



Session 3 Exercise:
Determining the Upper Bound
of a Discrete Uniform Density
or
How Many Taxis are there ?

How Many Taxis ?



Suppose one takes independent observations y_1, \dots, y_n from a uniform distribution on the set $\{1, 2, \dots, N\}$, where the upper bound N is unknown. Suppose one places a uniform prior for N on the values $1, \dots, B$, where B is known. Then the posterior probabilities for N are given by

$$g(N|y) \propto \frac{1}{N^n}, \quad y_{(n)} \leq N \leq B,$$

where $y_{(n)}$ is the maximum observation. To illustrate this situation, suppose a tourist is waiting for a taxi in a city. During this waiting time, she observes five taxis with the numbers 43, 24, 100, 35, and 85. She assumes that taxis in this city are numbered from 1 to N , she is equally likely to observe any numbered taxi at a given time, and observations are independent. She also knows that there cannot be more than 200 taxis in the city.

- Use R to compute the posterior probabilities of N on a grid of values.
- Compute the posterior mean and posterior standard deviation of N .
- Find the probability that there are more than 150 taxis in the city.