Project 2 - Modeling Tangent Planes of A Noodle
MATH 147, Calculus III, Honors

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1   Conclusions

The tangent plane for the orecchiette pasta was developed from two vectors that shared an initial point at the point of tangency and were orthogonal to one another on the plane. These two vectors were derived from taking the partial derivatives with respect to the two parametric variables (t and a) of each of the three parametric equations that described the pasta. Taking the components from each of these vectors, a set of three parametric equations was created to describe the tangent plane. By starting at a point defined by the function and continuing on a path defined by the scaled components of each of the vectors, the tangent plane was created. The point at which the tangent plane was created was accomplished through trial and error, in which multiple points were tested until $(0.8, \pi)$ was chosen. Due to the nature of the pasta parametrization (how it curves based off the value of a), for many of the points, the tangent plane would cut through another part of the noodle, making it unusable for 3D printing. Selecting $(0.8, \pi)$ as the point of tangency gave the best viewing angle of the noodle, while still being possible to print. In order to have the print be physically strong, the tangent base was shifted upwards to be in contact with more of the pasta's surface.

2   Appendix

Plots   All equations describing curves were parametric, hence the "ParametricPlot3D" command was used to graphically represent the equations. "Show" was used to combine these graphical plots together.

Vector Field   The normal vector was created by using "Cross" to take the cross product of the two vectors found by taking the partial derivatives of the parametric equations by using "D". Graphics3D was used in conjunction with "Table", "Arrowheads", "Arrow" to create the vector field. To make all vectors equal length, "Norm" was used to get the magnitude of each vector.

STL File   The pasta with its base was then exported to an STL file through the "Export" command. "Import" allowed for the generated STL file to be viewed in Mathematica.

3   Communications and Group Work Plan

Alfred   Alfred's role in the group was the creation of the parametric equations for the tangent plane used as a base and finalizing the position of the tangent plane on the pasta. Alfred was also responsible for importing the STL file into CURA and verifying that it would print. Alfred also worked to create the final Latex file.

Paul   Paul's role was to help with project consistency, especially during the early phases of the project when the group was working together in lab.

Garrett   Garrett facilitated group communication and provided assistance with notation and writing the final report. Garret also was responsible for managing the files throughout the project.

Sagindyk   Sagindyk created the equations for the pasta (from the first project) and also created the normal equation, vector field, and coordinate curves. Sagindyk also provided assistance with Latex syntax and structure.
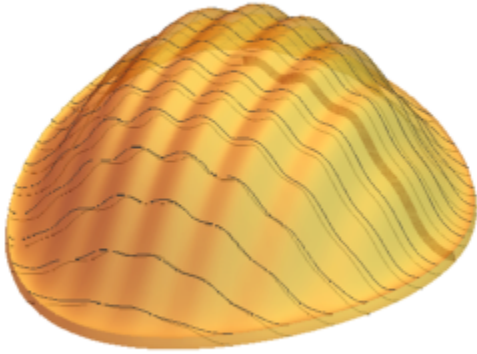
4   Parameterized Pasta

Assume that the surface S of this pasta is parametrized by the equations

$$\vec{r} = \begin{cases} \boldsymbol{X}(t,a) = 0.9t\cos(a) \\ \boldsymbol{Y}(t,a) = t\sin(a) \\ \boldsymbol{Z}(t,a) = 0.06\sqrt{t}(1.21-t)\sin(22.5t\cos(a)) - 0.08t^2\sin(2a) + \frac{1}{2}t^6 - t^4 - 0.1t^2 + 1 \end{cases} \quad 0 \le t \le 1.21, 0 \le a \le 2\pi$$
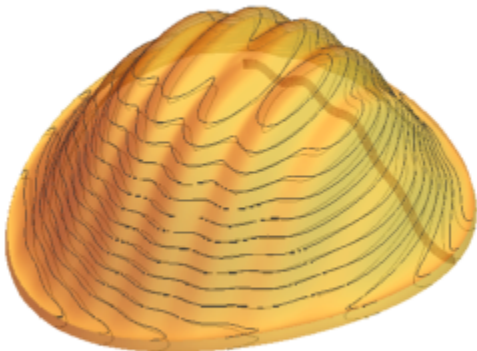
The *a* curves are given by:

$$\vec{r} = \begin{cases} \boldsymbol{X}(t,a) = 0.9t\cos(a) \\ \boldsymbol{Y}(t,a) = t\sin(a) \\ \boldsymbol{Z}(t,a) = 0.06\sqrt{t}(1.21 - t)\sin(22.5t\cos(a)) - 0.08t^2\sin(2a) + \frac{1}{2}t^6 - t^4 - 0.1t^2 + 1 \end{cases} \qquad 0 \le t \le 1.21, a_0 \text{ fixed}$$
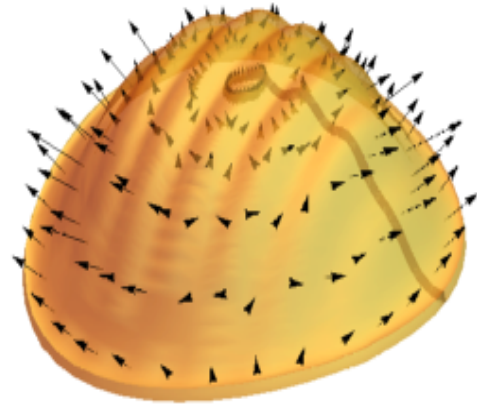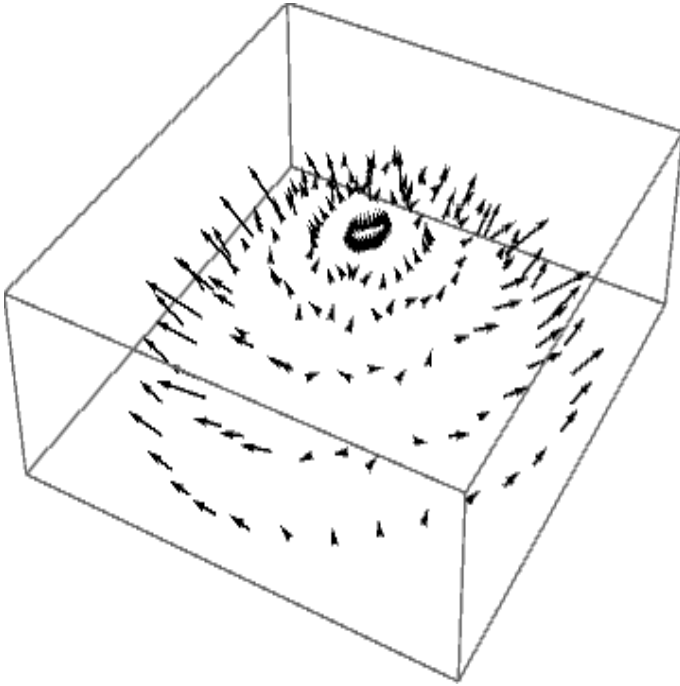
The *t* curves are given by:

$$\vec{r} = \begin{cases} \boldsymbol{X}(t,a) = 0.9t\cos(a) \\ \boldsymbol{Y}(t,a) = t\sin(a) \\ \boldsymbol{Z}(t,a) = 0.06\sqrt{t}(1.21 - t)\sin(22.5t\cos(a)) - 0.08t^2\sin(2a) + \frac{1}{2}t^6 - t^4 - 0.1t^2 + 1 \end{cases} \qquad 0 \le t \le 1.21, t_0 \text{ fixed}$$

3

The normal vector to the surface S at the point $\vec{r}(t_0, a_0)$ is given by

$$\vec{N}(t_0, a_0) = \vec{r}_t(t_0, a_0) \times \vec{r}_a(t_0, a_0) =$$
$$(1.35t - 1.6335)t^{3/2}\cos(22.5t\cos(a)) + 0.16t^2\sin(a) + \cos(a)$$
$$\left((0.09t - 0.0363)\sqrt{t}\sin(22.5t\cos(a)) - 3.t^6 + 4.t^4 + 0.2t^2\right),$$
$$0.144t^2\cos^3(a) + 0.144t^2\sin^2(a)\cos(a) + t^2\left(-2.7t^4 + 3.6t^2 + 0.18\right)$$
$$\sin(a) + (0.081t - 0.03267)\sqrt{t}\sin(a)\sin(22.5t\cos(a)), 0.9t + 0.$$

The tangent plane to the surface S at the point $\vec{r}(t_0, a_0)$ is

$$\vec{N}(t_0, a_0) \cdot ((\boldsymbol{X}(u,v), \boldsymbol{Y}(u,v), \boldsymbol{Z}(u,v)) - \vec{r}_a(t_0, a_0)) = 0$$

$$\vec{T} = \begin{cases} \boldsymbol{X}(u,v) = -0.9tv\sin(a_0) + 0.9t_0\cos(a_0) + 0.9u\cos(a_0) \\ \boldsymbol{Y}(u,v) = t_0v\cos(a_0) + t_0\sin(a_0) + u\sin(a_0) \\ \boldsymbol{Z}(u,v) = v\left(-1.35t^{3/2}(1.21 - t)\sin(a)\cos(22.5t\cos(a)) - 0.16t^2\cos(2a)\right) + \\ u(1.35\sqrt{t}(1.21 - t)\cos(a)\cos(22.5t\cos(a)) + \frac{0.03(1.21 - t)\sin(22.5t\cos(a))}{\sqrt{t}} - 0.16t\sin(2a) - \\ 0.06\sqrt{t}\sin(22.5t\cos(a)) + 3.t^5 - 4t^3 - 0.2t) + \\ 0.06\sqrt{t}(1.21 - t)\sin(22.5t\cos(a)) - 0.08t^2\sin(2a) + 0.5t^6 - t^4 - 0.1t^2 + 1 \end{cases}$$

Now if $t_0 = 0.8$ and $a_0 = \pi$, the equation of tangent plane is

$$\vec{T} = \begin{cases} \boldsymbol{X}(u,v) = -0.9u - 0.72 \\ \boldsymbol{Y}(u,v) = 0. - 0.8v \\ \boldsymbol{Z}(u,v) = -1.58183u - 0.1024v + 0.673996t^6 - t^4 - 0.1t^2 + 1 \end{cases}$$
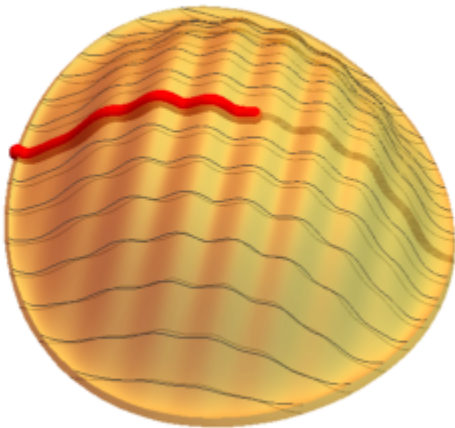
The t- coordinate curve corresponding to $t_0 = 0.8$ is

$$\vec{T} = \begin{cases} \boldsymbol{X}(t,a) = 0.72\cos(a) \\ \boldsymbol{Y}(t,a) = 0.8\sin(a) \\ \boldsymbol{Z}(t,a) = -0.0512\sin(2a) + 0.0220029\sin(18.\cos(a)) + 0.657472t^6 - t^4 - 0.1t^2 + 1 \end{cases} \quad 0 \le a \le 2\pi$$
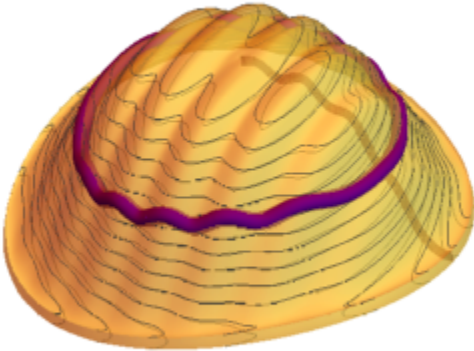
The t- coordinate curve corresponding to $a_0 = \pi$ is

$$\vec{T} = \begin{cases} \boldsymbol{X}(t,a) = -0.9t \\ \boldsymbol{Y}(t,a) = 0\sin(a) \\ \boldsymbol{Z}(t,a) = -0.06\sqrt{t}(1.21 - t)\sin(22.5t) + 0.5t^6 - t^4 - 0.1t^2 + 1.t^6 - t^4 - 0.1t^2 + 1 \end{cases} \quad 0 \le a \le 2\pi$$

The surface with the two coordinate curves and the tangent plane is below. The second picture corresponds to the STL file