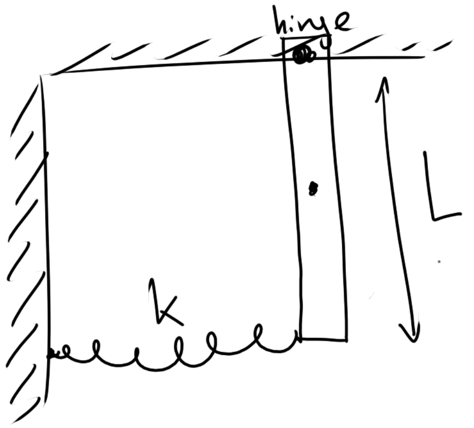
(1)

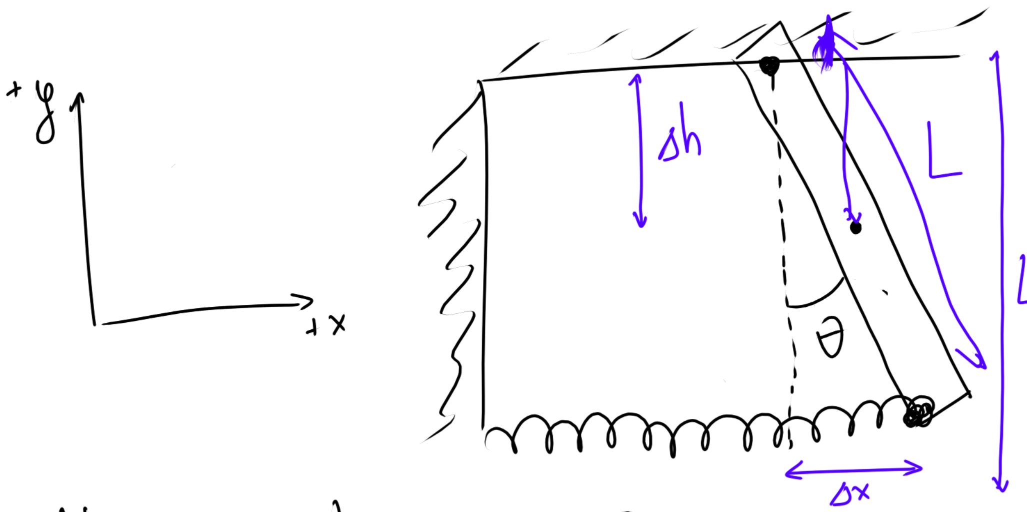


1.5m uniformly dense rigid rod with mass 3kg, where

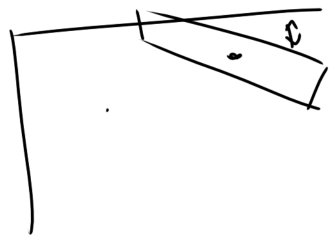
$k = 10\,N/m$, $L = 1.5\,m$

Suspended 10° from vertical and released.

Find the maximum angular velocity ($rad\,s^{-1}$).



$\theta = 10^\circ$



We need an Energy Equation

- Spring: $\frac{1}{2}k\cdot(\Delta x)^2$, Δx the displacement
- Potential: $mg\Delta h$, Δh is the y
- Inertia: $\frac{1}{3}mL^2 = I$; $\frac{1}{2}I\omega^2$ ← Energy

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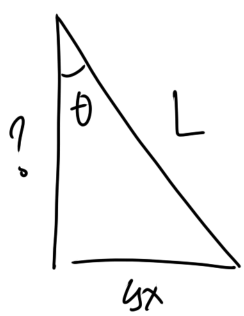

$\Delta x = \sin\theta\, L$ | $\Delta h = -\frac{L}{2}\cdot\cos\theta$

I

$$E = \frac{1}{2}k(\Delta x)^2 + mg\Delta h + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}k(L\sin\theta)^2 + mg\left(-\frac{L}{2}\cos\theta\right) + \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2$$

$$= \frac{1}{2}kL^2\sin^2\theta - \frac{mgL}{2}\cos\theta + \frac{1}{6}mL^2\omega^2$$

1. Find maximum possible energy in this system. This is when:

Before suspended:

- velocity = 0 $\Rightarrow$ Inertia = 0
- potential = max
- spring = max

2. After swinging (back to vertical)

- velocity = max $\Rightarrow$ Inertia = max
- potential = 0
- spring = 0

3. Isolated. (Potential + Spring) = Inertia

II

Let's try find max Energy at the top:

$$E_{max} = \frac{1}{2} k L^2 \sin^2\theta - \frac{mgL}{2}\cos\theta$$

$$= \frac{1}{2} \cdot (10\,N/m) \cdot (1.5m)^2 \sin^2 10° - \frac{(3kg)(-9.8ms^{-2})(1.5m)}{2} \cdot (\cos 10°)$$

$$\approx 22.05\,J$$

All this Energy will be converted into inertia ⇒ velocity, as

change in height = 0 ⇒ potential E = 0

change in spring length = 0 ⇒ spring E = 0

$$\frac{1}{2} I\omega^2 = 22.05\,J = E_{max}$$

$$\Rightarrow \frac{1}{6} mL^2\omega^2 = 22.05\,J$$

$$\Rightarrow \omega = \sqrt{\frac{6 \cdot 22.05\,J}{(3kg)(1.5m)^2}}$$

$$= 4.427\ rad\,s^{-1}$$

III

With Calculus: (Not finished) $\frac{d}{dx}(\sin^2(x)) = \sin(2x)$

$$E = \frac{1}{2}kL^2\sin^2\theta - \frac{mgL}{2}\cos\theta + \frac{1}{6}mL^2\omega^2 = 0$$

find $\frac{dE}{d\theta}$:

$$0 = \frac{dE}{d\theta} = \frac{1}{2}kL^2\left(\cancel{2\cos\theta}\sin\overset{(2\theta)}{\cancel{\theta}}\right) + \frac{mgL}{2}\sin\theta + \frac{1}{6}mL^2\frac{d\omega^2}{d\theta}$$

$$\frac{d\omega^2}{d\theta} = \alpha = \ddot{\theta}$$

$$\ddot{\theta} = \alpha = -\left[\frac{6}{mL^2}\left(\frac{1}{2}kL^2\left(\cancel{2\cos\theta}\sin\overset{2\theta}{\cancel{\theta}}\right) + \frac{mgL}{2}\sin\theta\right)\right]$$

$$= -\left[\frac{3k}{m}\left(\cancel{2\cos\theta}\sin\overset{2\theta}{\cancel{\theta}}\right) + \frac{3g}{L}\sin\theta\right]$$

Small Angle Approximation: $\begin{pmatrix}\theta\to 0\\ \sin\theta\to\theta\\ \cos\theta\to 1\end{pmatrix}$

$$\ddot{\theta} = \alpha = -\left(\frac{6k}{m}\theta\cdot 2 + \frac{3g}{L}\theta\right) = -\theta\left(-\frac{6k\cdot 2}{m} \cancel{+} \frac{3g}{L}\right)$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\theta\left(\frac{6k\cdot 2}{m} + \frac{3g}{L}\right) \qquad (\alpha = \theta\omega^2)$$

$$\Rightarrow \omega = \sqrt{\frac{6k\cdot 2}{m} + \frac{3g}{L}}$$

$$= 4.516 \text{ rad s}^{-1}$$

IV