6.21

(3) $T(x) = 50 \cos\left(\dfrac{\pi x}{20}\right)$

length is 10-cm

$$\text{Average} = \frac{\int_0^{10} T(x)\,dx}{10} = \frac{\int_0^{10} 50\cos\left(\frac{\pi x}{20}\right)dx}{10}$$

$$= \frac{50 \cdot \int_0^{10} \cos\left(\frac{\pi x}{20}\right)dx}{10} = 5\int_0^{10} \cos\left(\frac{\pi x}{20}\right)dx$$

$$= 5 \cdot \left[\sin\left(\frac{\pi x}{20}\right) \cdot \frac{20}{\pi}\right]\Bigg|_0^{10}$$

$$= 5 \cdot \frac{20}{\pi} \cdot \sin\left(\frac{\pi}{2}\right)$$

$$\therefore = \frac{100}{\pi}$$

(4) $f(x) = x^4$ on $[0,3]$. f_{avg} is the average,

find c such that $f(c) = f_{avg}$

Solution

① Find $f_{avg} = \dfrac{\int_0^3 x^4\,dx}{3}$

② Find $\sqrt[4]{f_{avg}} = \sqrt[4]{\dfrac{\int_0^3 x^4\,dx}{3}}$

Maybe there is a better way with MVT (Mean Value Theorem) or some other theorems. ¯_(ツ)_/¯