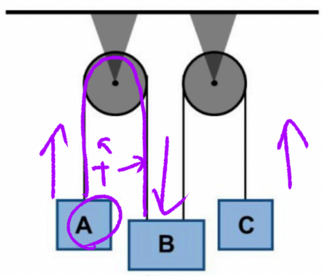
Three blocks are connected by massless ropes that move without slipping over massless and frictionless pulleys as shown in the figure; the mass of block A is 2 kg, the mass of block C is 3 kg, and the mass of block B is 6 kg. What is the magnitude of the tension (in N) in the rope connecting block A and block B?



(1) Solution Let ↓ be the positive force direction, negative otherwise.

Let's see how the forces react:

$$\vec{F}_{net} = m_B g - m_A g - m_C g$$

The effective

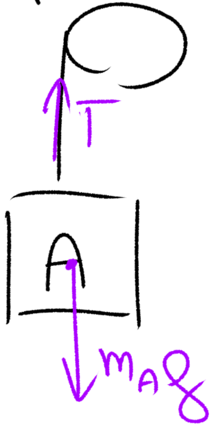
$$m_{effective} = m_A + m_B + m_C = m_{system}$$

From Newton's 2nd law.

$$\vec{a} = \frac{\vec{F}_{net}}{m_{effective}} \Rightarrow a = \frac{m_B g - m_A g - m_C g}{m_B + m_A + m_C}$$

$$= g\left(\frac{m_B - m_A - m_C}{m_B + m_A + m_C}\right) \quad ☆$$

Let's look at forces to find T



$$\Rightarrow T - m_A g = m_A a$$

$$\Rightarrow T = m_A g + m_A a = m_A (g + a)$$

I

Recall A, Plug it into T:

$$T = m_A\left(g + g\left(\frac{m_B - m_A - m_c}{m_B + m_A + m_c}\right)\right)$$

$$= m_A g\left(1 + \frac{m_B - m_A - m_c}{m_B + m_A + m_c}\right)$$

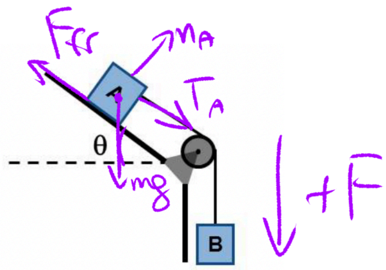
$$= 2 \cdot g\left(1 + \frac{6-2-3}{6+2+3}\right)$$

$$= 2g\left(1 + \frac{1}{11}\right) = 2g \cdot \frac{12}{11} = \frac{24g}{11}$$

$$\approx 21.38\ N.$$

$\therefore$ Tension in rope connecting $\boxed{A}$ and $\boxed{B}$ is 21.38 N.

(4) Two blocks, A and B, are attached to one another by a rope that passes over a frictionless pulley as shown in the figure; the mass of block A is 6 kg and the mass of block B is 4 kg. The coefficient of kinetic friction between the block A and the surface is 0.2. The pulley can be modeled as a uniformly dense solid cylinder with a radius of 2.5 cm and a mass of 4 kg. What is the magnitude of the tension (in N) in the rope between block A and the pulley if θ = 30°?



② Solution Let's find system's force

$$\vec{F}_{net} = m_B g + m_A g \sin\theta - \mu_k n_A$$

II

What's n_A?

$$n_A = m_A g \cos\theta$$

Then

$$\vec{F}_{net} = m_B g + m_A g \sin\theta - \mu_k m_A g \cos\theta$$
$$= g\left(m_B + m_A \sin\theta - \mu_k m_A \cos\theta\right)$$

What's the effective (system's) mass?

$$m_{effective} = m_A + m_B + \frac{1}{2} m_p$$

Let's call Newton

$$\vec{a}_{system} = \frac{\vec{F}_{net}}{m_{system}} = \frac{g\left(m_B + m_A \sin\theta - \mu_k m_A \cos\theta\right)}{m_A + m_B + \frac{1}{2} m_p}$$

$$= g\left(\frac{m_B + m_A \sin\theta - \mu_k m_A \cos\theta}{m_A + m_B + \frac{1}{2} m_p}\right)$$

All accelerations are the same because all objects are connected.

Let's find the tension.

$$\left(\vec{F}_{net}\right)_a = T_A + m_A g \sin\theta - \mu_k m_A g \cos\theta = m_A a_A$$

$$\Rightarrow T_A = m_A a_A - m_A g \sin\theta + \mu_k m_A g \cos\theta$$

$$= m_A \left(a_A - g \sin\theta + \mu_k g \cos\theta\right)$$

Plug in

$$T_A = m_a \left(g \left(\frac{m_B + m_A \sin\theta - \mu_k m_a \cos\theta}{m_A + m_B + \frac{1}{2} m_p} \right) - g \sin\theta + \mu_k g \cos\theta \right)$$

$$= m_a g \left(\frac{m_B + m_A \sin\theta - \mu_k m_a \cos\theta}{m_A + m_B + \frac{1}{2} m_p} - \sin\theta + \mu_k \cos\theta \right)$$

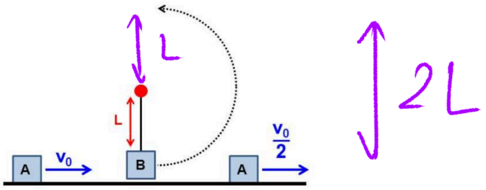
magic

$$= 9.99 N.$$

IV

QUESTION 3

Block A is sliding across a horizontal and frictionless surface when it collides with block B as shown in the figure.



Block B is attached to a massless string of length L = 1 m and is free to rotate as a pendulum. The speed of block A after the collision is half its speed before the collision. Block B was at rest before the collision. The mass of block A is 7 kg and the mass of block B is 2 kg. What is the minimum initial speed (in m/s) that block A must have for block B to swing through a complete vertical circle?

3 Attempt

Full Solution

$$(K_i)_A = \frac{1}{2} m_A v_0^2$$

$$(K_f)_A = \frac{1}{2} m_A \left(\frac{v_0}{2}\right)^2 = \frac{1}{8} m_A v_0^2$$

$$(U_i)_A = 0 = (U_f)_A$$

So all of ΔK goes into block B's potential energy.

Block $\boxed{B}$ must reach height $2L$,

$\iff$ Block $\boxed{B}$ must have enough energy like $U = m_B g h = m_B g 2L = 2 m_B g L$.

$\iff$ Block $\boxed{B}$ can only get this much energy from Block $\boxed{A}$. Then

$$2 m_B g L \le -\Delta K = \frac{1}{2} m_A v_0^2 - \frac{1}{8} m_A v_0^2$$

$$= m_A v_0^2 \left(\frac{3}{8}\right)$$

NOT cancelling out

$$\iff \frac{2 m_B g L}{m_A \, 3/8} \le v_0^2$$

$\boxed{V}$

$$\iff V_0^2 \geqslant \frac{16}{3} \cdot \frac{m_B g L}{m_A}$$

$$\iff V_0 \geqslant \pm\sqrt{\frac{16}{3} \cdot \frac{m_B g L}{m_A}}$$

Because Block $\boxed{A}$ moves to the right, $V_0 < 0$ is not considered. Then

$$V_0 \geqslant \sqrt{\frac{16}{3} \cdot \frac{m_B g L}{m_A}}$$

$$= \sqrt{\frac{16}{3} \cdot \frac{2 \cdot 9.8 \cdot 1}{7}}$$

$$\approx 3.86\,ms^{-1}$$

$\overline{\boxed{VI}}$