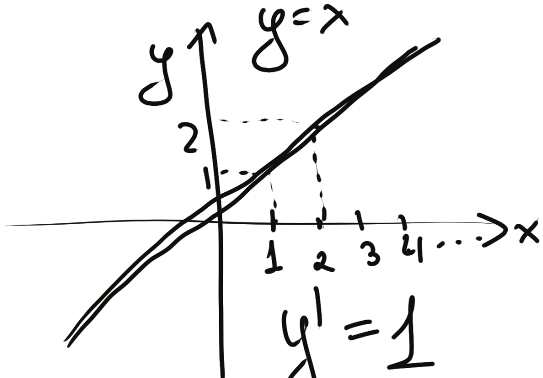
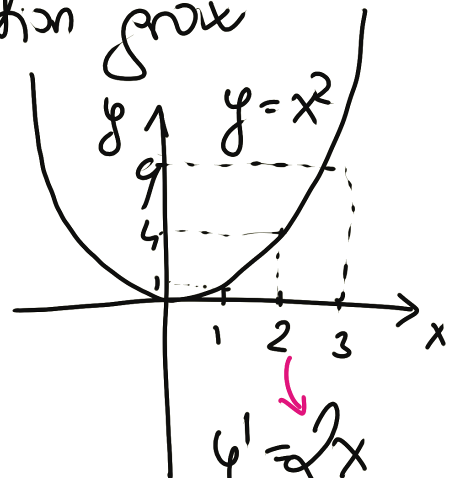
Derivatives 101

Derivative = Rate of change
= How fast does the function grow



$y = x$

$y' = 1$

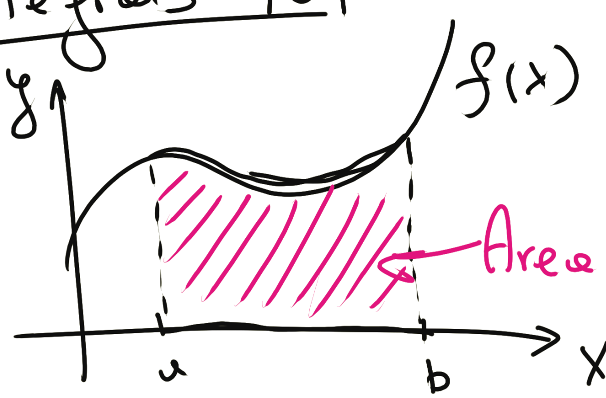


$y = x^2$

$y' = 2x$

$y = x^n \Rightarrow y' = n \cdot x^{n-1}$

Integrals 101



$= \int_a^b f(x)\,dx$

negative value

$$\lim_{x\to-\infty} x\tan\left(\frac{1}{x}\right) = \lim_{x\to-\infty} x\cdot\left(\frac{\sin\left(\frac{1}{x}\right)}{\cos\left(\frac{1}{x}\right)}\right)$$

$$\lim_{x\to-\infty} \frac{\tan\left(\frac{1}{x}\right) \to 0^-}{\frac{1}{x} \to 0^-}$$

$$\lim_{x\to-\infty} x = \infty$$

$$\lim_{x\to-\infty} \frac{1}{x} =$$ a very small negative.

L'Hôpital's iff $\lim_{x\to a} \frac{f(x)}{g(x)}$,

$f(x)=g(x)=0$ or $\pm\infty$

recognize L'Hôpital's

L'Hôpital's form

$$\lim_{x\to-\infty} x\tan\left(\frac{1}{x}\right) = \lim_{x\to-\infty} \frac{\tan\left(\frac{1}{x}\right) \to 0}{\frac{1}{x} \to 0}$$

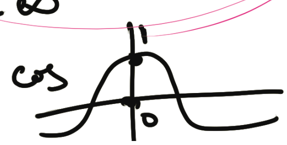
$$= \lim_{x\to-\infty} \frac{\left(\tan\left(\frac{1}{x}\right)\right)'}{\left(\frac{1}{x}\right)'} = \lim_{x\to-\infty} \frac{\sec^2\left(\frac{1}{x}\right)\cdot \cancel{\left(\frac{1}{x^2}\right)}}{-\cancel{\frac{1}{x^2}}}$$

cancelled

Apply L'Hôpital

$$= \lim_{x\to-\infty} \sec^2\left(\frac{1}{x}\right) = \lim_{x\to-\infty} \left(\frac{1}{\cos\left(\frac{1}{x}\right)}\right)^2 = 1$$

Cancel



=

simplify

$$\int \frac{1+\sqrt{x}-x^3}{x^{3/2}} + \frac{1}{x\sqrt{x^2-1}}\,dx$$ very famous

$$= \int \frac{1+\sqrt{x}-x^3}{x^{3/2}}\,dx + \int \frac{1}{x\sqrt{x^2-1}}\,dx$$

$$= \int \frac{1}{x^{3/2}}\,dx + \int \frac{\sqrt{x}}{x^{3/2}}\,dx - \int \frac{x^3}{x^{3/2}}\,dx +$$ solved

$\sqrt{x} = x^{1/2}$

$\frac{1}{x^{3/2}} = x^{-3/2}$

$\frac{x^3}{x^{3/2}} = x^{3/2}$

$$\frac{x^{1/2}}{x^{3/2}} = x^{\frac{1}{2}-\frac{3}{2}} = x^{-1} = \frac{1}{x}$$

$$= -\frac{2}{3} \cdot x^{-\frac{1}{2}} + \ln(|x|) + \frac{2}{3}x^{\frac{5}{2}} + C$$

$$\int x^{-1}\,dx = \ln x + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n} + C$$ famous

$$\int_{\sqrt{e}}^{e^2} \frac{1}{t\ln|t|}\,dt = \int_{\frac{1}{2}}^{2} \cancel{\frac{1}{t}} \cdot \frac{1}{u} \cdot \cancel{t} \cdot du = \ln u \Big|_{\frac{1}{2}}^{2}$$

$t \cdot du$ $\quad u$

$u' \quad u = \ln|t|$

$$\boxed{\begin{aligned} \ln|\sqrt{e}| &= \frac{1}{2} \\ \ln|e^2| &= 2 \end{aligned}}$$

$$u' = \frac{du}{dt} = \frac{1}{t} \Rightarrow dt = du \cdot t$$

$$= \ln(2) - \ln\left(\frac{1}{2}\right) = \ln\left(\frac{2}{\frac{1}{2}}\right)$$

$\sin^2 + \cos^2 = 1$

$$= \ln(4)$$

$$\boxed{\begin{aligned} &\int \sqrt{1-x^2}\,dx \\ &x = \sin\theta \\ &\int \sqrt{1-\sin^2\theta}\,dx \end{aligned}}$$

$= \cos\theta$

$$\ln a^b = b \cdot \ln a \quad \text{(log Property)}$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(ab) = \ln(a) + \ln(b)$$

Facts

$$\ln e = 1$$

$$\ln\sqrt{e} = \ln e^{\frac{1}{2}} = \frac{1}{2}\ln e = \frac{1}{2} \cdot 1 = \frac{1}{2}$$

$$\log_a b = c \Leftrightarrow a^c = b$$

$$\ln a = \log_e a$$

$$\log_a a = 1 \qquad \ln e = \log_e e = 1$$