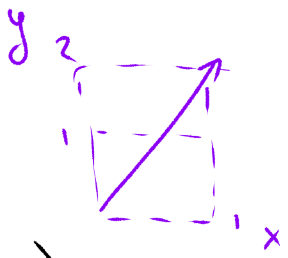
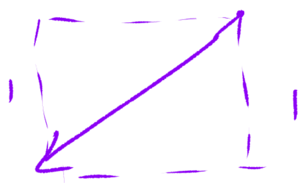
2D vectors!

Vector defines direction in space.

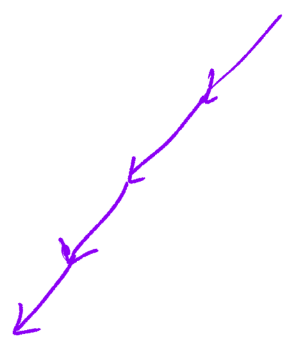
For example, $\begin{pmatrix}1\\2\end{pmatrix} = \langle 1,2\rangle = (1,2)$ tells us that we move twice as far in y direction than in x direction.



Vector $\begin{pmatrix}-1\\-1\end{pmatrix}$



No start, no finish. Take $\begin{pmatrix}-1\\-1\end{pmatrix}$



I

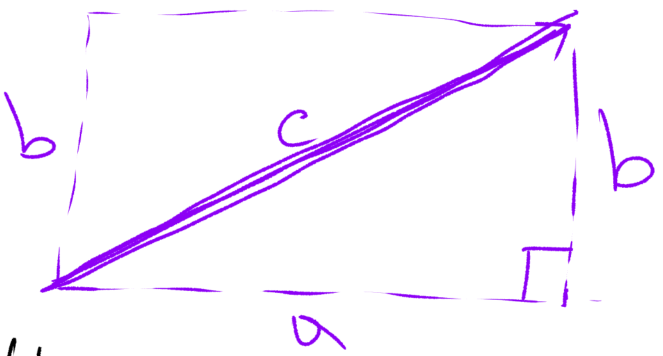
The magnitude of vectors tell you how fast you're going

$$\left\|\begin{pmatrix}a\\b\end{pmatrix}\right\| = \sqrt{a^2+b^2}$$

$$\left\|\begin{pmatrix}x\\y\\z\end{pmatrix}\right\| = \sqrt{x^2+y^2+z^2}$$

Why this

$$\begin{pmatrix}a\\b\end{pmatrix}$$



Pythagoras!!!

$$c^2 = a^2 + b^2$$
$$\left\|\begin{pmatrix}a\\b\end{pmatrix}\right\|^2 = a^2 + b^2$$
$$\left\|\begin{pmatrix}a\\b\end{pmatrix}\right\| = \sqrt{a^2+b^2}$$

Unit vector is a vector with magnitude = 1.

$\left\|\begin{pmatrix}1\\1\end{pmatrix}\right\| = \sqrt{2}$! No. | $\left\|\begin{pmatrix}0\\1\end{pmatrix}\right\| = 1$. Yes.

You can make any vector into a unit vector.

Let $\vec{v} = \begin{pmatrix} a \\ b \end{pmatrix}$,

$$\hat{v} = \frac{\vec{v}}{\|\vec{v}\|}$$

unit vector $\hat{v}$

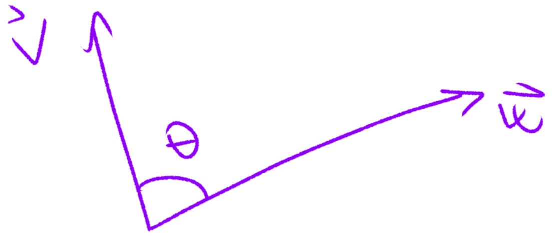
This vector goes in the same direction as $\vec{v}$, but with magnitude $= 1$.

Dot Product

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} c \\ d \end{pmatrix} = \overbrace{a \cdot c + b \cdot d}^{\text{scalar}}$$

$$\vec{v} \cdot \vec{w} = \|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos\theta$$

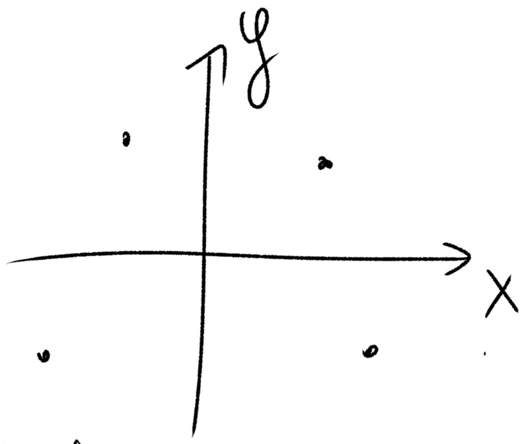
θ is the acute angle between $\vec{v}$ and $\vec{w}$



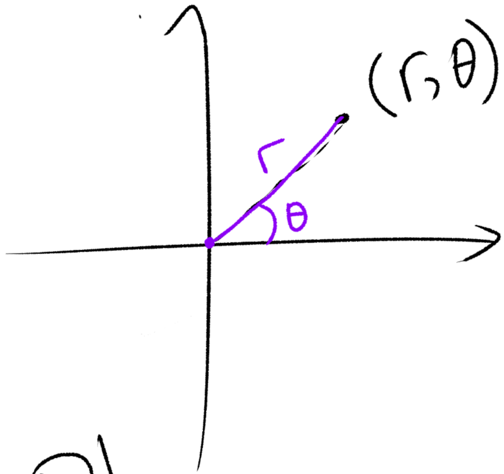
$$\Rightarrow \theta = \arccos\left(\frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \cdot \|\vec{w}\|}\right)$$

Polar Coordinates

Different way of thinking about space. Instead of x, y, we have r, θ.



Cartesian



Polar

How do we convert?

Relationship

All you need to know

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

- $\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{r\sin\theta}{r\cos\theta} = \dfrac{y}{x}$
- $r = \sqrt{x^2+y^2} = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta} = \sqrt{r^2} = \underline{r}$