Lec8-mwf

Tuesday, January 22, 2008 5:38 PM

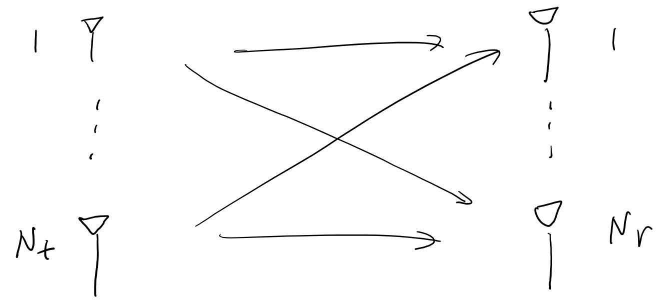
Project 1 is assigned
No class on Monday/Wednesday (due to travel)

Print 2-dim cellular patterns.

MIMO - 15min

Tuesday, January 28, 2020 4:19 PM

- Consider a system with N_t transmitting antennas and N_r receiving antennas.



- Let $\vec{S}$ be the vector of transmitting symbols, $\vec{S} \in R^{N_t}$
  $\vec{r}$ be the vector of receiving symbols, $\vec{r} \in R^{N_r}$
- The relationship between $\vec{r}$ and $\vec{S}$ is given by the channel model below:

$$\vec{r} = \sqrt{\rho} \cdot H \cdot \vec{S} + z$$

$$= \sqrt{\rho} \begin{array}{c|c} & N_t \\ \hline N_r & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array} \begin{array}{c} 1 \\ \begin{bmatrix} \\ \\ \end{bmatrix} N_t \end{array} + \begin{array}{c} 1 \\ \begin{bmatrix} \\ \\ \end{bmatrix} N_r \end{array}$$

$[H_{ij}]_{N_r \times N_t}$
- Assume to be normalized such that $\|H\|_F^2 = N_r \cdot N_t$
- For example, this is true if $H_{ij} = 1$ in all elements
- "The gain between each transmitter - receiver pair is about the same"

noise
- $z \in R^{N_r}$
- i.i.d. zero mean with variance $\sigma^2 = 1$
- "More antennas pick up more noise"

transmitted symbol
- $S \in R^{N_t}$
- $E[\|\vec{S}\|^2] = 1$
- "Total transmission power should not grow with N_t"

- If the power is uniform on all antennas, then $E(s_i^2) = \frac{1}{N_t} \triangleq \sigma_s^2$

- Thus, ρ can be interpreted as the received SNR, comparable to a SISO system.

Eigen-Beamforming

- Assume that both the sender & the receiver knows the channel matrix H
- Using SVD (Singular Value Decomposition), H can be written as

$$H = U \quad \Lambda \quad V^T$$

$$= \begin{array}{c} N_r \\ N_r \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array} \begin{bmatrix} \sqrt{\lambda_1} & & & \\ & \sqrt{\lambda_2} & & 0 \\ & & \ddots & \\ 0 & & & \sqrt{\lambda_N} \end{bmatrix} \begin{array}{c} N_t \\ \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} N_t \end{array}$$

- $N_r \times N_r$ unitary matrix

- $N_r \times N_t$ but diagonal
- $N = \min\{N_t, N_r\}$
- Each λ_i is also the eigenvalue of HH^T

- $N_t \times N_t$ unitary matrix

- By unitary matrix, we mean that

$$UU^T = U^TU = I_{N_r \times N_r} \qquad VV^T = V^TV = I_{N_t \times N_t}$$

$$UU^T = U^T U = I_{N_r \times N_r} \qquad VV^T = V^T V = I_{N_t \times N_t}$$

- Vx will rotate a vector x but won't change its length.

- In eigen-beamforming, the sender multiply the information symbols x by the matrix V, i.e.

$$\vec{S} = V \cdot x$$

$$= \begin{bmatrix} v_1 & \cdots & v_{N_t} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{N_t} \end{bmatrix}$$

- The information x_1 is now sent over all antennas as $v_1 x_1$

- Like the antenna array example, this roughly forms a beam for certain direction.

$\Rightarrow$ "beamforming"

- Note ~~that~~ the ~~total~~ xmit power doesn't change since $E\{\|Vx\|_2^2\} = E[x^T V^T V x] = E[x^T x] = E[\|x\|_2^2]$

- At the receiver end, multiply $\vec{r}$ by U^T.

$$y = U^T r = \sqrt{\rho}\, \underbrace{U^T U}_{I} \Lambda \underbrace{V^T V}_{I} x + U^T z$$

$$= \sqrt{\rho}\, \Lambda x + \tilde{z}$$

$\tilde{z} = U^T z$

- each element still has the variance $\sigma^2 = 1$

$$= \sqrt{\rho} \begin{bmatrix} \sqrt{\lambda_1} & & 0 \\ & \ddots & \\ 0 & & \sqrt{\lambda_N} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_N \\ \vdots \\ x_{N_t} \end{bmatrix} + \tilde{z}$$

- Thus, we get $N = \min\{N_t, N_r\}$ equations

$$y_i = \sqrt{\rho}\sqrt{\lambda_i}\, x_i + \tilde{z}_i \quad , \quad i = 1, \cdots, N$$

- In other words, through eigen-beamforming, the channel can be viewed as equivalent to N seperate channels, each of which has an SNR of

$$\frac{\rho\, E[x_i^2]\, \lambda_i}{\sigma^2} = \rho\, E[x_i^2] \cdot \lambda_i$$

  - The total rate is

$$\sum_{i=1}^{N} B \log_2 \left(1 + \rho\, E[x_i^2] \cdot \lambda_i\right)$$

The eigenvalues

- How much total data rate can we get depends on the values of $\lambda_1, \cdots, \lambda_N$

- Consider $N_t = N_r = N$. The following is true

$$\sum_{i=1}^{N} \lambda_i = \|H\|_F^2$$

  - since $\|H\|_F^2 = N^2$ by our assumption

$$\Rightarrow \sum_{i=1}^{N} \lambda_i = N^2$$

- Consider two possibility

① If $H_{ij} = 1$ for all i, j.

$$H = \underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{\text{all xmit-receive}} = \begin{bmatrix} \frac{1}{\sqrt{N}} & & \\ \vdots & & \\ \frac{1}{\sqrt{N}} & & \end{bmatrix} \underbrace{\begin{bmatrix} N^2 & & & \\ & 0 & & \\ & & \ddots & \\ & & & 0 \end{bmatrix}}_{\text{only 1 non-zero}} \begin{bmatrix} \frac{1}{\sqrt{N}} & \cdots & \frac{1}{\sqrt{N}} \\ & & \\ & & \end{bmatrix}$$

all xmit-receive pairs are highly correlated

only 1 non-zero eigenvalue

- only one effective channel

$$\text{total rate} = B\log_2\left(1+\rho N^2\cdot E[x_1^2]\right)$$

- Even though the total rate still increase with N, the growth is very slow

② If H is such that $\lambda_1 = \cdots = \lambda_N = N$,

- N effective channels

$$\text{total rate} = N\cdot B\log_2\left(1+\rho\cdot N\cdot E[x_i^2]\right)$$

- This growth is more desirable as it is linear in N.
- This is called the "spatial multiplexing" gain.

<u>When</u> will all λ_i's be approximately equal to N?

- One such case is when each element of H is i.i.d. zero-mean Gaussian with variance 1.

$$HH^T = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ & - & - & \end{bmatrix}\begin{bmatrix} h_{11} & h_{21} & & \\ \vdots & \vdots & \vdots & \vdots \\ h_{1N} & h_{2N} & & \end{bmatrix}$$

$\Rightarrow$ The $(1,1)$-element of HH^T is

$$\sum_{i=1}^{N} h_{1i}^2 \simeq N \quad \text{when } N \text{ is large}$$

The $(1,2)$-element of HH^T is

$$\sum_{i=1}^{N} h_{1i}\cdot h_{2i} \ll N \quad \text{when } N \text{ is large}$$

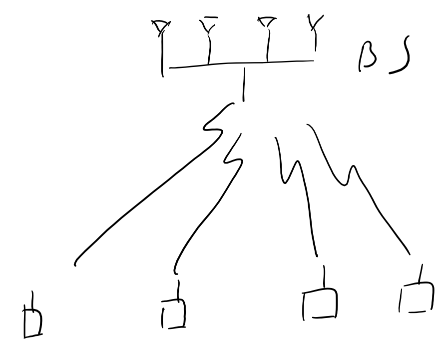
$\Rightarrow HH^T$ is roughly a diagonal matrix

$$\begin{bmatrix} N & & 0 \\ & N & \\ 0 & & \ddots \\ & & & N \end{bmatrix}$$

- Therefore, to get the spatial multiplexing gain, it is desirable that the channel of each transmit-receive pair is independent of others
  - Sufficient "spatial diversity"
  - More suitable when there are many multipaths
    → "rich scattering" environment.
- Unfortunately, this also means that there is more overhead to estimate the CSI of each xmit-receive pairs
  - overhead grows at N^2 (will see it again when we discuss LTE).

- Finally, in addition to the eigen-beamforming formulation above, there are other expressions of MIMO capacity that assume CSI only at the receiver, which may also produce spatial multiplexing gains.
- See reference on the course website.

- The above discussion is for single-user scenario
- In practice, often the receiver has less antennas, but there may be multiple receivers in the downlink



- It is not possible for apply eigen-beamforming (i.e., the multiplication by U^T at the receiver end)
- Instead, spatial multiplexing gain in such a MU-MIMO scenario can be attained by the so-called zero-forcing beamforming

Zero-Forcing Beamforming

- Suppose there are N antennas & N users

$$\underbrace{\vec{r}}_{\substack{\text{received}\\\text{signal at}\\N\text{ users}}} = \rho \underbrace{H}_{N\times N} \cdot \underbrace{\vec{s}}_{\substack{\text{xmit}\\\text{signal at}\\\text{the }N\text{ antennas (at BS)}}} + z$$

- Assume that H is invertable

$$H \cdot H^{-1} = I$$

$$\begin{bmatrix} H_1 \\ \vdots \\ H_N \end{bmatrix}_{N\times N} \begin{bmatrix} V_1 & \cdots & V_N \end{bmatrix}_{N\times N} = I$$

- The sender (the BS) multiplies the information of user k by V_k, i.e.

$$\vec{S} = \sum_{k=1}^{N} V_k \cdot x_k$$

- The signal received by user m is

$$r_m = \rho \cdot H_m \sum_{k=1}^{N} V_k \cdot x_k + z_m$$

$$= \rho \cdot \underbrace{H_m \cdot V_m}_{=1} x_m + \rho \cdot \sum_{k \neq m} \underbrace{H_m V_k}_{=0} x_k + z_m$$

$$= \rho \cdot x_m + z_m$$

- In other words, the beam V_m for user m will only produce signal at user k, and will zero-out at all other users. $\longrightarrow$ "Zero-forcing"

- On the other hand, the transmit power for each user m may differ based on V_m.

MIMO - handout1

Thursday, January 30, 2020 11:59 PM

Eigen-beamforming

receive

Xmit

$$\underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_{N_r} \end{bmatrix}}_{\vec{y}} = \begin{bmatrix} & & \\ & U^T & \\ & & \end{bmatrix} \underbrace{\begin{bmatrix} r_1 \\ \vdots \\ r_{N_r} \end{bmatrix}}_{\vec{R}}$$

$$\vec{r} = \sqrt{\rho}\, H \vec{s} + z$$

$$\underbrace{\begin{bmatrix} s_1 \\ \vdots \\ s_{N_t} \end{bmatrix}}_{\vec{S}} = \overbrace{\begin{bmatrix} v_1 & \cdots & v_{N_t} \end{bmatrix}}^{V} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_{N_t} \end{bmatrix}}_{\vec{x}}$$

$$H = U \Lambda V^T$$

$$= U \begin{bmatrix} \sqrt{\lambda_1} & & & 0 \\ & \sqrt{\lambda_2} & & \\ & & \ddots & \\ 0 & & & \sqrt{\lambda_N} \end{bmatrix} V^T$$

Altogether

$$\begin{bmatrix} y_1 \\ \vdots \\ y_{N_r} \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} \sqrt{\lambda_1} & & 0 \\ & \ddots & \\ 0 & & \sqrt{\lambda_N} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{N_t} \end{bmatrix} + \tilde{z}$$

Total rate :

$$\sum_{i=1}^{N} B \log_2 \left(1 + \rho\, E[x_i^2] \cdot \lambda_i \right)$$

Zero-forcing beamforming

receive Xmit

$$\vec{R} = \begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} \qquad \vec{r} = \sqrt{\rho}\, H \vec{s} + z \qquad \vec{S} = \begin{bmatrix} s_1 \\ \vdots \\ s_N \end{bmatrix} = \overbrace{\begin{bmatrix} v_1 & \cdots & v_N \end{bmatrix}}^{V} \underbrace{\begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}}_{\vec{x}}$$

$$V = H^{-1}$$

(assuming that $N_t = N_r = N$)

$$H = \begin{bmatrix} H_1 \\ \vdots \\ H_N \end{bmatrix}$$

Altogether

$$\begin{bmatrix} r_1 \\ \vdots \\ r_N \end{bmatrix} = \sqrt{\rho} \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix} + z$$

Zero-forcing:

$$H_i v_j x_j = 0 \quad \text{if } i \neq j$$

Voice system vs data systems

Tuesday, January 28, 2020 4:13 PM

- "Working against fading" is more common for voice systems.
  - e.g. power control

- "Exploiting fading & diversity" is more common for data systems
  - opportunistic scheduling
  - OFDM
    MIMO

- Why?
  - voice is fixed rate.
  - voice is more sensitive to delay.

- However, these may change for IoT / Machine-type communications, where the high overhead of CSI collection is undesirable.

Learning Objectives

Tuesday, January 23, 2018 9:07 AM

1. Explain the three components of probabilistic channel models
2. Derive the free-space and 2-ray models for the path-loss component. Identify settings where such models are more or less accurate
3. Explain the log-normal shadow fading model
4. Derive the Raleigh/Ricean model for multipath fading
5. Explain the notion of delay spread and doppler shift. Explain how they affect the time/frequency selectivity of the channel
6. Compute the coherence time and coherence bandwidth of the channel, and use them to identify different types of channel.
7. Explain the different approaches to deal with channel fading.

Cellular - 10min

Monday, February 18, 2008 4:50 PM

Recall that the cellular concept significantly increases the capacity of wireless networks

- channels are <u>reused</u> at multiple cells

- A channel can be a frequency band, a time-slot, or a code.

Questions:

(1) How far apart do cells with the same channels need to be?

- determined by inter-cell interference (co-channel interference)

(2) How big is the area of the cell?

- related to traffic density.

(3) How to allocate channels to cells

- Fixed/Dynamic channel allocation.

- The approaches that we will discuss now are more geared towards voice systems

- We will discuss their limitation for data services.

1-dim - 15min

Tuesday, February 19, 2008 3:52 PM

Channel Reuse Patterns

1-dimensional case (e.g. highway)

1	2	3	4	1	2	3	4	1

4-reuse

cells with the same channel are seperated by 3 other cells.

Example: AMPS has totally 832 channels $\left(\frac{25M}{30k}\right)$
Assume N cells
At 4-reuse, total capacity is

$$\frac{832}{4} \cdot N = 208 \cdot N$$

On the other hand, at 3-reuse

1	2	3	1	2	3	1	2	3

3-reuse

the total capacity is

$$\frac{832}{3} \cdot N = 277 \cdot N$$

Which one should we choose?

It depends on SINR.

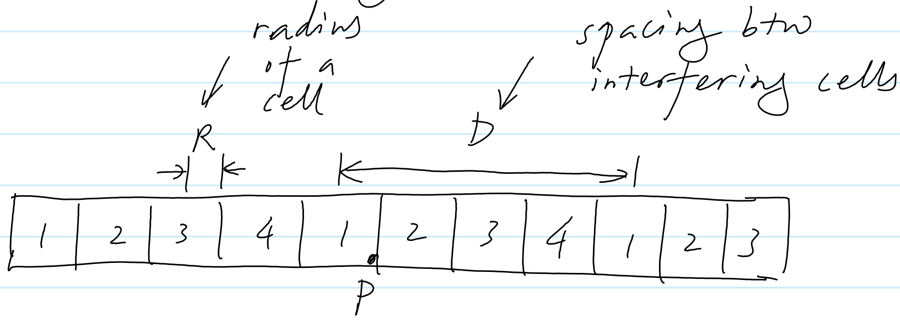
SINR (Signal to Interference and Noise Ratio): the ratio of the desired signal power at the receiver to the total interference + noise power.

Typical modulation schemes need SINR above certain threshold to achieve acceptable bit error prob.

AMPS : 18 dB
GSM : 7 – 12 dB

– Let us make a number of simplifications:

Ignoring background noise & fading, SINR can be determined by the reuse distance.



3 – reuse : $D/R = 6$
4 – reuse : $D/R = 8$

Focus on the downlink. Assume basestations at the center of each cell.

Consider the worst case receiver P at the edge of an arbitrary cell (cell 1 in figure)

Assume that all BS transmits at the same power P_T :

received signal strength: P_T / R^n

Interference from 1st-tier interferers:

$D_=$ P_T

Interference from 1st-tier interferers:

$$\frac{P_T}{(D-R)^n} + \frac{P_T}{(D+R)^n}$$

Interference from 2nd-tier interferers

$$\frac{P_T}{(2D-R)^n} + \frac{P_T}{(2D+R)^n}$$

In practice, only the 1st-tier interferers are considered (esp, when n is large)

$$S2NR = \frac{R^{-n}}{(D-R)^{-n} + (D+R)^{-n}}$$

$$= \frac{1}{\left(\frac{D}{R}-1\right)^{-n} + \left(\frac{D}{R}+1\right)^{-n}}$$

reuse factor	S2NR (dB)	
	$n=3$	$n=4$
3-reuse	19.6	27
4-reuse	23	32

Reexamine the assumptions: Is the above optimistic or pessimistic?

- noise = 0. otherwise S2NR ↓
- no fading
  - when there is fading, only part of the cell (or at part of the time) will receive acceptable service.

The prob. or fraction of time can be calculated from fading distribution (p74 of Schwartz).

- Alternatively, may design the reuse pattern according to the worst-case fading
  $\Rightarrow$ may be too pessimistic

- Revisit this when we discuss CDMA.

- Equal power:

  - Is it a good idea to keep all power the same?
    or use power control.

- Downlink:

  - What about uplink?

(25)

- Voice vs. data traffic

  - Data service will be able to take advantage of varying SINR and diversity.