Pre-midterm meeting: Tue Mar 9 2021

1 (Directional Derivatives)

$$f(x,y) = e^x - x^2 y, \quad a = (1,2)$$

$$\vec{u} = 2\vec{i} + \vec{j} = \langle 2,1 \rangle$$

Solution

$$f_x = e^x - 2xy$$

$$f_y = -x^2$$

$$\nabla f = \langle e^x - 2xy, -x^2 \rangle$$

$$D_{\vec{u}} f(a) = \nabla f(a) \cdot \vec{u}$$

$$= \nabla f(1,2) \cdot \langle 2,1 \rangle$$

$$= \langle e-4, -1 \rangle \cdot \langle 2,1 \rangle$$

$$= 2e - 8 - 1 = 2e - 9$$

② $f(x,y,z) = e^{-(x^2+y^2+z^2)}$

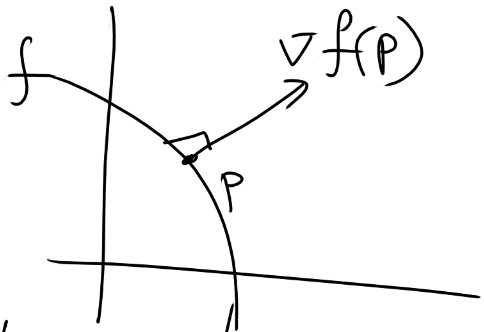
$a = (1, 2, 3) \qquad \vec{u} = \vec{i} + \vec{j} + \vec{k} = \langle 1, 1, 1 \rangle$

Solution
$$f_x = -2x\, e^{-(x^2+y^2+z^2)}$$
$$f_y = -2y f$$
$$f_z = -2z f$$

$$D_{\vec{u}} f(a) = \nabla f(a) \cdot \vec{u} = \langle -2 \cdot e^{-14}, -4e^{-14}, -6e^{14} \rangle \cdot \langle 1, 1, 1 \rangle$$
$$= -2e^{-14} + (-4e^{-14}) + (-6e^{-14})$$
$$= -12e^{-14}$$

③ Given f, find rate of change at P.

Solution
- Find ∇f



- Rate of change $= \|\nabla f(P)\|$ (scalar value)
- Direction $= \dfrac{\nabla f(P)}{\|\nabla f(P)\|}$ (unit vector)

4) Chain Rule

$$(f(g))' = f'(g) \cdot g'$$

Example

$$f(x,y,z) = e^{x^2y+y^3} \cdot \cos(xz^4)$$

Product Rule: $(fg)' = f'g + fg'$

$$f_x = \left(e^{x^2y+y^3}\right)' \cos(xz^4) + e^{x^2y+y^3} \cdot \left(\cos(xz^4)\right)'$$

$$= e^{x^2y+y^3} \cdot \frac{d}{dx} \cdot 2xy\cos(xz^4) + e^{x^2y+y^3} \cdot \left(-\sin(xz^4)\right) \cdot z^4$$

$$f_y = \cos(xz^4) \cdot e^{x^2y+y^3} \cdot \frac{d}{dy} (x^2 + 3y^2)$$

$$f_z = e^{x^2y+y^3} \cdot \left(-\sin(xz^4)\right) \cdot 4z^3x$$

Let $F(a,b,c,\theta) = a^2+b^2-c^2-2ab\cos\theta = 0$

$$F_a = 2a - 2b\cos\theta$$

$$F_b = 2b - 2a\cos\theta$$

$$F_c = -2c$$

$$F_\theta = 2ab\sin\theta$$

$$\theta_a^{(10,16,22)} = -\frac{F_a}{F_\theta} = -\frac{41}{80\sqrt{21}} \qquad \frac{da}{dt} = 2$$

$$\theta_b^{(10,16,22)} = -\frac{F_b}{F_\theta} = \frac{-5}{8\sqrt{21}} \qquad \frac{db}{dt} = 2$$

$$\theta_c(10,16,22) = -\frac{F_c}{F_\theta} = \frac{11}{16\sqrt{21}} \qquad \frac{dc}{dt} = -3$$

$$\frac{d\theta}{dt} = \frac{\partial\theta}{\partial a}\cdot\frac{\partial a}{\partial t} + \frac{\partial\theta}{\partial b}\cdot\frac{\partial b}{\partial t} + \frac{\partial\theta}{\partial c}\cdot\frac{\partial c}{\partial t}$$

$$\approx -0.946$$

⑥ $\sin(z) + y\cos(z) + xyz = 10$

Solution (Find $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$)

Let $F(x,y,z) = \sin(z) + y\cos(z) + xyz$
$= 10$

$F_x = yz \qquad F_y = \cos(z) + xz$

$F_z = \cos(z) - y\sin(z) + xy$

Fact: $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$

$$\therefore \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{-yz}{-y\sin(z)+xy}$$ ← answer

$$\therefore \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{\cos(z)+xz}{-y\sin(z)+xy}$$ ←

Say you have $f(x,y,z)$.

Problem: find crit. point & categorize

Step 1: find f_x, f_y, f_z

Step 2: Solve for x, y, z, the system

$$\begin{cases} f_x = 0 \\ f_y = 0 \\ f_z = 0 \end{cases}$$

(Crit. points found)

Only find

Step 3: Categorize:

Find: $f_{xx}, f_{yy}, f_{zz},$
$f_{xy}, f_{yx}, f_{xz},$
f_{zx}, f_{yz}, f_{zy}

Hint: $f_{xy} = f_{yx}$

Find $D(x,y,z)$.

if all $<0 \Rightarrow$ max

if all $>0 \Rightarrow$ min

otherwise, $\Rightarrow$ saddle