1) $\int \arccos^2(t)\, dt$

Recap:

$\int u\,dv = uv - \int v\,du$

Subproblem:

$\int \arccos(t)\,dt$: let $u = \arccos(t)$, $v = 1$

$\Rightarrow t \cdot \arccos(t) - \int -\frac{\cancel{x}t}{\sqrt{1-\cancel{x}_t^2}}\, d\cancel{x}t$

$= t \times \arccos(t) - \sqrt{1-t^2} + C$

Then

$\int \arccos^2(t)\, dt$, let $u = \arccos^2(t)$, $v' = 1$

$= t \cdot \arccos^2(t) - \int - \frac{2t \arccos(t)}{\sqrt{1-t^2}}\, dt$

$= t \arccos^2(t) - 2\left(\arccos(t)\sqrt{1-t^2} + t\right) + C$

I

② $\int t \arcsin(t)\,dt$, let

$\frac{du}{dt} = \frac{1}{\sqrt{1-t^2}}$

$u = \arcsin(t)$

$v' = t$

$v = t^2/2$

$$= \frac{t^2}{2}\arcsin(t) - \int \frac{t^2}{2}\cdot\frac{1}{\sqrt{1-t^2}}\,dt + C$$

Subproblem:

$\int \frac{t^2}{2}\cdot\frac{1}{\sqrt{1-t^2}}\,dt$, let $t = \sin\theta$

$\frac{dt}{d\theta} = \cos\theta$

$dt = d\theta\cos\theta$

$$= \int \frac{1}{2}\cdot\sin^2\theta\cdot\frac{1}{\cancel{\cos\theta}}\cdot d\theta\cdot\cancel{\cos\theta}$$

(Trig Sub)

$$= \frac{1}{2}\int \sin^2\theta\,d\theta$$

Integration by Parts (Next Page)

$$= \frac{1}{4}\left(\theta - \frac{1}{2}\sin(2\theta)\right)$$

next page

$\theta = \arcsin(t)$

$$= \frac{1}{4}\left[\arcsin(t) - \frac{1}{2}\sin(2\arcsin(t))\right]$$

II

$$\int \sin^2(x)\,dx,$$

$$\cancel{\text{let}}\quad \frac{du}{dx} = 2\cos(x)\sin(x)$$

$$u = \sin^2(x)$$

$$\cancel{v' = 1}$$

$$v = x$$

$$= \cancel{\sin^2(x)\cdot x - \int v\cdot 2\cos\sin(x)}$$

$$\int \sin^2(x)\,dx = \int \frac{1-\cos(2x)}{2}\,dx$$

$$= \frac{1}{2}\int 1 - \cos(2x)\,dx$$

$$= \frac{1}{2}\left(x - \sin(2x)\cdot\frac{1}{2}\right)$$

$$\Rightarrow \frac{1}{2}\int \sin^2(x)\,dx = \frac{1}{4}\left(x - \frac{1}{2}\sin(2x)\right)$$

$$\int \sec(x)\, dx$$

$$= \int \frac{\sec(x)(\tan(x) + \sec(x))}{\tan(x) + \sec(x)}\, dx$$

$$= \int \frac{\sec(x)\tan(x) + \sec^2(x)}{\tan(x) + \sec(x)}\, dx$$

Sub $u = \tan(x) + \sec(x)$

$$\frac{du}{dx} = \sec(x)\tan(x) + \sec^2(x)$$

$$dx = \frac{1}{\sec(x)\tan(x) + \sec^2(x)}$$

$$= \int \frac{\cancel{\sec(x)\tan(x) + \sec^2(x)}}{u \cdot (\cancel{\sec(x)\tan(x) + \sec^2(x)})}\, du$$

$$= \int u^{-1}\, du$$

$$= \ln|u|$$

$$= \ln|\sec(x) + \tan(x)|$$

$\arcsin(\sin(x)) = x$

$\sin(\arcsin(x)) = x$

$t = \sin\theta$

$\arcsin(t) = \arcsin(\sin(\theta))$

$\arcsin(t) = \theta$

$\Rightarrow \theta = \arcsin(t)$

$\int_0^{\pi/4} t\sec(t)\tan(t)\,dt,$ $\quad u = t$

$v' = \sec(t)\tan(t)$

$v = \sec(t)$

$= t\sec(t) - \int \sec(t)\,dt$

$= t\sec(t) - \ln|\tan(t) + \sec(t)|$

$\approx 2.81 \cdot 10^{-3}$

IV