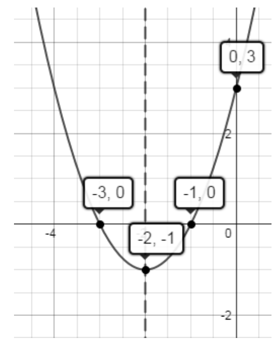
Chapter 2.1: Quadratic functions – How do you sketch graphs and write equations of parabolas?

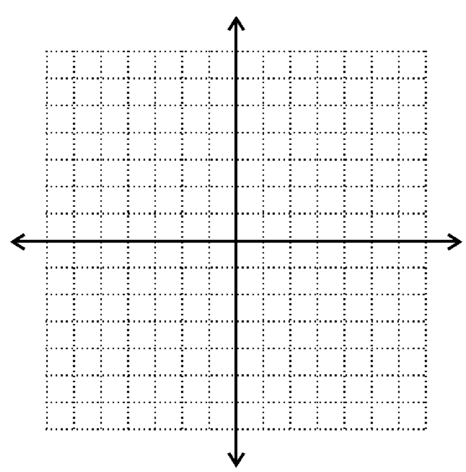
Polynomials and quadratic functions – review:

Polynomial Function: Degree: Quadratic function:

Parabolas: Standard (vertex) form, vertex, and a value: Axis of symmetry: Solutions/$x - intercepts$/roots/zeros: **Example:** Write the function in vertex form: $f(x) = -3x^2 + 12x + 2$.	Example: Write the function's equation. 

Practice:

Write the function in vertex form: $f(x) = 2x^2 + 8x - 10$. Then graph it and identify the vertex, AOS, domain/range, and the x and y intercepts.



Writing equations, max/min values:

How many parabolas can go through the same vertex?

What feature makes these parabolas different?

What features will all these parabolas have in common?

Example: find the equation for the quadratic function with a vertex at $(1, -3)$ that goes through the point $(2,5)$.

Max and min values:

Fact: All parabolas have either an absolute max or an absolute min.

Question: When will a parabola have an absolute max? Min?

Example: A local newspaper has daily production costs of $C = 55{,}000 - 108x + 0.06x^2$ where c is the total cost in dollars and x is the # of newspapers printed. How many newspapers should be printed to yield a minimum cost? What is the minimum cost?

Practice:

Find the standard form equation for the parabola described:

1. Vertex at $(2,3)$, zero at $(-1,0)$

2. Vertex at $(-2,0)$ and through the point $(-4,6)$

3. Find any two quadratic functions, one that opens upwards and one that opens downward through the given zeros $\mathbf{(3,0)}$ **and** $\mathbf{(9,0)}$

Chapter 2.2: Polynomial functions of higher degree – How do you sketch the graphs of polynomial functions?

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Basics of the graphs of polynomials:
All polynomial graphs are continuous and contain only smooth curves.
End behavior is based on the leading coefficient and the degree.

Even degree:
$a > 0$

$a < 0$

Odd degree:
$a > 0$

$a < 0$
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Roots and factors:
If $x = a$ solves a polynomial function for $P(a) = 0$, then…

$x = a$ is a zero of the function
$x = a$ is a solution to $P(x) = 0$
$(x - a)$ must be a factor of $P(x)$
$(a, 0)$ is an x-intercept of the graph of $P(x)$.
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Examples:

1. Determine the end behavior of the polynomial.
$f(x) = -3x^5 + 2x^2$

2. Find the zeros of the polynomial $f(x) = x^4 - 4x^2$.

3. If $3, -2$, and $\frac{1}{4}$ are zeros of a polynomial, find a possible equation for the polynomial.

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Multiplicity and irrational roots:
Multiplicity –

Odd:

Even:
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<td>
Dealing with algebraic irrationals (roots)
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Practice:

4. Find the real zeros of $\boldsymbol{P(x) = x^5 + 2x^4 + x^3}$.

5. Write a polynomial function with zeros at $\boldsymbol{-1\ (m\ 2), \sqrt{3}, -\sqrt{3}}$.

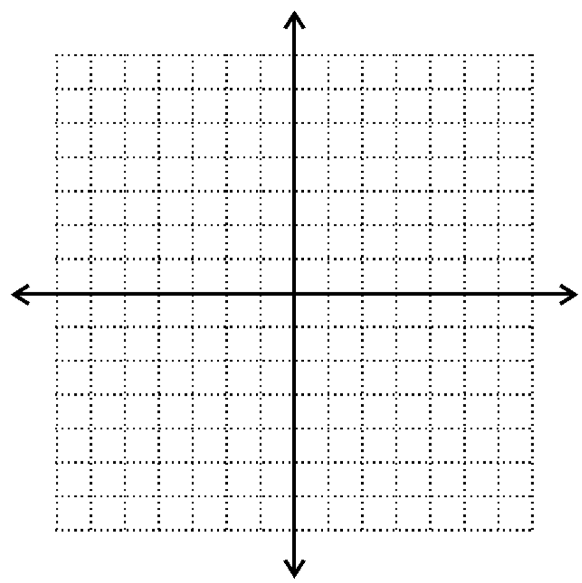
6. Write a polynomial function with zeros at $2, 1 + \sqrt{2}$.

Sketching graphs of polynomials:
What are some main features we can find?

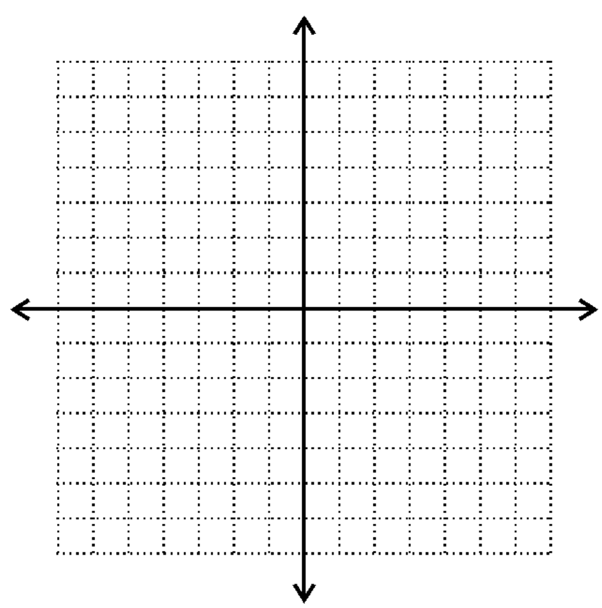
How can we figure out behavior *between* the zeros?

Practice: Sketch a reasonable graph of the function.

1. $f(x) = x^4 - 4x^2$



2. $f(x) = x^5 - 4x^3 + 8x^2 - 32$



Chapter 2.3 Using synthetic and polynomial division – How do you divide a polynomial by another polynomial and use polynomial division to find the rational and real zeros of polynomials?

Synthetic division:
When the *divisor* in a polynomial division problem is *linear* $(x-c)$, there is a very nice algorithm we can use. It is called *Synthetic division*.

Synthetic division:
Example -
The set up: The *coefficients* of the dividend are written out on the top row.

Put the constant term of the divisor, c in the box. **Put a place holder 0 for any missing exponent.**

$$\frac{x^6-3x^5+14x+4}{x-2}$$

The utility of synthetic division:
Synthetic division is an *algorithm* that works under certain constraints. It is not the only way to divide, but it's quick.

Factor theorem:
If $(x-k)$ is a factor of a polynomial, $P(x)$, then the quotient $\frac{P(x)}{(x-k)}=$

If we can divide evenly by a linear factor, what will happen to the degree of the quotient compared to the degree of the dividend?

Example:
Factor $f(x)=2x^3+11x^2+18x+9$ completely if $(x+3)$ is a factor of $f(x)$.

Practice: Factor the polynomial equation completely using the given factors.

1. $f(x)=3x^3+2x^2-19x+6;(x+3)$
2. $f(x)=x^4+4x^3-x^2-16x-12;(x+3),(x+1)$

Polynomial long division: When we want to divide by a non-linear factor, we have to use long division.

$Ex: \frac{x^4+5x^3+6x^2-x-2}{x+2}$

$Ex: \frac{x^4+5x^3+12x^2+17x+5}{x^2+3x+1}$

Rational zero test:

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_1 x + a_0$ is a polynomial with integer coefficients and $\frac{p}{q}$ is a rational root of $f(x)$…

Practice: Determine the possible rational zeros of the polynomial.

1. $f(x) = 2x^5 + 4x^2 + 3$

2. $f(x) = 4x^4 + 3x^2 + x + 6$

Use the rational zero test, and synthetic division, to factor the polynomial completely.

3. $f(x) = 2x^3 - 3x^2 - 3x + 2$

4. $f(x) = 3x^4 + 4x^3 - 11x^2 - 16x - 4$

Imaginary and complex numbers:
The imaginary Number: $i = \sqrt{-1}$, we will frequently use a property of radicals, $\sqrt{ab} =$

When a real number is multiplied by i, it is a number in the *complex number system.*
A *complex number* has two parts. Since real numbers and imaginary numbers are not like terms, then for any real numbers a and b, a complex number in standard form is defined as $\boldsymbol{a + bi}$

When doing operations with complex numbers, they must be treated as…
Examples:
1. $(3 + 6i) - (4 + 2i)$

2. $(3 + 6i)(4 + 2i)$

Practice:
1. $(9 + 2i) + (1 - 7i)$
2. $(6 - 11i)^2$
3. $(3 + 7i)(2 - 4i)$

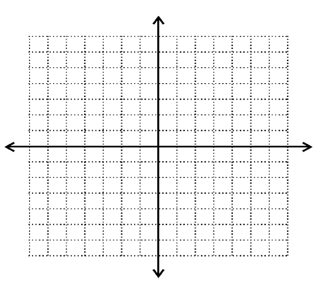
4. $\sqrt{-4} + (-4 - \sqrt{-4})$
5. $\sqrt{-4} \cdot \sqrt{-16}$

The complex conjugate, rationalization and graphing:
If $a + bi$ is a complex number, what is its conjugate?

Stylistically, we are not allowed to have a complex number in a denominator of a fraction. So, you must *rationalize* any denominators that have an i in them.
Example: rationalize the following:

$$\frac{6+i}{2-i}$$

Graphing a complex number is easy, instead of using a Cartesian plane, you use a similar set up with axes labeled $(real, imaginary)$, so to plot $a + bi$, you plot the ordered pair…

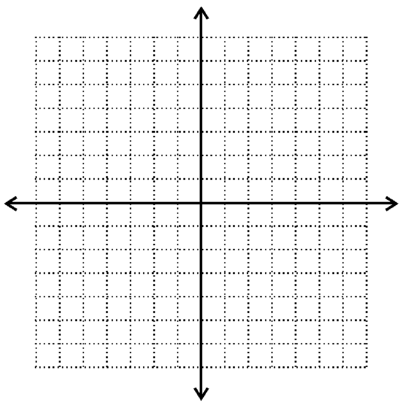


Practice:

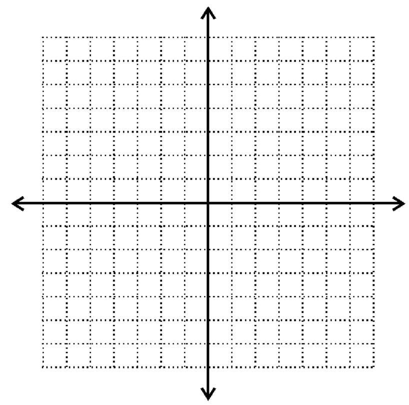
1. $\frac{4+i}{3-2i}$

2. Find the reciprocal of $1 + 3i$

3. Graph $2 - 5i$



4. Graph $(3 - i)(2 + 3i)$



Chapter 2.5: The fundamental theorem of algebra – How do you find all the zeros of a polynomial function?

The fundamental theorem of algebra and linear factorization theorem:
The fundamental theorem of algebra states that every nth degree polynomial has exactly n complex roots.

Linear factorization theorem states that every nth degree polynomial with n complex roots has EXACTLY n linear factors.

If a complex number is a zero of a polynomial, then…

Examples: Find the zeros of the function and write as a product of *linear* factors.

1. $f(x) = x^2 + 10x + 23$

2. $f(x) = x^4 - 81$

3. $f(x) = x^4 - 5x^2 - 66$

Use the given zeros to write a polynomial function with real coefficients

5. $3, 4i,$

6. $-1, -1, 2 + 5i$

Factoring and specific polynomials: $f(x) = x^4 - 5x^2 - 66$

Completely factoring over rational numbers:

Given zeros and a point on the function:

Completely factoring over real numbers:

Completely factoring over complex numbers:

Examples: Given the zeros, multiplicities, and a point on the function, determine the equation of the polynomial.

1. Zeros at $3 - i, -2, 1$, degree 4, $f(2) = 4$

2. Zeros at $\sqrt{3}, i, 2$, degree 5, $f(-1) = 4$

Factor each polynomial **a)** Over the rational numbers **b)** over the real numbers **c)** over the complex numbers.

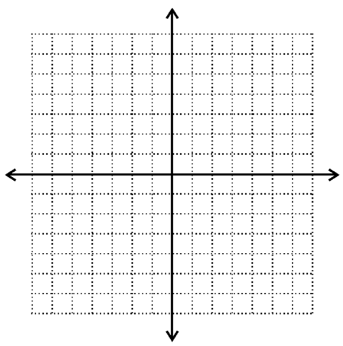
3. $f(x) = x^4 - 3x^3 + 6x^2 + 2x - 60$, $1 + 3i$ is a root

4. $f(x) = x^5 + x^3 + 2x^2 - 12x + 8$, $-2i$ and 1 are roots

Chapter 2.6: Rational functions and asymptotes – How do you find the domain and asymptotes of a rational function?

Rational function: $f(x) = \dfrac{N(x)}{D(x)}$

Parent function: $f(x) = \frac{1}{x}$



Domain:

Range:

Vertical asymptote:

Horizontal asymptote:

Discontinuous:

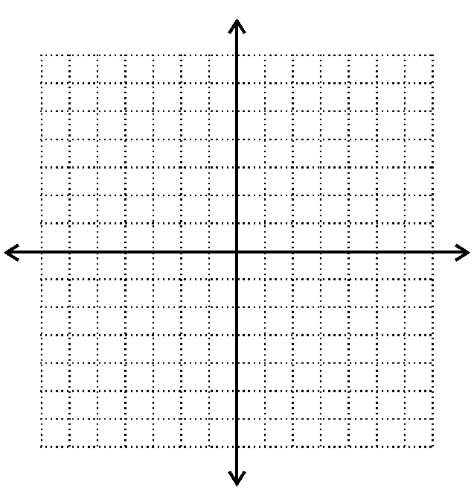
Asymptotes:

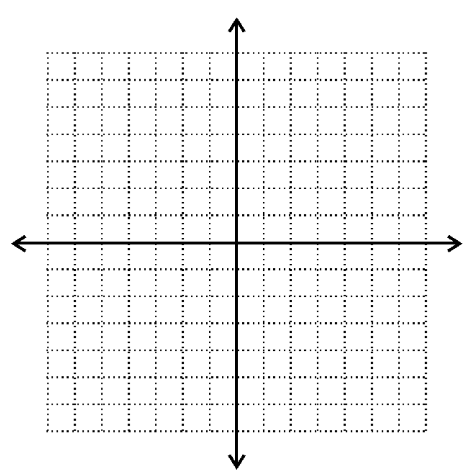
Vertical asymptotes: Horizontal asymptotes: Holes:

Examples: Find any asymptotes, zeros, holes and use the behavior at critical points and extremes to graph the function.

1. $f(x) = \frac{3}{x-2}$

2. $f(x) = \frac{x^2+2x+1}{2x^2-x-3}$



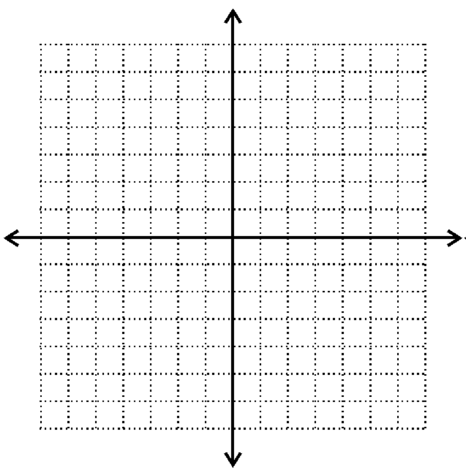


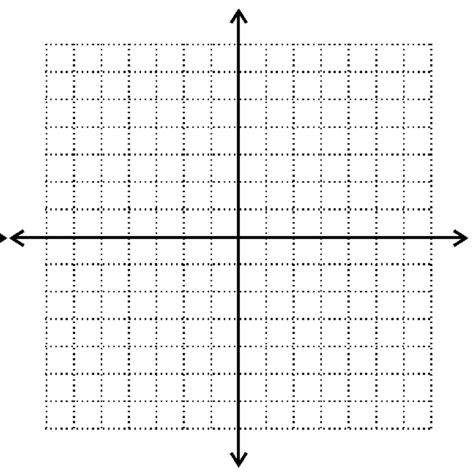
Practice: Find any asymptotes, zeroes, holes and use the behavior at critical points and extremes to graph the function.

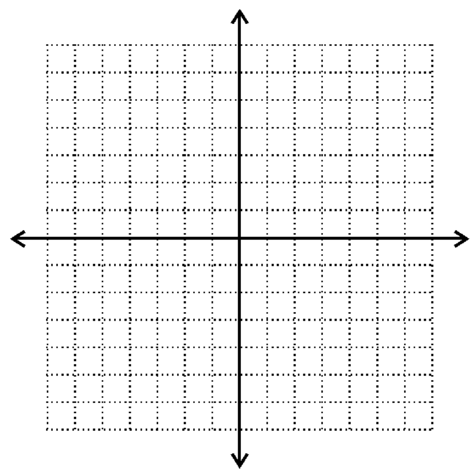
1. $f(x) = \frac{2x+3}{x-1}$

2. $f(x) = \frac{x^2+x-2}{x^2-x-6}$

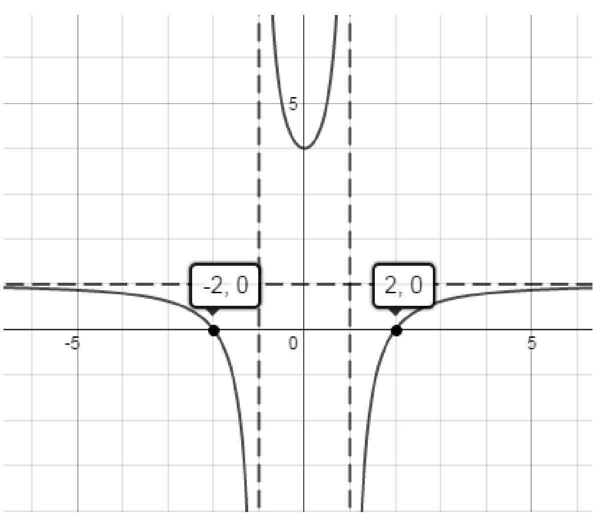
3. $f(x) = \frac{x-1}{x^2-9}$







3. What could be the equation of the function?



4. Write the equation of any function that fits the following criteria:
a) Horizontal Asymptote: $y = 0$
b) Vertical Asymptote: $x = -1$
c) Zero: $x = 1$

Chapter 2.7: More complex rational functions – How do you stretch the graph of a rational function?

The slant asymptote:
Third case: The degree of the numerator is ***exactly one more*** than the degree of the denominator. Use long division!

Describe the slant asymptotes:

1. $f(x) = \frac{2x}{1}$

2. $f(x) = \frac{2x^2+1}{x}$

3. $f(x) = \frac{2x^3}{2x^2+1}$

Graphing complex rational functions:
Things to note:

All horizontal and slant asymptotes can be found by finding the quotient through division and disregarding the remainder.
Ex:

1. $\boldsymbol{f(x) = \frac{4x+5}{2x-1}}$ **2.** $\boldsymbol{f(x) = \frac{x^2+2x+1}{x-3}}$

1. We need to check behavior in between all critical $x - values$ which include **zeros** and **vertical asymptotes.**

2. There will always be either a horizontal **<u>or</u>** a slant asymptote, not both.

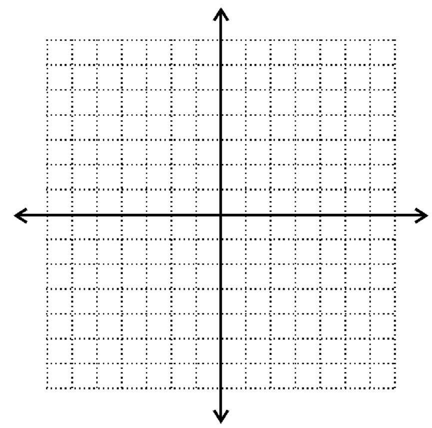
3. Graphs of rational functions *can* cross a **horizontal** or **slant** asymptote when near the center of the graph, asymptotes only describe what happens at the extremes.

4. Each non removable domain restriction will give you a unique vertical asymptote.

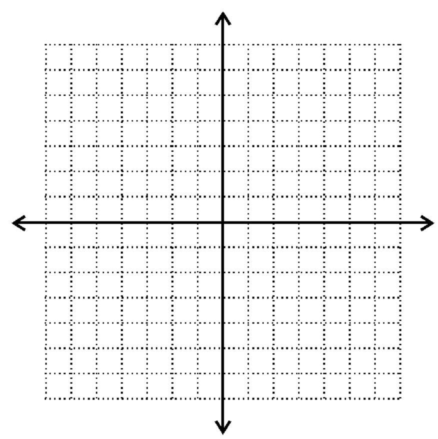
5. Always find the y-value of a hole by plugging the removable domain restriction's x-value into the simplified function.

Practice: Sketch a graph of the rational function by finding the key features.

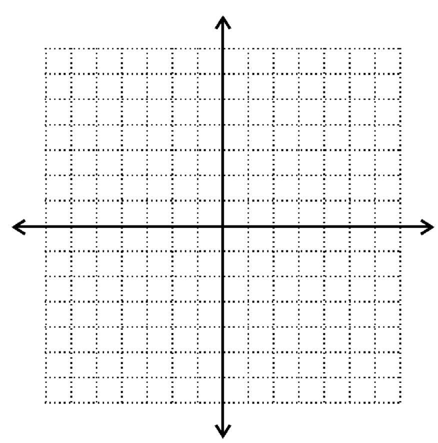
1. $f(x) = \frac{2x^2+1}{x}$



2. $f(x) = \frac{x^3}{2x^2-8}$



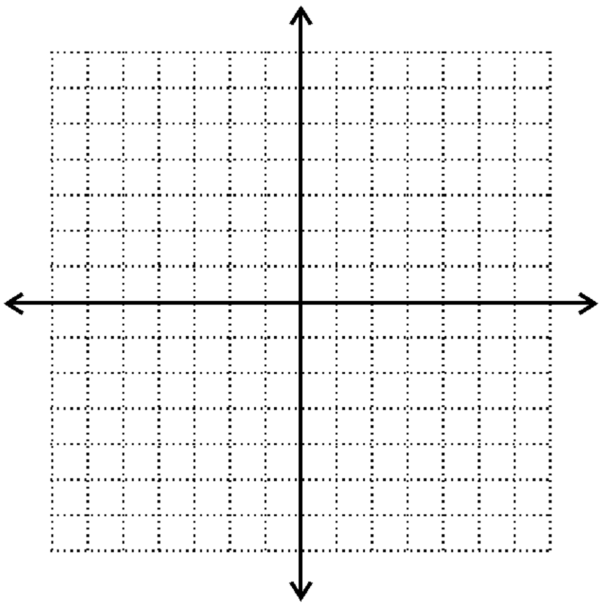
3. $f(x) = \frac{x}{x^2-4}$

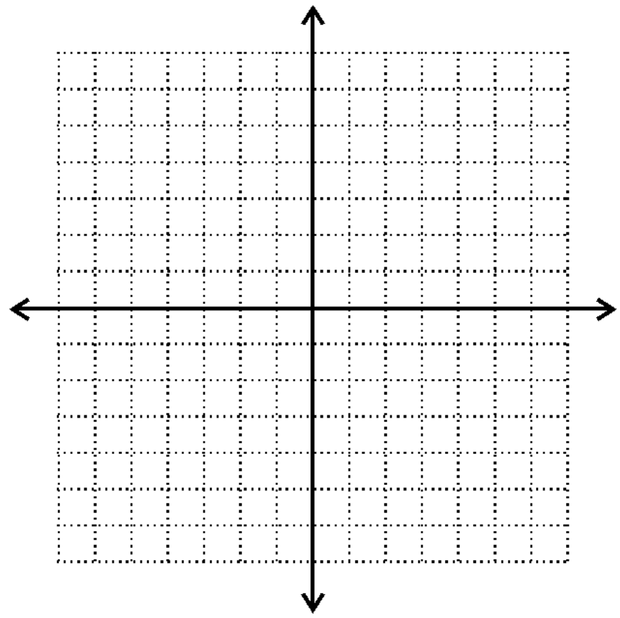


Practice: Graph each rational function.

1. $f(x) = \frac{x^2}{x^3 - x}$

2. $f(x) = \frac{x^2 - 4x + 4}{2x - 1}$





3. $f(x) = \frac{x^2 - 1}{2x - 3}$

4. $f(x) = \frac{x - 2}{x^2}$