

# Homework 7 Oracle

MATH 220 Spring 2021

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## Section 3.7

### Problem 1 [FOR GRADE]

We wish to write the two sinusoidal terms as one.

$$\begin{aligned}3 \cos 2t + 4 \sin 2t &= R \cos(\omega_0 t - \delta) \\&= R[\cos \omega_0 t \cos \delta + \sin \omega_0 t \sin \delta] \\&= (R \cos \delta) \cos \omega_0 t + (R \sin \delta) \sin \omega_0 t\end{aligned}$$

Matching the coefficients, we obtain the following system of equations for  $\omega_0$ ,  $R$ , and  $\delta$ .

$$R \cos \delta = 3 \quad (1)$$

$$\omega_0 = 2 \quad (2)$$

$$R \sin \delta = 4 \quad (3)$$

Square both sides of the first and third equations

$$R^2 \cos^2 \delta = 9$$

$$R^2 \sin^2 \delta = 16$$

and add their respective sides.

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$$R^2 \cos^2 \delta + R^2 \sin^2 \delta = 9 + 16$$

$$R^2 (\cos^2 \delta + \sin^2 \delta) = 25$$

$$R^2 = 25$$

$$R = 5$$

Divide the respective sides of equations (1) and (3).

$$\frac{R \sin \delta}{R \cos \delta} = \frac{4}{3} \rightarrow \tan \delta = \frac{4}{3} \rightarrow \delta = \tan^{-1} \frac{4}{3}$$

Therefore,

$$3 \cos 2t + 4 \sin 2t = 5 \cos \left( 2t - \tan^{-1} \frac{4}{3} \right)$$

## Problem 5

$$20u'' + 400u' + 3920u = 0$$

$$20r^2 + 400r + 3920 = 0$$

then

$$r = -10 \pm 4\sqrt{6}i$$

We see that

$$u(t) = C_1 e^{-10t} \cos(4\sqrt{6}t) + C_2 e^{-10t} \sin(4\sqrt{6}t)$$

$$u'(t) = 4\sqrt{6}C_1 e^{-10t} \sin(4\sqrt{6}t) - 10C_1 e^{-10t} \cos(4\sqrt{6}t) \\ + 4\sqrt{6}C_2 e^{-10t} \cos(4\sqrt{6}t) - 10C_2 e^{-10t} \sin(4\sqrt{6}t)$$

By knowing that  $u(0) = 2$  and  $u'(0) = 2$ , we find that  $C_1 = 2$  and  $C_2 = \frac{5}{\sqrt{6}}$ . Finally,

$$u(t) = 2e^{-10t} \cos(4\sqrt{6}t) + \frac{5}{\sqrt{6}} e^{-10t} \sin(4\sqrt{6}t)$$

Therefore

$$\text{Quasi-frequency : } 4\sqrt{6}$$

$$\text{Quasi-period : } \frac{\pi}{2\sqrt{6}}$$

## Section 7.1

### Problem 1 [FOR GRADE]

Let  $u = x_1$ .

$$x_1'' + 0.5x_1' + 2x_1 = 0$$

Finally, let  $x_2 = x_1'$ .

$$x_2' + 0.5x_2 + 2x_1 = 0$$

By making these substitutions, the original second-order ODE has become a system of first-order ODEs.

$$\begin{cases} x_1' = x_2 \\ x_2' = -2x_1 - 0.5x_2 \end{cases}$$

### Problem 5

Let  $u = x_1$ .

$$x_1'' + p(t)x_1' + q(t)x_1 = g(t), \quad x_1(0) = u_0, \quad x_1'(0) = u_0'$$

Finally, let  $x_2 = x_1'$ .

$$x_2' + p(t)x_2 + q(t)x_1 = g(t), \quad x_1(0) = u_0, \quad x_2(0) = u_0'$$

By making these substitutions, the original initial value problem has become a system of first-order ODEs,

$$\begin{cases} x_1' = x_2 \\ x_2' = -q(t)x_1 - p(t)x_2 + g(t) \end{cases}$$

subject to the initial conditions,

$$x_1(0) = u_0 \quad \text{and} \quad x_2(0) = u_0'.$$

## Section 7.2

### Problem 4

$A^T$  is the transpose of  $A$ ,  $\overline{A}$  is the complex conjugate of  $A$ , and  $A^* = \overline{A}^T$  is the adjoint of  $A$ .

$$A^T = \begin{pmatrix} 3-2i & 2-i \\ 1+i & -2+3i \end{pmatrix}$$

$$\overline{A} = \begin{pmatrix} 3+2i & 1-i \\ 2+i & -2-3i \end{pmatrix}$$

$$A^* = \begin{pmatrix} 3+2i & 2+i \\ 1-i & -2-3i \end{pmatrix}$$

### Problem 8 [FOR GRADE]

Start by calculating the determinant.

$$\det \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} = (1)(3) - (4)(-2) = 11$$

Since it's not zero, an inverse for the given matrix exists.

$$\left( \begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ -2 & 3 & 0 & 1 \end{array} \right)$$

The aim is to make the left side of the augmented matrix 1's and 0's as the right side is now. Since the top left entry is 1 already, we move on to the bottom left entry. To make it zero, multiply both sides of the first row by 2 and add it to the second row.

$$\left( \begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 11 & 2 & 1 \end{array} \right)$$

To make the bottom right entry 1, divide the bottom row by 11.

$$\left( \begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & \frac{2}{11} & \frac{1}{11} \end{array} \right)$$

To make the top right entry 0 , multiply the bottom row by  $-4$  and add it to the first row.

$$\left( \begin{array}{cc|cc} 1 & 0 & \frac{3}{11} & -\frac{4}{11} \\ 0 & 1 & \frac{2}{11} & \frac{1}{11} \end{array} \right)$$

Therefore, the inverse matrix is

$$\left( \begin{array}{cc} \frac{3}{11} & -\frac{4}{11} \\ \frac{2}{11} & \frac{1}{11} \end{array} \right).$$