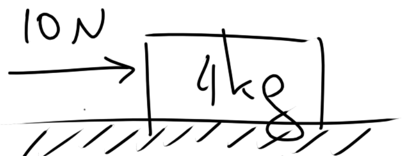
Section 8.2

initially at rest

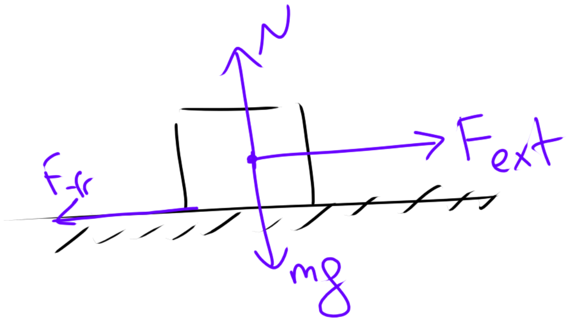
$F_{fr} = 0.2$ ← coefficient of kinetic friction

(1)



Question: what's the mag. of acc.?

Solution:



$$(F_{net})_x = ma_x \Rightarrow F_{ext} - F_{fr} = ma_x$$

$$\Rightarrow ma_x = F - \mu_x N$$

$$(F_{net})_y = ma_y \Rightarrow N - mg = ma_y$$

As $a_y = 0 \Rightarrow N = mg$

$$\Rightarrow ma_x = F - \mu_x N = F - \mu_x mg$$

$$\Rightarrow a_x = \frac{F}{m} - \mu_x g$$

$$\therefore = 0.54\,\mathrm{m\,s^{-2}}$$

I

② Same question, but find W_{net} if it's been sliding for 4s

Solution:

$$\left.\begin{array}{l} v = at \\ \Delta K = \frac{1}{2} m v_f^2 \end{array}\right\}$$

This is the total net work done in the system

$$\Rightarrow W_{net} = \Delta K = \frac{1}{2} m (at)^2$$

$$\therefore \approx 9.3 \text{ J}$$

③ Work done on the block by 10N force after sliding for 4s?

Solution:

The work done just by the external force

$$\Delta W = F \Delta S$$

$$\Delta S = \frac{1}{2} a (\Delta t)^2 \approx 4.32 \text{ m}$$

$$\therefore \Delta W = 10\text{N} \cdot 4.32\text{m} = 43.2 \text{ J}$$

let it be W_F

$+W$

④ Find work done by friction.

$$\therefore W_F + W_{fr} = W_{net} \Rightarrow W_{fr} = W_{net} - W_F \approx -33.9 \text{ J}$$

II)

Alternative Solution

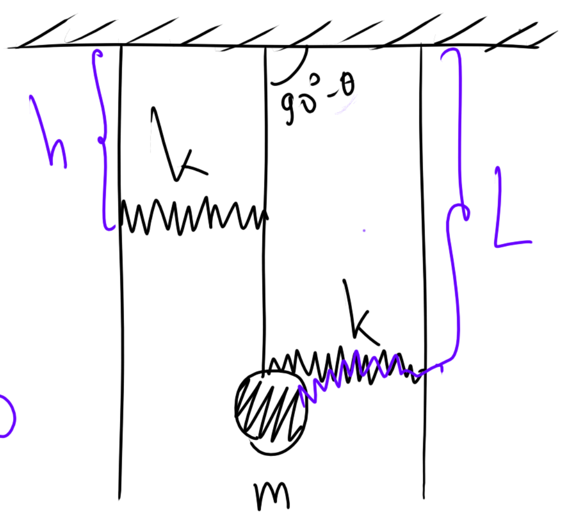
$$W_{fr} = -F_{fr}\Delta x = -\mu_k m g \Delta x$$

$$\approx -33.9\,\text{J}$$

III

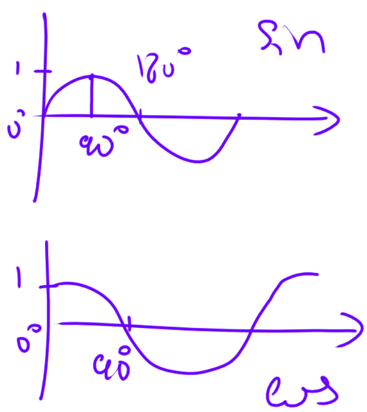
Section 6.3

(1) What is the period for small amplitude oscillations of this system? Assume that rods are massless.



Hint: the system is isolated
$\frac{dE}{d\theta} = 0$

Detour: Trig



Solution

Detour: Small Angle Approximation

$$\theta \longrightarrow 0 \implies \sin\theta = \theta$$

Proof:

$$\lim_{\theta \to 0} \frac{\sin\theta}{\theta} \underset{\text{L'Hôpital}}{=} \lim_{\theta \to 0} \frac{\frac{d}{d\theta}\sin\theta}{\frac{d}{d\theta}\theta} = \lim_{\theta \to 0} \frac{\cos\theta}{1}$$

$$= \cos 0$$

$$= 1$$

$$\implies \theta \to 0 \implies \sin\theta = \theta$$

$$\cos\theta = 1$$

① Build the energy equation

$$E=\frac{1}{2}I\omega^2-mgL\cos\theta+\frac{1}{2}k(h\sin\theta)^2+\frac{1}{2}k(L\sin\theta)^2$$

$$E=\frac{1}{2}I\omega^2-mgL\cos\theta+\frac{1}{2}k(h^2+L^2)(\sin\theta)^2$$

② The system is isolated

$$\frac{dE}{d\theta}=0 \Rightarrow I\alpha+mgL\sin\theta+k(h^2+L^2)\cos\theta\sin\theta\overset{0}{=}$$

③ Let's solve for angular acceleration

$$\alpha=\frac{-mgL\sin\theta-k(h^2+L^2)\sin\theta\cos\theta}{mL^2}$$

④ Small Angle Approximati

$$\alpha=\frac{-mgL\theta-k(h^2+L^2)\theta}{mL^2}\Rightarrow\frac{d^2\theta}{dt^2}=-\theta\cdot\left(\frac{mgL+k(h^2+L^2)}{mL^2}\right)$$

$$\Rightarrow \omega=\sqrt{\frac{mgL+k(h^2+L^2)}{mL^2}}$$

$$\Rightarrow \omega=\sqrt{\frac{g}{L}+\frac{k}{m}\left(1+\frac{h^2}{L^2}\right)}$$

$$\therefore\ T=\frac{2\pi}{\omega}=\frac{2\pi}{\sqrt{\frac{g}{L}+\frac{k}{m}\left(1+\frac{h^2}{L^2}\right)}}$$