Lec14-mwf

Thursday, February 1, 2018 2:59 PM

Model - 20min

Thursday, January 17, 2008 4:44 PM

To make progress, let us consider a smaller problem:

What is the optimal capacity of a wireless system, and how to achieve it?

To simplify, let us assume

- each source/destination has a single path through the network, and the path is given

  A source/destination pair is called a flow.

  Each flow $s = 1, 2, \cdots, S$ has one path that consists of a subset of the links

$$\begin{cases} H_s^l = 1 & \text{if the path of flow } s \text{ passes through link } l \\ H_s^l = 0, & \text{otherwise.} \end{cases}$$

- Assume a set of possible transmission scheme is chosen, and therefore each node can only choose its action among a set.

  Such actions can be time-varying.

- Let $p_l(t)$ denote the action chosen by the transmitting node of link l. Let $\vec{p}(t) = [p_1(t), \cdots p_L(t)]$

  Let Π denote the set of all feasible $\vec{p}(t)$.

- Once the action $\vec{P}(t)$ is chosen, certain service rates for all links can be determined.

  Let $r_l(t)$ = rate of link l

  $$\vec{r}(t) = [r_1, \cdots, r_L]$$ ← rate-power function

  Due to interference across links, in general $r_l(t)$ not only depends on $p_l(t)$, but also the action chosen by other links.

  Further, it depends on the time-varying channel condition.

  $$\vec{r}(t) = g(\vec{p}(t), K(t))$$

  global action vector — $\vec{p}(t)$

  $K(t)$ — denote the global channel state at time t.

  The function g is assumed to be given, and it is determined by the coding/modulation/transmission scheme that is chosen.

This is a very general model.

① Collision channel.

All nodes share a common frequency band.

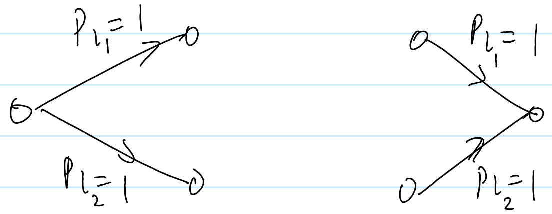
The action of a node is either to transmit ($p_l = 1$), or not to transmit ($p_l = 0$).

Achieve a ~~constant~~ rate if no collision.

<u>Example A:</u>

A node cannot transmit to two other nodes at the same time. Neither can a node receive from two other nodes at the same time.

- Primary conflicts



- e.g. if a unique spreading code is used on each link (e.g. in Bluetooth), but one radio per node.
- Determines the set Ⓗ

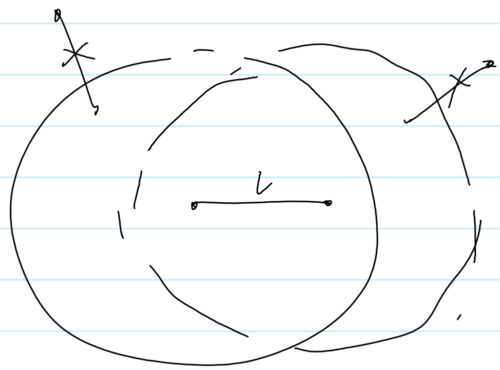
Example B:

Each link l has a set of interfering links. I_l

$r_l = 0$ if any of the interfering links is transmitting

r_l = Constant , otherwise and
if $p_l = 1$

- Example A as a special case.
- Each link interferes with other nodes in a certain radius (e.g.. 802.11).



② SINR-based Model

$$SINR_{ij} = \frac{P_i \, g_{ij}}{\sum_{k \neq i} P_k \cdot g_{kj} + n_0}$$

$SINR_{ij}$: SINR level at the link from node i to node j

P_{ij} : transmission power on this link

g_{ij} : Path loss

n_0 : background noise

- $r_{ij} = R$ if $SINR_{ij} > \gamma$
- Or, allowing adaptive coding/modulation Shannon bound:

$$r_{ij} = B \log_2 (1 + SINR_{ij})$$

- Or, CDMA:

– SINR is small (e.g. in CDMA systems with moderate gains)

$$r_{ij} \approx B \cdot SINR_{ij}$$

$$= B \cdot \frac{p_i g_{ij}}{\sum_k p_k g_{kj} + n_0} \quad \text{before de-spreading}$$

(For example, if $n_0 = 0$, CDMA maintains a certain target SINR $\hat{\gamma}$ after de-spreading to get a symbol rate R_s

$$\frac{p_i g_{ij}}{\frac{1}{W}\sum_k p_k g_{kj}} = \gamma \Rightarrow W = \frac{\gamma}{\frac{p_i g_{ij}}{\sum_k p_k g_{kj}}}$$

$$\Rightarrow \text{rate} = \frac{1}{W} \cdot R_s = \frac{R_s}{\gamma} \frac{p_i g_{ij}}{\sum_k p_k g_{kj}}$$)

(H) corresponds to the power constraints.

Time-varying channel gains can be incorporated into such SINR models easily.

For simplicity, assume

$K(t)$ takes one of the values in $1, \cdots, K$

$\pi_k : P\{K(t) = k\}$

(3) FDMA / OFDM
Each link's action is a vector over all channels / sub-carriers

(4) MIMO

(50) (4) MIMO

Each link's action is a vector over all transmitting antennas.

More model - 5min

Sunday, January 27, 2008 10:54 AM

In summary: $\vec{r}(t) = g(\vec{P}(t), K(t))$

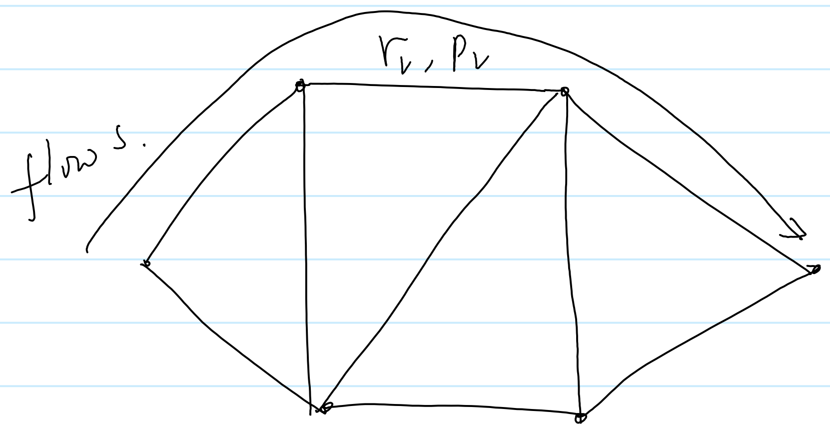
Θ : set of feasible actions.

$\pi_K : P\{K(t) = K\}$

We assume that the channel state is i.i.d over time

We assume that the set $\{g(\vec{P}), \vec{P} \in \Theta\}$ is bounded.

Go back to flows



Recall that

there are S flows in the system.
Each flow has a path through the network.

Let $H_s^l = 1$ is flow s uses link l

$= 0$ otherwise.

Let X_s = average rate of flow l

- Arrivals could be random

$$E[A_s(t)] = X_s$$

The offered load on link l is then

$$\sum_s H_s^l X_s$$

Note that for cellular systems, each flow is associated with a particular link.

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The capacity problem - 15min

Sunday, January 27, 2008 11:01 AM

Q What is the largest set of $\vec{X} = [X_s]$ that the network can support?

How to choose the actions of the links to achieve this capacity?

A We will show that the set of possible $\vec{X}$ must satisfy

$$\left[\sum_s H_s^l X_s \right] \in \sum_k \lambda_k \, \text{Conv\_hull} \{ g(\vec{p}, k) \mid \vec{p} \in H \}$$

Λ : capacity region

and we will develop a scheduling algorithm that can compute the action of the links for any given $\vec{X}$ within this set Λ. and does not require the knowledge of λ_k.

Capacity Region

Due to interference, typically not all links can be active at the same time.

To support the end-to-end rate $\vec{x}$, it is preferable to use a set of actions alternatively.

Suppose at each channel state K, the network can use

$$\vec{P}_K^1, \vec{P}_K^2, \cdots \vec{P}_K^M \in \textcircled{H}$$

actions, each for

$$\alpha_K^1, \alpha_K^2 \cdots \alpha_K^M$$

fraction of time, where $\sum_{m=1}^{M} \alpha_K^m = 1$ for all K.

The long-term rate that can be supported at each link l is then

$$\sum_K \lambda_K \sum_m \alpha_K^m g_l(\vec{P}_K^m, k)$$

Using these actions, the network can support the offered load $\vec{x} = [x_s]$ as long as the above quantity is larger than $\sum_s H_s^l x_s$

$$\Rightarrow \quad \left[\sum_s H_s^l x_s\right] \leq \sum_K \lambda_K \sum_m \alpha_K^m g(\vec{P}_K^m, k) \qquad (*)$$

Conversely, to support an offered load $\vec{x}$, there must exist

$$\vec{p}_k^{1}, \cdots \vec{p}_k^{m} \in \Theta, \sum_{m=1}^{M} \alpha_k^m = 1 \quad \text{for each } k$$

such that (*) is true.

Note that when we vary $\vec{p}, \vec{\alpha}$, the inner summation of the RHS of (*) forms the convex-hull of

$$\{ g(\vec{p}, k), \ \vec{p} \in \Theta \}$$

Hence, the offered load $\vec{x}$ that can be supported by the network must belong to

$$\Lambda = \sum_k \pi_k \ \text{Conv\_hull} \{ \vec{g}(\vec{p}, k) \mid \vec{p} \in \Theta \}.$$

If we know $\vec{x}$, π_k, and $\vec{g}(\cdot, \cdot)$, we may be able to find the actions $\vec{p}_k^{1}, \cdots \vec{p}_k^{M}$ and fractions $\alpha_k^1, \cdots \alpha_k^m$ offline. However,

- Prior knowledge of $\vec{x}$ & π_k may not be available
- or it can be inaccurate.

(Q) Can we develop an adaptive scheduling scheme that do not require a-prior knowledge of $\vec{x}$ & π_k, yet is able to support any $\vec{x} \in \Omega$?

(A) Yes. We will present such a throughput-optimal scheme that is queue-length based

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Convex hulls - 15min

Friday, March 27, 2009 9:30 AM

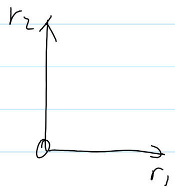
<u>Example:</u>

- One BS
- Two users
- BS can only transmit to one user at a time
- Four states

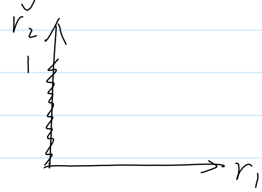
  (0, 0) (0, 1) (1, 0) (1, 1)

  ON for user 1   OFF for user 2

- The convex-hull for each state

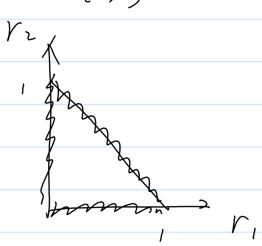


(0, 0)

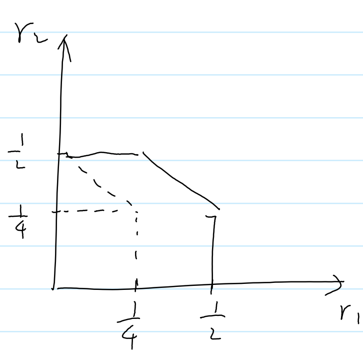


(0, 1)

(1, 0)



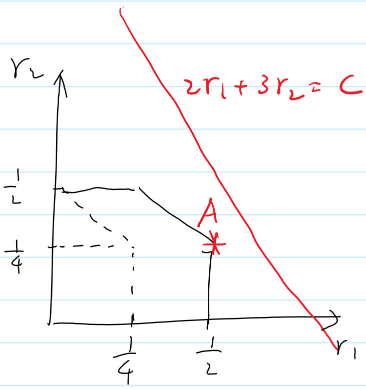
- Suppose $p_{00} = p_{01} = p_{10} = p_{11} = \frac{1}{4}$.
- What does the overall capacity region looks like.



Maximizing a weighted-sum

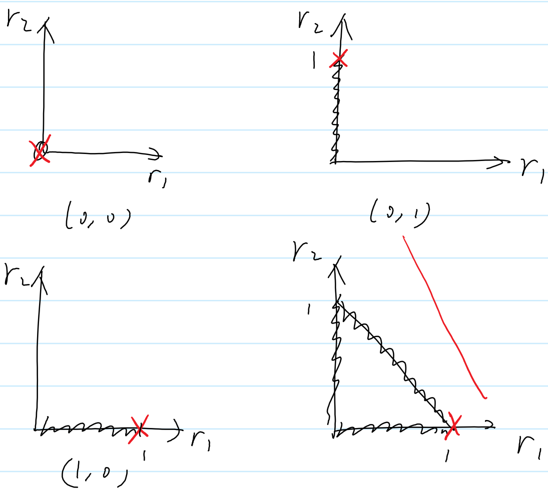
- Over this capacity region, suppose we want to maximize a weighted-sum

$$2r_1 + 3r_2$$



Three important facts

① The point A can be seen as the average of the maximizing rate vector at each channel state



$$(0,0)\times\frac{1}{4} + (0,1)\times\frac{1}{4} + (1,0)\times\frac{1}{4} + (1,0)\times\frac{1}{4} = \left(\frac{1}{2}, \frac{1}{4}\right)$$

- In general,

$$\max \sum_i w_i r_i$$

sub to $\quad [r_l] \in \sum_k \lambda_k \, \text{Conv-hull}\{ g(\vec{p}, k) \mid \vec{p} \in \Phi \}$

$$= \quad \sum_k \lambda_k \cdot \left\{ \begin{array}{l} \max \sum_l w_l r_l \\ \text{sub to } [r_l] \in \text{Conv-hull}\{ g(\vec{p}, k) \mid \vec{p} \in \Phi \} \end{array} \right.$$

② The maximizer of a weighted-sum over a convex hull always occurs at the extreme points

$$\max \sum_l w_l r_l$$
$$\text{sub to} \quad [r_l] \in \text{Conv-hull}\{ g(\vec{p}, k) \mid \vec{p} \in \Phi \}$$

$$= \quad \max \sum_l w_l r_l$$
$$\text{sub to} \quad [r_l] \in \{ g(\vec{p}, k) \mid \vec{p} \in \Phi \}$$

③ For any $[r_l^*] \in \Lambda$

$$\sum_l w_l r_l^* \leq \quad \begin{array}{l} \max \sum_l w_l r_l \\ \text{sub to} \quad [r_l] \in \Lambda \end{array}$$