Lec12

Saturday, January 27, 2018 10:22 AM

Cell sizing - 10min

Tuesday, February 19, 2008 4:41 PM

The reuse factor C immediately determines the # of available channels in each cell.

The size of the cell should then depend on the traffic intensity in the region.

Performance measures of interest are: call blocking probability, delay, etc

Simple One-cell Model without Handoff

Assumptions:

① Calls arrive to a cell according to a Poisson process with rate λ.

– which in turn depends on the traffic density per area and the size of cell.

– For very small interval Δt, prob. of one arrival is $\lambda \Delta t$. prob. of two or more arrival is $o(\Delta t)$.

② Call holding time is exponentially distributed with mean $1/\mu$

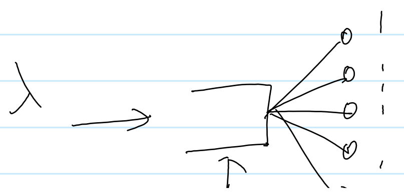
– $\mu \Delta t$.

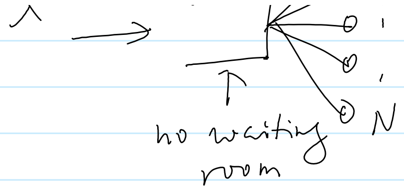
– $P\{\tau \geq t\} = e^{-\mu t}$

③ N channels in a cell. A call is immediately dropped if all N channels are in use.

④ No handoff. Calls complete in the same cell.

$\Rightarrow$ Simple M/M/N/N N-server queue with no waiting room.



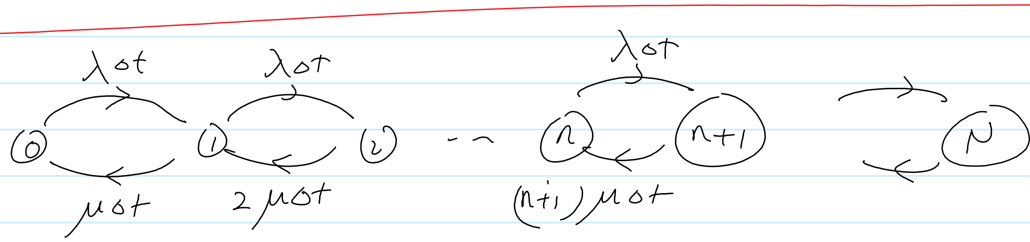


Traffic intensity (or offered load)

$$\rho = \frac{\lambda}{\mu} \quad \text{(in Erlangs)}$$

Probability of blocking is given by the Erlang-B formula

$$P_B = \frac{\rho^N / N!}{\sum_{n=0}^{N} \rho^n / n!}$$



$$P\{n \to n+1\} \simeq P_n \cdot \lambda \Delta t$$

$$P\{n+1 \to n\} \simeq P_{n+1} \cdot (n+1)\Delta t$$

must be equal

$$\Rightarrow \quad P_n \cdot \lambda = P_{n+1}(n+1)\mu \quad \text{(local balance eqn).}$$

$$\Rightarrow \quad P_{n+1} = \frac{\rho}{n+1} P_n$$

$$P_n = \frac{\rho}{n} P_{n-1} = \frac{\rho^2}{n \cdot (n-1)} P_{n-2} = \cdots = \frac{\rho^n}{n!} P_0$$

$$\Rightarrow \quad 1 = \sum_{n=0}^{N} P_i = P_0 \left[\sum_{n=0}^{N} \frac{\rho^n}{n!} \right] \qquad P_0 = \frac{1}{\sum_{n=0}^{N} \frac{\rho^n}{n!}}$$

$$\Rightarrow \quad \text{Loss prob. } P_N = \frac{\rho^N}{N!} \cdot P_0 \rightarrow \text{Erlang-B formula}$$

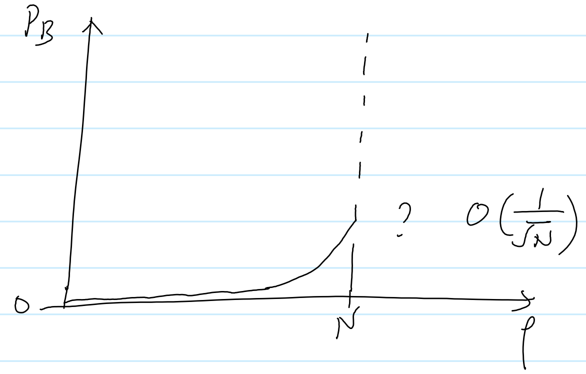
Example 1:

$N = 100$ channels, $\rho = 84$ Erlang

$\Rightarrow P_B = 1\%$

$N = 100$ $\rho = 95$ Erlangs

$\Rightarrow P_B = 5\%$



Example 2:

A call lasts 200 seconds on average.
A user makes a call every 15 minutes, on average.
$N = 100$, desired $P_B = 1\%$
How many users can the cell accommodate?

(A)

$$1/\mu = 200$$

$$\lambda = \frac{n}{15 \times 60}$$

$$\rho = \frac{\lambda}{\mu} = \frac{2}{9} n$$

At $N = 100$, $P_B = 1\% \Rightarrow \rho = 84$

$\Rightarrow n = 378$ users.

If user density is 2 terminals per km^2
the cell can cover area

$$\frac{378}{2} = 189 \text{ km}^2$$

$\Rightarrow$ radius $= 7.75$ km

If user density is 1000 terminals per km^2
the cell can cover area

$$\frac{378}{1000} = 0.378\ km^2$$

$\Rightarrow$ radius $= 0.35\ km$.

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Handoff - 15min

Thursday, February 21, 2008 4:08 PM

When a user moves to a neighboring cell in the middle of a call, the call must be seamlessly transfered to the neighboring cell.

The neighboring cell must have a channel available for the handoff call.

Model for Handoff

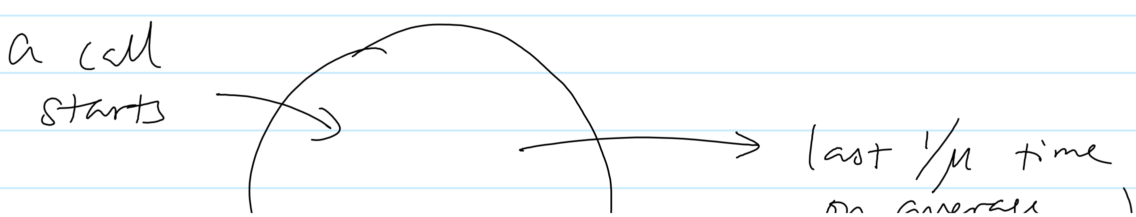
T_n : call holding time = the amount of time a call lasts

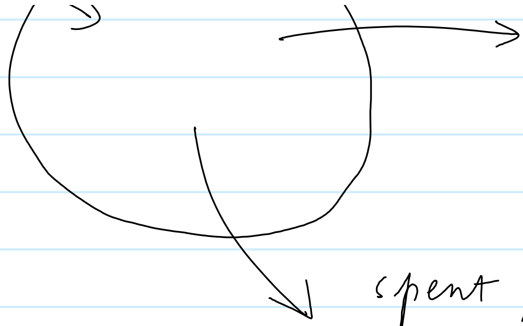
T_H : dwelling time = the amount of time a user (a call) spent in a given cell, before hand-off

T_C : channel holding time = the amount of time a channel is held (or occupied) in a given cell.

$$T_C = \min(T_n, T_H)$$

Assume: T_n is an exponentially distributed r.v. with mean $1/\mu$. T_H is an exponentially distributed r.v., with mean $1/\eta$!



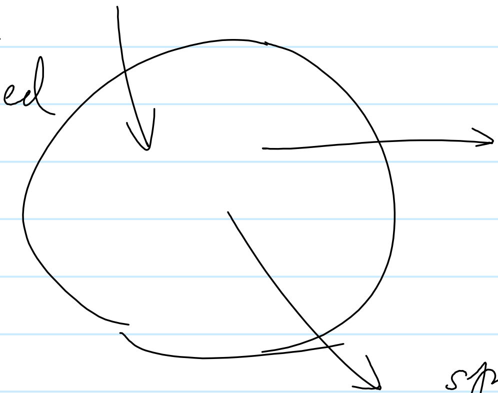


last $1/\mu$ time on average)
(and completes

spent $1/\eta$ time on average
and handoff to the next
cell.

(e) What happens at the next cell?

A call
handoffed



exp. dist with mean
$1/\mu$

last ? time
(and completes)

spent ? time and
handoff to the next
cell.

exp. dist. with
mean $1/\eta$

Exponential distribution has the memoryless property: The prob. of a call ending at a given time in the future is always the same, no matter how long the call has already lasted.

$$P\{X \geq t+a \mid X \geq a\} = P\{X \geq t\} = e^{-\mu t}$$

See Schwartz p262: how values of η change with the vehicle speed.

Once μ, η are given

The channel holding time $T_c = \min(T_n, T_H)$

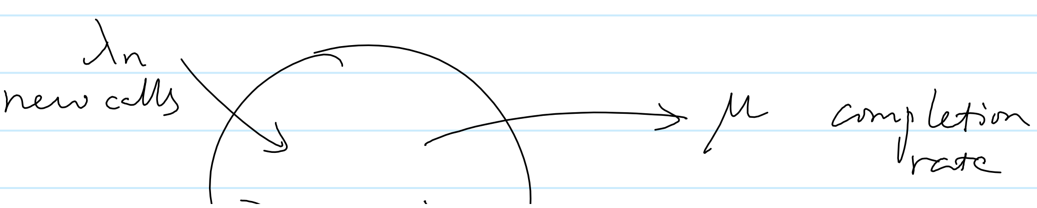
$\Rightarrow T_c$ is exp. distributed with mean $\frac{1}{\mu+\eta}$

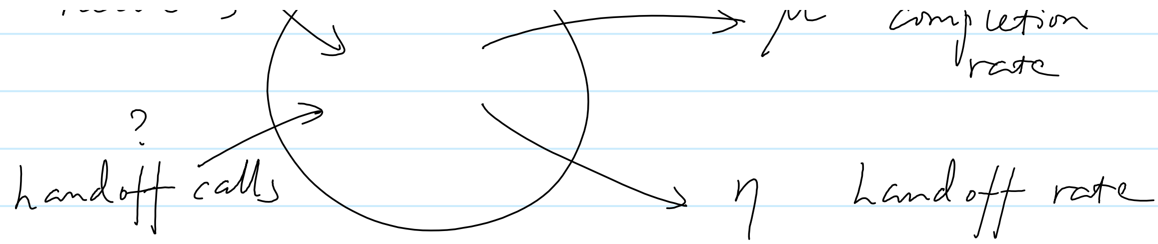
Prob. of handoff P_h

$= P_r\{T_H \leq T_n\}$

$= \frac{\eta}{\eta+\mu}$

Assume that new calls are generated in a given cell according to a Poisson process with rate λ_n.





Is it true?

Assume external handoff calls arrive according to a Poisson process with rate λ_H

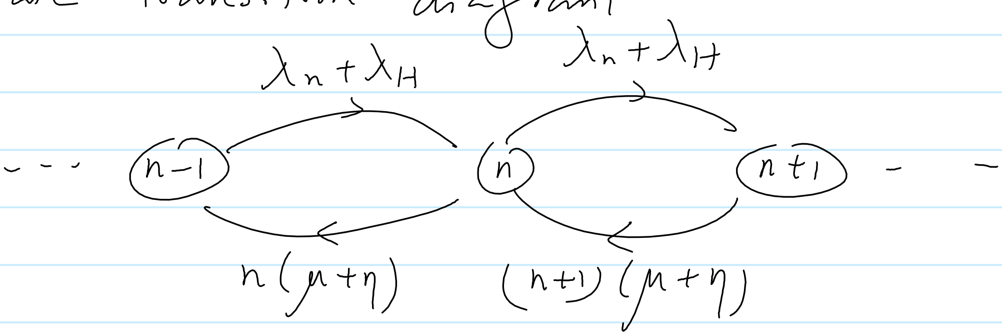
See Schwartz p264: how handoff rates are determined from μ, η, λ_n

- Assume prob. of blocking is small
- Assume uniform mobile density at all cells.

$$(\lambda_n + \lambda_H) \cdot P_h = \lambda_H$$

$$\Rightarrow \lambda_H = \frac{P_h}{1 - P_h} \lambda_n = \frac{\eta}{\mu} \lambda_n .$$

In this case, we can write down the state-transition diagram



Same state-transition diagram as M/M/N/N.
Erlang-B formula applies

$\Rightarrow$ prob. of new call blocking $= P_N$

prob. of handoff call blocking $= P_N$

Same treatment of handoff calls as new calls!

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Channel reservation - 10min

Thursday, February 21, 2008 4:29 PM

In practice, we want handoff calls receive a smaller amount of blocking.

- Users find calls blocked in mid-progress a far greater irritation than unsuccessful call attempts

Channel Reservation

Reserve a certain portion of the total channel pool in a cell for handoff uses only.

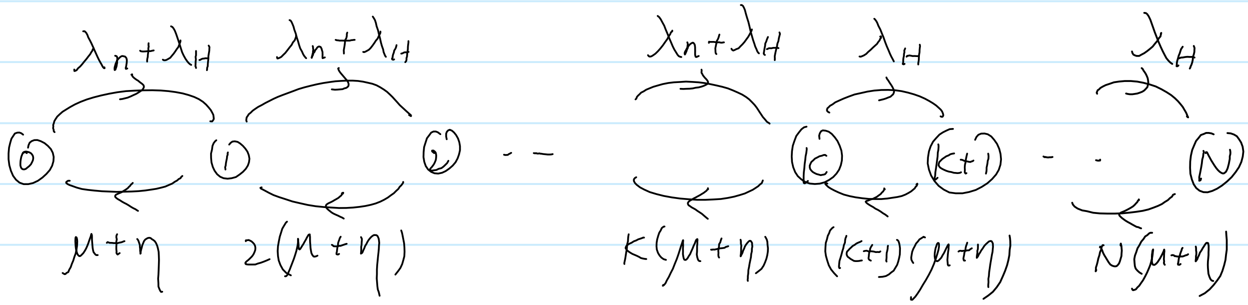
N : total # of channels
K : # of channels accessible by new calls
$K \le N$

All channels can be accessed by handoff calls.

When $n < K$, both new calls & handoff calls are accepted.

When $K \le n < N$, only handoff calls are accepted

When $n = N$, neither new calls nor handoff calls are accepted.



Let $\Omega = \lambda_n + \lambda_H$

$\mu_c = \mu + \eta$

Write down the balance equations

$$\begin{cases} P_{n-1}\Omega = P_n \cdot \mu_c \cdot n & \text{when } n \le K \\ P_{n-1}\lambda_H = P_n \cdot \mu_c \cdot n & n > K \end{cases}$$

$$\Rightarrow \quad P_n = \begin{cases} P_0 \left(\frac{\Omega}{\mu_c}\right)^n \frac{1}{n!} & \text{when } n \le K \\ P_0 \left(\frac{\Omega}{\mu_c}\right)^K \left(\frac{\lambda_H}{\mu_c}\right)^{n-K} \cdot \frac{1}{n!} & \text{when } n > K \end{cases}$$

Using $\sum_{n=0}^{N} P_n = 1$, solve P_0

$$P_0 = \frac{1}{\sum_{n=0}^{K} \left(\frac{\Omega}{\mu_c}\right)^n \frac{1}{n!} + \sum_{n=K+1}^{N} \left(\frac{\Omega}{\mu_c}\right)^K \left(\frac{\lambda_H}{\mu_c}\right)^{n-K} \cdot \frac{1}{n!}}$$

Prob. of handoff call blocking

$$P_{BH} = P_N$$

Prob. of new call blocking

$$P_{Bn} = \sum_{n=K}^{N} P_n$$

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