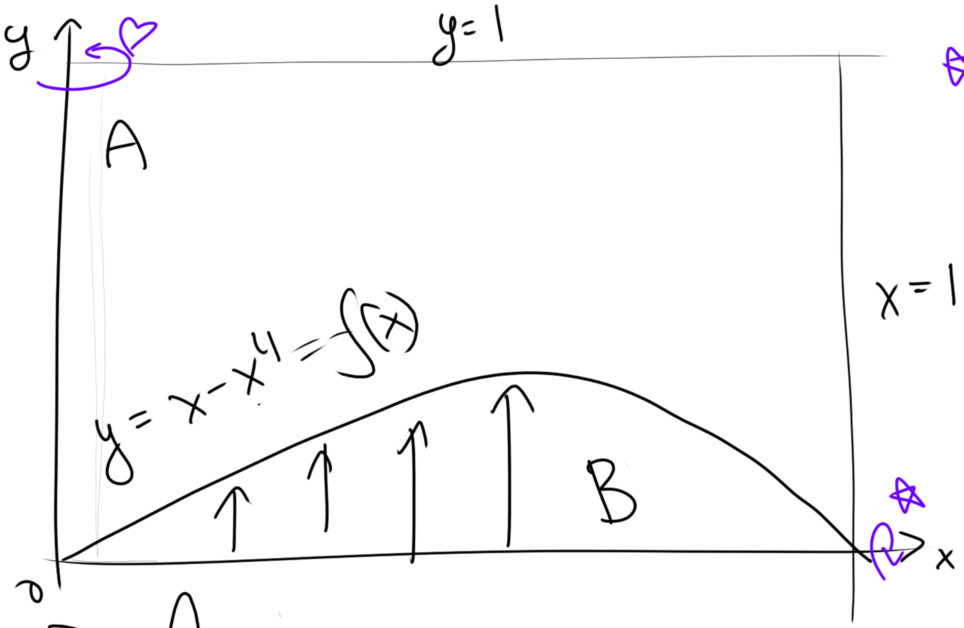


a) B about x-axis

b) A about y-axis

a) Area:

$$V = \pi \int_0^1 (x - x^4)^2 \, dx \quad \leftarrow \text{washer}$$

Washer: rotate along an axis parallel to the axis of revolution

integrating

$$\pi R^2 \Rightarrow V = \pi \int_{x_1}^{x_2} \left[R_o(x)^2 - R_I(x)^2 \right] dx$$

outer inner

In our case, $R_o(x) = f(x) = x - x^4$

$R_I(x) = 0$

Then

$$V = \pi \int_0^1 (x - x^4)^2 \, dx$$

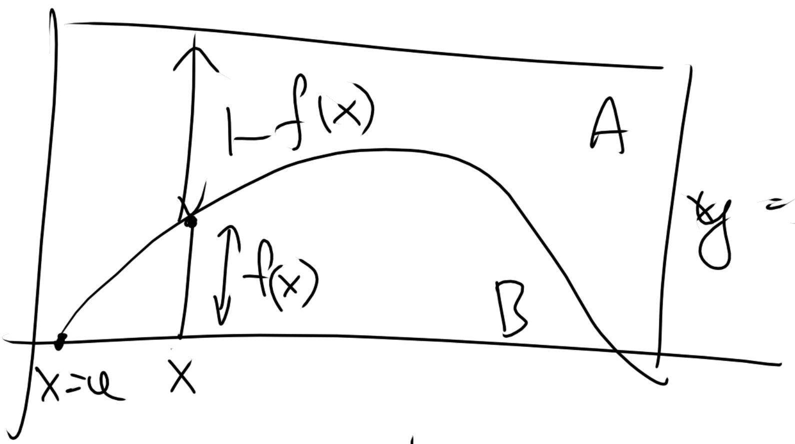
I

b) We will integrate with respect to x but we rotate around y.

$\Longrightarrow$ Shell method (a.k.a. Disc Method)

$$V = 2\pi \int_a^b \overset{\text{radius}}{x}\, f(x)\, dx, \quad \text{height}$$

where $f(x)$ is the height of the func



$\text{height}_A(x) = 1$

$y \to \text{height}_B(x) = 0$

Then our height is $1 - f(x) = 1 - (x - x^4)$
$= 1 - x + x^4$

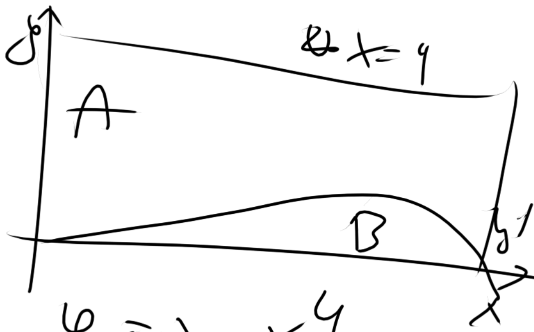
Then

$$V = 2\pi \int_0^1 x\,(1 - x + x^4)\,dx$$

II

c) B about $x=3$ Shell

d) A about $y=-1$ washer



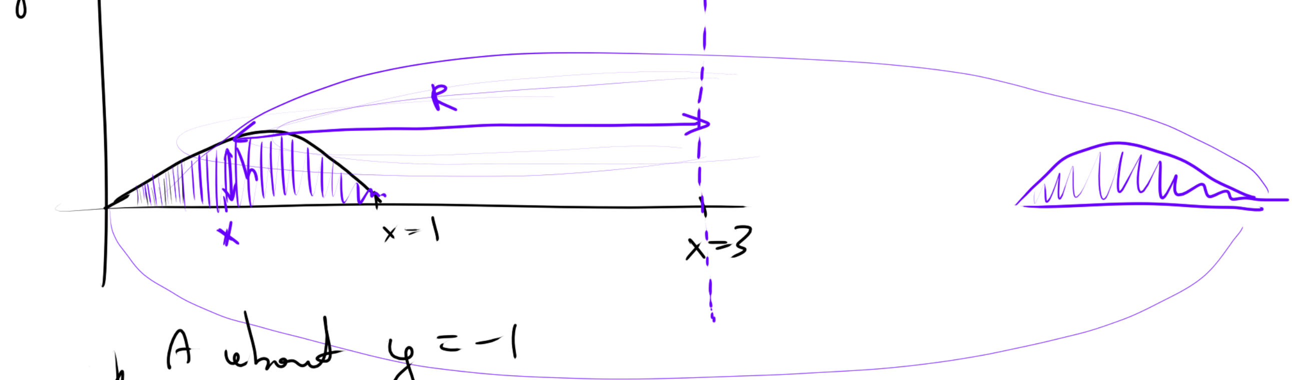
$y = x - x^4$

Solutions

c)

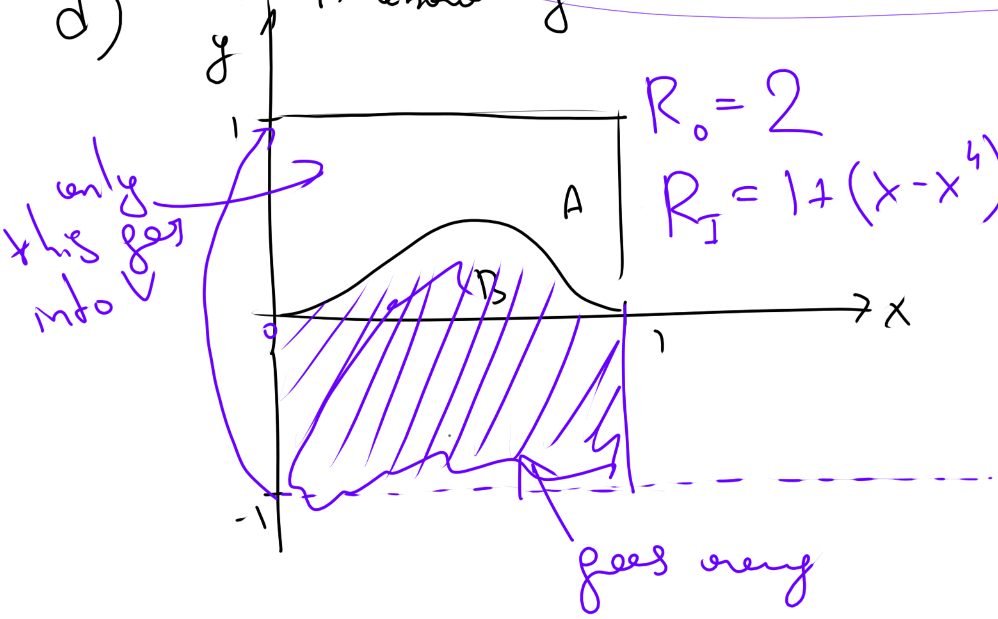
$h = f(x)$

$R = 3 - x$



$$V = 2\pi\int_0^1 (3-x)(x-x^4)\,dx$$

d) A about $y=-1$



$R_0 = 2$

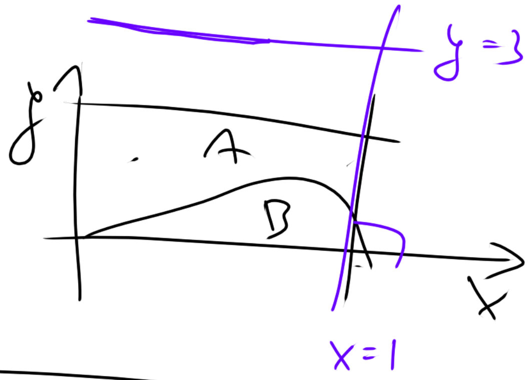
$R_I = 1 + (x - x^4)$

$$V = \pi\int_0^1 2^2 - (1+(x-x^4))^2\,dx$$

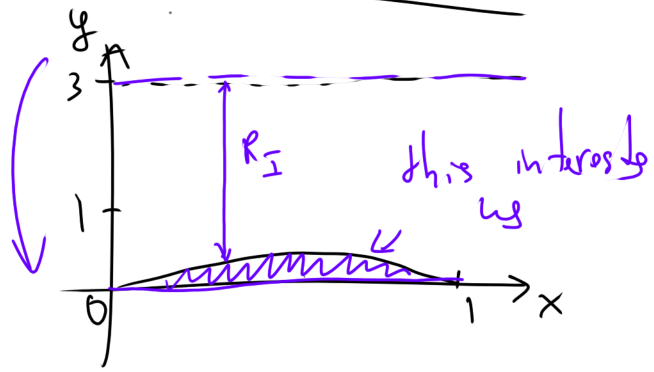
III

e) B about $y = 3$ washer

f) A about $x = 1$ shell



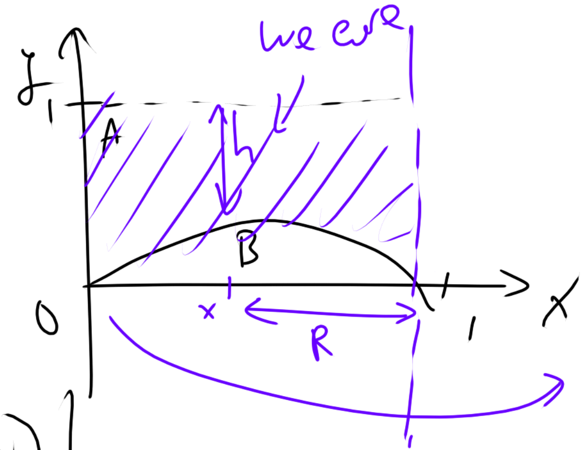
Solution



e) $R_o = 3$

$R_I = 3 - (x - x^4)$

$$V = \pi \int_0^1 9 - \left(3 - (x - x^4)\right)^2 dx$$



f) $h = 1 - (x - x^4)$

$R = 1 - x$

$$V = 2\pi \int_0^1 \left(1 - (x - x^4)\right)(1 - x)\, dx$$

IV