

Comparing five common classification algorithms on facial and numeral images

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Abstract—X

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I. INTRODUCTION

For humans, it doesn't take much effort to tell the difference between a picture of a dog or a cat. A natural number or a letter. A happy person or a sad person. For computers, these sort of problems are notorious hard to solve and often require many computational resources. Machine learning and computer vision deals with these issues as they encompass a range of algorithms and classification techniques to produce a model or scheme that can tell images apart and recognize similarities. In this report we study the recognition and classification of a data set containing human faces and one featuring hand-written numbers by implementing and testing five commonly used techniques (Nearest class centroid classifier, nearest subclass centroid, nearest neighbor and two perceptron variants) using MATLAB. We split the data in a training and a test set, then train our respective model or classifier and evaluate their ability to classify correctly on the testing set. A version using all the dimensions of the data and a principal component version will be applied. We use some visualization techniques to better communicate the data representation and tables to compare them. We start by going over the basic theory behind the classification schemes, then look at the data, briefly go over implementation details and then turn to results. At the end we review and argue which scheme would make the most sense to use with these two classification problems.

Initially, we split the ORL image data set, containing 400 40x30 pixel faces of 40 different persons, and the MNIST set containing 70.000 28x28 pixel hand-written numbers 0-9. The data is vectorized, so we use built-in MATLAB functions to construct the original pictures as such:



Figure 1: Ten random images of faces from the ORL set.



Figure 2: Ten random images of numbers from the MNIST set.

II. THEORY

This section will explain fundamental theory behind the dimensional reduction technique (PCA) and five schemes used in this report.

A. Principal Component Analysis

PCA is a procedure that transform a set of observations or samples in dimension D to a lower dimension $D - n$ while still preserving a smaller number of variables explaining the main features X_1, X_2, \dots, X_p in the original set¹. This is particularly useful when dealing with high dimension data, as this can be computationally hard and challenging to visualize. With PCA, we compute principal components d of n original samples with p features, where d is the desired output dimensionality and each dimension is a linear combination of the p features. In practice, we find eigenvectors of the covariance matrix of the original data set, sort them by highest eigenvalue score and use these as weights W in computing a linear transformation:

$$y_i = W^T * x_i, i = 1, \dots, N$$

. The scattering of the transformed data is the scatter matrix given by ²:

$$S_T = \sum_{i=1}^N [W^T(x_i - \mu)][W^T(x_i - \mu)]^T$$

. The weights W can be found by applying eigenanalysis and taking the eigenvectors with the highest score, more formally optimizing ³:

$$W^* = \arg \max_c Tr(W^T S_T)$$

subject to $W^T W = I$. We end with a data set containing fewer ($d < D$) dimensions.

¹[Casella et al.()Casella, Fienberg, and Olkin]

²[Iosifidis(2017)]

³[Iosifidis(2017)]

B. Nearest class centroid classifier

The NCC classifier assigns labels l_i to N observations determined by which class c_k 's mean (centroid) the observation x_i is closest to. We make the assumption that each class follow a normal distribution, as they are given equal importance in the classification algorithm. The mean class vector is given by ⁴:

$$\mu_k = \frac{1}{N_k} \sum_{i, l_i=k} x_i, k = 1, \dots, K.$$

To classify any observation x_i we find the smallest distance to any mean vector and assign the label of that vector to it, more formally:

$$d(x_i, \mu_k) = \|x_i - \mu_k\|_2^2$$

C. Nearest subclass centroid classifier

Similar to nearest class centroid, but each class c_k now has subclasses m that follow a normal distribution and has a mean vector μ_{km} :

$$\mu_{km} = \frac{1}{N_{km}} \sum_{i, l_i=k, q_i=m} x_i$$

where N_{km} denotes the number of observations in a given subclass and x_i is a observation with a subclass label qi . Like NCC, the distance to the subclass mean is used to classify each observation ⁵:

$$d(x_i, \mu_{km}) = \|x_i - \mu_{km}\|_2^2$$

The number of subclasses is a hyper parameter for NSC and must be decided prior. Nearest subclass classifier is a compromise between nearest mean and nearest neighbor and combines the flexibility of nearest neighbor with the robustness of nearest mean ⁶. When the number of subclasses m is equal to N samples, we have the nearest neighbor classifier.

D. Nearest neighbor classifier

The nearest neighbor (NN) classifier is a simple algorithm where each sample is assigned to the class of its closest neighbor, or the most common class among its k nearest neighbors in the k -NN variant. Pure NN is when $k = 1$, but often $k > 1$ where a majority vote takes place. The algorithm uses euclidean distance between a test sample x_i with class c_k and a training one y_i :

$$d(x_i, y_i) = \|x_i - y_i\|_2^2$$

E. Perceptron learning with backpropagation

A perceptron is a supervised algorithm of binary classifiers, that classify whether or not an input sample x_i belong to a class c_k . It uses the linear discriminant function $f(x) = w^T * x + w_0$, where w are weights, $x \in \mathbb{R}^D$ a feature vector and w_0 the bias⁷. w represent the orientation of the discriminant hyperplane, and w_0 a offset from the origin. Given weights w , sample x_i and

classes c_1 and c_2 , the function splits the feature space into these two classes. If $g(w, x_i) > 0$, x_i belongs to c_1 and if $g(w, x_i) < 0$, x_i belongs to c_2 . If $g(w, x_i) = 0$, it can be classified to either class. The resulting scalar is the distance of x_i to the hyperplane. The function is used in our perceptron to follow, where we define a binary label l_i for each sample to express the criterion function:

$$f(w^*, x_i) = l_i g(w, x_i) = l_i w^{*T} x_i \geq 0, i = 1, \dots, N$$

All samples are correctly classified, if $f(w_*, x_i) \geq 0$. To produce such result, we need to optimize weights w_* by the perceptron criterion function for w_* , where $j_p(w_*) = 0$ would be a solution for⁸ w_* :

$$j_p(w) = \sum_{x_i \in \mathcal{X}} -f(w, x_i) = \sum_{x_i \in \mathcal{X}} -l_i w^T x_i$$

To optimize $j_p(w)$, we use the gradient at w and follow it to update w , where η is the rate of change (learning rate) and expresses how fast it converges, and ε express a vector set of mislabeled samples:

$$w(t+1) = w(t) - \eta(t) \nabla j_p = w(t) + \eta(t) \sum_{x_i \in \mathcal{X}} l_i x_i$$

At each learning iteration, the weights $w(t)$ are "punished" by the sum of misclassified samples scaled, leading to convergence as the result of criterion function j_p will be higher. This is known as backpropagation. The algorithm to come will elaborate on this.

F. Perceptron learning with MSE

As an alternative to backpropagation, one can apply least-mean square regression to obtain the perceptron weights W . Here we use a criterion function j_{LSE} that involves all of the samples, not just the misclassified ones. Previously we made the inner products of $f(X) = W^T * X$ positive, now we look to find a solution to $W^T X = t_i, i = 1, \dots, N$, where t_i are arbitrarily set positive constants. This is written as $X^T * w = b$ in matrix form. If X is nonsingular, we could write $W = X^{-1}b$ and have a formal solution, but this is often not the case, however by obtaining the pseudo-inverse of X one can find a solution for the weights⁹:

$$X^\dagger = (X^T X)^{-1} X^T$$

Examining the criterion function, we look to minimize:

$$j_{LSE} = \|W^T X - B\|_F^2$$

By setting the derivative of this to zero with respect to W , we get:

$$\nabla j_{LSE} = 0 \Rightarrow 2X X^T W = 2X B^T$$

Hence:

$$W = (X X^T)^{-1} X B^T = W^\dagger B^T$$

If W is square and nonsingular, the pseudo-inverse is simply the regular inverse. Also it holds at $X^\dagger X = I$, but $X X^\dagger \neq I$, though a MSE always exists, hence $W = X^\dagger b$ is an MSE solution to $X^\dagger w = b$.

⁴[Iosifidis(2017)]

⁵[Iosifidis(2017)]

⁶[Veenman and Reinders(2005)]

⁷[Iosifidis(2017)]

⁸[Iosifidis(2017)]

⁹[Duda et al.(2012)Duda, Hart, and Stork]

III. IMPLEMENTATION

The implementation uses MATLAB to design and run the different schemes on the training data and evaluating on the test data. For understanding purposes, this section will briefly lay out the algorithms described in theory as pseudo-code how we implemented them.

A. PCA

The algorithm centers the data, then simply calculates eigenvectors from the variance matrix, sort them after highest eigenvalue and uses the n vector as principal components in representing the data. Listing 1 shows this.

```
pca_reduce(trainData , nDimensions) {
    trainData = trainData - mean(trainData) for
        rows in trainData
    eigVectors , eigValues = eig(cov(trainData))
    eigVectors = sort(eigVectors , 'descending')
    pc = eigVectors(from 1 to nDimensions)*
        trainData
}
```

Listing 1: Implementation of PCA.

B. NCC

We simply calculate a mean vector for c_k classes by summing the samples that belong to class and dividing that with the number of N samples in a given class, as expressed in listing 2.

```
train_ncc(trainData , trainLbls , nClasses) {
    mu = zero vector by size of trainData*
        nClasses
    n = zero vector by size of 1*nClasses
    index = 1;

    for (i = 1; i <= nClasses; i++) {
        n(i) = sum(num of classes i in trainLbls)
        mu(i) = sum(each training sample from index
            to n(i))/n(i)
        index = index+n(i)
    }
}
```

Listing 2: Implementation of NCC.

C. NSC

We make an initial k-means clustering of the training data for $k = m$ subclasses as hyper parameter and use the resulting centroids to construct a new centroid matrix for each subclass m spanning $c_k * m$. Listing 3 shows this.

```
train_nsc(trainData , trainLbls , nClasses ,
    nSubClasses) {
    centroids = zero vector by size of trainData
        *(nClasses*nSubClasses)
    n = zero vector by size of 1*nClasses
    index = 1;
    centroidIdx = vector to keep idx of centroids

    for (i = 1; i <= nClasses; i++) {
        n(i) = sum(num of classes i in trainLbls)
        [idx , clst] = kmeans(trainData for index+n(i)
            , nSubClasses)
        centroids(for idx in centroidIdx) = clst
        index = index+n(i)-1
    }
}
```

Listing 3: Implementation of NSC.

D. NNC

The algorithm takes as input the training and test data, as well as training labels. It then iterates through every test sample x_i and compares to every single training sample X_i . It classifies x_i to the class of X_i it is closest too in Euclidean distance. Listing 4 shows this.

```
train_nnc(trainData , trainLbls , testData) {
    distances = zero vector by size testData*
        trainData
    resultLabels = zero vector by size testData*1
    nTestImages = num of test images
    nTrainImages = num of train images

    for (i = 1; i <= nTestImages; i++) {
        for (j = 1; j <= nTrainImages; j++) {
            distances(i,j) = norm(testData(i)-
                trainData(j) , 2)^2.
        }
        trainIndex = min(distances(i))
        resultLabels = trainLbls(trainIndex)
    }
}
```

Listing 4: Implementation of NNC.

E. Perceptron with backpropagation

The algorithm trains a perceptron to correctly classify each sample x_i to class c_k . We start with an empty vector χ , then create a label l_i for sample x_i setting it to 1 if it corresponds to our current class, otherwise -1. We then use a one-vs-rest criterion function $f(w, x)$ for the x_i 's, save the wrongly classified samples and labels, then summing the wrong samples times their label and use this value to update weights w_k iteratively. The learning eta η is a hyper parameter. Listing 5 is the pseudo-code for this.

```

train_perp_bp(trainData , trainLbIs , eta ,
nClasses) {
w = random W matrix with size trainData*
nClasses

for (k = 1; k <= nClasses; k++) {
wrongLabels = empty matrix to save labels
done = false , i = 0
while (not done) {
X = empty matrix for wrong x_i's
for (i = 1; i <= nTrainImages , i++) {
x_i = trainData(i)
if(trainlbls(i) = k) label = 1 else label
= -1
if (label*w(k)*x_i) < 0
save x_i to X, save label to
WrongLabels
}
sumOfWrongs = sum(WrongLabels * X)
w(k) = w(k) + eta * sumOfWrongs
ifEmpty(X)
done = true
}
}
}

```

Listing 5: Implementation of NSC.

F. Perceptron with MSE

The algorithm for MSE is simplified, but possibly more computational demanding. We start by finding the pseudo-inverse of our training data set, then create a label vector for each class c_k with $l_i = 1$ if the label is in the class or $l_i = -1$ if it is not. Finally we find the MSE solution by multiplying the two for weight W_k . Listing 6 shows this.

```

train_perp_mse(trainData , trainLbIs , nClasses)
{
w = random W matrix with size trainData*
nClasses
X = pseudo-inverse of trainData

for (k = 1; k <= nClasses; k++) {
Labels = empty matrix to save labels
for (i = 1; i <= nTrainImages; i++)
if (trainlbls(i) = k)
save 1 to Labels
else save -1 to Labels
}
w(:,k) = X*Labels;
}
}

```

Listing 6: Implementation of NSC.

IV. RESULTS

The five classification schemes are trained and tested on the ORL and MNIST data set with scatter plots to visualize the resulting labels compared to the testing ones and a confusion matrix that depicts the predicated and true class. Each algorithm is tested with both the PCA and the full version of the data. All results were done with MATLAB R2018a on an Intel i5 4690K @ 3.50Ghz CPU. We start by showing the output of the PCA analysis, and then proceed to the classifications.

A. PCA

PCA was performed on both ORL and MNIST data set for number of dimensions $D = 2$ with MATLAB. Table I details the computation time for PCA and figure 3 and 4 shows a scatter plot with two principal components, i.e. the eigenvectors with highest eigenvalues.

Dataset	Method	Execution time in s
ORL	PCA	0.33s
MNIST	PCA	0.83s

Table I: PCA on ORL and MNIST data set.

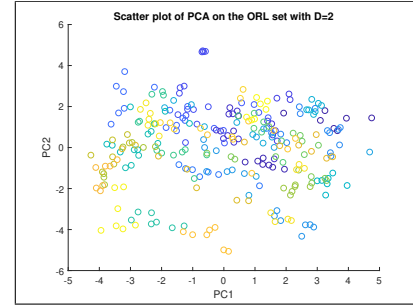


Figure 3: Plot of two PC's on ORL. Data labels are colors.

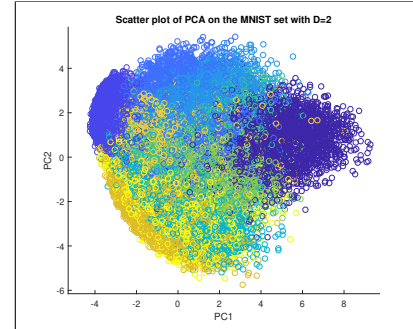


Figure 4: Plot of two PC's on MNIST. Data labels are colors.

B. NCC

Nearest Centroid was performed on ORL and MNIST. To test the algorithm, we find the class c_k 's mean to which a sample x_i is closest to by calculating the Euclidean distance (L2-norm) between a test sample and the mean vector. Table II details performance and accuracy, while figure 5 and 6 shows the resulting labels on scatter plots. The tests were performed on full-dimensional sets and PCA reduced sets. Accuracy is the number of resulting labels that equal the testing labels divided by the number of test samples.

Dataset	Accuracy in %	Execution time in s
ORL	94,1%	0.00s
ORL w. PCA	15,8%	0.00s
MNIST	75,2%	0.34s
MNIST w. PCA	14,9%	0.00s

Table II: Results for NCC performed on ORL and MNIST.

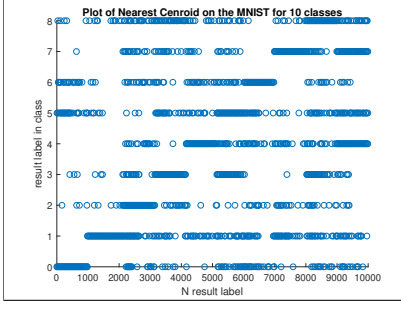


Figure 5: Plot results of NCC on ORL.

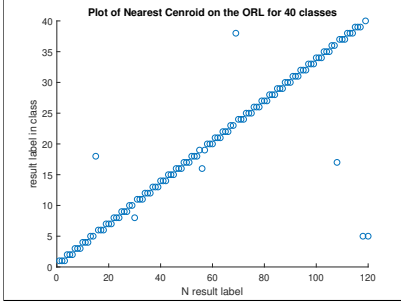


Figure 6: Plot results of NCC on MNIST.

C. NSC

Nearest Subclass Centroid was performed on ORL and MNIST with hyper-parameter $N_{sc} = 2, N_{sc} = 3, N_{sc} = 5$ for number of subclasses. Our algorithm returns centroids for number of classes $N_c = N_{ck} * N_{sc}$. We classify a testing sample to a class of which its centroid it is closest to. Table III details the results and figure X and X shows the resulting labels on plots. N_{sc} is number of subclasses.

Dataset	N_{sc}	Accuracy in %	Execution time in s
ORL	2	93.3	0.15s
ORL	3	95	0.15s
ORL	5	95.8	0.15s
ORL w. PCA	2	9.1	0.13s
MNIST	2	86.1	4.07s
MNIST	3	88	5.99s
MNIST	5	90	9.21s
MNIST w. PCA	2	14.3	0.14s

Table III: Results for NSC performed on ORL and MNIST.

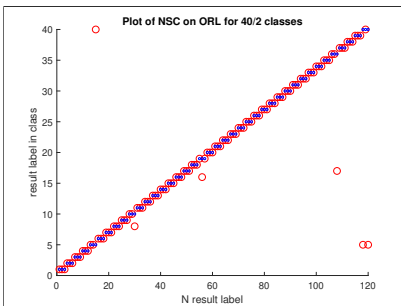


Figure 7: Plot results of NSC on ORL with $N_{sc} = 2$.

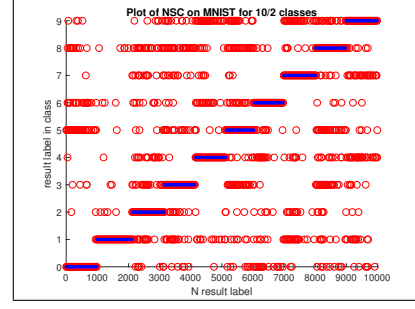


Figure 8: Plot results of NSC on MNIST with $N_{sc} = 2$.

D. NNC

Nearest Neighbor classifier was performed on ORL and MNIST. To classify a test sample, the algorithm calculates the Euclidean distance to every training sample and assigns the test to whichever training sample it is closest to. Table IV shows the results with accuracy and execution times.

Dataset	Accuracy in %	Execution time in s
ORL	95.8%	0.04s
ORL w. PCA	11.6%	0.01s
MNIST	96.9%	676.07s
MNIST w. PCA	14.9%	264.65s

Table IV: Results of NNC performed on ORL and MNIST.

E. Perceptron with BP

A perceptron was trained on both data sets with the learning rate η as hyper-parameter for $\eta = 0.01, \eta = 0.1, \eta = 1$ and $\eta = 10$. For each test sample, we calculate the decision function $y = w(k) * x_i$ for each class c_k and classify it to the class with maximizes the function. Table V details the results, figure X and X show plots of the resulting labels and 11 and 12 show the perceptron line in 1D and 2D with PCA.

Dataset	η	Accuracy in %	Execution time in s
ORL	0.01	86.6	1.39s
ORL	0.1	84.1	1.30s
ORL	1	82.5	1.36s
ORL	10	84.1	1.34s
ORL w. PCA	0.1	4.1	11.61s
MNIST	0.01	90.4	1127.68s
MNIST	0.1	86.6	1151.50s
MNIST	1	86.2	1155.60s
MNIST	10	79.3	1134.25s
MNIST w. PCA	0.1	7.4	638.61s

Table V: Results for a perceptron with BP performed on ORL and MNIST.

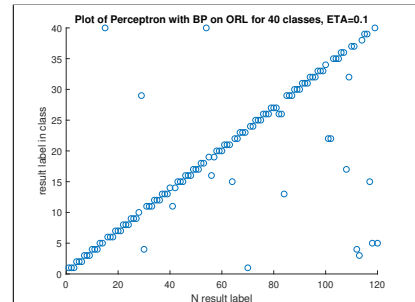


Figure 9: Plot results of training a perceptron with BP on ORL with $\eta = 0.1$.

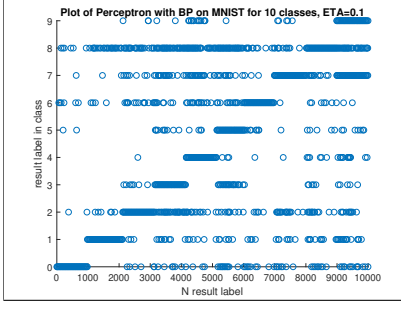


Figure 10: Plot results of training a perceptron with BP on MNIST with $\eta = 1$.

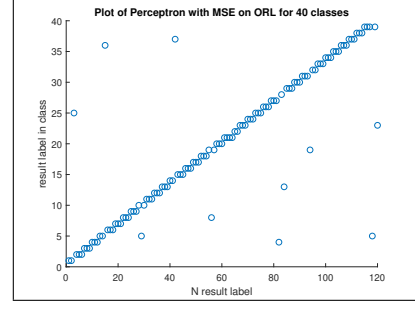


Figure 13: Plot results of training a perceptron with least-squares (MSE) on ORL.

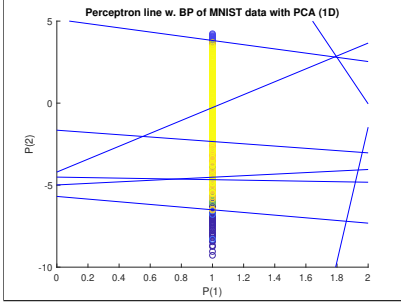


Figure 11: Plot lines of perceptron with BP for classes on MNIST in 1D.

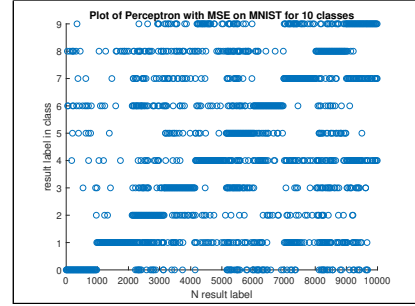


Figure 14: Plot results of training a perceptron with least-squares (MSE) on MNIST.

F. Perceptron with MSE

A perceptron was trained on both data sets. Like with backpropagation, we find the class that maximizes the decision function $y = w(k) * x_i$ for each sample and classify it to that class. Table VI details the results, figure 13 and 14 shows plots with the resulting labels and ?? show the perceptron in action on the PCA set.

Dataset	Accuracy in %	Execution time in s
ORL	90%	0.05s
ORL w. PCA	10%	0.01s
MNIST	86%	6.79s
MNIST w. PCA	18.4%	0.53s

Table VI: Results for a perceptron with least-squares (MSE) performed on ORL and MNIST.

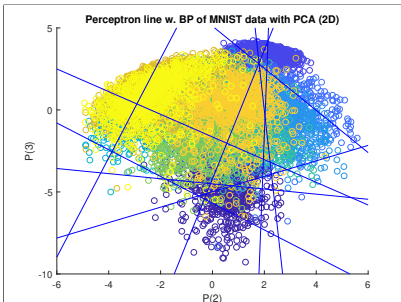


Figure 12: Plot lines of perceptron with BP for classes on MNIST in 2D.

V. DISCUSSION

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