

$$\hookrightarrow f(x, y) = x(y+1) - x^2$$

$$= xy + x - x^2$$

$$f_x(x, y) = y + 1 - 2x \quad (\text{with respect to } x)$$

$$f_y(x, y) = x + 0 - 0$$

$$= x \quad (\text{with respect to } y)$$

$$f(x) = 2x$$

$$f'(x) = 2x^0$$

$$f(y) = 7y$$

$$f'(y) = 7$$

$$f(x) = k$$

$$f'(x) = 0$$

$$f(x, y) = 3x^2y^5 + \cos(x)\sin(y) - (y \cdot e^y \cdot x)$$

$$f'_x(x, y) = 6xy^5 + \sin(y) \cdot (\cos(x) \cdot x' - (\cos(x)) \cdot x)$$

$$= 6xy^5 + \sin(y)(\cos(x) - x\sin(x))$$

Product Rule

$$(fg)' = f'g + f \cdot g'$$

$$f_y = 15x^2y^4 + \cos(x) \cdot \cos(y) \cdot x - x \cdot (e^y + y e^y)$$

$\cos(x)$   $\sin(y)$   $x$   
 $\uparrow$   $\uparrow$   $\uparrow$   
 first second third  
 fun fun fun

$$f(x, y) = \cos(x) \cdot \sin(y) \cdot x$$

$$\begin{aligned}
 f_y(x, y) &= (\cos(x) \cdot x) \cdot (\sin(y))' \\
 &= (\cos(x) \cdot x) \cdot (\cos(y)) \\
 &= x \cdot \cos(x) \cdot \cos(y)
 \end{aligned}$$

$$f(x_1, x_2, \dots, x_n) = \langle \text{something} \rangle$$

$$f_{x_k}(x_1, x_2, \dots, x_n) = \left( \begin{matrix} \text{everything} \\ \text{without } x_k \end{matrix} \right) \cdot \left( \begin{matrix} \text{everything} \\ \text{with } x_k \end{matrix} \right)'$$

$$f(x, y) = e^{xy}$$

$$f_x(x, y) = y e^{xy}$$

$$f_y(x, y) = x e^{xy}$$

Chain  
Rules

$$f(x, y) = y \cdot e^y \cdot x$$

$$f_x(x, y) = y e^y$$

$$\begin{aligned} f_y(x, y) &= (x) \cdot ((y)' \cdot e^y + y \cdot (e^y)') \\ &= (x) \cdot (e^y + y e^y) \end{aligned}$$

$$f(x, y) = e^{x^2 \cdot y} \cdot y \cdot \cos(x) + \sin(y) \cdot e^{\sin(y) \cos(y)}$$

$$f_x(x, y) = (y) \cdot (e^{x^2 y} \cdot \cos(x))'$$

Product Rule

$$= (y) \cdot ((e^{x^2 y})' \cdot \cos(x) + e^{x^2 y} (\cos(x))')$$

Chain Rule  
Product Rule

$$= (y) \cdot (e^{x^2 y} \cdot 2xy \cdot \cos(x) + e^{x^2 y} \cdot (-\sin(x)))$$

+ 0

$f_y$  is homework

$$f(x, y) = \sin(y) \cdot e^{\cos(y) \sin(y)}$$

$$f_x(x, y) = 0$$


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$$F(x, y, z, u, v, w, a, b) = xyzwv^2ab$$

$$F_x = yzwv^2ab$$

$$F_v = xyzw \cdot (2v)ab$$

$$F_y = xzwv^2ab$$