

## Assignment -1.

Ans. 1. Asymptotic notations describe the growth rate of Algorithms in terms of input size. They provide a way to describe time complexity of Algorithms.

- Big  $O()$ : Represent the upper bound of worst case scenario.
- Omega ( $\Omega$ ): Represents the lower bound or best case scenario.
- Theta ( $\Theta$ ): Represent both upper and lower bounds, indicating tight bounds.

Ans. 2  $O(\log n)$

$\therefore$  value of  $i$  will be doubled until it exceeds  $n$ .

Ans. 3  $3T(n-1)$   $n > 0$ ,  $T(0) = 1$ .

$$T(n) = 3T(n-1)$$

$$T(0) = 1$$

$$T(n-1) = 3T(n-2)$$

$$T(n-2) = 3T(n-3)$$

$$T(n-1) = 3T(n-2)$$

$$T(n-2) = 3T(n-3)$$

$$T_n = 3T(n-2) \cdot 3$$

$$T_n = T(n-3) \cdot 3 \cdot 3 \cdot 3$$

$$T(n) = \underbrace{3 \cdot 3 \cdot 3}_{n+1} \cdot T(0)$$

$$T_n = 3 \cdot 3 \cdot 3 \dots T(0)$$

$$T_n = 3^n$$

$$T.C = O(3^n)$$

Q.3.  $[2T(n-1) - 1]$  for  $n > 0$ , otherwise 1.

$$T(0) = 1$$

$$T(n) = 2T(n-1) - 1$$

$$T(n-1) = 2T(n-2) - 1$$

$$T(n-2) = 2T(n-3) - 1$$

~~$$T(n) = 2 \times 2 \times 2 \dots$$~~

$$T(n) = 2 \times [2T(n-2) - 1] - 1$$

$$T(n) = 2 \times [2 \times (2T(n-3) - 1) - 1] - 1$$

$$T(n) = 2 \times [8T(n-3) - 2] - 1$$

$$= 16T(n-3) - 4 - 1$$

$$T(3) = 16T(n-3) - 5$$

$$T(n) = \cancel{2+2 \times 2 \dots} \quad 2 + 2 \times 2 + \dots + 2 \cdot (T(0) - n)$$

$$T(n) = 2^n - (2^n - 1)$$

$$T(n) = 2^n - 2^n + 1 = 1$$

~~$$T(n) = 2^n$$~~

$$T.C = O(1)$$



5.

Time complexity:-

~~Time complexity~~

$$i = 1, 2, 3, 4, 5, \dots$$

$$s = 1, 3, 6, 10, 15, \dots$$

$$s = \frac{i \cdot (i+1)}{2} \leq n$$

$$i^2 + i \leq 2n$$

$$i \approx \sqrt{2n}$$

So,

$$T.C = O(\sqrt{n})$$

Q.6

$$i * i < n$$

$$i^2 < n$$

$$i \approx \sqrt{n}$$

T.C:-

$$O(\sqrt{n})$$

7. ~~Can~~ outer loop runs -  $n$  times

And.

T.C of nested loops -  $O(\log n)$

So.

T.C =  $O(n \log n)$ .

8. function(int n) {

if ( $n == 1$ ) return;

for ( $i = 1$  to  $n$ ) {  $O(n)$   
for ( $j = 1$  to  $n$ ) {  $O(n)$

print(" ");

}

}

function( $n/3$ );  $T(n/3)$

}

~~T.C =  $O(n^3)$~~

T.C =  $O(n^2) + T(n/3)$



9) void function (int n) {

for (i = 1 to n) { (n)

for (j = 1; j <= n; j = j + 1) (n)

{

print ( " " )

}

}

i = 1, j = 1, 2, 3, 4, ...

i = 2, j = 1, 3, 5, ...

i = 3, j = 1, 4, 7, ...

~~Time complexity~~

~~Time complexity~~

$$T(n) = n + n/2 + n/3 + n/4 + \dots$$

$$T.C = O(n \log n)$$

10)

Asymptotic relationship between  $n^k$  and  $c^n$  is that  $c^n$  grows faster than  $n^k$ . In other words,  $c^n$  is an exponential growth function, while  $n^k$  is a polynomial growth function.