KNAPSACK (repetition)

$\Updownarrow$

"LONGEST PATH" IN A DAG

INPUT: A set of items with (weights, values)

$(w_1, v_1), (w_2, v_2) \ldots (w_n, v_n)$

TOTAL WEIGHT = W

GOAL: FIND A SUBSET OF ITEMS with Total weight $< W$ and maximum value

$W = 30$

$\{A, A\}$

$15 + 15 \leq 30$

$43 + 43 = 86$

$\{A, C, D\}$

$15 \quad 7 \quad 8 \quad \leq 30$

$43 + 19 + 23 \approx 84$

ITEM	WEIGHT	VALUE
A	15	43
B	6	18
C	7	19
D	8	23
E	5	14
F	4	9

$W = 21$

KNAPSACK
(no repetition)

INPUT: Sequence of items with (weight, value)
$(w_1, v_1), (w_2, v_2) \ldots (w_n, v_n)$
Maximum Total Weight $= W$

GOAL: Bundle of items with maximum total value

ITEM	WEIGHT	VALUE
A	15	43
B	6	18
C	7	19
D	8	23
E	5	14
F	4	9

DEFINE "SUBPROBLEMS":

Imagine the optimal solution

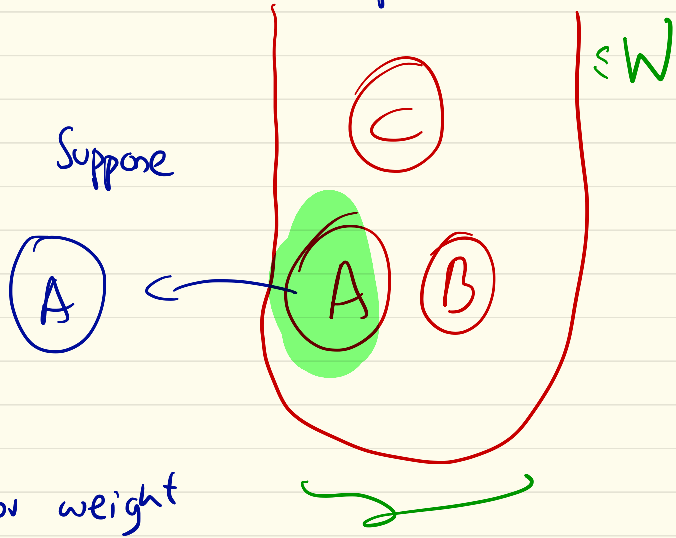
& break it up into smaller pieces.

Optimal Knapsack for **W** — {A}

= Optimal knapsack for weight $W - w_A$ without using {A}

Suppose

optimal knapsack



Trial 1:

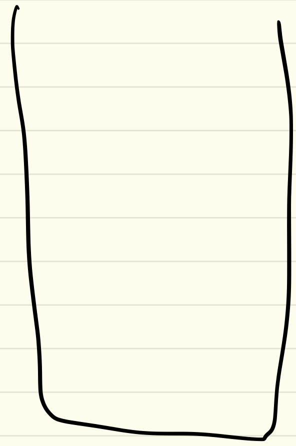
let

$K[w]$ = optimal knapsack for weight w

for each weight $w = 1 .. W$.

Imagine:

Is item (w_n, v_n) part of it??



$K[w,i]$ = optimal knapsack with total weight $\leq w$ that uses only elements from $\{1..i\}$

$K[W,n]$ = Answer

Attempt 2

$K[w, F]$ $\equiv$ optimal Knapsack with total weight $\leq w$ and using no forbidden elements.

$F \to 2^n$

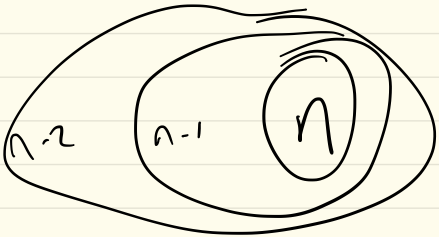
weight, forbidden elements

$\{1 \dots W\} \cdot 2^n$

$K[w\ \{n\}]$

$K[w, \{n-1, n\}]$

$\vdots$

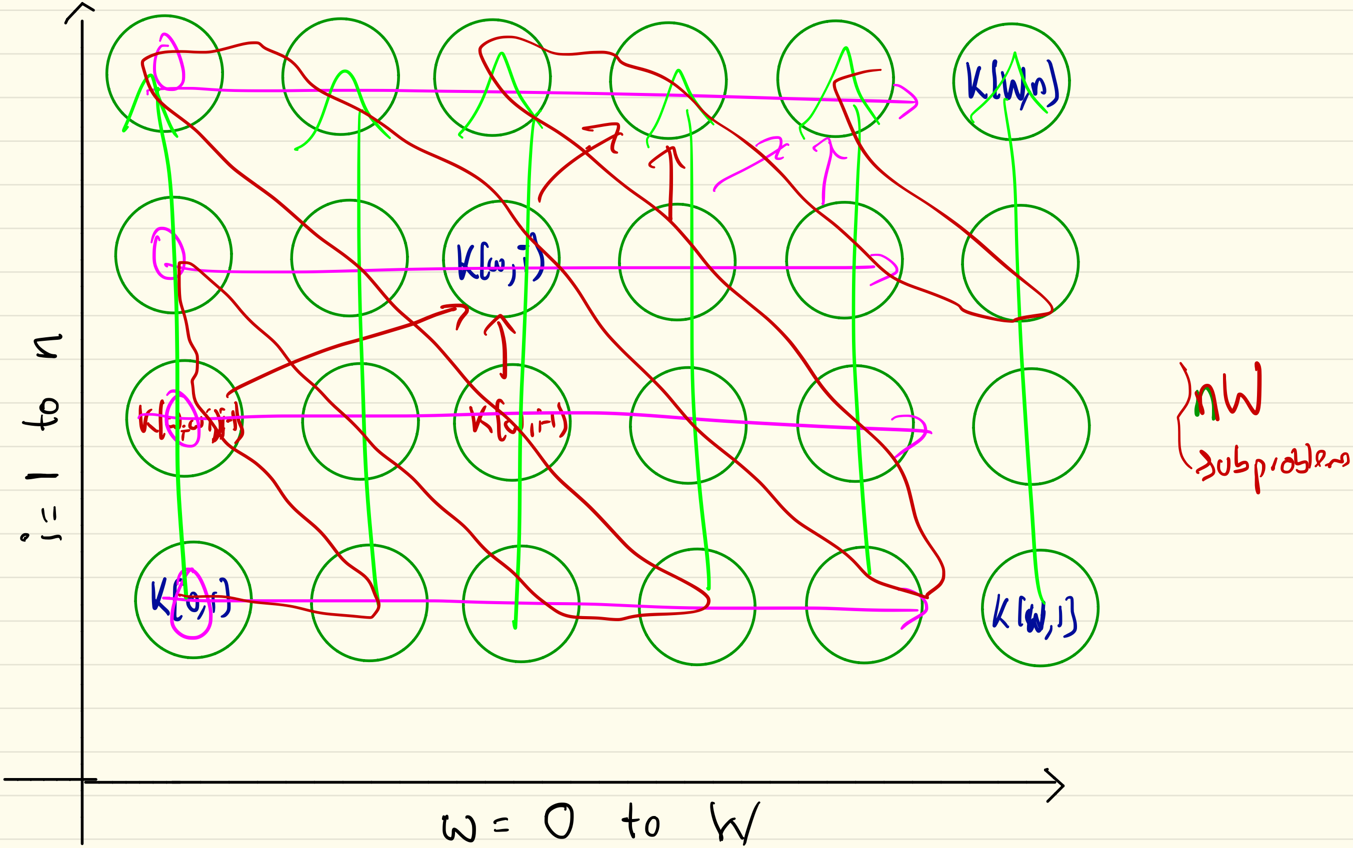


RECURRENCE RELATION:

$K[w,i]$ = optimal knapsack with total weight $\leq w$ that uses only elements from $\{1..i\}$

$$K[w,i] = \max \begin{cases} \text{contains } \{i\} & v_i + K[w-w_i, i-1] \\ \text{does not contain } \{i\} & K[w, i-1] \end{cases}$$

SUBPROBLEMS: (DAG of Subproblems)



shortest path

$$D[i,j] = \min_{k} \left(D[i,k] + D[k,j] \right)$$

$K[0, i] = 0 \qquad \forall\ i = \{1 .. n\}$

$K[\omega, 0] = 0 \qquad \forall\ \omega = \{1 .. W\}$

for $i = 1$ to n

  for $\omega = 1$ to W

$$K[\omega, i] = \text{maximum} \begin{cases} (\omega_i < \omega)\ K[\omega - \omega_i, i-1] + v_i \\ K[\omega, i-1] \end{cases}$$

return $K[W, n]$.