Newton-Euler Method

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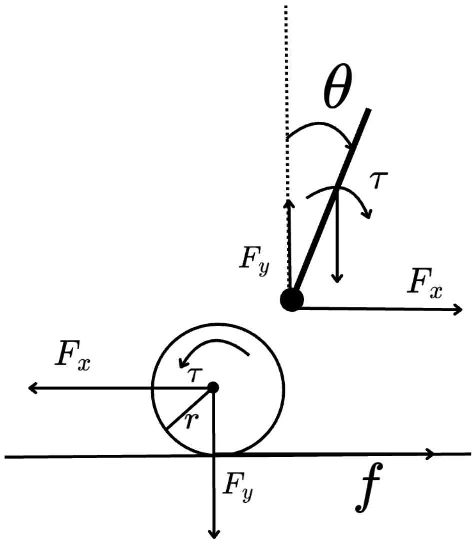

Figure 1: Newton Euler Method Diagram

Newton Euler Equations

$$\begin{cases} a_{bx} = \dfrac{F_x}{m_b} (1) \\ a_{by} = \dfrac{F_y - m_b g}{m_b} (2) \\ \ddot{\theta} = \dfrac{F_y r_b \sin(\theta) - F_x r_b \cos(\theta) + \tau}{I_b} (3) \end{cases} \qquad \text{(Body)}$$

$$\begin{cases} a_{wx} = \dfrac{f - F_x}{m_w} (4) \\ \beta_w = \dfrac{-\tau - fr}{I_w} (5) \end{cases} \qquad \text{(Wheel)}$$

$$\begin{cases} a_{bx} + \dot{\theta}^2 r_b \sin(\theta) - \ddot{\theta} r_b \cos(\theta) = a_{wx} (6) \\ a_{by} + \dot{\theta}^2 r_b \cos(\theta) - \ddot{\theta} r_b \sin(\theta) = 0 (7) \\ a_{wx} = \beta_w r (8) \end{cases} \qquad \text{(Constraint)}$$

Solve

$$a_{wx} = \frac{r(-fr - \tau)}{I_w} \quad \text{(from (5)(8))} \tag{9}$$

$$a_{wx} = \frac{r(-r(F_x + M_w a_{wx}) - \tau)}{I_w} \text{(from (4)(9))} \tag{10}$$

$$a_{wx} = \frac{F_x}{M_b} - r_b \ddot{\theta} \cos(\theta) + r_b \dot{\theta}^2 \sin(\theta) \text{(from (1)(6))} \tag{11}$$

$$0 = \frac{F_y}{M_b} + r_b \ddot{\theta} \sin(\theta) + r_b \dot{\theta}^2 \cos(\theta) - g \text{(from (2)(7))} \tag{12}$$

$$\begin{aligned} \ddot{\theta} = \frac{1}{I_b} \Big[& r_b M_b \left(-r_b \ddot{\theta} \sin(\theta) - r_b \dot{\theta}^2 \cos(\theta) + g \right) \sin(\theta) \\ & - r_b M_b \left(r_b \ddot{\theta} \cos(\theta) - r_b \dot{\theta}^2 \sin(\theta) + a_{wx} \right) \cos(\theta) + \tau \Big] \end{aligned} \tag{14}$$

$$a_{wx} = \ddot{x} = \frac{1}{I_w} r \left(-r \left(M_b \left(r_b \ddot{\theta} \cos(\theta) - r_b \dot{\theta}^2 \sin(\theta) + a_{wx} \right) + M_w a_{wx} \right) - \tau \right) \tag{15}$$

$$D = I_b I_w + I_b M_b r^2 + I_b M_w r^2 + I_w M_b r_b^2 + M_b^2 r^2 r_b^2 \sin^2(\theta) + M_b M_w r^2 r_b^2 \tag{16}$$

$$\begin{aligned} \ddot{x} = \frac{r}{D} \Big(& I_b M_b \dot{\theta}^2 r r_b \sin(\theta) - I_b \tau + M_b^2 \dot{\theta}^2 r r_b^3 \sin(\theta) \\ & - \frac{1}{2} M_b^2 g r r_b^2 \sin(2\theta) - M_b r r_b \tau \cos(\theta) - M_b r_b^2 \tau \Big) \end{aligned} \tag{17}$$

$$\begin{aligned} \ddot{\theta} = \frac{1}{D} \Big(& I_w M_b g r_b \sin(\theta) + I_w \tau - \frac{1}{2} M_b^2 \dot{\theta}^2 r^2 r_b^2 \sin(2\theta) \\ & + M_b^2 g r^2 r_b \sin(\theta) + M_b M_w g r^2 r_b \sin(\theta) \\ & + M_b r^2 \tau + M_b r r_b \tau \cos(\theta) + M_w r^2 \tau \Big) \end{aligned} \tag{18}$$

Linearized System

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\dfrac{M_b^2 g r^2 r_b^2}{D} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \dfrac{I_w M_b g r_b + M_b^2 g r^2 r_b + M_b M_w g r^2 r_b}{D} & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ \dfrac{r(-I_b - M_b r r_b - M_b r_b^2)}{D} \\ 0 \\ \dfrac{I_w + M_b r^2 + M_b r r_b + M_w r^2}{D} \end{bmatrix}$$

$$D = I_b I_w + I_b M_b r^2 + I_b M_w r^2 + I_w M_b r_b^2 + M_b M_w r^2 r_b^2$$