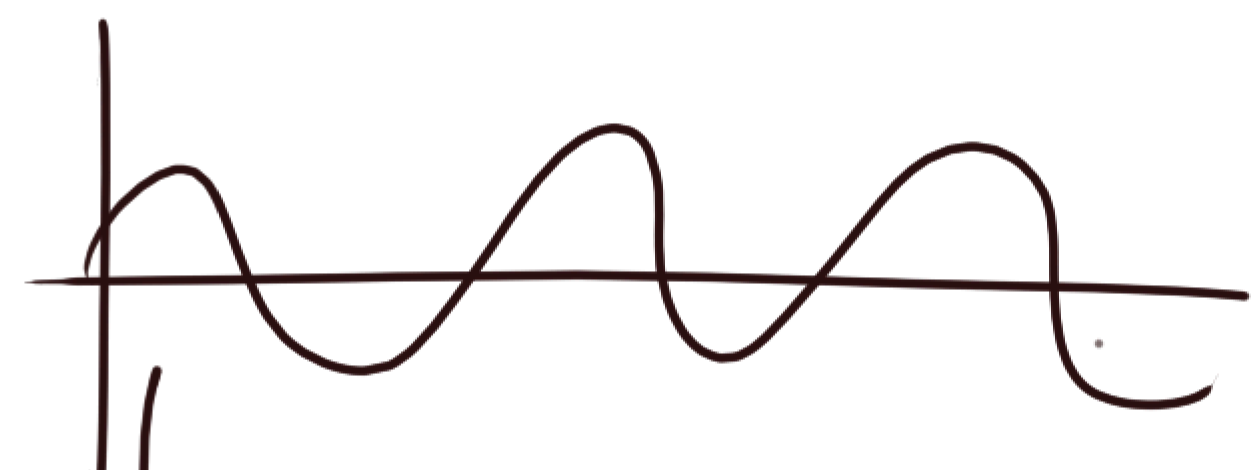
① Let $f:\mathbb{R}^2 \longrightarrow \mathbb{R}$ be defined as

$$f(x,y)=\sin(x)+\sin(y)+\sin(x+y)$$

a) Is f differentiable? b) If so compute gradient $= \nabla f$

Solution

a) Yes! Is sin diff?



Sum of diff. fn diff!

b) $f_x = \cos(x) + \cancel{\sin(y)} + \cos(x+y)$ | $f_y = \cos(y)+\cos(x+y)$

$$\nabla f = \langle \cos(x)+\cos(x+y), \cos(y)+\cos(x+y) \rangle$$

c) Is f cont. at $(0,0)$?

$$\nabla f(0,0) = \langle 2,2 \rangle$$

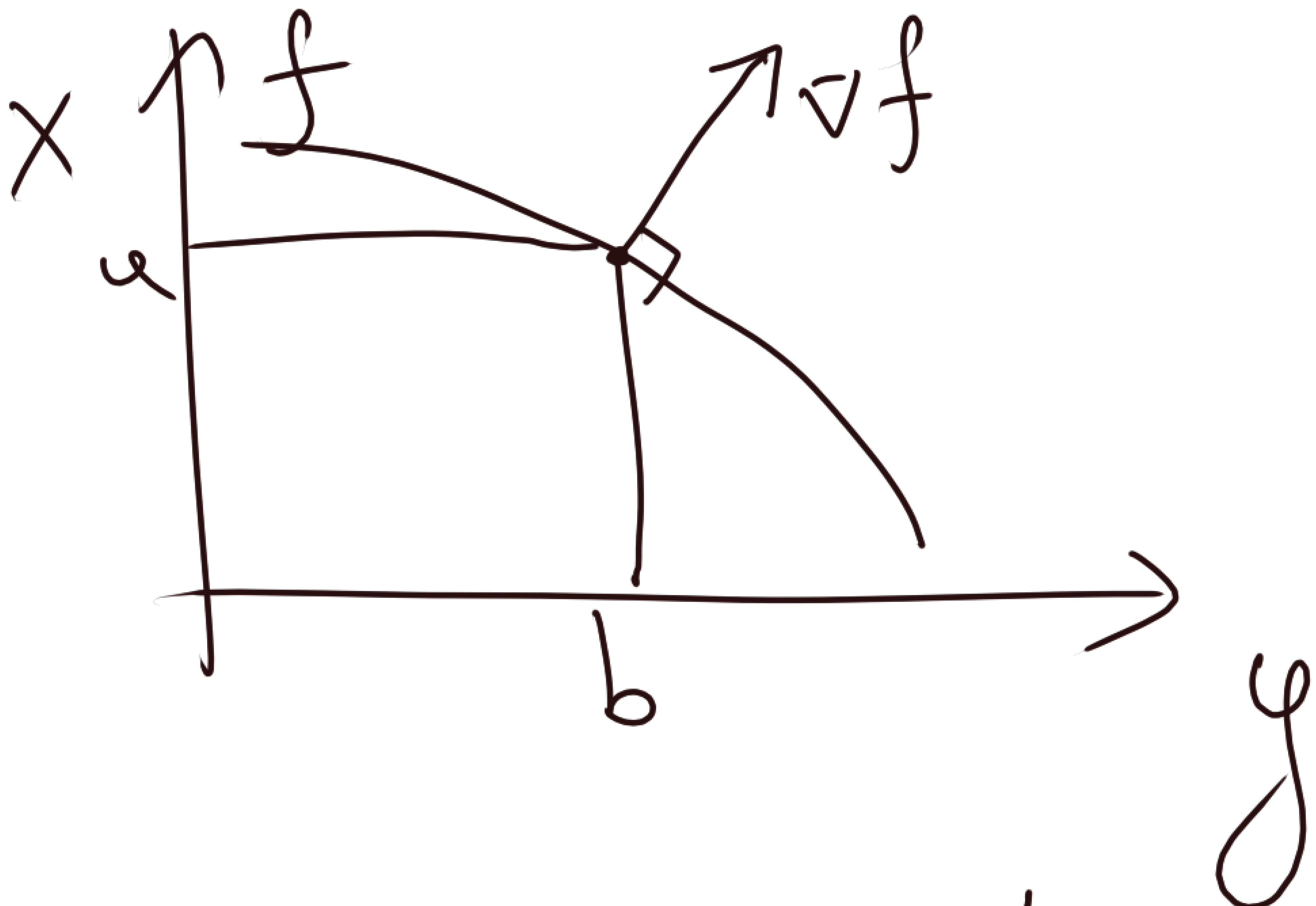
d) Compute $D_{\vec{v}} f\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$, where $\vec{v} = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

$$D_{\vec{v}} f\left(\frac{\pi}{2}, \frac{\pi}{2}\right) = \nabla f\left(\frac{\pi}{2}, \frac{\pi}{2}\right) \cdot \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$

$$= \left\langle \cos\left(\frac{\pi}{2}\right) + \cos(\pi), \cos\left(\frac{\pi}{2}\right) + \cos(\pi) \right\rangle \cdot \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$

$$= \langle 0-1, 0-1 \rangle \cdot \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle = \langle -1, -1 \rangle \cdot \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$$

$$= -\frac{\sqrt{3}}{2} - \frac{1}{2} = -\left(\frac{\sqrt{3}+1}{2}\right)$$



Draw dir. of $\nabla f(a,b)$

$\|\nabla f\|$ - highest growth point

e) Find the largest rate of change of f at $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ and the direction at which it occurs.

<u>Solution</u>: Direction is $\langle -1, -1 \rangle$

Rate of change: $\|\langle -1,-1\rangle\| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{1+1} = \sqrt{2}$

(2) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4-y^4}{x^2+y^2}$, let $f(x,y)=\dfrac{x^4-y^4}{x^2+y^2}$

$$f(r\cdot\cos\theta,\ r\cdot\sin\theta)=\frac{r^{\cancel{4}\,2}\cos^4\theta-r^4\sin^4\theta}{\cancel{r^2}\cos^2\theta+\cancel{r^2}\sin^2\theta}$$

bleume 1

$$=r^2\left(\cos^4\theta-\sin^4\theta\right)$$

$$\lim_{(x,y)\to(0,0)} f(x,y)=\lim_{r\to 0} f(r\cos\theta,\ r\sin\theta)$$

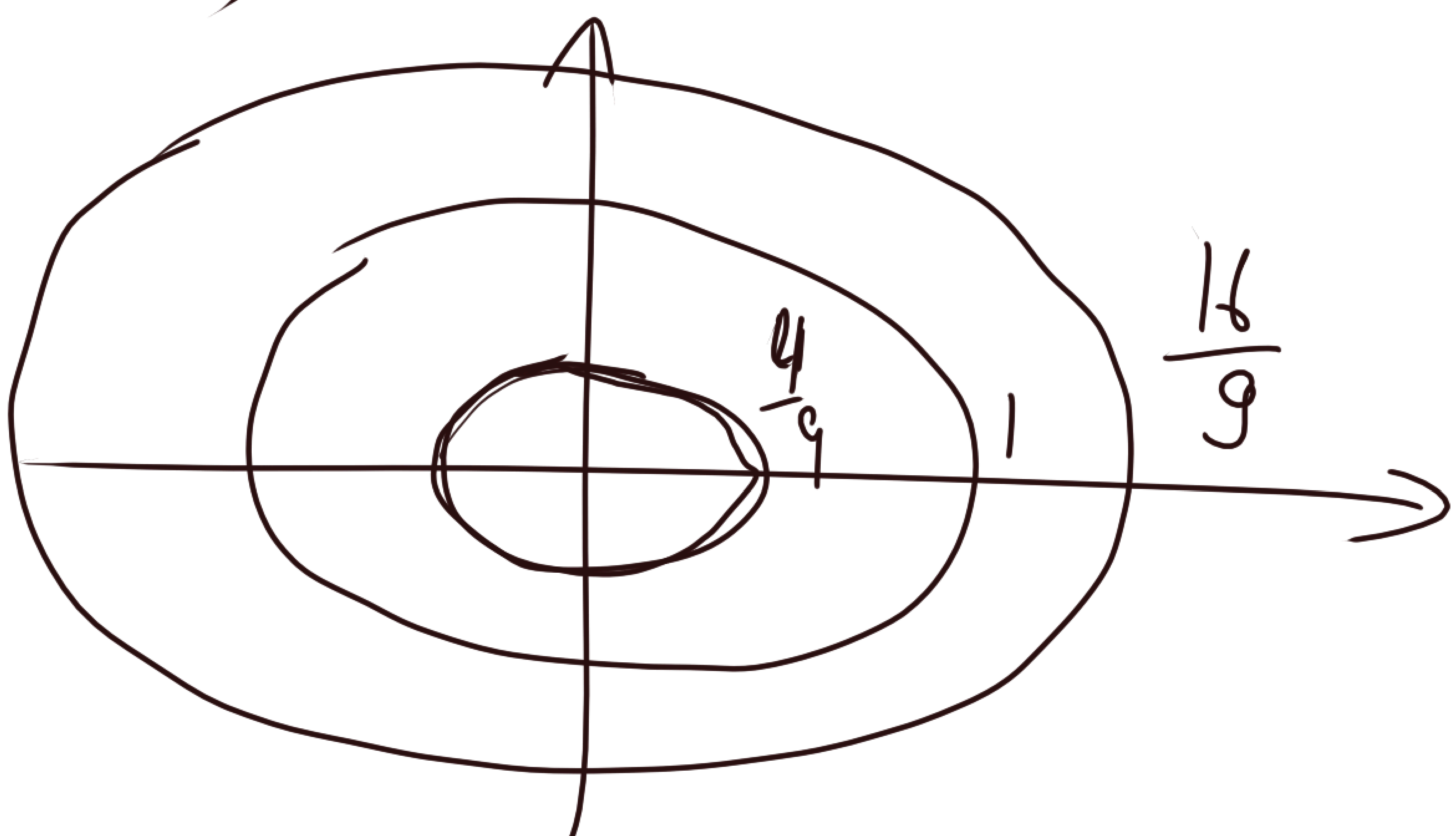
$$=\lim_{r\to 0}\left[r^2\left(\cos^4\theta-\sin^4\theta\right)\right]$$

$$=0$$

③ Let $f : \mathbb{R} \setminus (0,0) \rightarrow \mathbb{R}$,

$$f(x,y) = \frac{x^2}{9} + \frac{y^2}{4}$$

a) Sketch 3 level curves, for $f(x,y) = k$, $k \in \mathbb{R}$



$$f(x,y) = \frac{x^2}{a} + \frac{y^2}{4}$$

b) Compute $\lim\limits_{(x,y)\to(a,0)} f(x,y)$

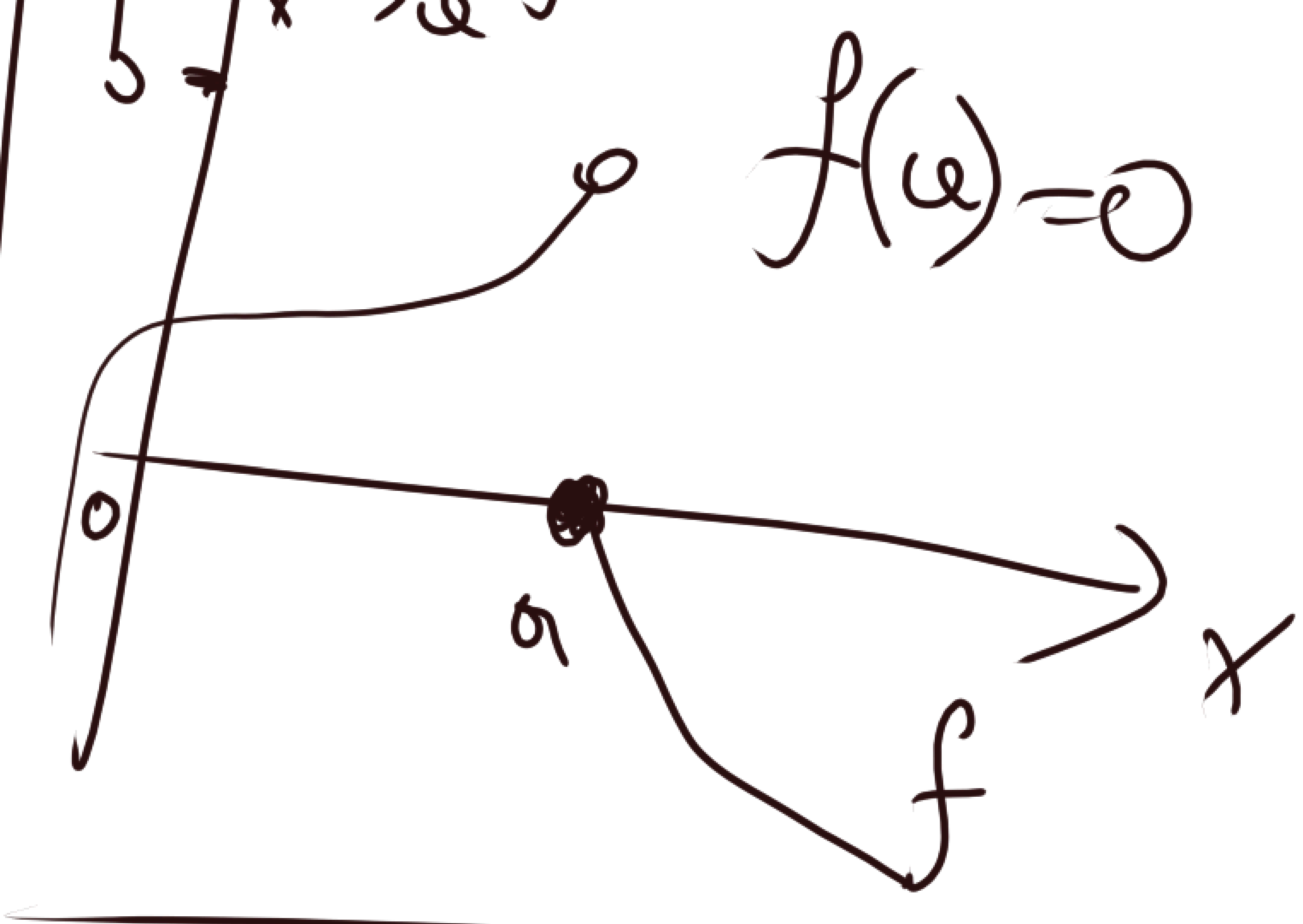
$$\frac{1}{x}$$

$= 0$

(Optional) $f(x, \lambda x)$,

λ is constant

$$\lim_{x\to a} f(x) = b$$



$$\frac{d}{dx}\left(\cos(x+a)\right) =$$

$$= \left(\cos(x+a)\right)' \cdot (x+a)'$$

$$= -\sin(x+a) \cdot 1$$

hos fen

$$= -\sin(x+a)$$

? $0/0$

$$\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2} = \lim \frac{(x^2-y^2)\cancel{(x^2+y^2)}}{\cancel{x^2+y^2}}$$

$$= \lim x^2 - y^2$$

Let $f(x,y)$ be a function

f_x - partial derive. of f_x w.r.t. x.

$$\nabla f = \langle f_x, f_y \rangle$$ (Gradient

$f_x \Leftrightarrow \frac{\partial}{\partial x} f$

e.g., $f(x,y) = 2x + 4y$

$$\nabla f = \langle f_x, f_y \rangle = \langle 2, 4 \rangle$$