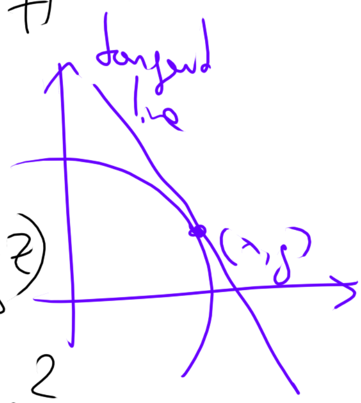
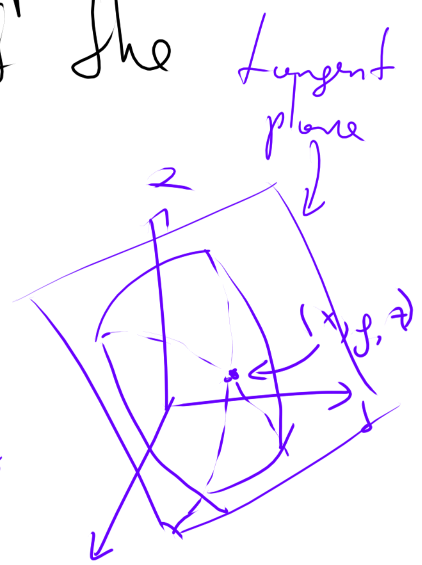
Wednesday March 3rd · Math 127

(1) I give you a surface

$$x^3z + x^2y^2 + \sin(yz) = -3$$

a) Find the equation of the plane tangent to this surface at the point $(-1, 0, 3) = A$





Solution

Let $f(x,y,z) = x^3z + x^2y^2 + \sin(yz)$

$$f_x(x,y,z) = 3x^2z + 2xy^2$$

$$f_y(x,y,z) = 2x^2y + \cos(yz)\cdot z$$

$$f_z(x,y,z) = x^3 + y\cos(yz)$$

$$\nabla f = \langle 3x^2z + 2xy^2,\ 2x^2y + z\cos(yz),\ x^3 + y\cos(yz)\rangle$$

Let's find the normal to A.

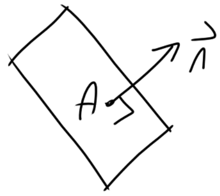
$$\vec{n} = \nabla f(A) = \nabla f(-1,0,3) = \langle 9, 3, -1\rangle$$

Find plane by knowing A and $\vec{n}$:

$$\vec{x}\cdot\vec{y} = \|\vec{x}\|\cdot\|\vec{y}\|\cdot\cos(\theta),\ \text{if } \vec{x}\perp\vec{y} \Rightarrow \vec{x}\cdot\vec{y} = 0$$

$\vec{n} = \langle 9, 3, -1 \rangle \qquad A = (-1, 0, 3)$

Tangent plane is defined as



$$\vec{n} \cdot \left[(x, y, z) - A \right] = 0$$

$$\langle 9, 3, -1 \rangle \cdot (x+1, y, z-3) = 0$$

$$\Rightarrow \quad 9(x+1) + 3y - (z-3) = 0$$

$$\Rightarrow \quad 9x + 9 + 3y - z + 3 = 0$$

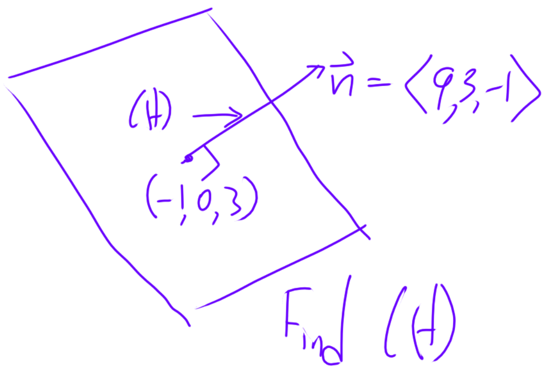
$$\Rightarrow \quad 9x + 3y - z = -12$$

this is the answer!!!

b) Find the parametric equation of the normal line to the surface at $A = (-1, 0, 3)$

Solution

$$l(t) = \underbrace{(-1, 0, 3)}_{A} + t \underbrace{\langle 9, 3, -1 \rangle}_{\vec{n}}$$

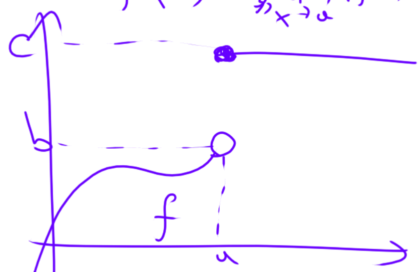


② Let $f: \mathbb{R}^2 \setminus (0,0) \to \mathbb{R}$ defined as

$$f(x,y) = \frac{x^a y^b}{x^2+y^2}$$

where $a, b \in \mathbb{R}^+$. Determine all values of a, b, such that $f(x,y)$ has a limit as $(x,y) \to (0,0)$. When it exists, what's the limit?

$f(a) \overset{?}{=} c$; $\lim_{x \to a}(f(x)) = b$



Solution (Hint: Use polar coordinates)

$$f(r\cos\theta, r\sin\theta) =$$

$$= \frac{[r\cos(\theta)]^a [r\sin(\theta)]^b}{r^2\cancel{\cos^2\theta}+r^2\cancel{\sin^2\theta}} = \frac{r^{a+b} \cdot \cos^a(\theta) \cdot \sin^b(\theta)}{r^2}$$

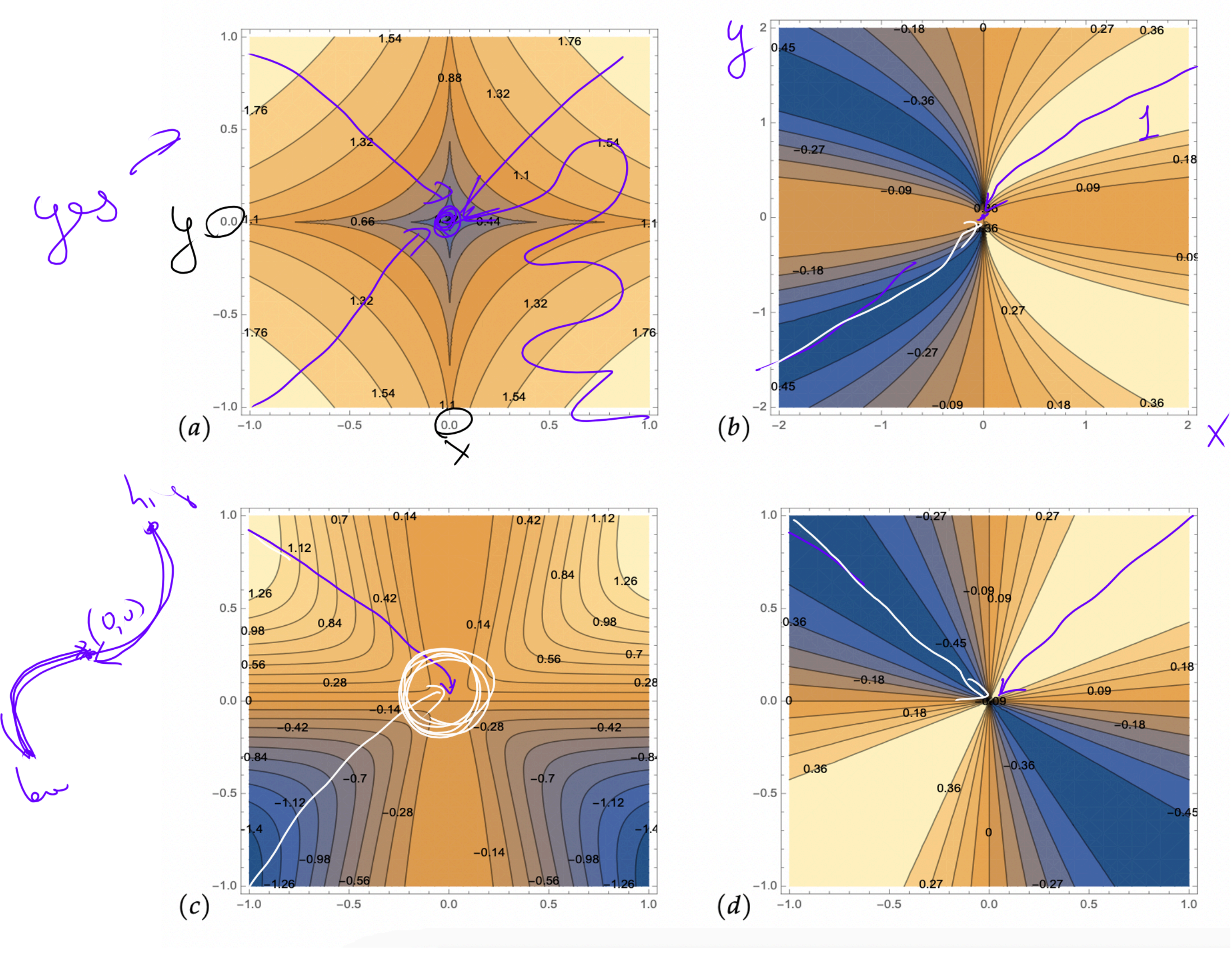
$$= r^{a+b-2} \cdot \cos^a(\theta) \cdot \sin^b(\theta)$$

$$\lim_{r \to 0} \left[r^{a+b-2} \cdot \cos^a(\theta) \cdot \sin^b(\theta) \right] = 0$$

iff r is present with positive power $\iff a+b-2>0$
$\iff a+b>2$ ← answer

What happens if $a+b \leq 2$?
Then the $\lim \to \infty$

③ Argue if the corresponding function's limits on contour maps exist, and limit is $(x, y) \to (0, 0)$



(Provided by Dr. Gaosh)