Homework 2 Oracle

MATH 220 Spring 2021

Sandy Urazayev*

52; 12021 H.E.

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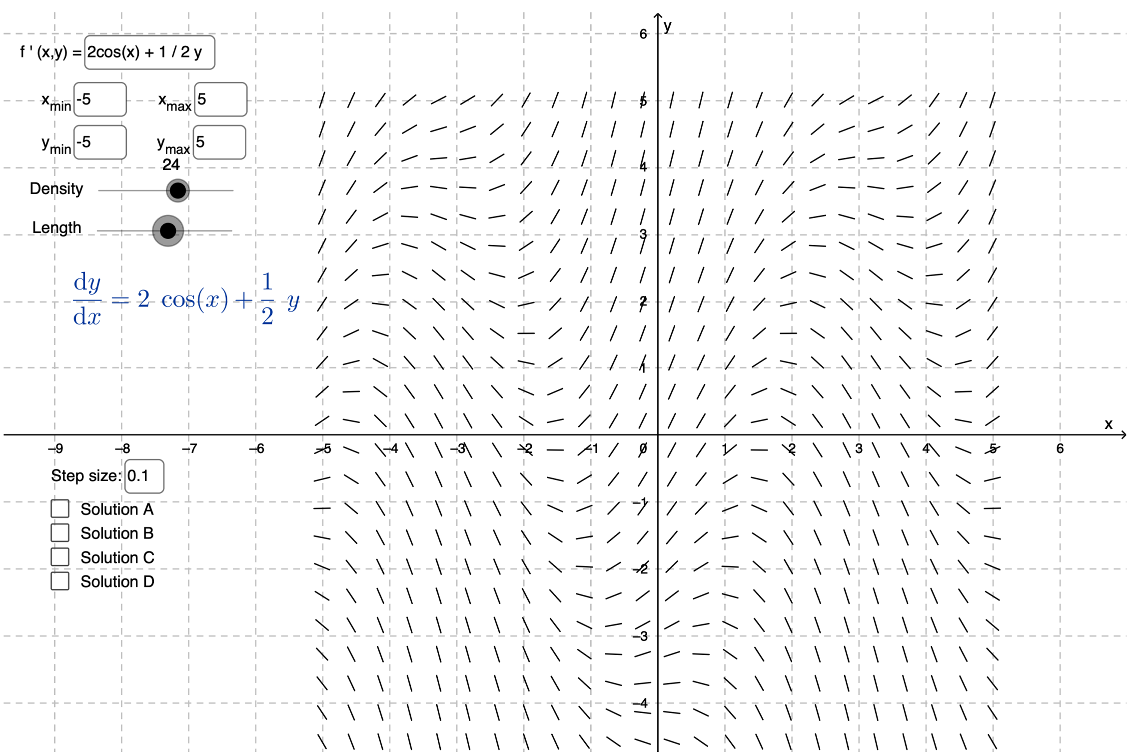
Chapter 2.1

Problem 13



*University of Kansas (ctu@ku.edu)

Part a

As t gets infinitely large, it simply oscillates in an inverse cosine fashion. a does give the function an initial starting point, to which it starts oscillating from. That would probably be $a+\pi$ because $2\cos(t)$ changes its behaviour every π revolution.

Part b

This is a first-order linear differential equation of the form $y'+p(t)y=q(t)$. Find $\mu(t)=e^{\int -\frac{1}{2}}$ and then solve $\frac{d}{dt}(\mu(t)y)=q(t)\mu(t) \implies y=\frac{\int q(t)\mu(t)dt}{\mu(t)}$. You should get

$$y(t)=ce^{t/2}+\frac{8}{5}\sin(t)-\frac{4}{5}\cos(t)$$

Then solving for $y(0)$ and c, we have the full solution to be

$$y(t)=(a+\frac{4}{5}e^{t/2})+\frac{8}{5}\sin(t)-\frac{4}{5}\cos(t)$$

Part c

y oscillates for $a=a_0$

Problem 15 [FOR GRADE]

Part a

This is again, a first-order linear differential equation, so we do our μ and integration from both sides trick. Recognize that we have to divide everything by t, so that our lead y' doesn't have a coefficient and the method for solving this type of equations is applicable.

$$ty'+(t+1)y=2te^{-t} \iff y'+(\frac{t+1}{t})y=2e^{-t}$$

After cleaning it up, the actual solution process becomes more or less trivial, $\mu(t)=e^{\int \frac{t+1}{t}}=te^{t}$. Then we find for $t>0$

$$y(t)=\frac{ce^{-t}}{t}+e^{-t}t$$

Applying $y(1) = a$, then we get

$$y(t) = te^{-t} + \frac{(ea-1)e^{-t}}{t}$$

We need $ea-1$ to be equal to zero, then $a_0 = \frac{1}{e}$

Part c

As $t \to 0$, then $y \to 0$.

Problem 17 [FOR GRADE]

Recall the solution to Problem 13. We need to swap the sign on $p(t)$ and update the initial value constant solution. We will get

$$y(t) = -\frac{9}{5}e^{t/2} + \frac{8}{5}\sin(t) + \frac{4}{5}\cos(t)$$

Set the derivative of y to 0 and solve for t.

$$0 = -\frac{9}{5} \times (-\frac{1}{2}) \times e^{t/2} + \frac{8}{5}\cos(t) - \frac{4}{5}\sin(t)$$

You can check the nature of the point by taking y''. Finally, we find that the local maximum is at $(t, y) = (1.36, 0.82)$. Better approximated values are accepted.

Problem 20

The solution process is similar to the problem of 17, you should get a general solution for y:

$$y = -1 - \frac{3}{2}(\sin t + \cos t) + Ce^{t}$$

where C is a constant. Solving $y(0) = y_0$ for y_0 yields that $C = y_0 + \frac{5}{2}$ so then the solution is $y_0 = -\frac{5}{2}$.

Problem 28

Part a

Recall the form $y' + p(t)y = g(t)$ and solution form of

$$\frac{d}{dt}(\mu(t)y) = g(t)\mu(t)$$

Then if $g(t) = 0$, solution is $y = Ae^{-\int p(t)dt}$

Part b

Simply substitute (50) into (48), perform some trivial Chain Rule and confirm that

$$A'(t) = g(t)\exp\left(\int p(t)dt\right)$$

Part c

Substitution is mechanical. Prove that variation of parameters works.

Chapter 2.2

Problem 1

$$\frac{dy}{dx} = \frac{x^2}{y}$$

then

$$\int y dy = \int x^2 dx$$

So the solution is

$$3y^2 - 2x^3 = C$$

It's OK to leave the solution implicitly here, otherwise, the explicit solution for y can be very nasty.

Problem 7

$$\frac{dy}{dx} = \frac{y}{x}$$

then

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

Then

$$\ln(y) = \ln(x) + \ln(C) = \ln(C \times x)$$

C is any constant, then $\ln(C)$ is also a constant. Finally, $y = Cx$

Problem 8 [FOR GRADE]

$$\frac{dy}{dx} = \frac{-x}{y}$$

then

$$\int y dy = -\int x dx$$

Therefore

$$y^2 + x^2 = C$$

It's fine if you wrote $y = \pm\sqrt{C - x^2}$

Problem 21

$$y' = \frac{ty(4-y)}{3}, \quad y(0) = y_0$$

Part a

As $t \to \infty$, then $y \to 4$

Part b

First, you will have to solve the system, which is a first-order separable ordinary differential equation. The implicit solution is

$$\frac{3}{4}\ln(\frac{4}{4-5}) = \frac{t^2}{2} + C$$

where $C = \frac{3}{4}\ln(\frac{y_0}{4-y_0})$.

Solve for t, so

$$t = \sqrt{\frac{3}{2}\ln\left(\frac{y(4-y_0)}{y_0(4-y)}\right)}$$

Use $y = 3.98$ and $y_0 = 0.5$, then $t \approx 3.29527$.

Problem 25

Part a

Simple divide both the numerator and the denominator by x.

Part b

You should get

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

Part c

This is simply to show.

Part d

Yet another separable equation, you should get the implicit solution

$$x^4 |2 - v| |v + 2|^3 = C$$

Part e

Rearrange to get

$$|y + 2x|^3 |2x - y| = C$$

Part f

It's like a 1/x star.