April 13th | Calculus III : Midterm II Review

① "Pitted Cherry"

Defined by a solid sphere

$$x^2+y^2+z^2 \le 9 = 3^2$$

cylinder

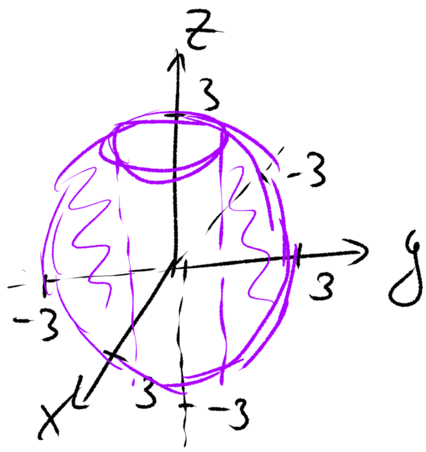
with the "cone" $x^2+y^2 \le 1$ removed.

Assuming density of mass is

$$\delta(x,y,z) = \sqrt{9-x^2-y^2}$$

a) Compute the total mass of the pitted cherry.

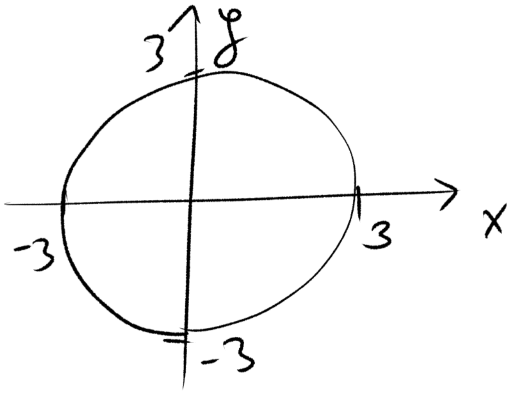
$$M = \iiint \delta \, dV$$



For this, $dV = dz\,dy\,dx$

Range of x: $[-3,3]$

Let's look at the 'shadow'



$x^2+y^2 \leq 9$

for sim

$x^2+y^2=9=3^2$

$\Rightarrow y^2 = 9-x^2$

Range $y: \left[-\sqrt{9-x^2}, \sqrt{9-x^2}\right]$

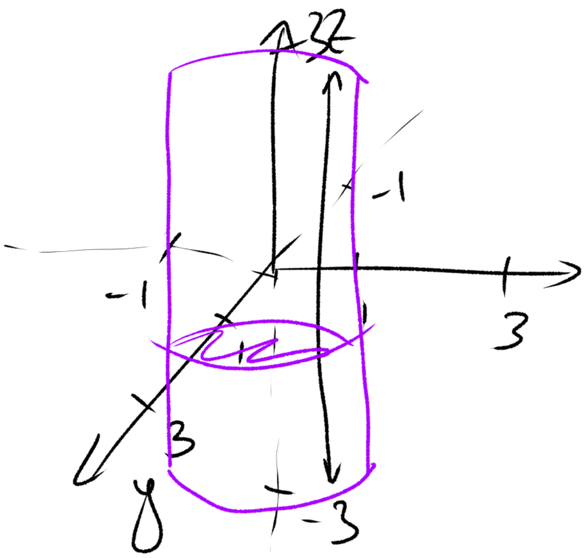
$x^2+y^2+z^2=9 \Rightarrow z^2 = 9-x^2-y^2$

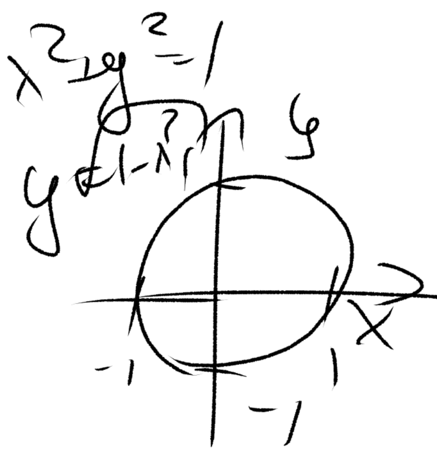
$\Rightarrow$ Range $z: \left[-\sqrt{9-x^2-y^2}, \sqrt{9-x^2-y^2}\right]$

$$M_{\text{sphere}} = \int_{x=-3}^{x=3} \int_{y=-\sqrt{9-x^2}}^{y=\sqrt{9-x^2}} \int_{z=-\sqrt{9-x^2-y^2}}^{z=\sqrt{9-x^2-y^2}} \delta \, dz\, dy\, dx$$

$M_{\text{cherry}} = M_{\text{sphere}} - M_{\text{cone}}$ "ore"

II





Range: $x \in [-\cancel{3}1, \cancel{3}1]$

Range: $y \in [-1, 1]$

Range: $z \in [-\sqrt{9-x^2-y^2}, \sqrt{9-x^2-y^2}]$

$$M_{cone} = \int_{x=-\cancel{3}1}^{x=\cancel{3}1} \int_{y=-1}^{y=1} \int_{z=-\sqrt{9-x^2-y^2}}^{z=\sqrt{9-x^2-y^2}} \delta \, dz\,dy\,dx$$

$\therefore$ $\text{Mass}_{\text{pitted}}$ dong =

$$\underbrace{\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} \delta \, dz\,dy\,dx}_{\text{sphere}} - \underbrace{\int_{-\cancel{3}1}^{\cancel{3}1} \int_{-1}^{1} \int_{-\sqrt{9-x^2-y^2}}^{\sqrt{9-x^2-y^2}} \delta \, dz\,dy\,dx}_{\text{cone}}$$

III

b) Find mass using $\begin{cases} x^2+y^2+z \le 9 \\ x^2+y^2 \le 1 \\ \delta(x,y,z) = \sqrt{9-x^2-y^2} \end{cases}$

i) Cylindrical

ii) Spherical.

Solution

I) $x^2+y^2=r^2$

$$\int_{\theta=0}^{\theta=2\pi} \int_{r=1}^{r=3} \int_{z=-\sqrt{9-r^2}}^{z=\sqrt{9-r^2}} \sqrt{9-r^2} \cdot r \, dz \, dr \, d\theta$$

$$= 81\pi.$$

$$\int_{\theta=0}^{\theta=2\pi} \int_{\varphi=0}^{\varphi=2\pi} \int_{\rho=0}^{\rho=3} \delta \cdot \rho^2 \sin\varphi \, d\rho \, d\varphi \, d\theta$$

$$\int_{\theta=0}^{\theta=2\pi} \int_{\varphi=0}^{\varphi=\frac{1}{\sin\theta}} \int_{\rho=0}^{\rho=1} \delta \, \rho^2 \sin\varphi$$

II

(8) Parametrize the intersection of the surfaces

$$y^2+z^2=4 \quad \text{and} \quad x=5y^2$$

Solution

Dream is to express x, y, z in terms of t (time, here varied))

Look $y^2+z^2=4$, Huh...

Looks like a circle with a radius of 2.

I know! $\langle \cos(t), \sin(t) \rangle$ is a circl!

Let $y=2\cos(t)$, $z=2\sin(t)$

$$y^2+z^2=4\cos^2(t)+4\sin^2(t)=4.$$

How to find x???

We have $x=5y^2=5\cdot(2\cos(t))^2=20\cos^2(t)$.

V

$\Rightarrow$ $\langle 20\cos^2(t), \cancel{\ }\cos^2(t), \cancel{\ }\sin^2(t)\rangle$

$(3+2e^t, 1+4e^t, 2+5e^t)$

the same as

$\begin{pmatrix}3\\1\\2\end{pmatrix} + t\begin{pmatrix}2\\4\\5\end{pmatrix}$

$x+2=4$

find x

$x+2+(-2) = 4+(-2)$

$x+0 = \cancel{\ } 2$

$x=2$

$\langle 3+2e^t, 1+4e^t, 2+5e^t\rangle$

$0 \cdot a = 0$

$= \langle 3,1,2\rangle + \langle 2e^t, 4e^t, 5e^t\rangle$

$= \langle 3,1,2\rangle + e^t\langle 2,4,5\rangle$

$\neq$

$\langle 3,1,2\rangle + t\langle 2,4,5\rangle$

VI

$$0 \cdot a = (0+0) \cdot a = 0 \cdot a + 0 \cdot a$$

Subtra $0 \cdot a$

$$\underbrace{0 \cdot a - 0 \cdot a}_{0} = 0 \cdot a + \underbrace{0 \cdot a - 0 \cdot a}_{0}$$

$$\Rightarrow 0 = 0 \cdot a$$

$$\Rightarrow 0 \cdot a = 0.$$

$$(-1)^2 = \underline{1}$$