Tuesday Feb 23

① $\lim_{n\to\infty}\left(\frac{56}{\pi}\right)^n = \infty$, $\frac{56}{\pi} > 1$

Note: $\left(\frac{1}{2}\right)^n$, $n \to \infty$ " 0

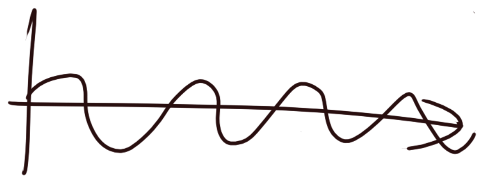
② $\lim_{n\to\infty}\left(\frac{e}{\pi}\right)^n = 0$

Dr. Brennan

- Sequences: ∞ list of values, $a_n = \dots, n \to \infty$
- Series: $S_k = \sum_{n=0}^{k} a_n$
- Convergence: $\lim_{n\to\infty} a_n = L \neq \pm\infty$
- Divergence: otherwise, $\lim_{n\to\infty} a_n$ is undefined or it's $\pm\infty$

Give me a sequence, $\lim_{n\to\infty} a_n \neq \pm\infty$ but it's diverging

- $a_n = \sin(n)$



- $a_n = (-1)^n = \{1, -1, 1, -1, \dots\}$

③ $\lim_{n\to\infty} \frac{\ln(n)}{n}$ has the form $\frac{\infty}{\infty}$

$= \lim_{n\to\infty} \frac{\frac{d}{dn}(\ln(n))}{\frac{d}{dn}(n)} = \lim_{n\to\infty} \frac{1/n}{1} = \lim_{n\to\infty} \frac{1}{n} = 0$ (L'Hop.)

(4) $\lim_{n\to\infty} \frac{2n}{n} = \lim_{n\to\infty} \frac{(2n)'}{(n)'} = \lim_{n\to\infty} \frac{2}{1} = 2$

(5) $\lim_{n\to\infty} \frac{n^n}{n!} = \lim_{n\to\infty} \frac{n\times n\times n\times \dots \times n}{1\times 2\times 3\times \dots \times n}$

$= \lim_{n\to\infty} \frac{n}{1}\times \frac{n}{2}\times\frac{n}{3}\times \dots \times \frac{n}{n} = 1$

$n>1 \quad \frac{n}{2}>1 \quad \frac{n}{3}>1 \quad >1$

$> \lim_{n\to\infty} n = \infty$

By Squeeze Theorem, $\lim_{n\to\infty} \frac{n^n}{n!} = \infty$

(6) $\sum_{n=1}^{\infty} \frac{(3-2x)^n}{4^n \cdot n}$, find x for which converges $a<x<b$

Solution: Ratio test $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1.$

$a_n = \frac{(3-2x)^n}{4^n \cdot n} \qquad a_{n+1} = \frac{(3-2x)^{n+1}}{4^{n+1}\cdot(n+1)}$

$L = \lim_{n\to\infty} \left| \frac{(3-2x)^{n+1}}{4^{n+1}\cdot(n+1)} \cdot \frac{4^n\cdot n}{(3-2x)^n} \right| = \lim_{n\to\infty} \frac{|3-2x|}{4} \cdot \frac{n}{n+1}$

$= \frac{|3-2x|}{4}$

The sum converges, iff

$$\boxed{L = \frac{|3-2x|}{4} < 1}$$

Then

$$-1 < \frac{3-2x}{4} < 1$$

$$\Rightarrow -4 < 3-2x < 4$$

$$\Rightarrow -7 < -2x < 1$$

$$\Rightarrow 7 > 2x > -1$$

$$\Rightarrow -1 < 2x < 7$$

$$\Rightarrow -\frac{1}{2} < x < \frac{7}{2}$$

$$\therefore \sum_{n=1}^{\infty} \frac{(3-2x)^n}{4^n n} \text{ conv if } -\frac{1}{2} < x < \frac{7}{2}$$

Fact!!!

$$|x| < a$$

$$\Leftrightarrow -a < x < a$$

Fact 2!!!

$$a < x < b$$

$$\Leftrightarrow -a > -x > -b$$

7) Determine the limit of seq.

$$\frac{(-5)^n}{n!}$$

Fact!!!

$$\lim_{n\to\infty} \frac{R^n}{n!} = 0, R \in \mathbb{R}$$

<u>Solution</u>

$$\frac{(-5)^1}{1!}, \frac{(-5)^2}{2!}, \frac{(-5)^3}{3!}$$

$$\frac{-5}{1}, \frac{25}{2}, \frac{-125}{6}, \ldots$$

$n=1\ldots\infty$

$$\frac{-(5)^n}{n!} \le \frac{(-5)^n}{n!} \le \frac{5^n}{n!}$$

$$0 = \lim_{n\to\infty} \frac{-5^n}{n!} \le \lim_{n\to\infty} \frac{(-5)^n}{n!} \le \lim_{n\to\infty} \frac{5^n}{n!} = 0$$

By Squeeze Theorem, $\lim_{n\to\infty} \frac{(-5)^n}{n!} = 0$