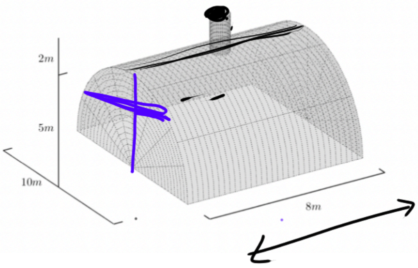
(10 points) A tank shaped like a semicircular cylinder is shown below. The length of the cylinder is 8 meters and the semicircle has radius 5. **Set-up**, do not solve, an integral which represents the work required to empty the tank by pumping all of the water to the top spout on the tank. The spout is $2\,m$ long.

Note: The density of water is 1000 $\frac{kg}{m^3}$ and the acceleration of gravity is 9.8 $\frac{m}{s^2}$.

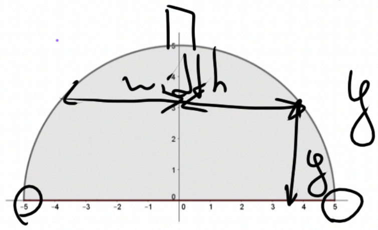
$x^2+y^2=R^2$

$y^2+\text{width}=5^2$

2



y



① Work = Force × Displacement

= Mass × Gravity × Displacement

= Volume × Density × Gravity × Displacement

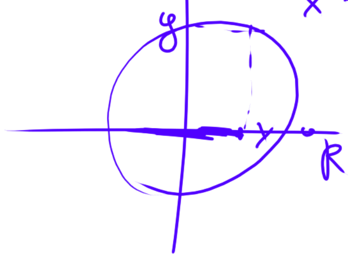
= Area × Height × Density × Gravity × Displacement

= Length × Width × Height × Density × Gravity × Displacement

<u>Length</u>

$L = 8m$

$x^2+y^2=R^2$



<u>Width</u>

$\text{width} = 2\sqrt{25-y^2}$

<u>Height</u>

Δy

I]

Density

let $\rho = 1000 \frac{kg}{m^3}$

Gravity

let $g = 9.8 \frac{m}{s^2}$

Displacement

let $d = 7 - y$

Let's build the integral

$$W = \int_{y=0}^{y=5} 8 \cdot 2\sqrt{25-y^2} \cdot dy \cdot 1000 \cdot 9.8 \cdot (7-y)$$

$$= 9.8 \cdot 16000 \int_0^5 \sqrt{25-y^2} \cdot (7-y)\, dy$$

II -)

② $L = 3m$

$$\rho(x) = 2x(4-x)\ kg\ m^{-1}$$

Here's what's happen

Solution

$$\text{Work} = \text{Force} \times \text{Displacement}$$
$$= \text{Mass} \times \text{Gravity} \times \text{Displacement}$$
$$= \text{Length} \times \text{Density} \times \text{Gravity} \times \text{Displacement}$$

Δx $\quad 2x(4-x) \quad 9.8 \quad 3-x$

$$W = \int_0^3 2x(4-x) \cdot 9.8 \cdot (3-x)\, dx$$

$$= 220.5\ J$$