Lec5-mwf

Thursday, January 17, 2008 4:13 PM

Annoucement:

- HW1 is on the web

Rational of the multipath fading model - 10min

Monday, January 14, 2008 4:41 PM

Let the transmitted signal be

$$E_T e^{j\omega_c t}$$

The received signal through the k-th path can be written as

$$\underset{\text{attenuation}}{\underset{\uparrow}{a_k}} E_T e^{j(\omega_c t - \overset{\text{phase shift}}{\underset{\uparrow}{\phi_k}})}$$

attenuation phase shift

The total received signal is then

$$\sum_k a_k E_T e^{j(\omega_c t - \phi_k)}$$

$$= E_T e^{j\omega_c t}\left[\sum_k a_k e^{-j\phi_k}\right]$$

whose magnitude is determined by

$$\left|\sum_k a_k e^{-j\phi_k}\right|$$

Note:

Note:

$$\sum_k a_k e^{j\phi_k} = \underbrace{\sum_k a_k \cos\phi_k}_{X} + j\Big[\underbrace{\sum_k a_k \sin\phi_k}_{Y}\Big]$$

When none of the multipath components dominate, X & Y can be approximated by independent Gaussian r.v.'s with zero mean

$$\Rightarrow \quad |X + jY| = \sqrt{X^2 + Y^2}$$

can be shown to follow Raleigh distribution

If there is a dominant term, w.l.o.g. assume the dominant term is $a_0 e^{j\phi_0} = A$.

$$\Rightarrow \quad |(X+A) + jY| = \sqrt{(X+A)^2 + Y^2}$$

can be shown to follow Ricean distribution.

Consult ECE 600 or any probability textbook.

(20)

Stochastic channel characterization - 15min

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Recall

$$P_R(d) = \alpha^2 \, 10^{X/10} \, g(d) \, G_T G_R P_T.$$

The received signal power is in general a stochastic process, both

- in time domain, and
- in frequency domain

The above equation characterizes the marginal distribution.

In practice, it is also important to model the joint pdf, or correlation

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In general, let X_1, X_2 be two random variables. The auto correlation is

$$E(X_1 X_2)$$

Define the correlation-coefficient

$$\rho = \frac{E\left[\left(X_1 - E(X_1)\right)\left(X_2 - E(X_2)\right)\right]}{\sqrt{Var(X_1) \cdot Var(X_2)}}$$

If $\rho = 0$, X_1, X_2 are uncorrelated
dependence is low

$\rho = 1$, X_1, X_2 are highly correlated
dependence is high

Suppose we observe the received signal at time t, and $t + \Delta t$
or at frequency f and $f + \Delta f$

<u>Why two frequencies:</u> the bandwidth B of the signal is reversely portional to the symbol duration T_s. For wideband signal with a small T_s, B can be quite large (1MHz in IS-95). ($\sim$20MHz in LTE)

Two extreme frequencies can be taken as $f_c - \frac{B}{2}$ and $f_c + \frac{B}{2}$. If the fading characteristics change substantially over such a freq. range, the signal can be seen as passing through a filter, and thus can be distorted.

We may then calculate the correlation coefficient either in time or in frequency.

If the correlation coefficient is close to 1
— in time, the signal power

changes slowly

- in frequency, different frequency component of the signal experiences same level of fading.

If the correlation coefficient is close to 0.

- in time, the signal power changes rapidly

- in frequency, different component experiens frequency-selective fading (distortion)

(Q) Which is better? Is it better to have a channel whose fading levels change slowly (or quickly) in time (or in freq)?

- If signal power changes slowly in time
  - transmitter will have enough time to estimate the channel
    $\Rightarrow$ can design effective control schemes

(e.g. power control)

- however, there may be long periods of deep fades

- If signal power changes slowly in freq.

  - signal distortion will be small easy to do equalization

  - The entire band may run into deep fades

  On the contrary, for frequency-selective channel the chance of having some frequency component with a good signal power is higher

  ⇒ good for using multiple narrow band carriers (e.g. OFDM)

As we can see, understanding these properties is important for designing appropriate communication/multiple-access schemes.

We will discuss the factors that contribute to time- and frequency-selectivity of the channel, and also typical approaches to deal with it.

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Main results - 20min

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More specifically, following the multipath model earlier

xmitted signal $E_T e^{j\omega_c t} = S(t)$

k-th path $a_k E_T e^{j(\omega_c t - \phi_k)}$

Both a_k, ϕ_k are stochastic processes.

Total received signal

$$a(t) = E_T \sum_k a_k e^{j(\omega_c t - \underline{\underline{\phi_k}})}$$

As in 2-ray model, the phase randomness dominates

Two components for ϕ_k:

(1) travel distance

Different paths travel different distance, which leads to dispersion in the time-shift (or delay) τ_k

$$\Rightarrow \theta_k = \omega_c \tau_k = 2\pi \underline{\underline{f_c}} \tau_k$$

Define the delay-spread as the difference in delay between the first path & the last path

Typical values for delay-spread:

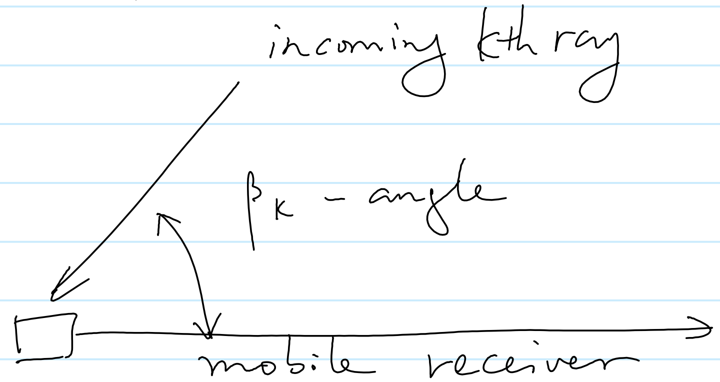
0.2 ~ 0.5 μsec in suburban areas

3 ~ 8 μsec in urban areas

The effect of this time-dispersion will be manifested by frequency-selectivity, since different frequency-component sees different amount of phase shift.

It also leads to inter-symbol interference

② doppler effect



Doppler effect leads to a frequency - shift

$$f_K = \frac{V \cos\beta_K}{\lambda}$$

$$\Rightarrow \quad \omega_K t = 2\pi f_K t = 2\pi \frac{V\cos\beta_K}{\lambda} t.$$

$$\Rightarrow \quad \omega_k t = 2\pi f_k t = 2\pi \frac{v\cos\beta_k}{\lambda} t.$$

Different paths can have diffent f_k due to angle of arrival.

Maximum doppler shift is given by

$$f_m = \frac{v}{\lambda}$$

e.g. $\quad v = 100\,km/hr = 28\,m/s$

$f_c = 1\,GHz \Rightarrow \lambda = 0.3\,m$

$f_m = 90\,Hz$

phase change by 2π every $\frac{1}{90}$ sec

if $\quad v = 0.1\,m/s \quad \rightarrow \quad f_m = 0.33\,Hz$

if $\quad f_c = 30\,GHz \Rightarrow \lambda = 0.01\,m, \quad v = 0.1\,m/s$

$\rightarrow \quad f_m = 10\,Hz$

The effect of this doppler shift will be manifested by time-selectivity, since the value of the phase shift changes in time.

In summary:

- Delay spread determines the frequency-

- Delay spread determines the frequency-selectivity of the channel
- Doppler shift determines the rate of fading in time.

Sketch of the main intuition:

As a rough estimate, consider the case when τ_k & f_k are deterministic (non-random)

Effect of τ_k:

Consider two frequency component f_c & $f_c + \Delta f$

The phase difference (of two paths) will vary by $2\pi \Delta f \cdot \tau_k$ at these two frequencies.

If $2\pi \Delta f \tau_k = \pi$, significant variations of the fading levels will be observed by the two frequency components

$$\Leftrightarrow \quad \Delta f = \frac{1}{2\tau_k} \quad \text{coherenent bandwidth}$$

It determines the frequency-selectivity of the channel.

× If signal BW $B > \Delta f$, will experience freq-selective channel.

Effect of f_K

Consider two time instants
$t, \quad t + \Delta t$

The phase difference (of two paths) will vary by $2\pi f_K \Delta t$ at these two time instants.

If $2\pi f_K \Delta t = \pi$, significant variations of the fading levels will be observed at these times.

$$\Leftrightarrow \quad \Delta t = \frac{1}{2 f_K} \quad \text{coherent-time}$$

It determines the time-selectivity (or rate of fading)

When f_K, τ_K are random, the correct approach is to study correlation in frequency & time

$f, \quad f + \Delta f$
$t, \quad t + \Delta t$

$\rho_a(\Delta t, \Delta f)$ — correlation coefficient as a function of Δt & Δf.

See Schwartz ch 2.5, in particular (2.46)

① Delay spread τ_{av}

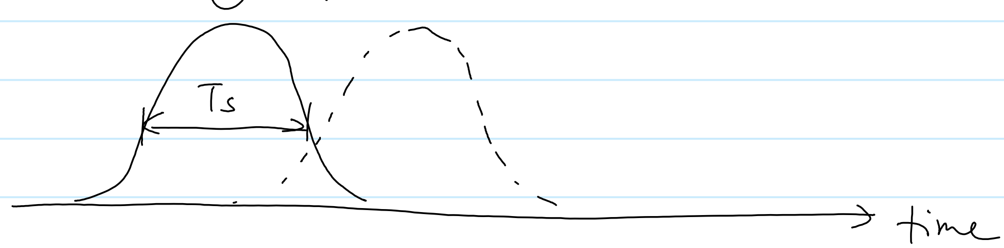
— standard deviation of τ_k

Coherent bw: $\Delta f = \dfrac{1}{2\pi \tau_{av}}$

If $B > \dfrac{1}{2\pi \tau_{av}}$, freq-selective fading

$\Leftrightarrow$ $T_s < 2\pi \tau_{av} \approx 6\tau_{av}$

Inter-symbol interference occurs when the symbol duration is less than 6 times of the delay spread.



If $B < \dfrac{1}{2\pi \tau_{av}}$ flat fading

If $B \gg \dfrac{1}{2\pi \tau_{av}}$ $\Leftrightarrow$ $T_s \ll 2\pi \tau_{av}$
- wide-band signal
- different path can be individually resolved (in CDMA)

— like "echos"

(2) Doppler shift

— f_m : max doppler shift

$$f_m = \frac{v}{\lambda}$$

Coherence time :

$$\delta t = \frac{9}{8\omega_m} = \frac{9}{16\pi f_m} \simeq 0.18/f_m$$

If $T_B > \Delta T = \frac{1}{5.6 f_m}$, distortion within duration of a block occurs

— time-selective fading

Summary

	flat	freq-selective
slow		
time-selective (fast)		

Depending on the value of the coherent bw & coherent time, 55 the channel can fall into one of these four categories