Homework 4 Oracle

MATH 220 Spring 2021

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70; 12021 H.E.

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Section 2.4

Problem 2

tan is discontinuous at odd multiples of $\frac{\pi}{2}$, since $\frac{\pi}{2} < \pi < \frac{3\pi}{2}$, the interval is $(\frac{\pi}{2}, \frac{3\pi}{2})$.

Problem 4

Dividing both sides by $\ln(t)$ yields

$$y' + \frac{y}{\ln(t)} = \frac{\cot(t)}{\ln(t)}$$

for $\ln(t) \neq 0 \iff t \neq 1$. $\cot(t)$ forces out t to be in the range $(0, \pi)$. By finding the intersection of those constraints, we get an interval $(1, \pi)$.

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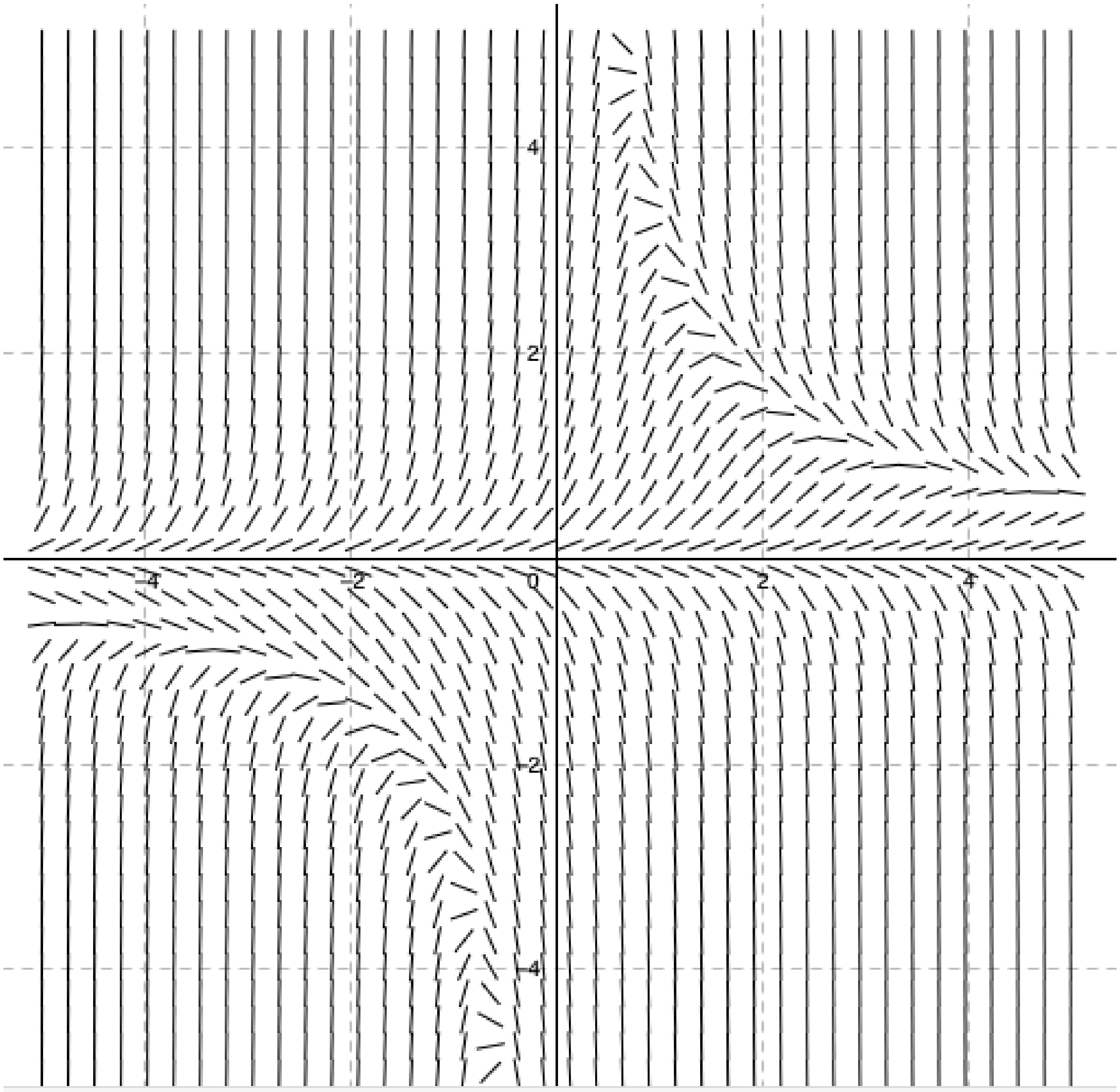
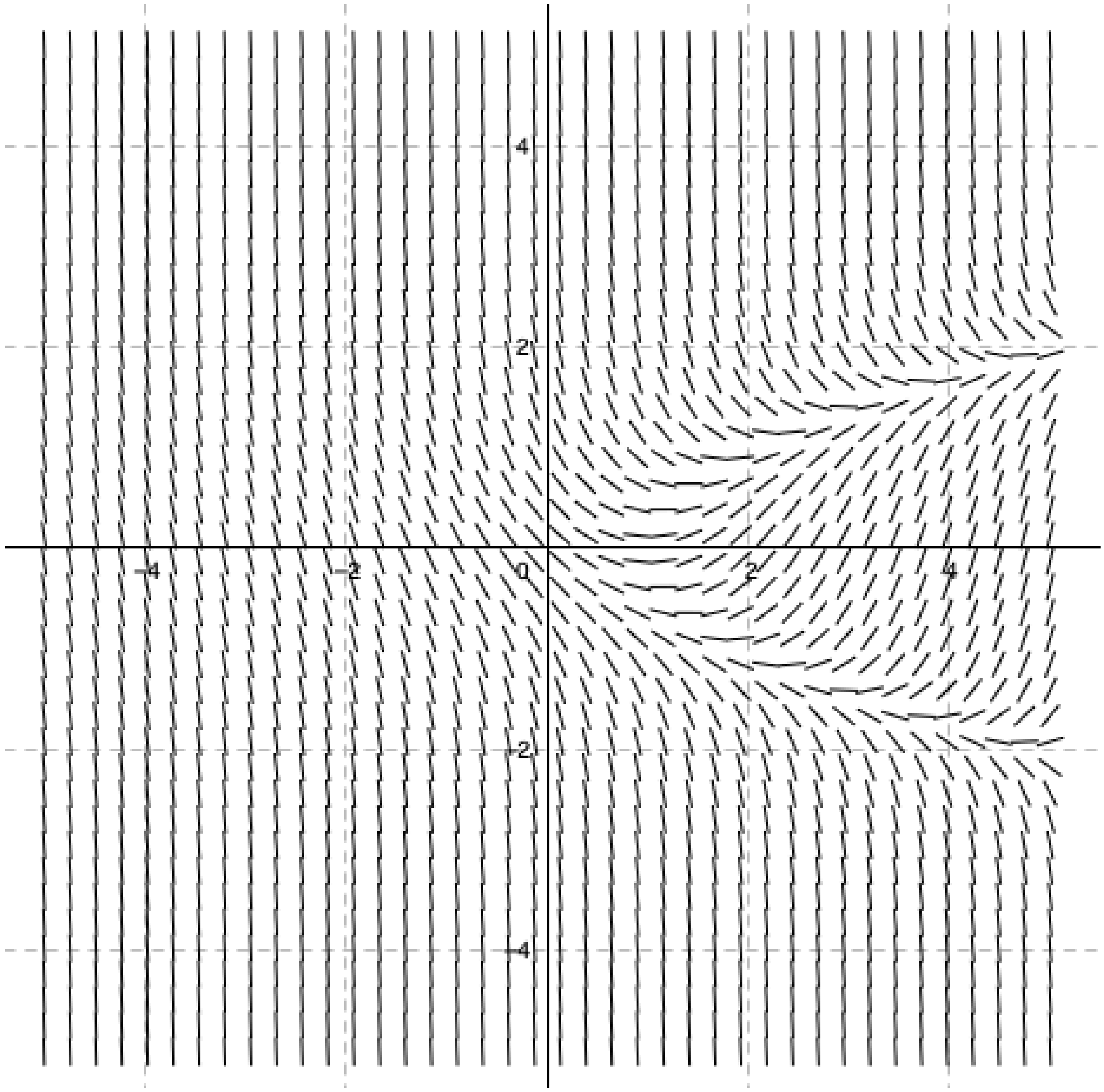
Problem 14



Based on the direction field and on the differential equation, for $y_0 < 0$, the slopes eventually become negative, therefore tend to $-\infty$. If $y_0 = 0$, then we get an equilibrium solution. Note that slopes are zero along the curves $y = 0$ and $ty = 3$.

Problem 16



Solutions with $t_0 < 0$ all tend to $-\infty$. Solutions with initial conditions (t_0, y_0) to the right of the parabola $t = 1 + y^2$ asymptotically approach the parabola as $t \to \infty$. Integral curves with initial conditions above the parabola (and $y_0 > 0$) also approach the curve. The slopes for solutions with initial conditions below the parabola (and $y_0 < 0$) are all negative. These solutions tend to $-\infty$.

Problem 27 [FOR GRADE]

The solution of the initial value problem

$$y_1' + 2y_1 = 0, \quad y_1(0) = 1$$

is $y_1(t) = e^{-2t}$. Therefore by approaching to 1 from the left side (1^- notation), we get $y(1^-) = y_1(1) = e^{-2}$. On the interval $(1, \infty)$, the differential equation is $y_2' + y_2 = 0$ with $y_2(t) = ce^{-t}$. Therefore by approaching 1 from the right side (notationally 1^+), we see $y(1^+) = y_2(1) = ce^{-1}$. Equating both the limits of the function

$$y(1^-) = y(1^+) \iff c = e^{-1}$$

Therefore the global solution is

$$y(t) = \begin{cases} e^{-2t}, & 0 \leq t \leq 1 \\ e^{-1-t}, & t > 1 \end{cases}$$

Problem 28

The Eleventh Edition (latest) of the book doesn't have this problem.

Section 2.6

Problem 3 [FOR GRADE]

They have the form $M(x,y) + N(x,y)\frac{dy}{dx} = 0$. So

$$M(x,y) = 3x^2 - 2xy + 2 \quad \text{and} \quad N(x,y) = 6y^2 - x^2 + 3$$

Then we see $\frac{\partial M}{\partial y} = -2x$ and $\frac{\partial N}{\partial x} = -2x$. Therefore, our equation is of exact form. So our solution $F_x = M \implies F = \int M dx = x^3 - x^2 y + 2x + g(y)$. Then

$$F_y = -x^2 + g'(y) = N \implies g'(y) = 6y^2 + 3 \implies g(y) = 2y^3 + 3y$$

Finally,

$$F = x^3 - x^2 y + 2x + 2y^3 + 3y = C$$

Problem 5

$$\begin{aligned} \frac{dy}{dx} &= -\frac{ax - by}{bx - cy} \\ \iff (ax - by)dx + (bx - cy)dy &= 0 \end{aligned}$$

Now, $M = ax - by$ and $N = bx - cy$. See that

$$M_y = -b \neq N_x = b$$

The differential equation is not exact.

Problem 13

Integrating $\psi_y = N$, while holding x constant, yields $\psi(x,y) = \int N(x,y)dy + h(x)$ Taking the partial derivative with respect to $x, \psi_x = \int \frac{\partial}{\partial x} N(x,y)dy + h'(x)$. Now set $\psi_x = M(x,y)$ and therefore $h'(x) = M(x,y) - \int \frac{\partial}{\partial x} N(x,y)dy$. Based on the fact that $M_y = N_x$, it follows that $\frac{\partial}{\partial y}[h'(x)] = 0$. Hence the expression for $h'(x)$ can be integrated to obtain

$$h(x) = \int M(x,y)dx - \int \left[\int \frac{\partial}{\partial x} N(x,y)dy \right] dx$$

Problem 15 [FOR GRADE]

$$\begin{aligned} M = x^2y^3, \qquad & N = x(1+y^2) \\ \implies M_y = 3x^2y^2, \qquad & N_x = 1 + y^2 \end{aligned}$$

Trivially, not exact. Let $\mu(x,y) = \frac{1}{xy^3}$, then

$$M \times \mu = x, \qquad N \times \mu = \frac{1+y^2}{y^3} \implies (M \times \mu)_y = 0, \qquad (N \times \mu)_x = 0$$

Now they're exact!

So then just find that $F = \frac{x^2}{2} - \frac{1}{2y^2} + \ln(y)$

Problem 18

$$\begin{aligned} M = 3x^2y + 2xy + y^3, \qquad & N = x^2 + y^2 \\ \implies M_y = 3x^2 + 2x + 3y^2, \qquad & N_x = 2x \end{aligned}$$

Let us find the integrating factor

$$\begin{aligned}
\mu(y) &= \exp\left(\int \frac{M_y - N_x}{N} dx\right) \\
&= \exp\left(\int \frac{3x^2 + 2x + 3y^2 - 2x}{x^2 + y^2} dx\right) \\
&= \exp\left(\int 3dx\right) \\
&= e^{3x}
\end{aligned}$$

Simply confirm that $M\mu$ and $N\mu$ are now exact. Find $F(x, y) = e^{3x}y(3x^2 + y^2) = C$