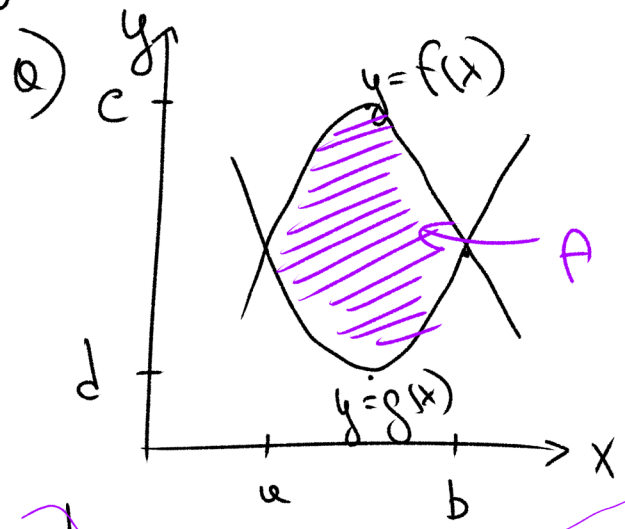
April 12th, Midterm II Review.

(1) Set-up, do <u>not</u> solve volumes for the following. Clearly state the method you used

a)

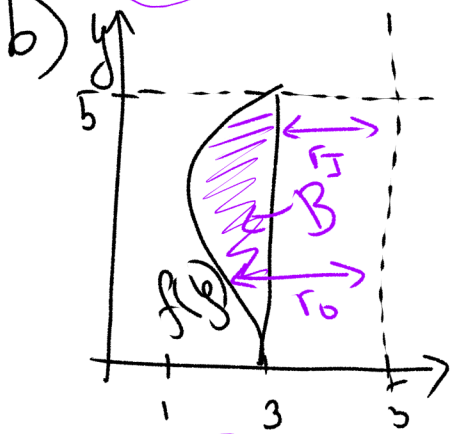


Rotate A about the y-axis.

<u>Solution</u> Shell Method

$$\text{Volume} = \int_a^b 2\pi(x)(f(x)-g(x))\,dx$$

b)



Rotate B when $x=5$

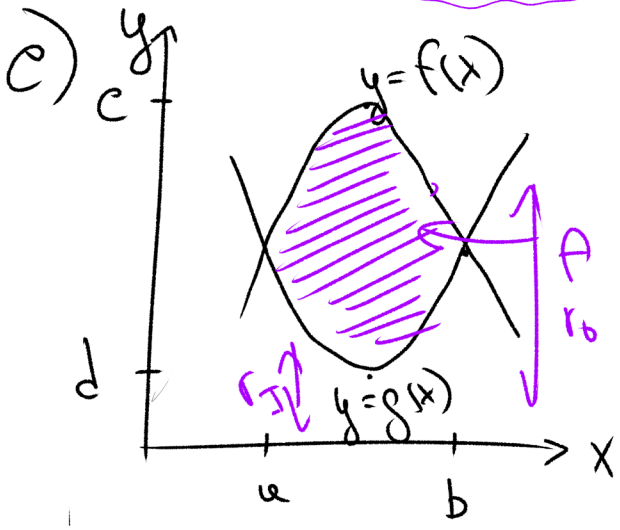
<u>Solution</u> Washer Method

$r_I = 5 - f(y)$

$r_o = 5-3$

$$\text{Volume} = \int_0^5 \pi(r_o^2 - r_I^2)\,dy$$

c)



Rotate A about $y=-1$.

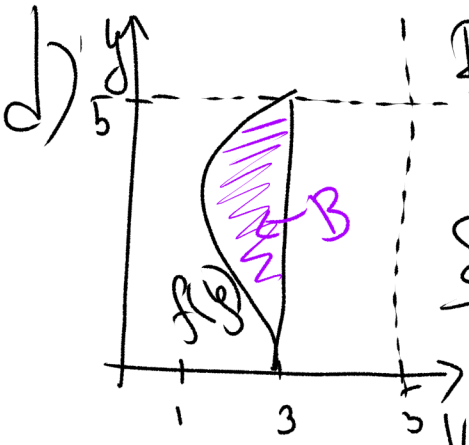
<u>Solution</u> Washer

$r_o = 1+f(x)$

$r_I = 1+g(x)$

$$\text{Volume} = \int_a^b \pi(r_o^2 - r_I^2)\,dx$$

d)



Rotate B about $y=6$

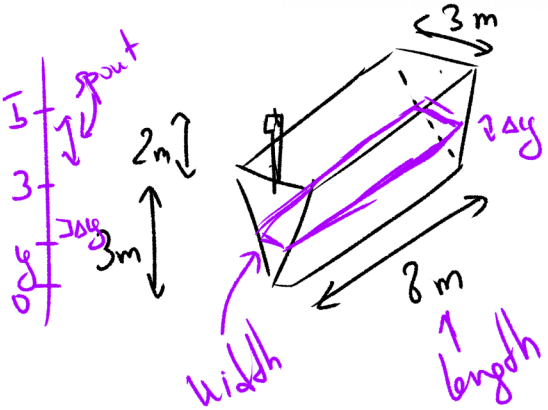
Solution Shell

$$V=\int_0^5 2\pi(6-y)(3-f(y))\,dy$$

(2)

4. *(10 points)* A tank shaped like a triangular prism is shown below. If the tank is filled to a height of 3 meters. **Set-up**, do not solve, an integral which represents the work required to empty the tank by pumping all of the water to the top spout on the tank.

Note: The density of water is 1000 $\frac{kg}{m^3}$ and the acceleration of gravity is 9.8 $\frac{m}{s^2}$.



Work = Force × Displacement

= Mass × Gravity × Displacement

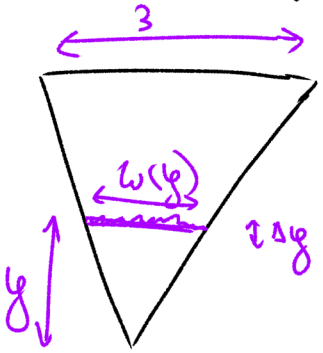
= Volume × Density × Gravity × Displacement

= Area × Height × Density × Gravity × Displ

= Width × Length × Height × Density × Gravity × Displacement

??? 8 Δy 1000 9.8 (5−y)

Check out this side view, let $w(y)$ be the width of the rect. at y



Similar Triangles:

$$\frac{w(y)}{y}=\frac{3}{3}\iff w(y)=y$$

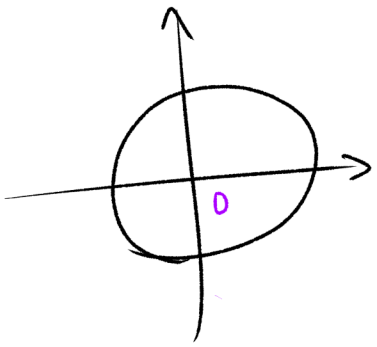
Building the integral, we get:

$$\text{Work} = \int_0^3 78400\, y(5-y)\, dy \quad J$$

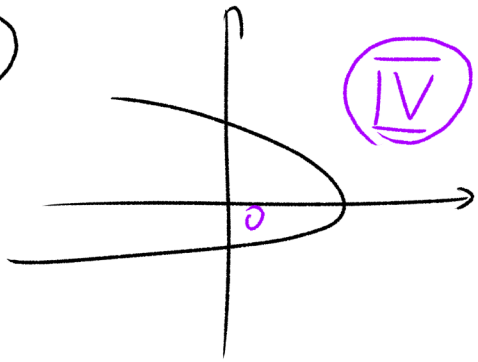
$$= 78400 \int_0^3 (5-y)\, y\, dy \quad J$$

3

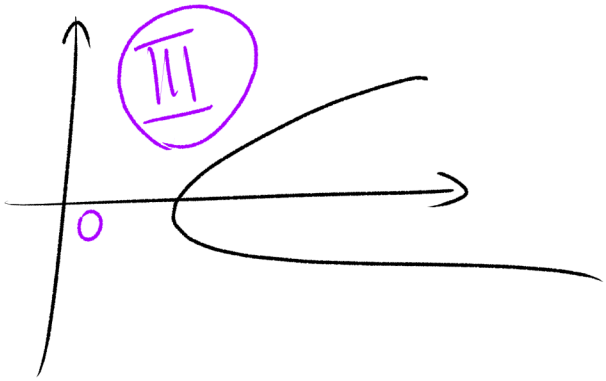
(A) (I)



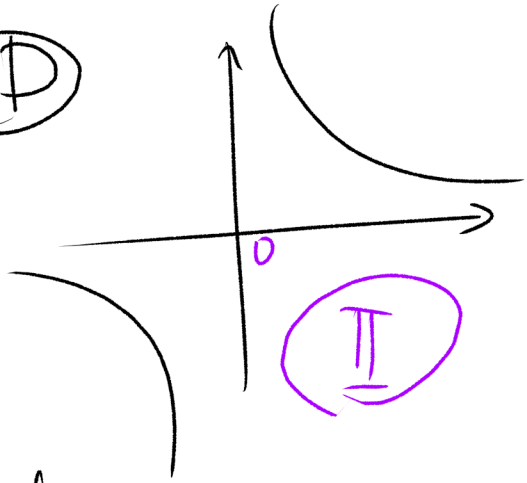
(B) (IV)



(C) (III)



(D) (II)



Match to param. eq. below

(I) $m(t) = (\cos(t), \sin(t))$ (A)

(II) $n(t) = (\sec(t), \cos(t))$ (D) (Similar to $y = 1/x$)

(III) $o(t) = (\sec^2(t), \tan(t))$ (C)

(IV) $p(t) = (\sin^2(t), \cos(t))$ (B)

III

(4) Find the improper integrals

$$\int_0^{\infty} e^{-x}\,dx \qquad \int_0^{\pi} \sin(x)\,dx \qquad \int_0^{1} \frac{1}{\sqrt{3-x^2}}\,dx$$

$$\int_0^{\pi} \sec(x)\,dx \qquad \int_{-1}^{1} \ln|x|\,dx \qquad \int_{-\infty}^{6} x^2\,dx$$

IV