Spring 2022

ECE 562: Embedded Systems

Lecture #6: Real-Time Scheduling (Fixed Priority Assignment)

Vijay Raghunathan

Modeling for Scheduling Analysis

- Need to specify workload model, resource model, and algorithm model of the system in order to analyze the timing properties

- Workload model: Describes the applications/tasks executed
  - Functional parameters: What does the task do?
  - Temporal parameters: Timing properties/requirements of the task
  - Precedence constraints and dependencies

- Resource model: Describes the resources available to the system
  - Number and type of CPUs, other shared resources

- Algorithm model: Describes how tasks use the available resources
  - Essentially a scheduling algorithm/policy

General Workload Model

- Set Γ of "n" tasks τ_1, τ_2, …, τ_n that provide some functionality
- Each task may be invoked multiple times as the system functions
  - Each invocation is referred to as a task instance
  - $\tau_{i,j}$ indicates the j^{th} instance of the i^{th} task
- Time when a task instance arrives (is activated/invoked) is called its release time (denoted by $r_{i,j}$)
- Each task instance has a run-time (denoted by $C_{i,j}$ or $e_{i,j}$)
  - May know range $[C_{i,jmin}, C_{i,jmax}]$
  - Can be estimated or measured by various mechanisms
- Each task instance has a deadline associated with it
  - Each task instance must finish before its deadline, else of no use to user
  - Relative deadline ($D_{i,j}$): Span from release time to when it must complete
  - Absolute deadline ($d_{i,j}$): Absolute wall clock time by which task instance must complete

Simplified Workload Model

- Tasks are periodic with constant inter-request intervals
  - Task periods are denoted by $T_1, T_2, \ldots, T_n$
  - Request rate of τ_i is $1/T_i$
- Tasks are independent of each other
  - A task instance doesn't depend on the initiation/completion of other tasks
  - However, task periods may be related
- Execution time for a task is constant and does not vary with time
  - Can be interpreted as the maximum or worst-case execution time (WCET)
  - Denoted by $C_1, C_2, \ldots, C_n$
- Relative deadline of every instance of a task is equal to the task period
  - $D_{i,j} = D_i = T_i$ for all instances $\tau_{i,j}$ of task τ_i
  - Each task instance must finish before the next request for it
  - Eliminates need for buffering to queue tasks
- Other implicit assumptions
  - No task can implicitly suspend itself, e.g., for I/O
  - All tasks are fully preemptible
  - All kernel overheads are zero

Resource Model

- Tasks need to be scheduled on one CPU
  - Referred to as uni-processor scheduling
  - Will relax this restriction later to deal with multi-processor scheduling

- Initially, will assume that there are no shared resources
  - Will relax this later to consider impact of shared resources on deadlines

Scheduling Algorithm

- Set of rules to determine the task to be executed at a particular moment
- One possibility: Preemptive and priority-driven
  - Tasks are assigned priorities
    - Statically or dynamically
  - At any instant, the highest priority task is executed
    - Whenever there is a request for a task that is of higher priority than the one currently being executed, the running task is preempted and the newly requested task is started
- Therefore, scheduling algorithm == method to assign priorities

Assigning Priorities to Tasks

- Static or fixed-priority approach
  - Priorities are assigned to tasks once
  - Every instance of the task has the same priority, determined apriori

- Dynamic approach
  - Priorities of tasks may change from instance to instance

- Mixed approach
  - Some tasks have fixed priorities, others don’t

Deriving An Optimum Fixed Priority Assignment Rule

Critical Instant for a Task

- Overflow is said to occur at time t, if t is the deadline of an unfulfilled request
- A scheduling algorithm is feasible if tasks can be scheduled so that no overflow ever occurs
- Response time of a request of a certain task is the time span between the request and the end of response to that task
- Critical instant for a task = instant at which a request for that task will have the maximum response time
- Critical time zone of a task = time interval between a critical instant and the absolute deadline for that task instance

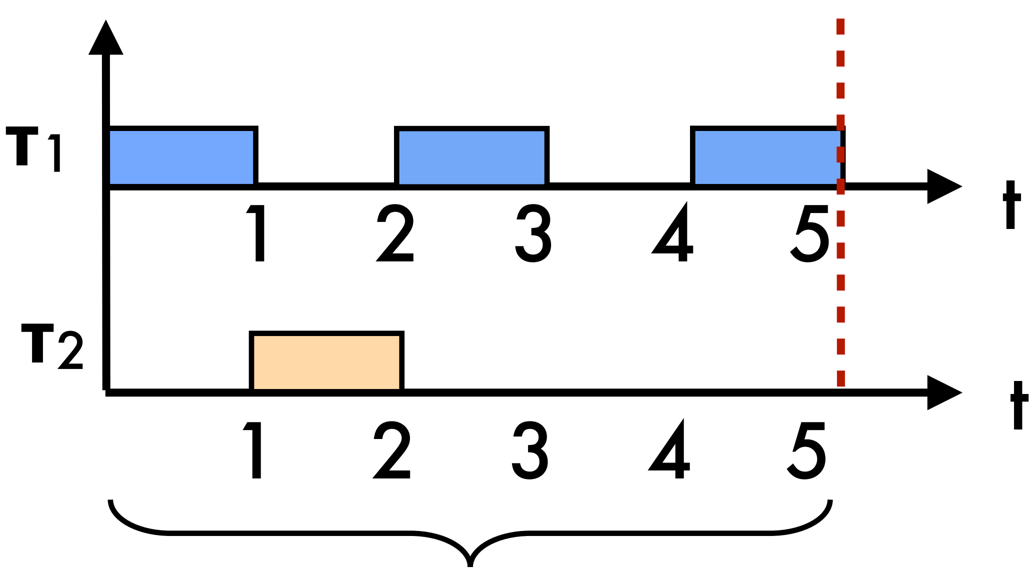
When does Critical Instant occur?

- Theorem 1: A critical instant for any task occurs whenever the task is requested simultaneously with requests of all higher priority tasks

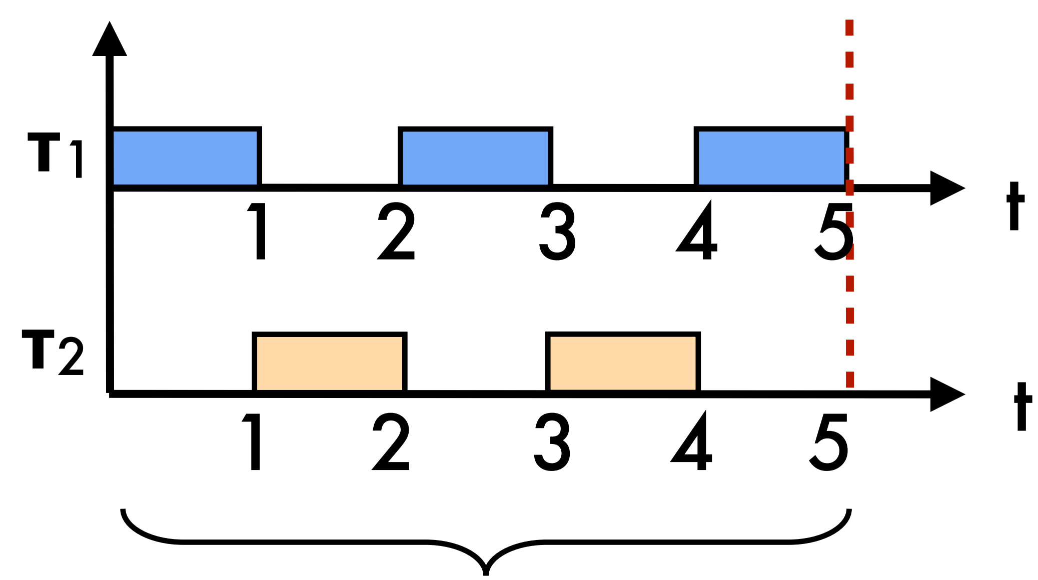
- Can use this theorem to determine whether a given priority assignment will yield a feasible schedule or not
  - If requests for all tasks at their critical instants are fulfilled before their respective absolute deadlines, then the scheduling algorithm is feasible

Example

- Consider two tasks τ_1 and τ_2 with T_1=2, T_2=5, C_1=1, C_2=1
- Case 1: τ_1 has higher priority than τ_2
  - Priority assignment is feasible
  - Can increase C_2 to 2 and still avoid overflow



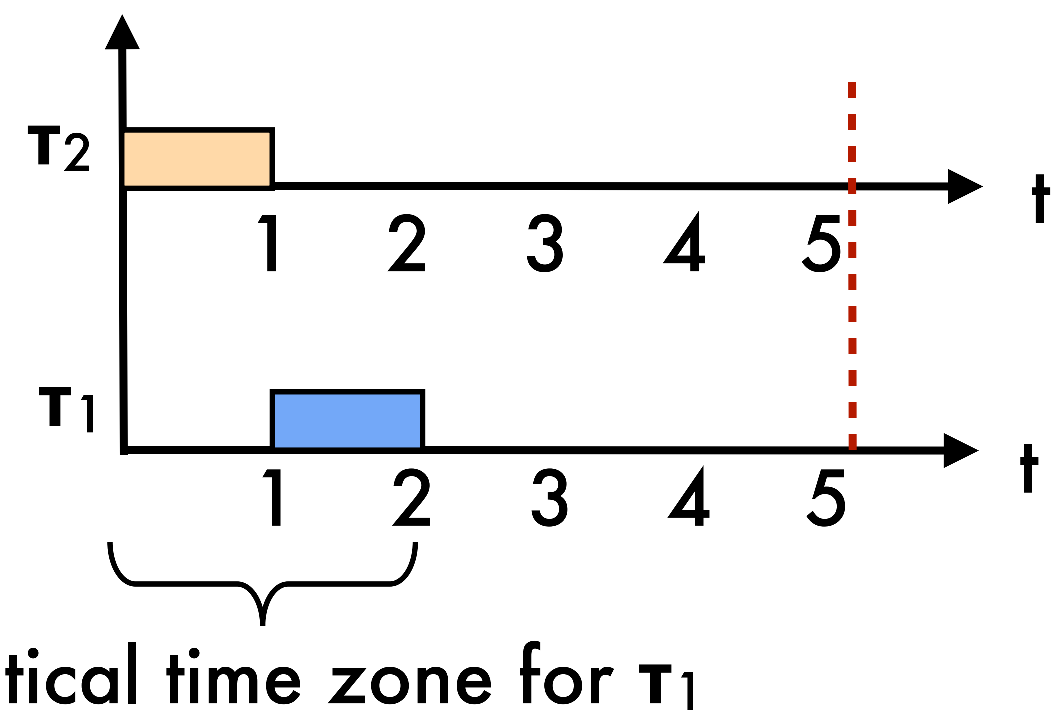
Critical time zone for τ_2
C_2=1



Critical time zone for τ_2
C_2=2

Example (contd.)

- Case 2: τ_2 has higher priority than τ_1
  - Priority assignment is still feasible
  - But, can't increase beyond $C_1=1$, $C_2=1$



Case 1 seems to be the better priority assignment for schedulability… can we formalize this?

Rate-Monotonic Priority Assignment

- Assign priorities according to request rates, independent of execution times
  - Higher priorities for tasks with higher request rates (shorter time periods)
  - For tasks τ_i and τ_j, if $T_i < T_j$, Priority(τ_i) > Priority(τ_j)

- Called Rate-Monotonic (RM) Priority Assignment
  - It is optimal among static priority assignment based scheduling schemes

- Theorem 2: No other fixed priority assignment can schedule a task set if RM priority assignment can't schedule it, i.e., if a feasible priority assignment exists, then RM priority assignment is feasible

Proof of Theorem 2 (RM optimality)

- Consider n tasks {τ_1, τ_2, … τ_n} ordered in increasing order of time periods (i.e., $T_1 < T_2 < \ldots . < T_n$)

- Assumption 1: Task set is schedulable with priority assignment {Pr(1), …, Pr(n)} which is not RM
  - Therefore, ∃ at least one pair of adjacent tasks, say τ_p and τ_{p+1}, such that Pr(p) < Pr(p+1) [higher value is higher priority]
  - Otherwise, assignment becomes RM (violates assumption)

- Assumption 2: Instances of all tasks arrive at t=0
  - Therefore, t=0 is a critical instant for all tasks. From Theorem 1, we only need to check if first instance of each task completes before deadline

Proof (contd.)

- Swap the priorities of tasks τ_p and τ_{p+1}
  - New priority of τ_p is Pr(p+1), new priority of τ_{p+1} is Pr(p)
  - Note that Pr(p+1) > Pr(p) (by assumption 1)

- Tasks {τ1, …, τ_{p-1}} should not get affected
  - Since we are only changing lower priority tasks

- Tasks {τ_{p+2}, …, τ_n} should also not get affected
  - Since both τ_p and τ_{p+1} need to be executed (irrespective of the order) before any task in {τ_{p+2}, …, τ_n} gets executed

- Task τ_p should not get affected
  - Since we are only increasing its priority

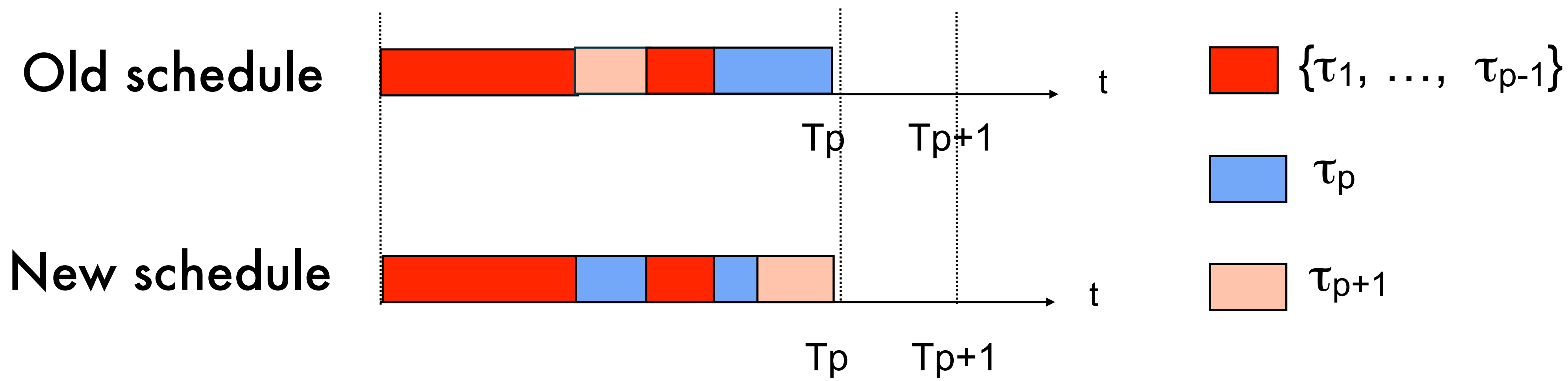
Proof (contd.)

- Consider τ_{p+1}:

- Since original schedule is feasible, in the time interval $[0, T_p]$, exactly one instance of τ_p and τ_{p+1} complete execution along with (possibly multiple) instances of tasks in $\{\tau_1, \ldots, \tau_{p-1}\}$
  - Note that τ_{p+1} executes before τ_p

- New schedule is identical, except that τ_p executes before τ_{p+1} (start/end times of higher priority tasks is same)
  - Still, exactly one instance of τ_p and τ_{p+1} complete in $[0, T_p]$. As $T_p < T_{p+1}$, task τ_{p+1} is schedulable

Proof (contd.)



We proved that swapping the priority of two adjacent tasks to make their priorities in accordance with RM does not affect the schedulability (i.e., all tasks $\{\tau_1, \tau_2, \ldots \tau_n\}$ are still schedulable)

Proof (contd.)

- If τ_p and τ_{p+1} are the only such non RM tasks in original schedule, we are done since the new schedule will be RM
- If not, starting from the original schedule, using a sequence of such re-orderings of adjacent task pairs, we can ultimately arrive at an RM schedule (Exactly the same as bubble sort)
- E.g., Four tasks with initial priorities [3, 1, 4, 2] for [τ_1, τ_2, ... τ_n]

[3 1 4 2] is schedulable

[3 4 1 2] is schedulable

[4 3 1 2] is schedulable

[4 3 2 1] is schedulable

RM priority assignment

Hence, Theorem 2 is proved.

Processor Utilization Factor

- Processor Utilization: fraction of processor time spent in executing the task set (i.e., 1 - fraction of time processor is idle)
  - Provides a measure of computational load on CPU due to a task set
  - A task set is definitely not schedulable if its processor utilization is > 1
- For n tasks, τ_1, τ_2, … τ_n the utilization "U" is given by:

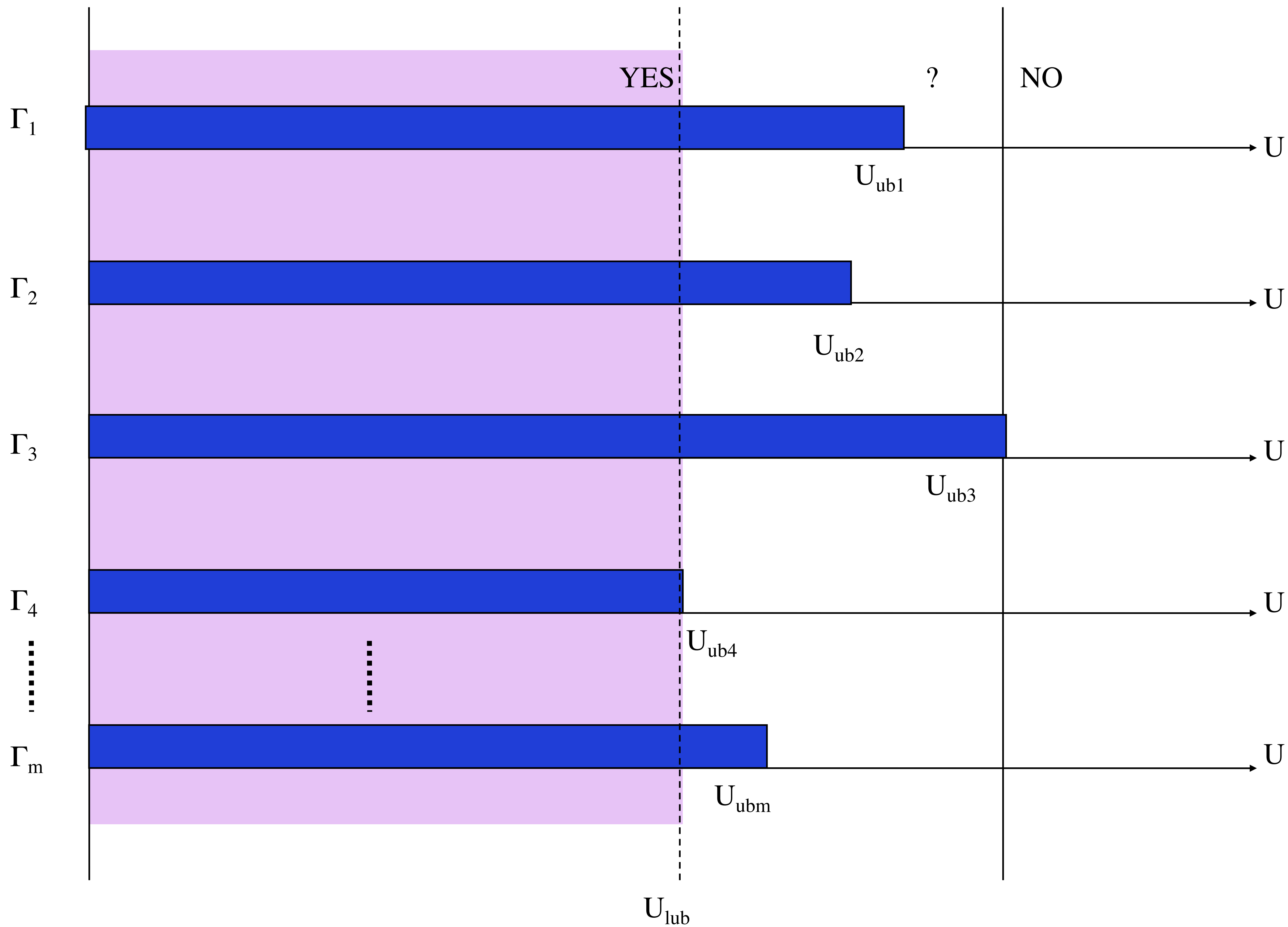
$$U = C_1/T_1 + C_2/T_2 + \ldots + C_n/T_n$$

- U for a task set Γ can be increased by increasing C_i's or by decreasing T_i's as long as tasks continue to satisfy their deadlines at their critical instants
- There exists a minimum value of U below which Γ is schedulable and above which it is not
  - Depends on scheduling algorithm and the task set

How Large can U be?

- For a given priority assignment, a task set fully utilizes a processor if:
  - The priority assignment is feasible for the task set (i.e., no deadline misses)
  - And, if an increase in the execution time of any task in the task set will make the priority assignment infeasible (i.e., cause a deadline miss)
- The U at which this happens is called the upper bound $U_{ub}(\Gamma,A)$ for a task set Γ under scheduling algorithm A
- The least upper bound of U is the minimum of the U's over all task sets that fully utilize the processor (i.e., $U_{lub}(A) = \min_{\Gamma} [U_{ub}(\Gamma,A)]$)
  - For all task sets whose U is below this bound, there exists a fixed priority assignment which is feasible
  - U above this can be achieved only if task periods Ti's are suitably related
- $U_{lub}(A)$ is an important characteristic of a scheduling algorithm A as it allows easy verification of the schedulability of a task set
  - Below this bound, a task set is definitely schedulable
  - Above this bound, it may or may not be schedulable

Least Upper Bound of U



Utilization Bound for RM

- RM priority assignment is optimal; therefore, for a given task set, the U achieved by RM priority assignment is ≥ the U for any other priority assignment

- In other words, the least upper bound of U = the infimum of U's for RM priority assignment over all possible T's and all C's for the tasks

Two Tasks Case

- Theorem 3: A set of two tasks is schedulable with fixed priority assignment if the processor utilization factor is $U \leq 2(2^{1/2}-1)$

- Given any task set consisting of only two tasks, if the utilization is less than 0.828, then the task set is schedulable using RM priority assignment

General Case

- Theorem 4: A set of n tasks with fixed priority assignment is schedulable if processor utilization factor $U \leq n(2^{1/n} - 1)$

- Equivalently, a set of "n" periodic tasks scheduled by the RM algorithm will always meet deadlines for all task start times if $C_1/T_1 + C_2/T_2 + \ldots + C_n/T_n \leq n(2^{1/n} - 1)$

General Case (contd.)

- As $n \rightarrow \infty$, the U rapidly converges to ln 2 = 0.69
- However, note that this is just the least upper bound
  - A task set with larger U may still be schedulable
  - e.g., if $(T_n \% T_i) = 0$ for i=1,2,…,n-1, then U=1

- How to check if a specific task set with n tasks is schedulable?
  - If $U \leq n(2^{1/n}-1)$ then it is schedulable
  - Otherwise, need to use Theorem 1!

- Two ways in which this analysis is useful
  - For a fixed CPU, will a set of tasks work or not? How much background load can you throw in without affecting feasibility of tasks?
  - During CPU design, you can decide how slow/fast a CPU you need

Theorem 1 Recalled

- Theorem 1: A critical instant for any task occurs whenever the task is requested simultaneously with requests of all higher priority tasks

- Can use this to determine whether a given priority assignment will yield a feasible scheduling algorithm
  - If requests for all tasks at their critical instants are fulfilled before their respective deadlines, then the scheduling algorithm is feasible

- Applicable to any static priority scheme... not just RM

Example #1

- Task τ_1 : $C_1 = 20$; $T_1 = 100$; $D_1 = 100$
  Task τ_2 : $C_2 = 30$; $T_2 = 145$; $D_2 = 145$
  Is this task set schedulable?

- $U = 20/100 + 30/145 = 0.41 \leq 2(2^{1/2}-1) = 0.828$

- Yes!

Example #2

- Task τ_1 : $C_1 = 20$; $T_1 = 100$; $D_1 = 100$
  Task τ_2 : $C_2 = 30$; $T_2 = 145$; $D_2 = 145$
  Task τ_3 : $C_3 = 68$; $T_3 = 150$; $D_3 = 150$
  Is this task set schedulable?

- $U = 20/100 + 30/145 + 68/150 = 0.86 > 3(2^{1/3}-1) = 0.779$

- Can't say! Need to apply Theorem 1

Example #2 revisited

- The utilization based test is only a sufficient condition. Can we obtain a stronger test (a necessary and sufficient condition) for schedulability?

- Task τ_1 : C_1 =20; T_1 =100; D_1 =100
  Task τ_2 : C_2 =30; T_2 =145; D_2 =145
  Task τ_3 : C_3 =68; T_3 =150; D_3 =150

- Consider the critical instant of τ_3, the lowest priority task
  - τ_1 and τ_2 must execute at least once before τ_3 can begin executing
  - Therefore, completion time of τ_3 is ≥ C1 +C2 +C3 = 20+68+30 = 118
  - However, τ_1 is initiated one additional time in (0,118)
  - Taking this into consideration, completion time of τ_3 = $2C_1 + C_2 + C_3$ = 2*20+68+30 = 138
- Since 138 < D_3 = 150, the task set is schedulable

Response Time Analysis for RM

- For the highest priority task, worst case response time R is its own computation time C
  - R = C

- Other lower priority tasks suffer interference from higher priority processes
  - $R_i = C_i + I_i$
  - I_i is the interference in the interval $[t, t+R_i]$

Response Time Analysis (contd.)

- Consider task i, and a higher priority task j
- Interference from task j during R_i:
  - # of releases of task j = $\lceil R_i/T_j \rceil$
  - Each release will consume C_j units of processor
  - Total interference from task j = $\lceil R_i/T_j \rceil * C_j$
- Let hp(i) be the set of tasks with priorities higher than that of task i
- Total interference to task i from all tasks during R_i:

$$I_i = \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

Response Time Analysis (contd.)

- This leads to:

$$R_i = C_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

- Smallest R_i that satisfies the above equation will be the worst case response time

- Fixed point equation: can be solved iteratively

$$w_i^{n+1} = C_i + \sum_{j \in hp(i)} \left\lceil \frac{w_i^n}{T_j} \right\rceil C_j$$

Algorithm


```
for i in 1..N loop -- for each process in turn
  n := 0
  w_i^n := C_i
  loop
    calculate new w_i^{n+1} from Equation
    if w_i^{n+1} = w_i^n then
      R_i := w_i^n
      exit {value found}
    end if
    if w_i^{n+1} > T_i then
      exit {value not found}
    end if
    n := n + 1
  end loop
end loop
```

Deadline Monotonic Assignment

- Relax the $D_i = T_i$ constraint to now consider $C_i \leq D_i \leq T_i$
- Priority of a task is inversely proportional to its relative deadline
  - $D_i < D_j \Rightarrow P_i > P_j$
- DM is optimal; Can schedule any task set that any other static priority assignment can
- Example: RM fails but DM succeeds for the following task set

	Period T	Deadline D	Comp Time, C	Priority P	Response Time, R
Task_1	20	5	3	4	3
Task_2	15	7	3	3	6
Task_3	10	10	4	2	10
Task_4	20	20	3	1	20

- Schedulability Analysis: One approach is to reduce task periods to relative deadlines
  - $C_1/D_1 + C_2/D_2 + \ldots + C_n/D_n \leq n(2^{1/n}-1)$
  - However, this is very pessimistic
- A better approach is to do critical instant (response time) analysis

Task Synchronization

- So far, we considered independent tasks

- In reality, tasks do interact: semaphores, locks, monitors, rendezvous, etc.
  - shared data, use of non-preemptable resources

- Jeopardizes systems ability to meet timing constraints
  - e.g., may lead to an indefinite period of “priority inversion” where a high priority task is prevented from executing by a low priority task

Priority Inversion Example

- Let τ_1 and τ_3 be two tasks that share a resource (protected by semaphore S), with τ_1 having a higher priority. Let τ_2 be an intermediate priority task that does not share any resource with either. Consider the following sequence of actions:

- τ_3 gets activated, obtains a lock on the semaphore S, and starts using the shared resource

- τ_1 becomes ready to run and preempts τ_3. While executing, τ_1 tries to use the shared resource by trying to lock S. But S is already locked and therefore τ_1 is blocked

- Now, τ_2 becomes ready to run. Since only τ_2 and τ_3 are ready to run, τ_2 preempts τ_3.

Priority Inversion Example (contd.)

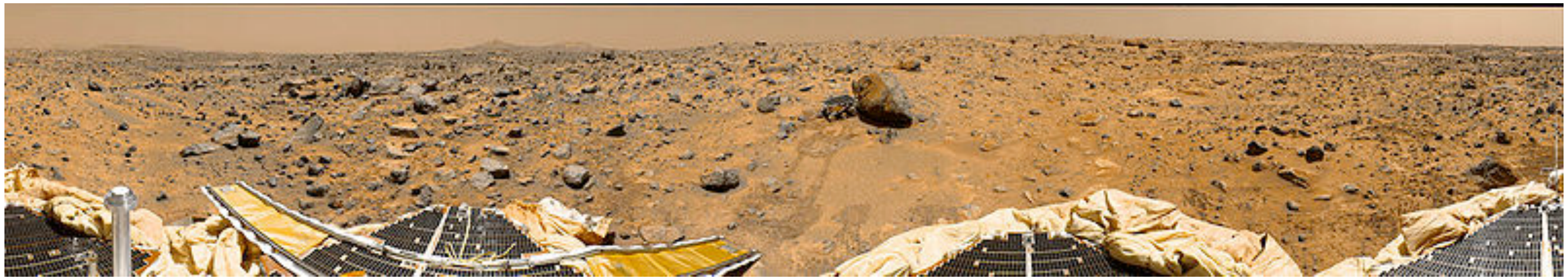
- What would we prefer?
  - τ_1, being the highest priority task, should be blocked no longer than the time τ_3 takes to complete its critical section

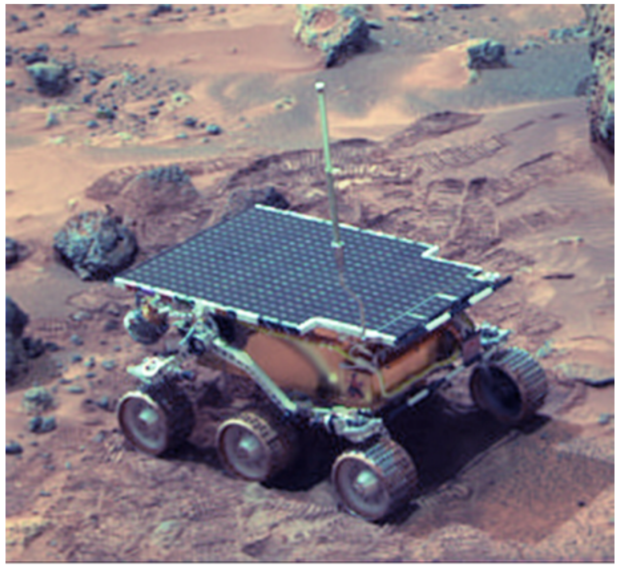
- But, in reality, the duration of blocking is unpredictable
  - τ_3 can remain preempted until τ_2 (and any other pending intermediate priority tasks) are completed

- The duration of priority inversion becomes a function of the task execution times, and is not bounded by the duration of critical sections

Just another theoretical problem?

- Recall the Mars Pathfinder from 1997?
  - Unconventional landing - bouncing onto Martian surface with airbags
  - Deploying the Sojourner rover: First roving probe on another planet
  - Gathering and transmitting voluminous data, including panoramic pictures that were such a hit: http://en.wikipedia.org/wiki/Mars_Pathfinder
  - Used VxWorks real-time kernel (preemptive, static-priority scheduling)
- But…
  - A few days into the mission, not long after Pathfinder started gathering meteorological data, the spacecraft began experiencing total system resets, each resulting in losses of data
  - Reported in the press as "software glitches" and "the computer was trying to do too many things at once"





What really happened on Mars?

- The failure was a priority inversion failure!

- A high priority task *bc_dist* was blocked by a much lower priority task *ASI/MET* which had grabbed a shared resource and was then preempted by a medium priority communications task

- The high priority *bc_dist* task didn't finish in time

- An even higher priority scheduling task, *bc_sched,* periodically creates transactions for the next bus cycle

- *bc_sched* checks whether *bc_dist* finished execution (hard deadline), and if not, resets the system

Why was it not caught before launch?

- The problem only manifested itself when *ASI/MET* data was being collected and intermediate tasks were heavily loaded
- Before launch, testing was limited to the "best case" data rates and science activities
- Did see the problem before launch but could not get it to repeat when they tried to track it down
  - Neither reproducible or explainable
  - Attributed to “hardware glitches”
  - Lower priority - focus was on the entry and landing software

What saved the day?

- How did they find the problem?
  - Trace/log facility + a replica on earth

- How did they fix it?
  - Changed the creation flags for the semaphore so as to enable "priority inheritance"
  - VxWorks supplies global configuration variables for parameters, such as the "options" parameter for the semMCreate used by the select service
    - Turns out that the Pathfinder code was such that this global change worked with minimal performance impact
  - Spacecraft code was patched: sent "diff"
    - Custom software on the spacecraft (with a whole bunch of validation) modified the onboard copy

Diagnosing the Problem

- Diagnosing the problem as a black box would have been impossible

- Only detailed traces of actual system behavior enabled the faulty execution sequence to be captured and identified

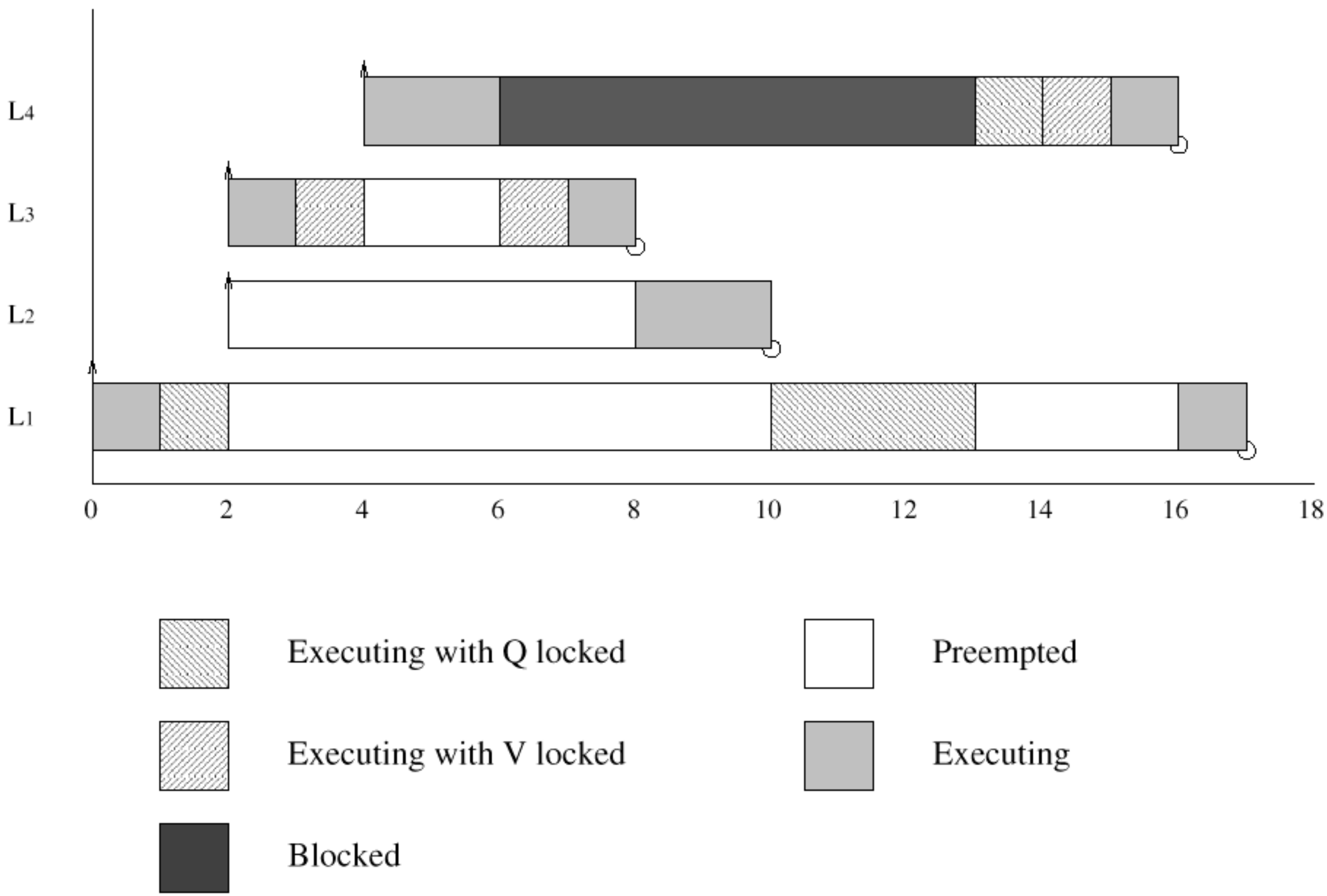
- See http://research.microsoft.com/en-us/um/people/mbj/mars_pathfinder/ for a description of how things were diagnosed and fixed

Process Interactions and Blocking

- Priority inversions
- Blocking
- Priority inheritance

Process	Priority	Execution Seq	Release Time
L_4	4	EEQVE	4
L_3	3	EVVE	2
L_2	2	EE	2
L_1	1	EQQQE	0

Example: Priority Inversion



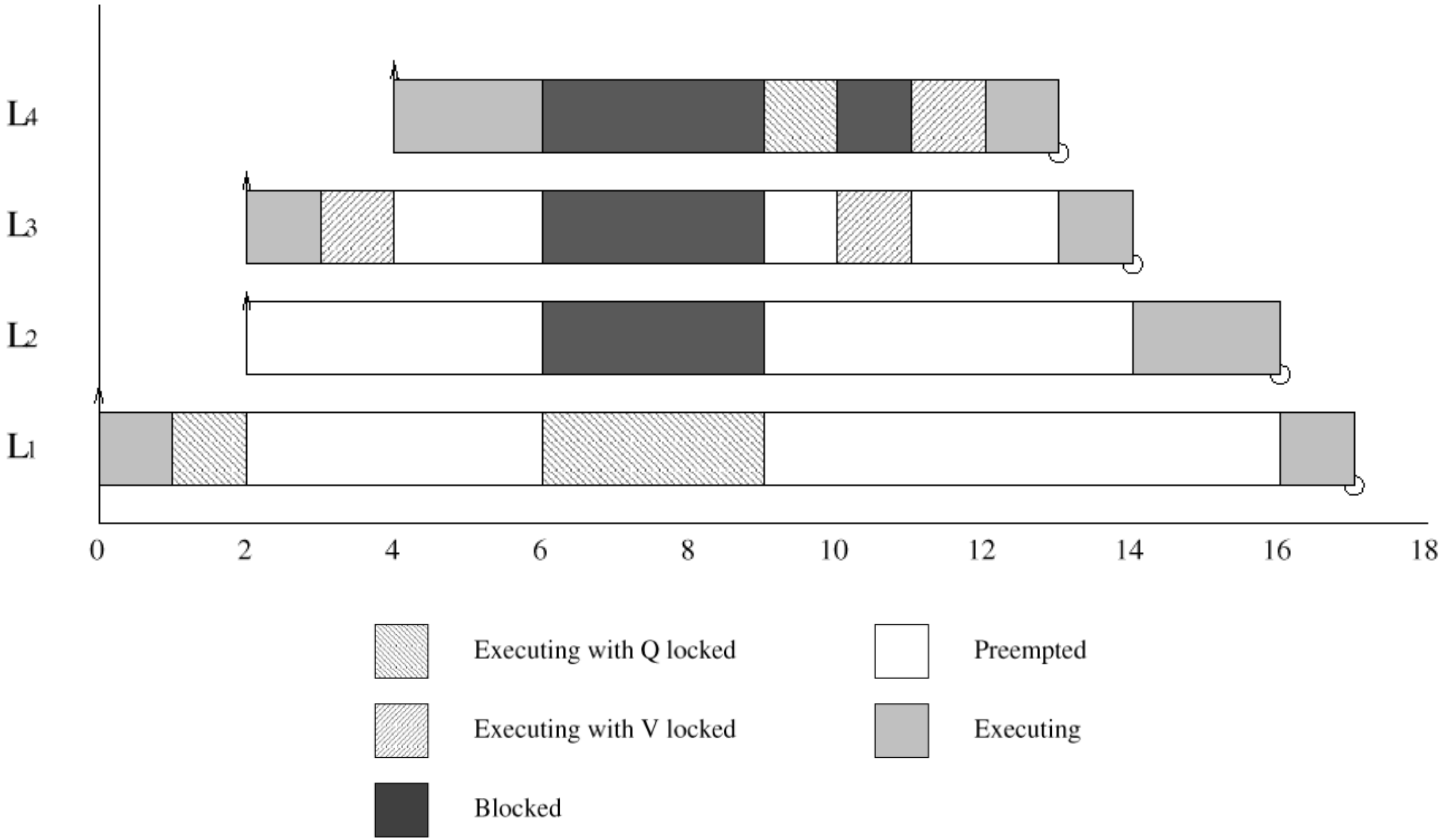
Priority Inheritance

- Simple method for eliminating priority inversion problems

- Basic Idea: If a high priority task H gets blocked while trying to lock a semaphore that has already been locked by a low priority task L, then L temporarily *inherits* the priority of H while it holds the lock to the semaphore
  - The moment L releases the semaphore lock, its priority drops back down

- Any intermediate priority task, I, will not preempt L because L will now be executing with a higher priority while holding the lock

Example: Priority Inheritance



Response Time Calculations

- R = C + B + I
  - solve by forming recurrence relation

- With priority inheritance:

$$R_i = C_i + B_i + \sum_{j \in hp(i)} \left\lceil \frac{R_i}{T_j} \right\rceil C_j$$

$$B_i = \sum_{k=1}^{K} usage(k, i) CS(k)$$

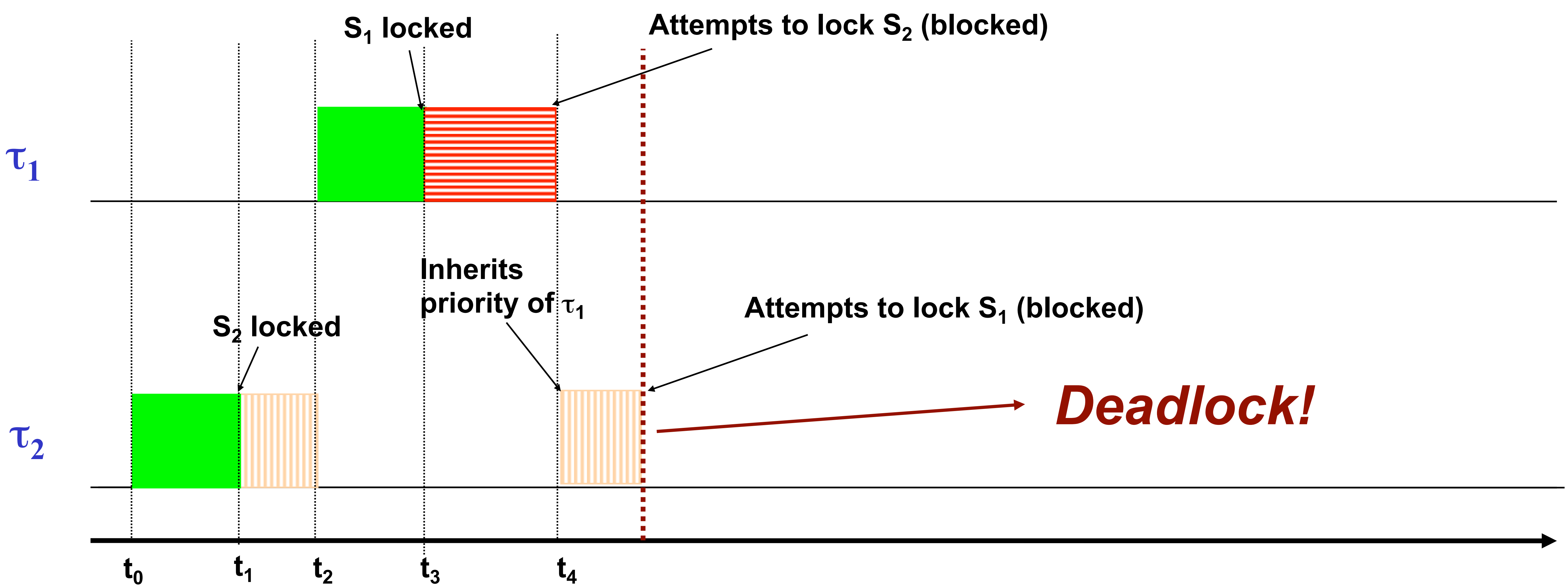
Response Time Calculations

- Where usage is a 0/1 function:
  - usage(k, i) = 1 if resource k is used by at least 1 process with priority < i, and at least one process with a priority greater or equal to i.
  - = 0 otherwise

- CS(k) is the computational cost of executing the critical section associated with resource k

Priority Inheritance Can Lead to Deadlock

- Two tasks τ_1 and τ_2 with two shared data structures protected by binary semaphores S_1 and S_2.
  - τ1: {… Lock(S_1)… Lock(S_2) … Unlock (S_2) … Unlock (S_1) … }
  - τ2: {… Lock(S_2)… Lock(S_1) … Unlock (S_1) … Unlock (S_2) … }
- Assume τ_1 has higher priority than τ_2



Priority Ceiling Protocols

- Basic idea:
  - Priority ceiling of a binary semaphore S is the highest priority of all tasks that may lock S
  - When a task τ attempts to lock a semaphore, it will be blocked unless its priority is > than the priority ceiling of all semaphores currently locked by tasks other than τ
  - If task τ is unable to enter its critical section for this reason, the task that holds the lock on its semaphore with the highest priority ceiling is
    - Said to be blocking τ
    - Hence, inherits the priority of τ

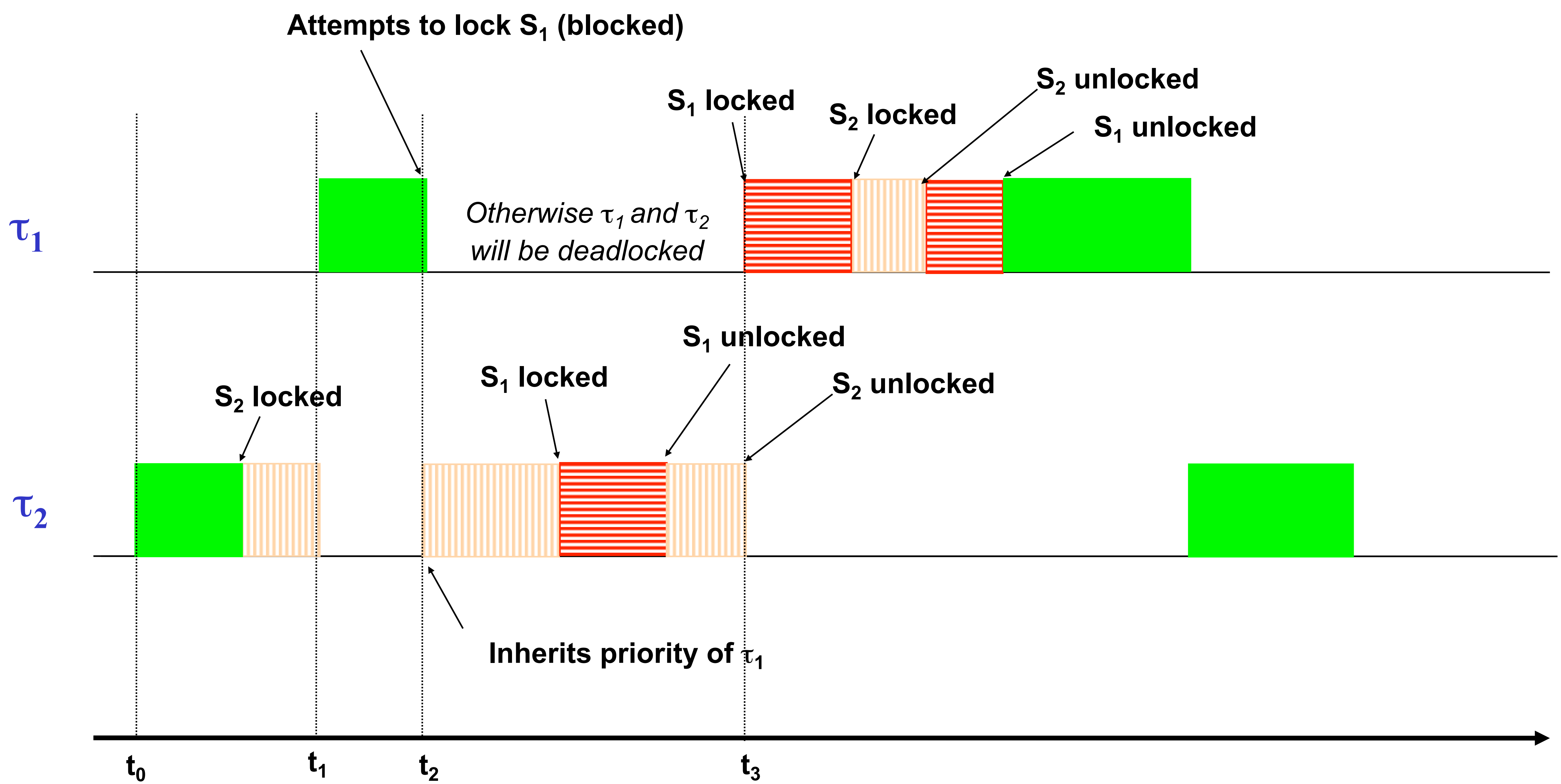
Example of Priority Ceiling Protocol

- Two tasks τ_1 and τ_2 with two shared data structures protected by binary semaphores S_1 and S_2.
  - τ_1: {… Lock(S_1)… Lock(S_2) … Unlock (S_2) … Unlock (S_1) … }
  - τ_2: {… Lock(S_2)… Lock(S_1) … Unlock (S_1) … Unlock (S_2) … }

- Assume τ_1 has higher priority than τ_2

- Note: priority ceilings of both S_1 and S_2 = priority of τ_1

Example of Priority Ceiling Protocol



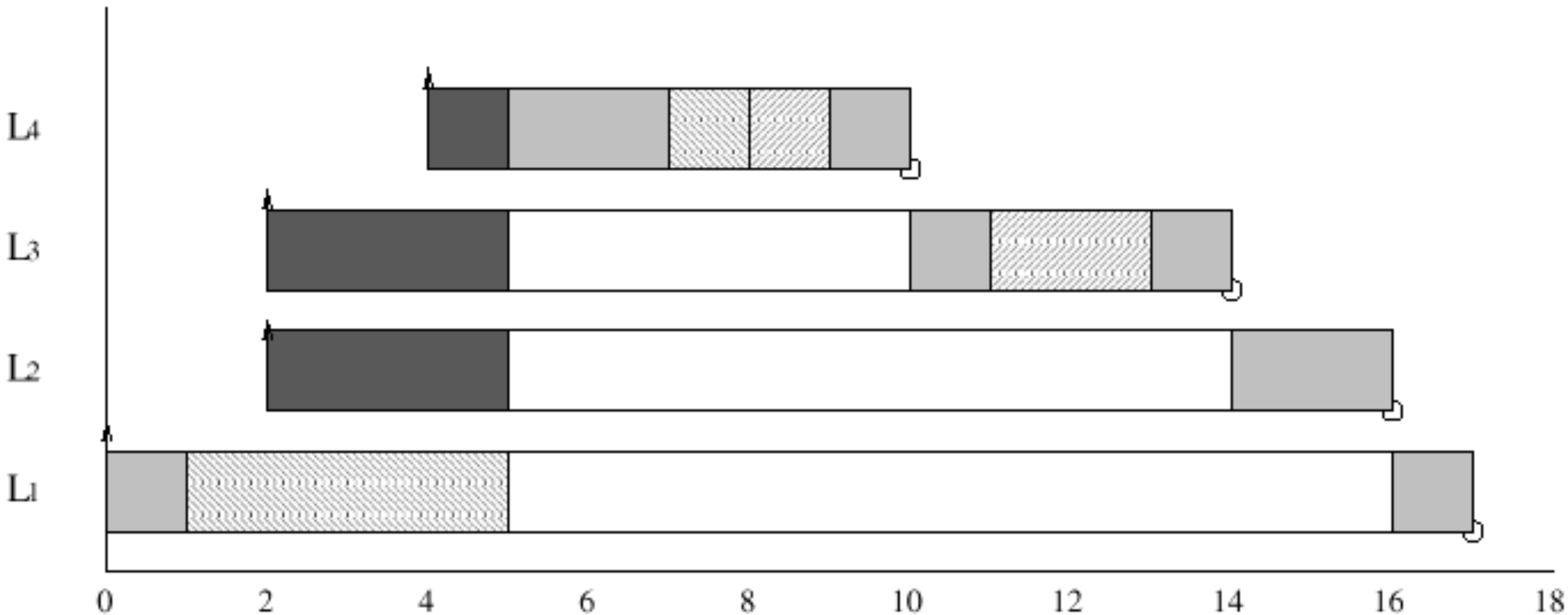
Priority Ceiling Protocols (contd.)

- Two forms
  - Original ceiling priority protocol (OCPP)
  - Immediate ceiling priority protocol (ICPP)
- On a single processor system
  - A high priority process can be blocked at most once during its execution by lower priority processes
  - Deadlocks are prevented
  - Transitive blocking is prevented
  - Mutual exclusive access to resources is ensured (by the protocol itself)

ICPP

- Each process has a static default priority assigned (perhaps by the deadline monotonic scheme)
- Each resource has a static ceiling value defined, this is the maximum priority of the processes that use it
- A process has a dynamic priority that is the maximum of its own static priority and the ceiling values of any resources it has locked.

Example of ICPP

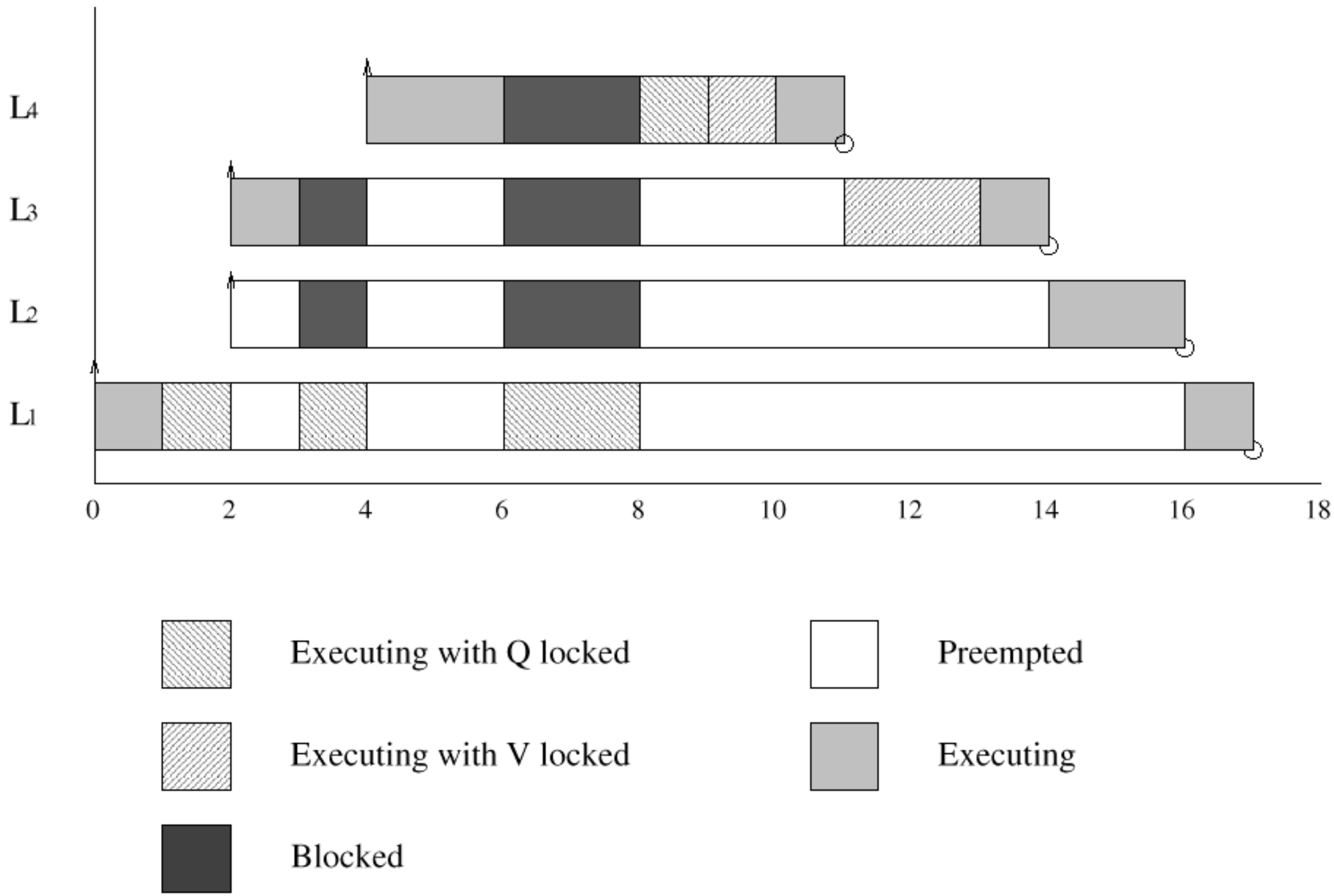


OCPP

- Each process has a static default priority assigned (perhaps by the deadline monotonic scheme)
- Each resource has a static ceiling value defined, this is the maximum priority of the processes that use it
- A process has a dynamic priority that is the maximum of its own static priority and any it inherits due to it blocking higher priority processes
- A process can only lock a resource if its dynamic priority is higher than the ceiling of any currently locked resource (excluding any that it has already locked itself).

$$B_i = \max_{k=1}^{K} usage(k,i)CS(k)$$

Example of OCPP



OCPP vs. ICPP

- Worst case behavior identical from a scheduling point of view
- ICCP is easier to implement than the original (OCPP) as blocking relationships need not be monitored
- ICPP leads to less context switches as blocking is prior to first execution
- ICPP requires more priority movements as this happens with all resource usages; OCPP only changes priority if an actual block has occurred.

Schedulability Impact of Task Synchronization

- Let B_i be the duration in which τ_i is blocked by lower priority tasks

- The effect of this blocking can be modeled as if τ_i's utilization were increased by an amount B_i/T_i
- The effect of having a deadline D_i before the end of the period T_i can also be modeled as if the task were blocked for $E_i=(T_i-D_i)$ by lower priority tasks
  - As if utilization increased by E_i/T_i

- Theorem: A set of n periodic tasks scheduled by RM algorithm will always meet its deadlines if:

$$i, 1 \leq i \leq n, \frac{C_1}{T_1} + \frac{C_2}{T_2} + \ldots + + \frac{C_i + B_i + E_i}{T_i} \leq i(2^{1/i} - 1)$$

Arbitrary Deadlines

- Case when deadline $D_i < T_i$ is easy…
- Case when deadline $D_i > T_i$ is much harder
  - Multiple iterations of the same task may be alive simultaneously
  - May have to check multiple task initiations to obtain the worst case response time
- Example: consider two tasks
  - Task 1: $C_1 = 28$, $T_1 = 80$
  - Task 2: $C_2 = 71$, $T_2 = 110$
  - Assume all deadlines to be ∞

Arbitrary Deadlines (contd.)

- Response time for task 2:

Task 1: $C_1 = 28$, $T_1 = 80$
Task 2: $C_2 = 71$, $T_2 = 110$

Initiation	Completion time	Response time
0	127	127
110	226	116
220	353	133
330	452	122
440	551	111
550	678	128
660	777	117
770	876	106

- Response time is maximum not for the first initiation of the task!
  - Not sufficient to consider just the first iteration
  - Theorem 1 (critical instant definition) no longer holds

Schedulability for Arbitrary Deadlines

- Analysis for situations where D_i (and hence potentially R_i) can be greater than T_i

$$w_i^{n+1}(q) \;=\; B_i + (q+1)C_i + \sum_{j \in hp(i)} \left\lceil \frac{w_i^n(q)}{T_j} \right\rceil C_j$$

- The number of releases that need to be considered is bounded by the lowest value of q = 0,1,2,... for which the following relation is true:

$$\begin{aligned} R_i(q) \;&=\; w_i^n(q) \;-\; qT_i \\ R_i(q) \;&\leq\; T_i \end{aligned}$$

Arbitrary Deadlines (contd.)

- The worst-case response time is then the maximum value found for each q:

$$R_i = \max_{q=0,1,2,\ldots} R_i(q)$$

- Note: for D ≤ T, the relation $R_i(q) \leq T_i$ is true for q=0 if the task can be scheduled, in which case the analysis simplifies to original
  - If any R>D, the task is not schedulable

Dynamic Priority Scheduling

- With dynamic-priority scheduling, priorities are assigned to individual instances (jobs) of a task

- One of the most used algorithms of this class is EDF, or Earliest Deadline First priority assignment
  - Priorities assigned to tasks are inversely proportional to absolute deadlines of active jobs
  - It is optimal among all preemptive scheduling algorithms
    - If there exists a feasible schedule, then schedule given by EDF is also feasible

- Another optimal algorithm: Least Laxity First (LLF)
  - Assigns processor to the active task with smallest laxity
  - Larger overhead than EDF due to higher number of context switches
    - Less studied than EDF due to this reason

Preemptive Earliest Deadline First

- Processor executes the task whose absolute deadline is the earliest
- Priorities change with the closeness of a task to its absolute deadline
- Example:

Task	Arrival Time	Execution Time	Absolute Deadline
T1	0	10	30
T2	4	3	10
T3	5	10	25

EDF Schedulability

- Shown to be optimal for single processor
  - If EDF cannot schedule a task set on a single processor, then no other scheduling algorithm can

- Simple schedulability test if tasks are periodic, and have relative deadlines greater than or equal to their time periods
  - If the total utilization U of the task set is no greater than 1, the task set can be feasibly scheduled by EDF on a single processor

- If the relative deadlines are less than the time periods, there is no simple schedulability test
  - One will have to develop a schedule using EDF to see whether all the deadlines are met over a given time interval

EDF Schedulability (D == T case)

- Theorem: Suppose we have a set of n periodic tasks (Ti, Ci, Di) , each of whose relative deadline Di equals its period Ti. The tasks can be feasibly scheduled by EDF on a single processor iff:

$$C_1/T_1 + C_2/T_2 + \ldots + C_n/T_n \leq 1$$

Summary

- Real-Time Scheduling: Orchestrating the execution of multiple tasks (processes) so that timing constraints of tasks are satisfied

- Preemptive, priority based scheduling if the most commonly used approach
  - Static priority assignment: Rate Monotonic, Deadline Monotonic
  - Dynamic priority assignment: Earliest Deadline First

- Key results on schedulability exist for both RM and EDF. These results allow us to model and analyze an embedded software system to understand its feasibility before actually building it

- Several real-world effects (shared resources leading to priority inversion, etc.) can also be accounted for