Homework 1 Oracle

MATH 220 Spring 2021

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44; 12021 H.E.

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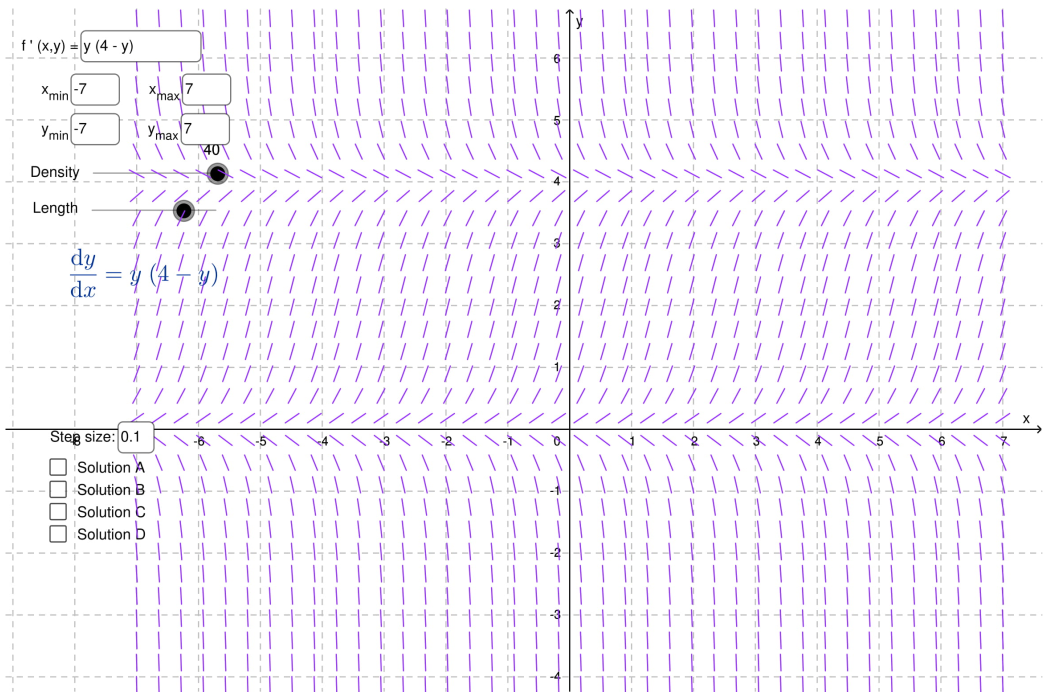
Chapter 1.1

Problem 7



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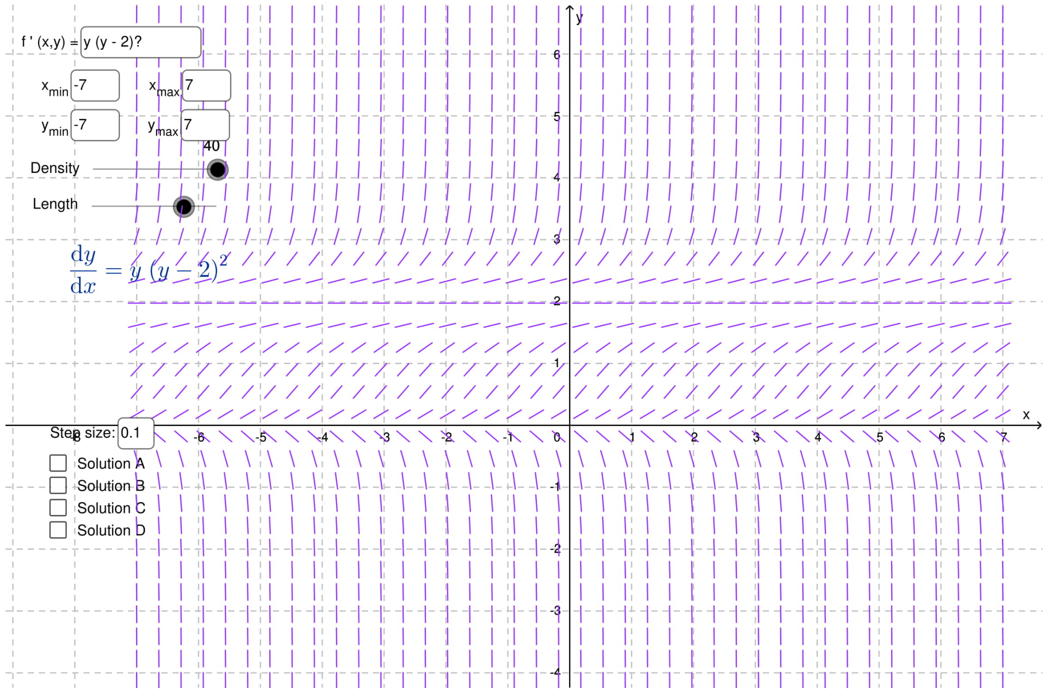
Problem 10



Problem 15 [FOR GRADE]

We can see that the direction field results in constant rate of change of zero at level where $y = 0$ and $y = 3$. Then we can also see that the direction of the graph decreases when $y < 0$ and $y > 3$, while also increasing in section of $y > 0$ and $y < 3$. That means, we are looking for a separate y to get constant change at 0 and $(3 - y)$ term, so that the constraints above are satisfied. $\therefore y' = y(3 - y)$. Answer is h.

Chapter 1.2

Problem 7 [FOR GRADE]

Our differential equation is as given

$$\frac{dp}{dt} = \frac{p}{2} - 450$$

a. Find the time at which the population becomes extinct if $p(0) = 850$
We find that $\mu(t) = e^{\int -\frac{1}{2} dt} = e^{-\frac{1}{2}t}$. Then

$$
\begin{aligned}
\frac{d}{dt}(\mu(t)p) &= -450\mu(t) \\
\implies p(t) &= \frac{\int -450\mu(t)dt}{\mu(t)} \\
&= \frac{\int -450e^{-\frac{1}{2}t}dt}{e^{-\frac{1}{2}t}} \\
&= \frac{-450\times(-2e^{-\frac{1}{2}t})+C}{e^{-\frac{1}{2}t}} \\
&= 900 + Ce^{\frac{1}{2}t}
\end{aligned}
$$

Then by using $p(0)=850$, we find the constant C to be -50. We need to find the time t_e, at which $p(t_e)=0$. We simply solve the equation $0=900-50e^{\frac{1}{2}t_e}$, where we find $t_e = \ln(324)\approx 5.78$.

b. Find the time of extinction if $p(0)=p_0$, where $0<p_0<900$

Using the above, we find that $t_e = 2\ln(\frac{900}{900-p_0})$

c. Find the initial population p_0 if the population is to become extinct in 1 year.

Recall from Example 1 that t is measured in months. So we simply solve

$$0=900-(900-p_0)e^6 \implies p_0 = 900 - \frac{900}{e^6} \approx 897.77$$

Problem 9

a. If the limiting velocity is 49m/s (the same as in Example 2), show that the equation of motion can be written as

$$\frac{dv}{dt} = \frac{1}{245}(49^2 - v^2)$$

Recall the base equation from Example 2, by using the given, we have

$$0 = 9.8 - C(49^2) \implies C = \frac{9.8}{49^2} = \frac{1}{245} \implies \frac{dv}{dt} = \frac{1}{245}(49^2 - v^2)$$

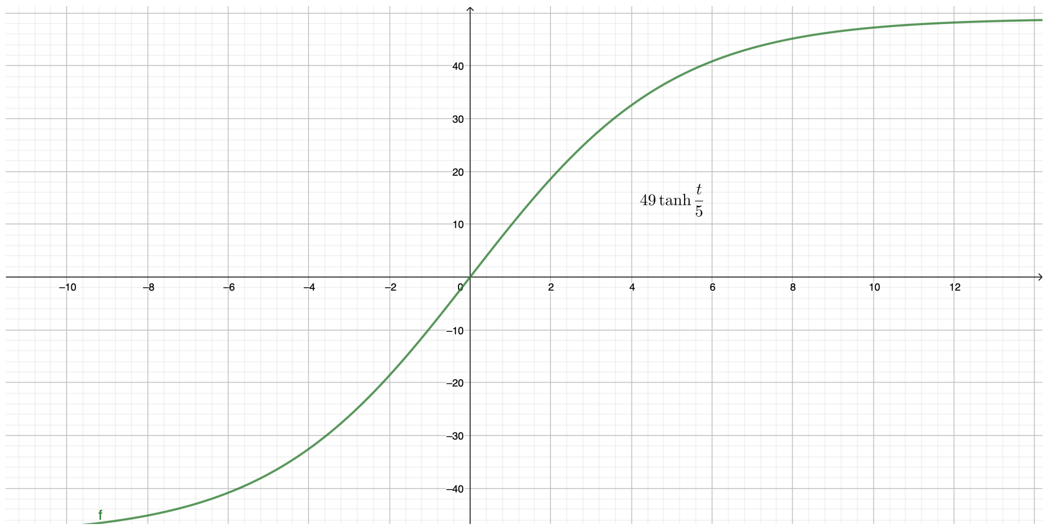
b. If $v(0)=0$, find an expression for $v(t)$ at any time.

We can view this problem as a first-order separable equation and rewrite it as

$$\int \frac{dv}{49^2 - v^2} = \int \frac{dt}{245}$$

After some computations, the expression for $v(t)$ is $49\tanh(\frac{t}{5})$

c. Plot your solution from part b and the solution (26) from Example 2 on the same axes.



d. Based on your plots in part **c**, compare the effect of a quadratic drag force with that of a linear drag force.

The quadratic drag damps the speed faster and has a bigger effect on the speed than the linear dependance.

e. Find the distance $x(t)$ that the object falls in time t.

We know that $\frac{dx}{dt} = v(t) = 49\tanh(\frac{t}{5})$ so then $x(t) = 245\ln(\cosh(\frac{t}{5})) + C$. If $x(0) = 0$, then $C = 0$.

f. Find the time T it takes the object to fall 300m.

Simply solve $300 = 245\ln(\cosh(\frac{t}{5}))$, it will be something like $t = 5\cosh^{-1}(e^{\frac{60}{49}}) \approx 9.477$

Chapter 1.3

Problem 1

2nd order and linear.

Problem 3

4th order and linear.

Problem 5

Plug them both in. They are solutions.

Problem 9

Plug them both in. They are solutions.

Problem 11 [FOR GRADE]

$y' + 2y = 0$ form yields solutions for $r = -2$. You can solve a characteristic polynomial of this equation to get it. *I guess you guys aren't ready for that yet. But your kids are gonna love it*

Problem 16

$u_{xx} + u_{yy} + u_{zz} = 0$ is linear and second order.

Problem 20

Plug them both in. They are solutions.