Exercise: Learning about a Mortality Rate using a Mixture Prior

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In the heart transplant surgery example in Section 3.3, suppose you are interested in estimating the mortality rate λ for a particular hospital. To construct your prior, you talk to two experts. The first expert's beliefs about λ are described by a gamma(1.5, 1000) distribution and the second expert's beliefs are described by a gamma(7, 1000) distribution. You place equal credence in both experts, so your prior beliefs are represented by the mixture prior

$$g(\lambda) = .5g_1(\lambda) + .5g_2(\lambda),$$

where g_1 and g_2 are respectively the gamma(1.5, 1000) and gamma(7, 1000) distributions.

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a) Using the `curve` function, construct a graph of the prior density for λ.

b) Suppose this hospital experiences $y_{\text{obs}} = 4$ deaths with an exposure of $e = 1767$. Using the function `poisson.gamma.mix` in the `LearnBayes` package, compute the posterior distribution of λ. The inputs to this function are similar to the inputs to the function `binomial.beta.mix` described in Section 3.5.

c) Plot the prior and posterior densities of λ on the same graph.

d) Find the probability that the mortality rate λ exceeds .005.

e) Based on the mixing probabilities, were the data more consistent with the beliefs of the first expert or the beliefs of the second expert? Explain.

`poisson.gamma.mix()` Function

Computes the posterior for Poisson sampling and a mixture of gammas prior

Description

Computes the parameters and mixing probabilities for a Poisson sampling problem where the prior is a discrete mixture of gamma densities.

Usage

```
poisson.gamma.mix(probs,gammapar,data)
```

`poisson.gamma.mix()` Function

Computes the posterior for Poisson sampling and a mixture of gammas prior

Arguments

`probs:` vector of probabilities of the gamma components of the prior.

`gammapar:` matrix of rate and shape parameters for a gamma component of the prior.

`data:` list with components y, vector of counts, and t, vector of time intervals.