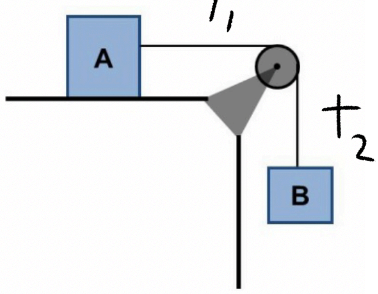
Two blocks, A and B, are attached to one another by a rope that passes over a frictionless pulley as shown in the figure; the mass of block A is 6 kg and the mass of block B is 4 kg. The coefficient of kinetic friction between the block A and the surface is 0.15. The pulley can be modeled as a uniformly dense solid cylinder with a radius of 2.5 cm and a mass of 4 kg. What is the magnitude of the tension (in N) in the rope between block A and the pulley?



$T_1 \neq T_2$

Solution

$$(\vec{F}_{external})_{net} = m_B g - \mu_k n_A$$

We have

$$n_A = m_A g \quad \text{(Balanced)}$$

$$\Rightarrow \vec{F}_{net} = m_B g - m_A g \mu_k$$

Now let's find mass effective.

$$m_{effective} = m_A + m_B + \tfrac{1}{2} m_p$$

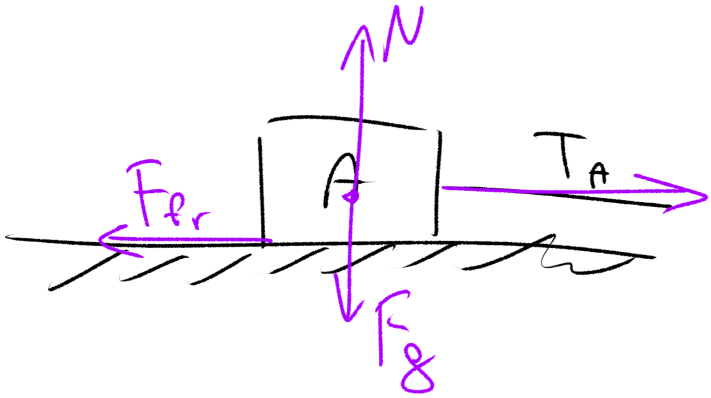
From Newton's 2nd law.

$$\vec{a}_{system} = \frac{\vec{F}_{net}}{m_{eff}} = \frac{m_B g - m_A g \mu_k}{m_A + m_B + \frac{1}{2} m_p}$$

I

$$= g\left(\frac{m_B - m_A \mu_k}{m_A + m_B + \frac{1}{2} m_p}\right) \leftarrow \text{scalar}$$

Let's find T_A



$$\Rightarrow T_A - F_{fr} = m_A a$$

$$\Rightarrow T_A = m_A a + m_A g \mu_k$$

$$= g\left(m_A\left(\frac{m_B - m_A \mu_k}{m_A + m_B + \frac{1}{2} m_p}\right) + m_A \mu_k\right)$$

$$\therefore \quad = 24.01\ N.$$

<u>Extra:</u> T_B? $(29.07N)$

II)

(2) Net force is given as follow

$$F_{net}(x) = \left(3\frac{N}{m^2}\right)x^2$$

for a 3kg mass. Starts at rest at $x=2m$ what's the magnitude of its linear momentum at $x=4m$?

Solution Recall

$$\Delta K = \int F_{net}\,dx$$

Substitution

$$\Delta K = \int_2^4 3x^2\,dx$$

$$= \left[x^3\right]\Big|_2^4$$

$$= 4^3 - 2^3$$

$$= 56\,J$$

But $v(2)=0$ (at rest), so

$v_i = 0 \Rightarrow K_i = 0 \Rightarrow K_f = 56J$

Then

$$K_f = \frac{1}{2} m v_f^2 = 56J$$

$$\Rightarrow V_f = \sqrt{\frac{112}{m}}$$

$$\approx 6.11 \ ms^{-1}$$

$$= V(x=4).$$

Finally,

$$p(x=4) = V(x=4) \cdot m$$

$$= 6.11 \cdot 3$$

$$= 18.33 \ kg\,ms^{-1}$$

3) Net torque is given

$$\tau_{net}(\theta) = \left(2 \frac{Nm}{rad}\right) \theta$$

The rod is free to rotate around a frictionless axle located at one end of the rod. The mass and the length of the rod are 3kg and 0.5m. If rod starts from rest, what's the mag. of its final angular momentum after has gone for 2 revolutions?

This problem is similar to 2

$$\Delta K_{rot} = \int_{\theta_i}^{\theta_f} \tau_{net}\, d\theta$$

Let $\theta_{max} = 4\pi$ (2 full rev)

$$\Delta K_{rot} = \int_{0}^{\theta_{max}} 2\theta\, d\theta$$

$$= \left[\theta^2\right]\Big|_0^{\theta_{max}}$$

$$= \theta_{max}^2$$

Because starts from rest $\Leftrightarrow K_{rot,i} = 0$

$$\Rightarrow K_{rot,f} = \Delta K = \theta_{max}^2$$

Recall

$$K_{rot,f} = \frac{1}{2} I \omega_f^2$$

Then

$$\omega_f = \sqrt{\frac{2\theta_{max}^2}{I}}$$

$\overline{V}$

Recall

$$L = I\omega$$

Then

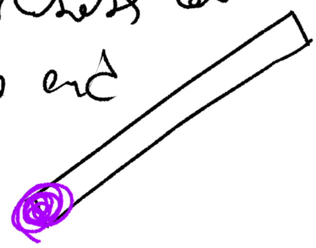
$$L_f = I\omega_f$$

$$= I\sqrt{\frac{2\theta_{max}^2}{I}}$$

$$= \sqrt{2I\theta_{max}^2}$$

What's the Inertia here?

rotates at one end



$$I = \frac{1}{3} m L^2$$

Therefore

$$L_f = \sqrt{\frac{2}{3} m L^2 \theta_{max}^2}$$

$$= \sqrt{\frac{2}{\cancel{3}} \cdot \cancel{3} (0.5)^2 (4\pi)^2}$$

$$= 0.5\, 4\pi \cdot \sqrt{2}$$

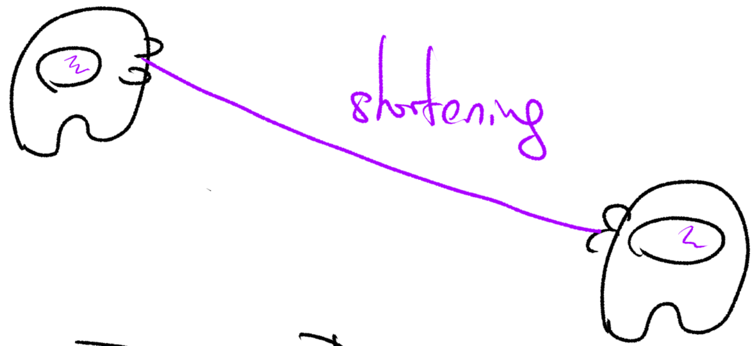
$$= 2\pi\sqrt{2}$$

$$\approx 8.89\ kg\,m^2 s^{-1}$$

VI

4

Two astronauts, each with a mass of 50 kg, are connected by a 7 m massless rope. Initially they are rotating around their center of mass with an angular velocity of 0.5 rad/s. One of the astronauts then pulls on the rope shortening the distance between the two astronauts to 4 m. What is angular speed (in rad/s) of the system at this new separation distance between the astronauts? You may model each astronaut as a point particle.



<u>Solution</u> $I_i\omega_i = I_f\omega_f$

$$\Rightarrow \left(m_A R_{A,i}^2 + m_B R_{B,i}^2\right)\omega_i = \left(m_A R_{A,f}^2 + m_B R_{B,f}^2\right)\omega_f$$

As $m_A = m_B$

$$\left(R_{A,i}^2 + R_{B,i}^2\right)\omega_i = \left(R_{B,f}^2 + R_{A,f}^2\right)\omega_f$$

As $R_A = R_B$

$$R_f^2\,\omega_f = R_i^2\,\omega_i$$

$$\Rightarrow \omega_f = \frac{R_i^2}{R_f^2}\cdot\omega_i$$

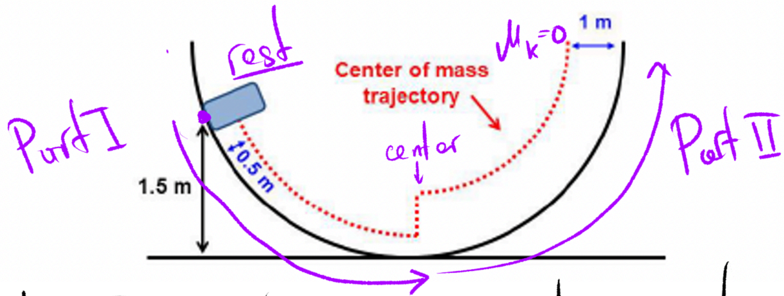
$$= \frac{7^2}{4^2}\cdot\frac{1}{2}$$

$$= 1.53 \text{ rad s}^{-1}$$

VII

5

A skateboarder is skating through a half-pipe with a radius of 4 m. Initially she is in a crouched position with her center of mass 0.5 m above the surface of the half-pipe. She starts from rest with her center of mass a height of 3 m above the base of the half-pipe and experiences no loss of energy from friction as she moves. At the bottom of the half-pipe she stands up and lifts her arms into the air, thereby raising her center of mass to 1.0 m. She then continues up the other side of the half-pipe. She had to perform work to lift her arms while skating. How much work (in J) did she do if her mass is 78 kg? You may model the skateboarder as a point particle.



Hint: Try to find the change in rotational kinetic energy

Solution: $W = \Delta E$

$$\Rightarrow W = \Delta K_{rot}$$

$$= \frac{1}{2} I_f \omega_f^2 - \frac{1}{2} I_i \omega_i$$

$$= \frac{1}{2} m R_f^2 \omega_f^2 - \frac{1}{2} m R_i^2 \omega_i^2$$

$$= \frac{1}{2} m \left(R_f^2 \omega_f^2 - R_i^2 \omega_i^2 \right)$$

$$= \frac{1}{2} \cdot (78) \left((4-1)^2 (2.98)^2 - (4-0.5)^2 \cdot (2.19)^2 \right)$$

$$\approx 825.7 \text{ J}$$

(A) Find ω_i and ω_f by solving ΔK_{rot}, using the relationship between energy and ~~Conversation~~ of momentum conservation

VIII