Learning Goal Activity 123

② a) $\langle$ Midterm 1, Midterm 2, Homework, Quizzes, All, Final $\rangle \in \mathbb{R}^6$

Roehy the Cod's grades:

$$\vec{\Omega} = \langle 70, 76, 84, 62, 100, x \rangle$$

↑ final grade

Here are the weight

$$\vec{\xi} = \langle 20\%, 20\%, 15\%, 10\%, 10\%, 25\% \rangle$$

$$= \langle 0.2, 0.2, 0.15, 0.1, 0.1, 0.25 \rangle$$

b) Dot product:

$$\vec{\Omega} \cdot \vec{\xi} =$$

$$= 70 \cdot 0.2 + 76 \cdot 0.2 + 84 \cdot 0.15 + 62 \cdot 0.1 + 100 \cdot 0.1 + x \cdot 0.25$$

$$= 58 + 0.25x,$$

where x is the grade on the final exam

I7

c) The maximum Rocky can get on the final is 100. But even if he gets 100

his final grade: $58 + 0.25 \cdot 100$
$= 58 + 25$
$= 83$

Which is below the A cutoff, which 89.

d) $58 + 0.25 \cdot x = 78$
$x = 4 \cdot (78 - 58)$
$= 4 \cdot 20$
$= 80$

Rocky needs to get 80% on the final exam to secure a B.

II

(7) Verify the Cauchy-Schwarz inequality.

$$|\vec{u}\cdot\vec{v}| \le \|\vec{u}\|\,\|\vec{v}\|$$

<u>Solution</u> let $\vec{u} = \langle 6,-3,2\rangle$, $\vec{v} = \langle 8,9,12\rangle$.

$$\begin{aligned}
LHS &= |\vec{u}\cdot\vec{v}| \\
&= |\langle 6,-3,2\rangle\cdot\langle 8,9,12\rangle| \\
&= |6\cdot 8 - 9\cdot 3 + 2\cdot 12| \\
&= |48 - 27 + 24| \\
&= |45| \\
&= 45.
\end{aligned}$$

$$\begin{aligned}
RHS &= \|\vec{u}\|\,\|\vec{v}\| \\
&= \|\langle 6,-3,2\rangle\|\,\|\langle 8,9,12\rangle\| \\
&= \sqrt{6^2+(-3)^2+2^2}\cdot\sqrt{8^2+9^2+12^2} \\
&= 7\cdot 17 \\
&= 119
\end{aligned}$$

$\overline{|||}$

Verify if $LHS \leq RHS$

$\Rightarrow 45 \leq 119$

$\Rightarrow$ Yes!

$\therefore$ Verified.

(6) $\vec{u}_1 = \langle 1,2,3 \rangle$ $\qquad$ $\vec{u}_2 = \langle 4,0,-1 \rangle$

a) $\vec{w} = \langle -2, 13, -8 \rangle$,

Is $\vec{w} \perp \vec{u}_1$? and $\vec{w} \perp \vec{u}_2$?

orthogonal/perpendicular

Recall $\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cdot \cos\theta$, θ is the acute angle in between

If orthogonal, $\Rightarrow \theta = 90^\circ$

$\Rightarrow \cos\theta = 0$.

1) $\vec{w} \cdot \vec{u}_1 = \langle -2, 13, -8 \rangle \cdot \langle 1,2,3 \rangle$

$= -2 + 26 - 24$

$= 0$ $\qquad \Longrightarrow \vec{w} \perp \vec{u}_1$.

2) $\vec{w} \cdot \vec{u}_2 = \langle -2, 13, -8 \rangle \cdot \langle 4,0,-1 \rangle$

$= -8 + 8$

$= 0$ $\qquad \Longrightarrow \vec{w} \perp \vec{u}_2$

IV

b) verify $\vec{u}_3 = \langle 1,0,0 \rangle$ is not a linear combination of $\vec{u}_1$ and $\vec{u}_2$.

If is a lin. comb. if for $a, b \in \mathbb{R}$

$$a\vec{u}_1 + b\vec{u}_2 = \vec{u}_3$$

$$\Rightarrow \quad a\langle 1,2,3\rangle + b\langle 4,0,-1\rangle = \langle 1,0,0\rangle$$

$$\langle 1\cdot a, 2\cdot a, 3\cdot a\rangle + \langle 4\cdot b, 0\cdot b, -1\cdot b\rangle = \langle 1,0,0\rangle$$

$$\langle 1\cdot a + 4\cdot b, 2\cdot a + 0\cdot b, 3\cdot a - 1\cdot b\rangle = \langle 1,0,0\rangle$$

Solve

$$\begin{cases} a + 4b = 1 \\ 2a = 0 \longrightarrow a = 0 \\ \underset{0}{3a} - \underset{0}{b} = 0 \end{cases}$$

If both $a, b = 0$ the $a + 4b = 1$ is <u>impossible</u>.

therefore suitable a and b <u>do not exist</u>.

$\Longrightarrow \vec{u}_3$ is <u>not</u> a lin. comb of $\vec{u}_1$ & $\vec{u}_2$.

VK

⑧ Cauchy - Bunyakovsky - Schwarz inequality.

$f \cdot g := \int_a^b f(x)g(x)\,dx$

$$\left(\int_a^b f(x)g(x)\,dx\right)^2 \leq \left(\int_a^b (f(x))^2 dx\right)\left(\int_a^b (g(x))^2 dx\right)$$

a) Verify for $f(x) = x$,

$g(x) = \sqrt{x^2+1}$,

$[a,b] = [0,1]$.

$u = x^2+1$ $\quad u_a = 0^2+1 = 1$ $\quad u_b = 1^2+1 = 2$

$\frac{du}{dx} = 2x \Rightarrow dx = \frac{du}{2x}$

$$LHS = \left(\int_0^1 x\sqrt{x^2+1}\,dx\right)^2$$

$$= \left(\int_1^2 \cancel{x}\sqrt{u}\,\frac{du}{2\cancel{x}}\right)^2$$

$$= \left(\frac{1}{2}\int_1^2 \sqrt{u}\,du\right)^2$$

$\sqrt{u} = u^{\frac{1}{2}}$

$\frac{2}{3}u^{\frac{3}{2}}$

$$= \left(\frac{1}{\cancel{2}} \cdot \left[\frac{\cancel{2}}{3}u^{\frac{3}{2}}\right]\Bigg|_1^2\right)^2$$

$$= \left(\frac{1}{3} \cdot \left(\sqrt{2^3} - \sqrt{1^3}\right)\right)^2$$

$$= \left(\frac{2\sqrt{2}-1}{3}\right)^2 = \frac{9-4\sqrt{2}}{9} \approx 0.37$$

VI

RHS:

1) $\int_0^1 x^2 dx = \frac{1}{3}\left[x^3\right]_0^1 = \frac{1}{3}$

2) $\int_0^1 x^2+1\, dx = \left[\frac{1}{3}x^3 + x\right]_0^1 = 1/3 + 1 = 4/3$

$RHS = 1/3 \cdot 4/3 = 4/9$

• Verate $LHS \leq RHS$

$\Rightarrow 0.37 \leq 4/9 \approx 0.44$

Yes!

VII

b) let $g(x) = \frac{1}{\sqrt{x^2+1}}$

LHS: $\int_0^1 x \cdot \frac{1}{\sqrt{x^2+1}}\, dx$

$u = x^2+1$

$dx = \frac{du}{2x}$

$= \frac{1}{2}\int_1^2 \frac{1}{\sqrt{u}}\, du$

$= \frac{1}{2}\int_1^2 u^{-\frac{1}{2}}\, du$

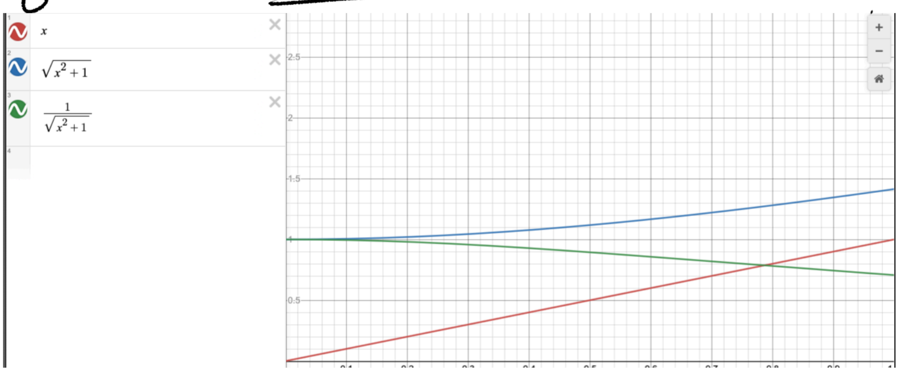
$= \cancel{\frac{1}{2}}\left[\cancel{2}\cdot u^{\frac{1}{2}}\right]\Big|_1^2$

$= \sqrt{2} - 1 \approx 0.41$

RHS: 1) $\int_0^1 x^2\, dx = 1$

2) $\int_0^1 \frac{1}{x^2+1}\, dx \approx 0.78$

Vengo: **True!**

c) 

$\sqrt{x^2+1}$ is closer to x on $[0,1]$.

$\boxed{\text{VIII}}$