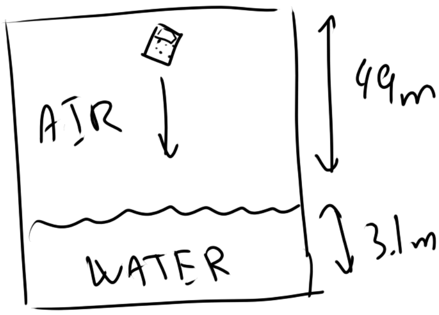
March 4th

[redacted]The cellphone starts from rest before falling 15 floors (49 m) and splashing into a pool. Afterward, the cellphone sinks to a depth of 3.1 m before coming to a stop. What is the average acceleration experienced by the cellphone as it comes to a stop in the water? You can neglect air resistance during the fall from the balcony.

1 Given:

PICTURE



a) Find V_f in Air before it hits the water

b) Find acceleration in water after 3.1m

Solution

a) Recall $V_f^2 = V_i^2 + 2aS$

Then $V_f = \sqrt{0^2 + 2 \cdot 9.8 \cdot 49} \approx 31 ms^{-1}$ ↓ Direction

b) $V_f^2 = V_i^2 + 2aS \Rightarrow a = \dfrac{V_f^2 - V_i^2}{2S}$

$$= \frac{0 - 31^2}{2 \cdot 3.1} = 155 ms^{-2}$$

Answer

2 Isolated oscillating system along x-axis

$\frac{dE}{dx} = 0$

$$E = (3kg) \cdot V_x^2 + \left(12 \frac{J}{m^2}\right) \cdot x^2$$

Question: what's the angular frequency in rad/s? ω?

(Calculus Solution):

because Isolated

$$E = 3v_x^2 + 12x^2$$

$$\frac{dE}{dx} = 2\cdot 3\cdot v_x \cdot \frac{dv_x}{dx} + 24x = 0$$

$$\Rightarrow v_x \frac{dv_x}{dx} = -\frac{24x}{2\cdot 3} = -4x$$

Recall: $a_x = \underbrace{v_x \frac{dv_x}{dx}} = -4x = \frac{d^2x}{dt^2}$

Recall: $\frac{d^2x}{dt^2} = -\omega^2 x \Rightarrow \omega = \sqrt{\frac{a_x}{x}} = 2\,\text{rad}\,s^{-1}$

answer

(Pattern finding)

$$E = (3\,kg)v_x^2 + \left(12\,\frac{J}{m^2}\right)x^2$$

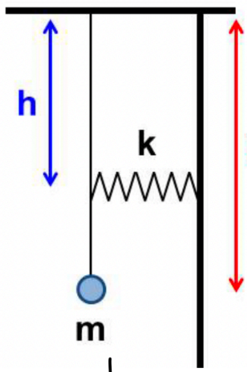
$$= m v_x^2 + k x^2$$ ← spring system energy

AND!!! We know that

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{12}{3}} = \sqrt{4} = 2\,\text{rad}\,s^{-1}$$

(3)

(2) A 5 kg mass is connected to a thin massless, but rigid rod of length $L = 1.3$ m to form a simple pendulum. The rod is connected to a nearby vertical wall by a spring with spring constant $k = 75$ J/m^2, connected to it at a distance $h = 1.1$ m below its point of suspension. What is the angular frequency (in rad/s) of the system for small amplitude oscillations?



$I = mL^2$

$\frac{dE}{d\theta} = 0$

Solution

$$E = \frac{1}{2}I\omega^2 - mgL\cos\theta + \frac{1}{2}k(h\sin\theta)^2$$

$$E = \frac{1}{2}I\omega^2 - mgL\cos\theta + \frac{1}{2}kh^2(\sin\theta)^2$$

$$\frac{dE}{d\theta} = 0 \Rightarrow I\alpha + mgL\sin\theta + kh^2 \cdot \sin\theta \cdot \cos\theta = 0$$

$$mL^2\alpha + mgL\sin\theta + kh^2\sin\theta\cos\theta = 0$$

Solve for angular acceleration

$$\alpha = \frac{-mgL\sin\theta - kh^2\sin\theta\cos\theta}{mL^2}$$

$\theta \to 0$

$\sin\theta \to \theta$

$\cos\theta \to 1$

Apply small angle approximation

$$\alpha = \frac{-mgL\theta - kh^2\theta}{mL^2} \Rightarrow \frac{d^2\theta}{dt^2} = -\alpha\theta$$

$$\Rightarrow \omega = \sqrt{\alpha}$$

$$\Rightarrow \omega = \sqrt{\frac{mgL + kh^2}{mL^2}} = \sqrt{\frac{5 \cdot 9.8 \cdot 1.3 + 75 \cdot (1.1)^2}{5 \cdot (1.3)^2}} = 4.3 \text{ rad/s}$$

answer