Apr 28th |

Parametrized Surface

<u>Motivation</u> If we can param. a line, why not a surface?

We would need 2 vars. to define a param. surface.

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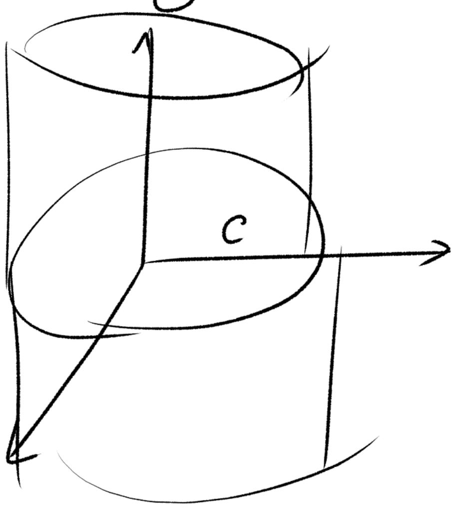
① Parametrize a cylinder $x^2+y^2=c^2$

Cylinder coord. sys.

$$\begin{cases} x = \cos\theta \cdot c \\ y = \sin\theta \cdot c \\ z \end{cases}$$



$G(\theta, z) = \langle c\cos\theta, c\sin\theta, z \rangle$

where $\theta \in [0, 2\pi]$, $z \in \mathbb{R}$.

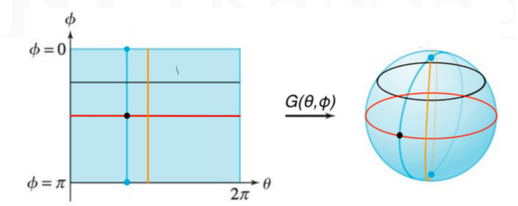
I

② Parametrize

$$x^2+y^2+z^2=c^2 \qquad \text{(Sphere)}$$

$$G=\langle c\cdot\cos\theta\cdot\sin\varphi,\ c\cdot\sin\theta\cdot\sin\varphi,\ c\cdot\cos\varphi\rangle$$

where: $\theta\in[0,2\pi)$, $\varphi\in[0,\pi]$.



Parametrization "converts" a domain of values into a surface.

II

③ Find param. of

$$x^2+y^2-z^2=\cancel{3}\ 1$$

Solution

1) Set $z=u$. Then $x^2+y^2=1+u^2$

We can let

$$x=\sqrt{1+u^2}\cos v \qquad y=\sqrt{1+u^2}\sin v$$

$$LHS = \left(\sqrt{1+u^2}\cdot\cos v\right)^2+\left(\sqrt{1+u^2}\sin v\right)^2$$

$$=(1+u^2)\cos^2 v+(1+u^2)\sin^2 v$$

$$=(1+u^2)(\cos^2 v+\sin^2 v)$$

$$=1+u^2=RHS$$, x,y work.

2) let $z=\tan u$

$$x^2+y^2=1+\tan^2 u=\sec^2 u$$

$$x=\sec u\cdot\cos v \qquad y=\sec u\cdot\sin v$$

III

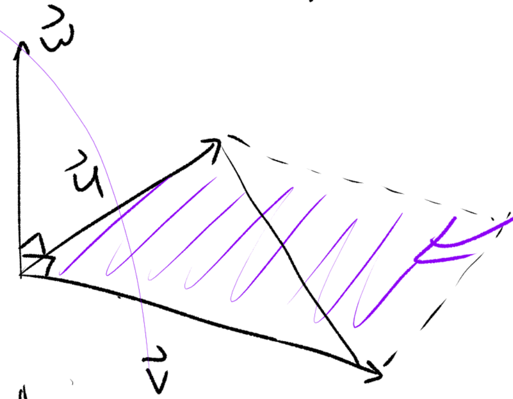
Surface Areas

$$\text{Area}(S) = \iint_D \|G_u(u,v) \times G_v(u,v)\| \, dA$$

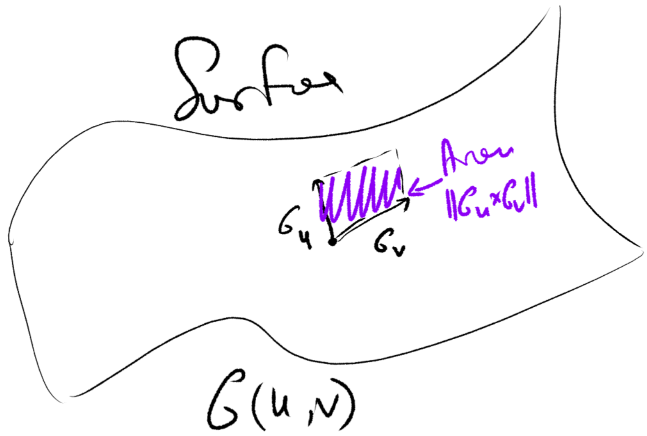
$\vec{u} \times \vec{v} = \vec{w}$

$\vec{u} \perp \vec{w}$

$\vec{v} \perp \vec{w}$



$\|\vec{w}\|$ = Area of the parallelogram



Given $x^2+y^2=4=2^2$, between $z=0$ & $z=5$

Find the surface area using the integral.

$$G(\cancel{\theta}\,\psi, \Omega) = \langle 2\cos \cancel{\theta}\,\psi, 2\sin\psi, \Omega \rangle$$

$$\psi \in [0, 2\pi], \quad \Omega \in [0,5].$$

$$G_\psi = \langle -2\sin\psi, 2\cos\psi, 0 \rangle$$

$\boxed{\text{IV}}$

$$G_\Omega = \langle 0, 0, 1 \rangle$$

$$\text{Area} = \int_{\psi=0}^{\psi=2\pi} \int_{\Omega=0}^{\Omega=5} \|\langle 2\cos\psi, 2\sin\psi, 0\rangle\| \, d\Omega \, d\psi = \int_0^{2\pi}\int_0^5 2 \, d\Omega \, d\psi$$

$$2 \cdot 5 \cdot 2\pi = 20\pi$$

$= 20\pi.$

Given $f(x,y)$, the trivial param. is

$$\sigma = \langle x, y, f(x,y) \rangle$$

$$\sigma_x = \langle 1, 0, f_x \rangle \qquad \sigma_y = \langle 0, 1, f_y \rangle$$

$$\sigma_x \times \sigma_y = \langle -f_x, -f_y, 1 \rangle$$

$$\Rightarrow \quad \|\sigma_x \times \sigma_y\| = \sqrt{1 + f_x^2 + f_y^2}$$

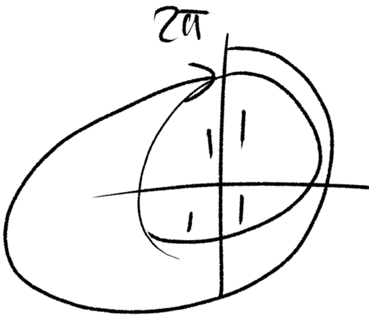
$$\text{Area} = \iint_D \sqrt{1 + f_x^2 + f_y^2}\, dA.$$

V

5. Find surface area of

$$f(x,y) = xy$$

over a unit circle.



$$\text{Area} = \iint_D \sqrt{1+x^2+y^2}\,dA$$

Let's switch to polar

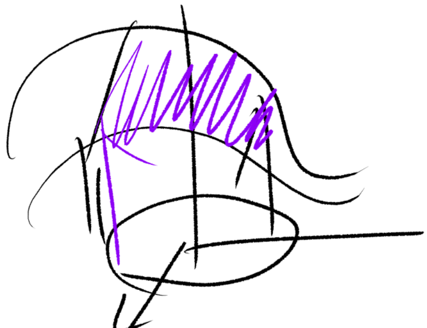
$$\text{Area} = \int_0^{2\pi}\int_0^1 \sqrt{1+r^2}\,dr\,d\theta$$

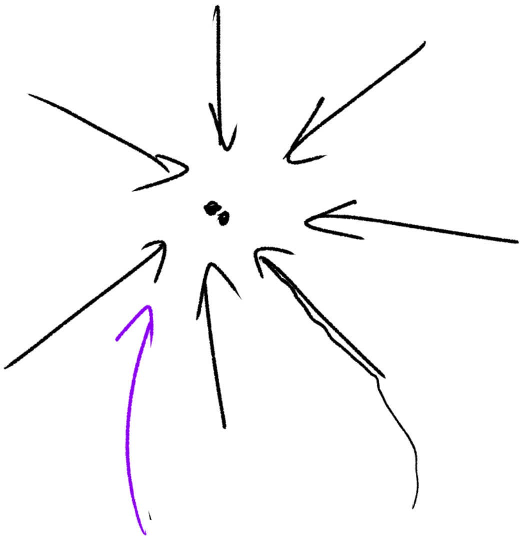
$$= 2\pi \int_0^1 \sqrt{1+r^2}\,dr$$

= (magic, sub integral)

$$= \underline{\underline{2\pi(2\sqrt{2}-1)}}$$

This is the surface area of $f = xy$ over a unit circle.





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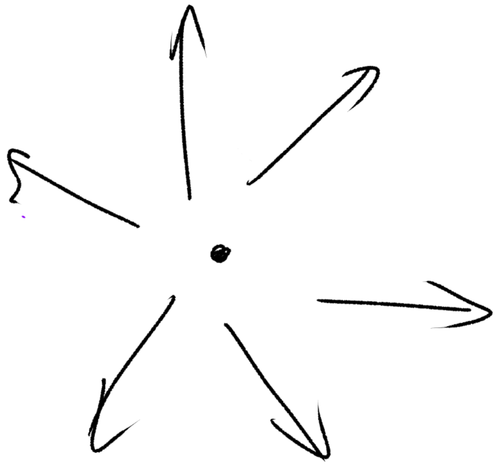
Sink



Source.

VII