,v) = \hat{f}(u,v) + \hat{g}(u,v)$$

Question 10: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.



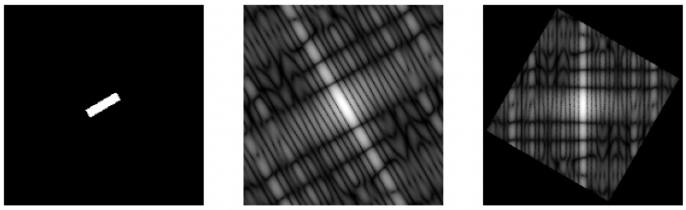
Answers:
We can obtain the same result by first taking the Fourier transform of both images and then convolve them together in the frequency domain. This work since convolution in the Fourier domain is equivalent to point wise multiplication in the spatial domain and vice versa.

Question 11: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.



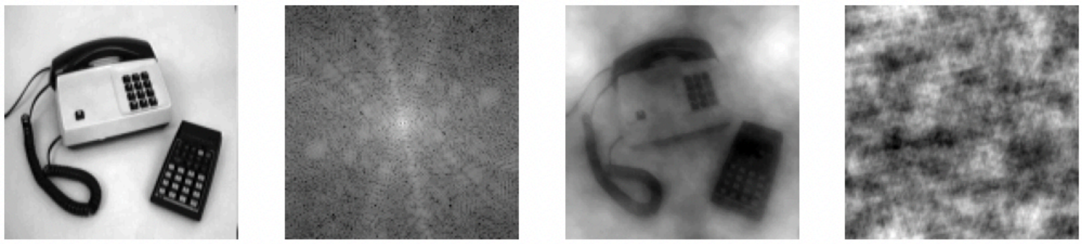
Answers:
Scaling down an image make the FFT grow larger since more high frequency are represented in the image. Scaling up an image make the FFT shrink since less high frequency are represented in the image.

Question 12: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.



Answers:
Since the rotation is rescaling the image some frequencies are remapped incorrectly and thus some information is lost about the original image. Also since you cannot have a perfectly straight edge at various angles, those edge contain multiple frequencies that would be different if the image was in the continuous domain or of higher resolution.

Question 13: What information is contained in the phase and in the magnitude of the Fourier transform?



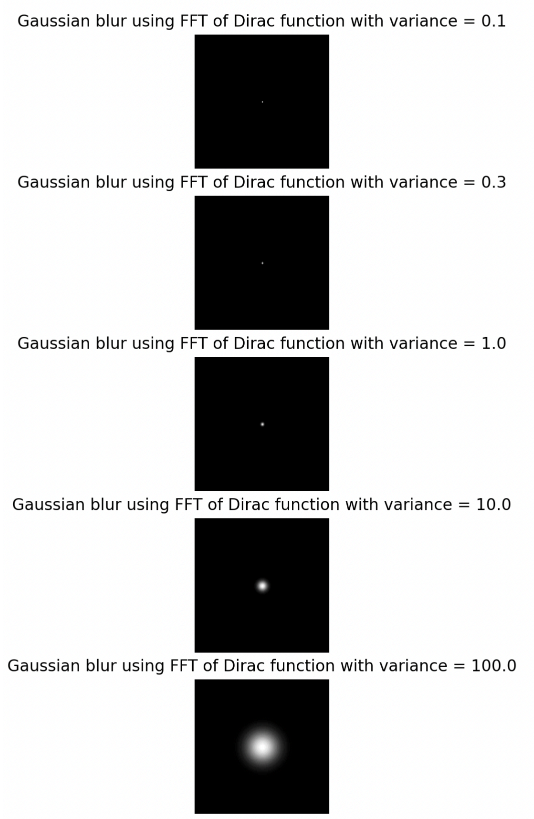
Answers:
The phase controls where the edges are in the image, the magnitude only says how much of a certain frequency is part of the image. This is why phase is a lot more important for images (not so much for sound) since the edges are what makes an image an image.

Question 14: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:

Variances for different sigma values:

Sigma	Variance
0.1	0.013
0.3	0.281
1.0	0.999
10	10.00
100	99.99

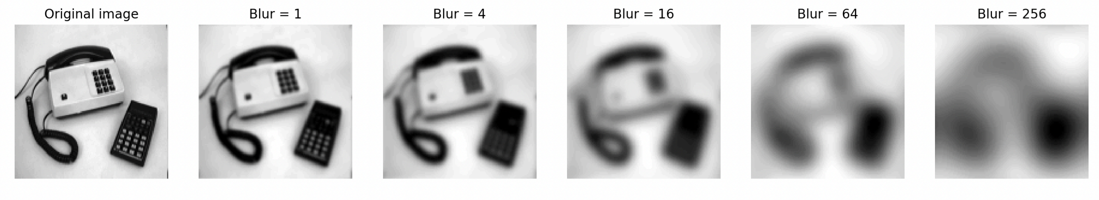


Question 15: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

The variance match the expected values for large variance values but not for smaller values. This is due sampling error not being able to capture the rapid variation on the gaussian function.

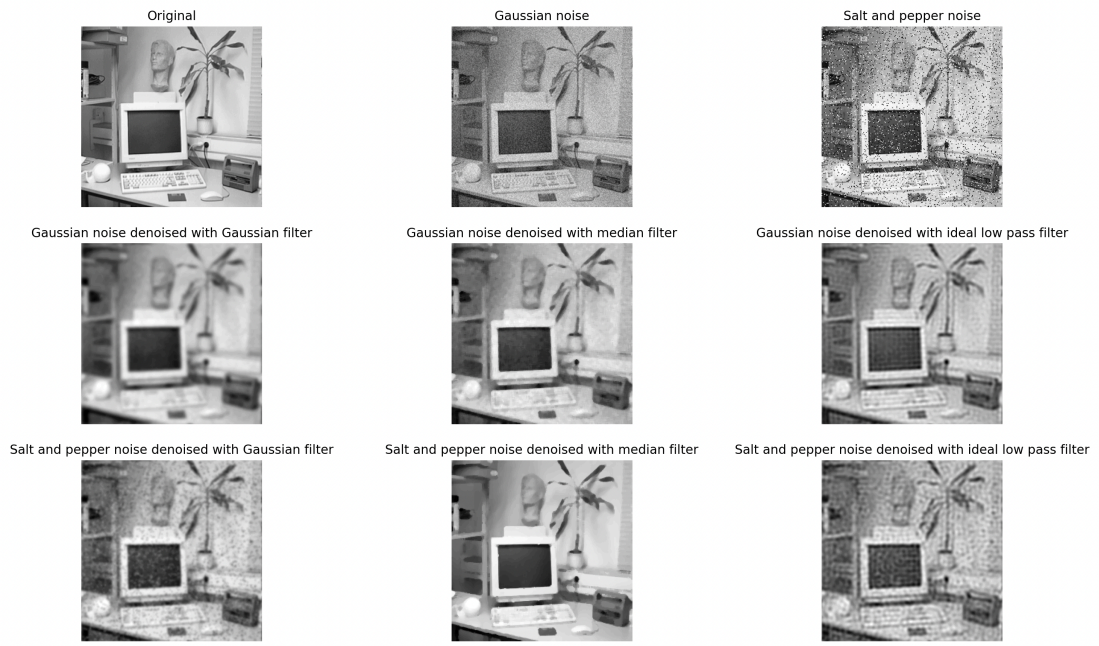
Question 16: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?



Answers:

The gaussian filter acts as a smooth low-pass filter. It removes high frequency components from the image and thus makes the image look smoother.

Question 17: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).



Answers:

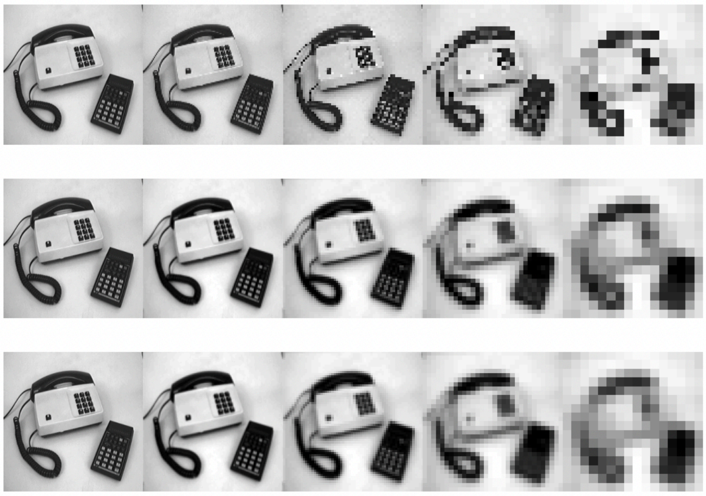
The gaussian filter will blur edges and thus can be more visually appealing but is not beneficial if you are trying to detect edges. The median filter is really good at removing salt and pepper noise but makes the image look like a painting and it is thus less visually appealing, but it preserves edges which is really good for edge detection. The low pass filter adds a lot of ringing to the output image and does not do as good a job as the gaussian filter does.

Question 18: What conclusions can you draw from comparing the results of the respective methods?

Answers:

It is important to understand what type of noise is in your image before your try and denoise it. If the noise is gaussian, then a gaussian filter is the best way to denoise it. If the noise is salt and pepper, then a median filter is the best way to denoise it.

Question 19: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.



On the first row, subsampling is done without any blurring involved.
On the second row, a gaussian blur with a variance of 0.63 is used in between each subsampling.
On the last row, a gaussian blur with a variance of 1 is used between each subsampling.

Answers:

If you subsample without smoothing, you get a very harsh picture that contains a lot of aliasing. If you subsample with smoothing, you get a smoother picture that is less aliased.

Question 20: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:
If you don't apply smoothing before subsampling you get what is called aliasing and it is caused by the high frequencies being remapped to lower frequencies and thus causing artifact in the image. On the other hand, if you smooth it out beforehand, you get a much nicer output since you are removing the high frequency component that anyway cant be represented in the smaller image and thus avoid aliasing. A good filter for that is:

1/16	1/8	1/16
1/8	1/4	1/8
1/16	1/8	1/16