March 1st

(1) A 7.5 kg block is attached to a horizontally-mounted spring with a spring constant of 750 J/m^2. The block is released from rest with the spring extended by 15 cm from its normal length of 27 cm. The horizontal surface along which the block moves is frictionless. How long (in s) will it take this system to complete one full oscillation?

(1) Solution

$$\omega = 2\pi f = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{m}}} = \frac{2\pi}{\sqrt{\frac{750}{7.5}}} \approx 0.63$$

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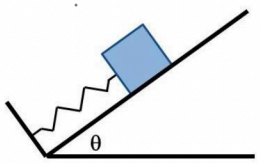
(2) A simple harmonic oscillator is created by attaching a spring with a spring constant of 8 J/m^2 to a 2 kg mass as shown in the figure. The angle of the ramp is $\theta = 55°$ and the surface of the ramp is frictionless. What is the angular frequency (in rad/s) of oscillations of this system?



(2) Solution

$$\omega^2 = \frac{k_{effective}}{m_{effective}} \Rightarrow \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8}{2}} = 2 \text{ rad s}^{-1}$$

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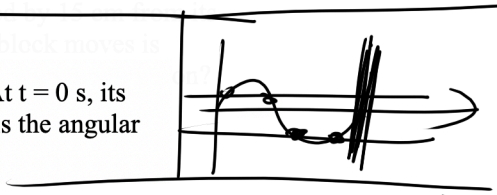
(1) An object oscillates in 1-dimension with an amplitude of 10 cm. At t = 0 s, its position is x = 5 cm and it is moving with a velocity of 2 cm/s. What is the angular frequency of the oscillation (in rad/s)?

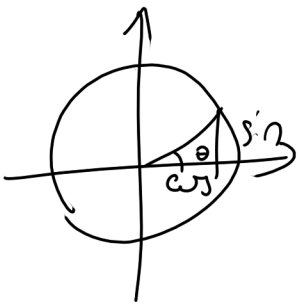


(3) Solution

$$x(t) = A\sin(\omega t + \phi_0)$$

$$v(t) = \frac{dx}{dt} = A\omega\cos(\omega t + \phi_0)$$

$$\phi_0 = \arcsin\left(\frac{5}{10}\right)$$



At $t = 0$, $5\text{cm} = (10\text{cm})\sin(\phi_0)$

$$\Rightarrow \phi_0 = 0.52 \text{ or } 2.62$$

$$v(t=0) > 0 \Rightarrow \phi_0 = 0.52$$

Therefore, $v(0) = 2 \Rightarrow 2 = 10 \cdot \omega \cdot \cos(.52)$

$$\Rightarrow \omega = \frac{2}{10 \cdot \cos(.52)} \approx 0.23 \text{ rad s}^{-1}$$

(2) An object oscillates in 1-dimension with an amplitude of 5 cm. At t = 0 s, its position is x = 2.5 cm and it is moving with a velocity of -4 cm/s. What is the position (in cm) of the object at t = 6 s?

(4) Solution

- $x(t) = A\sin(\omega t + \phi_0) \Rightarrow v(t) = A\omega\cos(\omega t + \phi_0)$
- $5 = 2.5 \cdot \sin\phi_0 \Rightarrow \phi_0 = 0.52$ or 2.62
- As $v(t) < 0 \Rightarrow \phi_0 = 2.62$
- Then $\omega = \dfrac{v(0)}{A\cos(\phi_0)} = 0.92\ \text{rads}^{-1}$
- $x(t=6) = A\sin(6\omega + \phi_0) = 4.8\text{cm}$

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(5) $E = (3\text{kg})\cdot v_x^2 + \left(12\dfrac{\text{J}}{\text{m}^2}\right)x^2$, v is a funct of x

This system is isolated, oscillating, along x-axis

Question: What's the angular frequency? (in rads^{-1})

Solution:

Isolated $\Rightarrow \dfrac{dE}{dx} = 0 \Rightarrow 2\cdot 3\cdot v_x\cdot\dfrac{dv_x}{dx} + 2\cdot 12\cdot x = 0$

Fact! $v\dfrac{dv}{dx} = a$

$\Rightarrow a_x = \dfrac{d^2x}{dt^2} = -\left(4\dfrac{1}{s^2}\right)x$

$\dfrac{d^2x}{dt^2} = -\omega^2 x \Rightarrow \omega = \sqrt{4} = 2\ \text{rads}^{-1}$

(4) A 7.5 kg block is attached to a horizontally-mounted spring with a spring constant of 750 J/m^2. The block is released from rest with the spring extended by 15 cm from its normal length of 27 cm. What is the maximum speed (in cm/s) of the block during its oscillation?

(6) Solution

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{750\ J m^{-2}}{7.5\ kg}} = 10\ rad\ s^{-1}$$

Cordy Theorem

$v(t) = A\omega \cos(\omega t + \phi_0)$: maximized iff $\cos = 1$

$$\Rightarrow V_{max} = A\omega = 15\ cm \cdot 10\ rad\ s^{-1} = 150\ cm\ s^{-1}$$