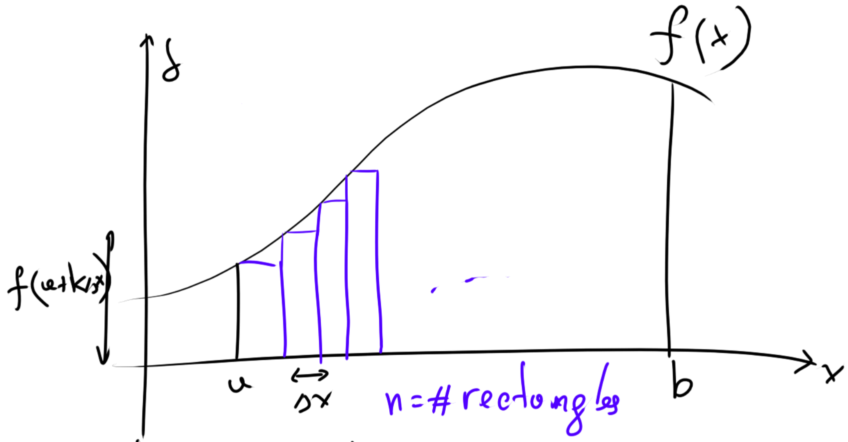
① What's the difference between **Riemann Sums** and **Integrals**?

Riemann Sums

$$\sum_{k=0}^{n} \underbrace{\Delta x}_{\text{length}} \cdot f(\underbrace{a+k\cdot\Delta x}_{\text{displacement}})$$



Integrals are the same! But $n \longrightarrow \infty$

$$\sum_{k=0}^{\infty} \Delta x \cdot f(a+k\cdot\Delta x) = \int_a^b f(x)\,dx$$

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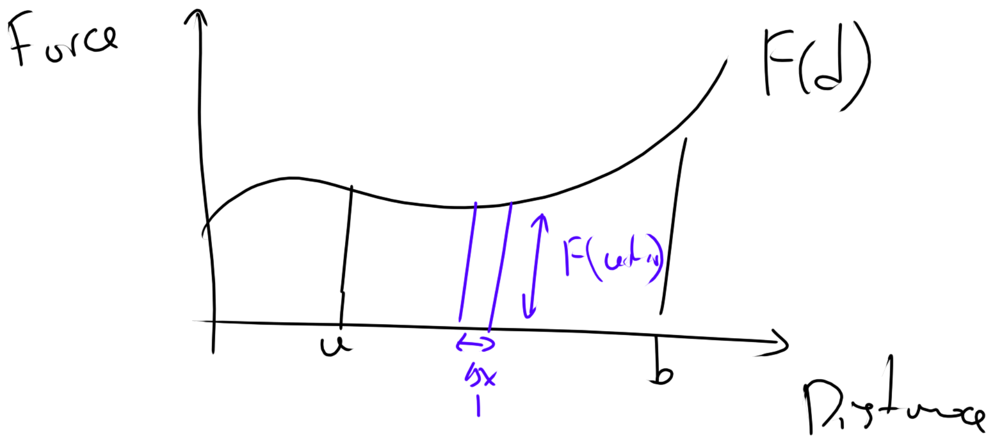
② Work = Force × Distance



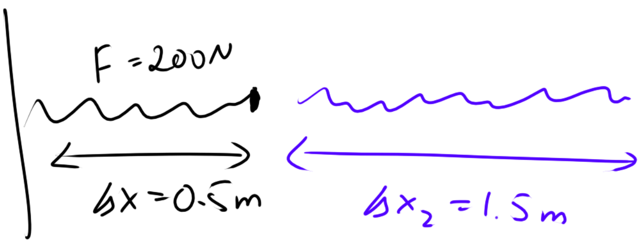
Work done from a to b is the Riemann sum from a to b of F(d).

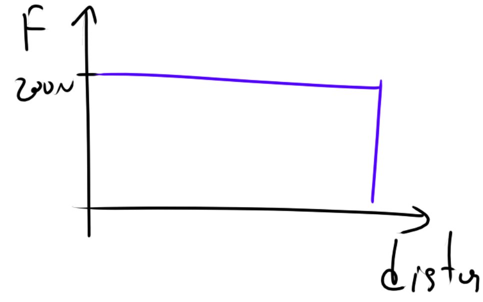
I

(3) $F = 200N$

$\Delta x = 0.5m$

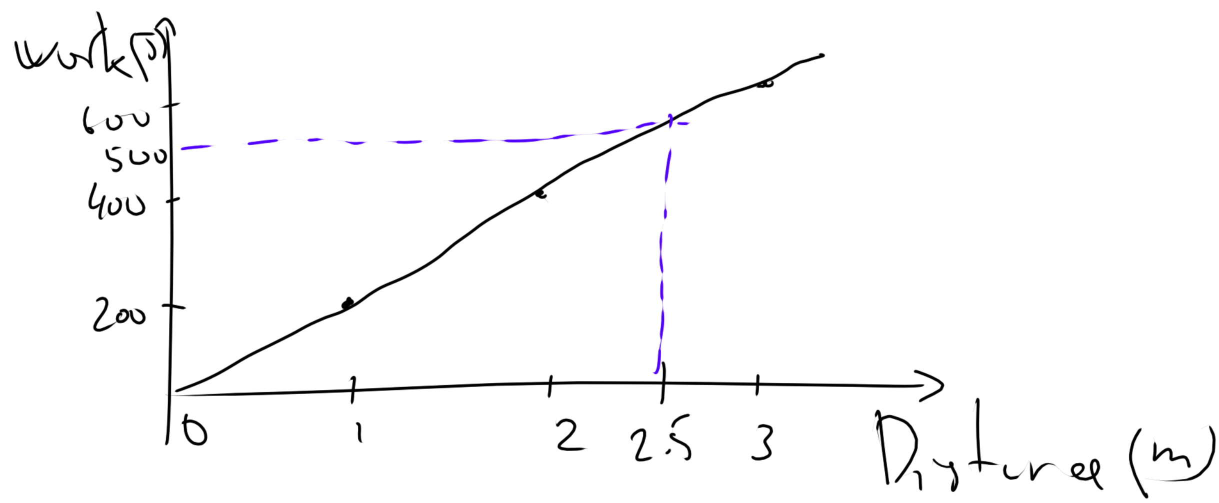
W if stretch 1.5 m more?





Work done $= 200N \cdot 0.5m = 100J$

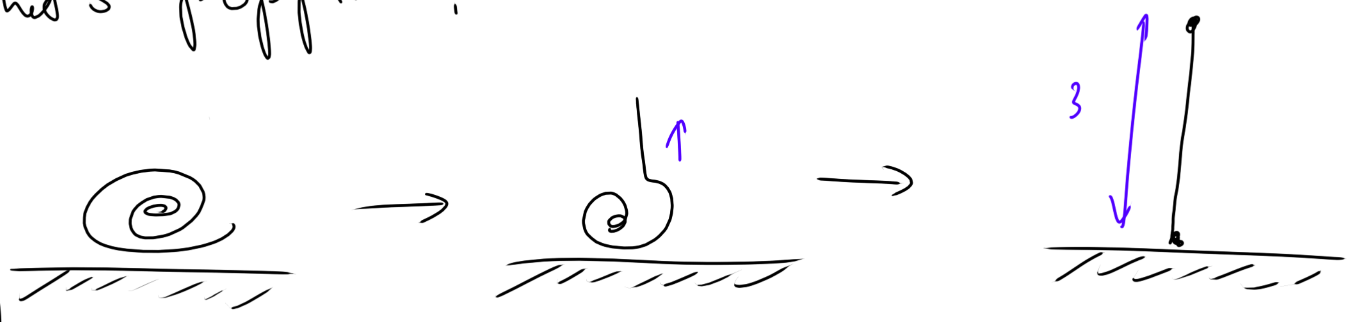
Need to apply 300 J more of work to stretch additional 1.5m.



(4) $l = 3m$

$\rho(x) = 2x(4-x)\ kg\,m^{-1}$

What's poppin':



II)

Work = Force × Displacement

= Length × Density × Gravity × Displacement

Quick Physic Detour:

Second Law of Newton: $F = ma$

On Earth, $a = g \approx 9.8 \, m s^{-2}$

Mass is length × density

Therefore $F = L \times \rho \times g$

Again,

Work = Length × Density × Gravity × Displacement

$\Delta x \qquad 2x(4-x) \qquad 9.8 \qquad 3-x$

Converting Riemann Sums into integrals

$$\sum_{i=1}^{n} f(d) \Delta x$$

When $n \to \infty \Rightarrow \Delta x \to 0$

$\Rightarrow \Delta x = dx$

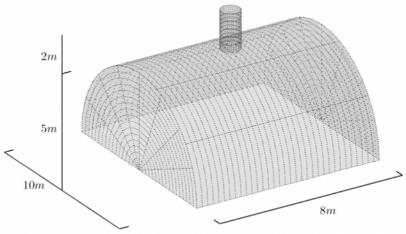
$$\text{Work} = \int_0^3 2x(4-x) \cdot 9.8 \cdot (3-x)\, dx = 220.5 \text{ J}$$

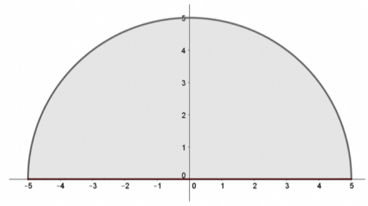
III

(b)

7. *(10 points)* A tank shaped like a semicircular cylinder is shown below. The length of the cylinder is 8 meters and the semicircle has radius 5. **Set-up**, do not solve, an integral which represents the work required to empty the tank by pumping all of the water to the top spout on the tank. The spout is $2\,m$ long.

Note: The density of water is 1000 $\frac{kg}{m^3}$ and the acceleration of gravity is 9.8 $\frac{m}{s^2}$.





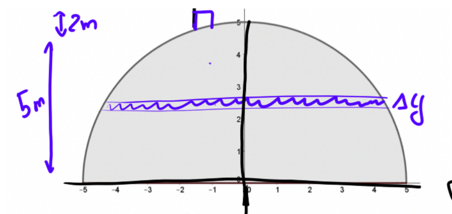
Work = Force × Displacement

= Mass × Gravity × Displacement

= Volume × Density × Gravity × Displacement

? constant constant ?

What's the Displacement?



Displacement $= 7 - y$

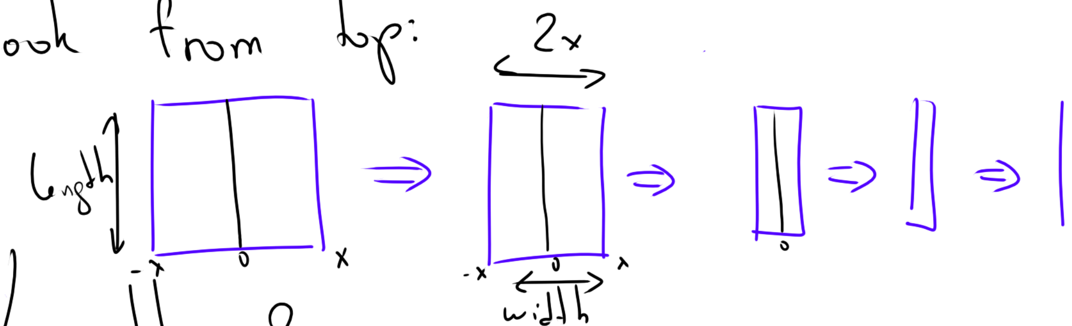
$x^2 + y^2 = 25 = 5^2$

$\Rightarrow x = \sqrt{25 - y^2}$ (✲)

What's the Volume?

Volume = Height × Width × Length

Look from top:



Length = 8

height $= \Delta y$ | Width $= 2\sqrt{25 - y^2}$ from (✲)

IV Width $= 2x$

$$\text{Volume} = \underset{8}{\text{length}} \times \underset{2\sqrt{25-y^2}}{\text{width}} \times \underset{\Delta y}{\text{height}}$$

The finally:

$$\text{Work} = \underset{8}{\text{length}} \times \underset{2\sqrt{25-y^2}}{\text{width}} \times \underset{\Delta y}{\text{height}} \times \underset{1000}{\text{density}} \times \underset{9.8}{\text{gravity}} \times \underset{7-y}{\text{displacement}}$$

$$= \int_0^5 8 \cdot 2\sqrt{25-y^2} \cdot 1000 \cdot 9.8 \cdot (7-y)\, dy \text{ J}$$

V