(1) The angular velocity of an object as a function of time is given by the equation

$$\omega(t) = \left(4\frac{\text{rad}}{\text{s}}\right) + \left(6\frac{\text{rad}}{\text{s}^2}\right)t$$

What is the angular displacement of the object (in rad) during the time interval from t = 1 s to t = 3 s?

(2) An object starts from rest and travels around a circle of radius 12 cm with constant angular acceleration. The object covers a tangential distance of 50 cm in 7 s. What is the angular speed of the object (in rad/s) after it has moved the 50 cm?

① $\omega(t) = 4\frac{rad}{s} + 6\frac{rad}{s^2}\cdot t \Leftrightarrow \omega(t) = 4 + 6t$

$\Delta\theta$? if $t_i = 1$ & $t_f = 3$

Solution

$$\omega = \frac{d\theta}{dt} \Rightarrow \omega\, dt = d\theta \Rightarrow \int_{t_i}^{t_f} \omega\, dt = \int_{\theta_i}^{\theta_f} 1\, d\theta$$

$$\Delta\theta = \int_1^3 4 + 6t\, dt = \left(4t + 3t^2\right)\Big|_1^3 = 32 \text{ rad}$$

(trivial)

② In the att.

Solution

$$S = R\theta \Rightarrow \Delta\theta = \frac{\Delta S}{R} = 4.2 \text{ rad}$$

($S$: tangent dist., $R$: radius)

$$\Delta\theta = \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow \alpha = \frac{2\Delta\theta}{(\Delta t)^2}$$

$$\omega_f = \alpha\,\Delta t = \frac{2\Delta\theta}{(\Delta t)^2}\cdot \Delta t = \frac{2\Delta\theta}{\Delta t} =$$

$$= 2\cdot\frac{4.2}{7} \approx 1.2\frac{rad}{s}$$

③ We have:

- $K_{rot}$ (Rotational Kinetic Energy) $= 860.\ 10^6\ J$
- $R$ (radius) $= 4m$
- $M$ (mass) $= 5000kg$

We need:

- $\omega$ ? (angular velocity)

Solution $K_{rot} = \frac{1}{2} I \omega^2$

$$I = \frac{1}{2} MR^2 \Rightarrow K_{rot} = \frac{1}{2} \cdot \frac{1}{2} \cdot M \cdot R^2 \cdot \omega^2$$

$$\Leftrightarrow \omega^2 = \frac{4K_{rot}}{MR^2}$$

$$\Leftrightarrow \omega = \sqrt{\frac{4K_{rot}}{MR^2}}$$

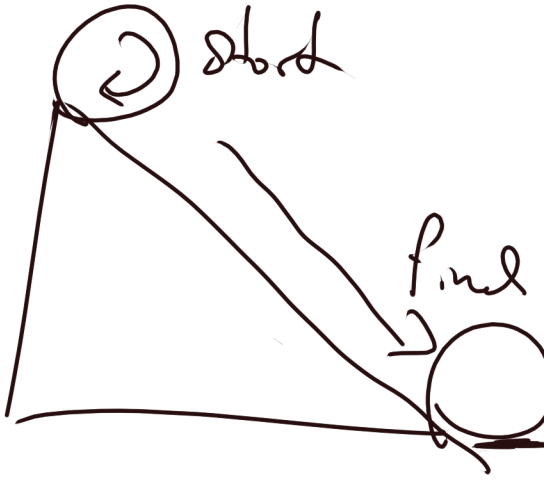
$$\Rightarrow \omega = 207.4 \frac{rad}{s}$$

(1) A typical wind turbine at the Smoky Hills Wind Farm in Lincoln and Ellsworth counties in Kansas generates 1 MW when the wind is blowing. During calm periods, however, no power is generated. Thus, to ensure a steady supply of power a mechanism must be created by which the energy generated by the turbine can be easily stored and retrieved. One option is to use batteries, but another and perhaps simpler approach would be to store the energy in a spinning flywheel. One percent of a day's maximum output from the turbine would correspond to 860 MJ of energy. With what angular speed (in rad/s) would a 5000 kg flywheel with a radius of 4 m spin in order to store 860 MJ of energy? You can model the flywheel as a uniform disk spinning about an axis perpendicular to the disk passing through its center.

(4) We have: $\omega_i = 1.5 \frac{rad}{s}$

$\omega_f = 0 \ rad\ s^{-1}$

$\Delta t = 20 s$




We need: #rev?

Solution

1. Find Acceleration: $\Delta\omega = \alpha \Delta t \Rightarrow \alpha = -\frac{\omega_i}{\Delta t}$

2. Find $\Delta\theta$

$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta \Rightarrow \Delta\theta = \frac{-\omega_i^2}{2\alpha} = \frac{-\omega_i^2}{2\cdot\left(\frac{-\omega_i}{\Delta t}\right)}$

($\omega_f$ → 0)

$= \frac{\omega_i \cdot \Delta t}{2} = 15 rad$

3. Find #rev

$\#rev = \frac{15 \cancel{rad}}{2\pi \cancel{rad}} \approx \frac{15}{6.28} \approx 2.39$

(2) A merry-go-round is spinning with an angular velocity of 1.5 rad/s. The merry-go-round then slows down with constant angular acceleration and comes to a rest in 20 s. Through how many revolutions will it move from the time it starts slowing down to the time it comes to a stop?