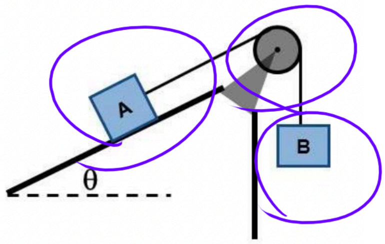
[redacted] Two blocks are attached to one another by a rope that passes over a frictionless pulley as shown in the figure. The mass of block A is 4 kg, the mass of block B is 3 kg, and the mass of the pulley is 3 kg. The pulley and can be modelled as a uniformly dense solid cylinder rotating around an axis through its center. The surface along which block A slides is frictionless, the rope passes over the pulley without slipping, and θ = 30°. What is the magnitude of the acceleration of block A (in m/s²)?



It's March 8th today

① The energy of the system

$$E = \underbrace{\frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 + \frac{1}{2}I_p \omega_p^2}_{\text{Kinetic Part}} + \underbrace{m_A g y_A + m_B g y_B + m_p g y_p}_{\text{Potential Part}}$$

Notice that A and B are connected, then

$$v_A = v_B \Rightarrow v_A^2 = v_B^2$$

Then

$$E = \frac{1}{2}(m_A + m_B)v_A^2 + \frac{1}{2}I_p\omega_p^2 + m_A g y_A + m_B g y_B + m_p g y_p$$

Since the rope doesn't slip,

$$R_p \omega_p = v_A \Rightarrow R_p^2 \omega_p^2 = v_A^2$$

Then

$$E = \frac{1}{2}\left(m_A + m_B + \frac{I_p}{R_p^2}\right)v_A^2 + m_A g y_A + m_B g y_B + m_p g y_p$$

Since puly is ideal, $I_p = \frac{1}{2}m_p R_p^2$, then

$$E = \frac{1}{2}\left(m_A + m_B + \frac{1}{2}m_p\right)v_A^2 + m_A g y_A + m_B g y_B + m_p g y_p$$

Because the system is at an angle,

$$y_A = S_A \sin\theta$$

Then

$$E = \frac{1}{2}\left(m_A + m_B + \frac{1}{2} m_p\right) v_A^2 + m_A g S_A \sin\theta + m_B g y_B + m_p g y_p$$

Since the system is isolated (frictionless), then

$$\frac{dE}{dS_A} = 0 \Rightarrow \left(m_A + m_B + \frac{1}{2} m_p\right) a_A + m_A g \sin\theta + m_B g \frac{dy_B}{dS_A} + m_p g \frac{dy_p}{dS_A}$$

Since pulley doesn't move vertically,

$$\frac{dy_p}{dS_A} = 0$$

Since B is pulled down,

$$\frac{dy_B}{dS_A} = -1$$

Finally,

we want this

$$\left(m_A + m_B + \frac{1}{2} m_p\right) a_A + m_A g \sin\theta - m_B g = 0$$

Consequently,

$$a_A = -\left(\frac{m_A \sin\theta - m_B}{m_A + m_B + \frac{1}{2} m_p}\right) g \simeq 1.15\ \mathrm{m\,s^{-2}}$$