Application of Multiplicative Weights.

Cheat

Scissors

Paper

Rock

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Scissors

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Cheat

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Paper

Rock S

Application of Multiplicative Weights.

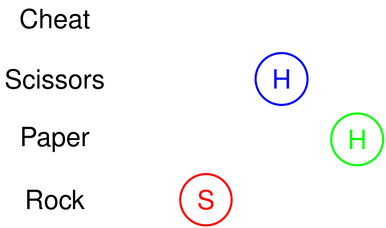
Cheat

Scissors H

Paper

Rock S

Application of Multiplicative Weights.



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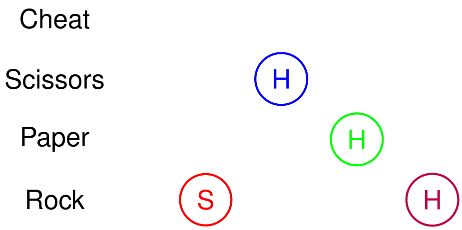

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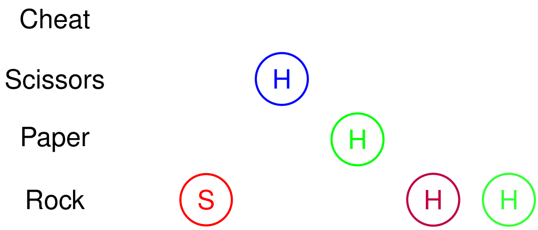

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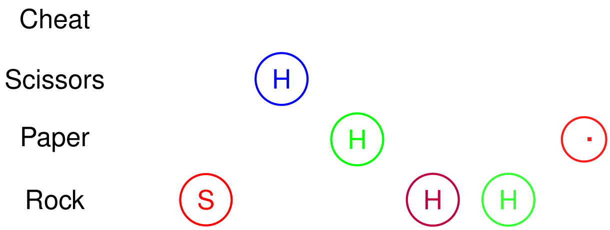

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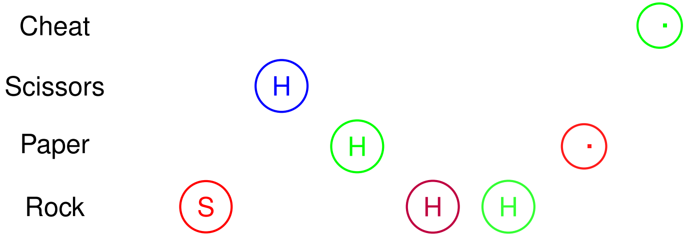

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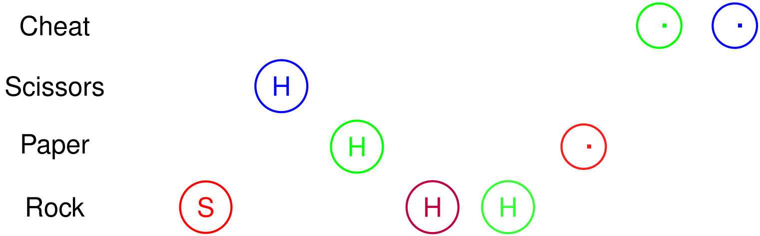

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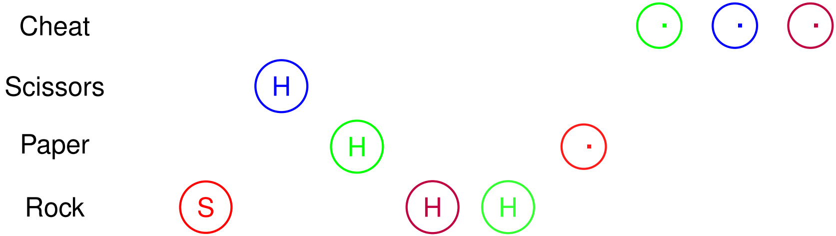

BTW: Only the circles mean anything.

Lecture in a minute

Multiplicative Weights
$\Longrightarrow$ strong duality for Zero-Sum Games.
Column player plays MW distribution.
Row player plays best response.
Output average of column player as y.
Output average of row player as x. MW Alg (Column strategy) $\rightarrow$
Close to best response against row.
Row x is best response against y.

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Boosting: (Extra.) Barely learning $\implies$ really good learning.
Alg that predicts $1/2+\varepsilon$ of input points.
plus multiplicative weights.

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Expert/Input points lose when alg predicts correctly.
Adversary every day is learning algorithm.
Predict majority.

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Predict majority.
MW analysis $\implies$ most points predicted correctly.

Matrix Reminders.

$m \times n$ matrix A.

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m-dimensional vector x.

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Ay is m-dimensional (column) vector.

Two person zero sum games.

$m \times n$ payoff matrix A.

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Payoff for strategy pair (x, y):

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That is,

$$\sum_i x_i \left(\sum_j a_{i,j} y_j \right) = \sum_j \left(\sum_i x_i a_{i,j} \right) y_j.$$

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Recall row maximizes, column minimizes.

Equilibrium.

Equilibrium pair: (x^*, y^*)?

---

[1] $A^{(i)}$ is ith row.

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$$p(x,y) = (x^*)^T A y^* = \min_y (x^*)^T A y = \max_x \; x^T A y^*.$$

(No better column strategy, no better row strategy.)

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$\max_i A^{(i)} \cdot y^* = (x^*)^T A y^*$.[1]

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(No better column strategy, no better row strategy.)

No row is better:
$\max_i A^{(i)} \cdot y^* = (x^*)^T Ay^*$.[1]

No column is better:
$\min_j (A^T)^{(j)} \cdot x^* = (x^*)^T Ay^*$.

[1] $A^{(i)}$ is ith row.

Equilibrium: play the boss...

$$A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Column player: every column payoff is $\frac{1}{6}$.

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Column player: every column payoff is $\frac{1}{6}$.

Both only play optimal strategies!

Equilibrium: play the boss...

$$A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equilibrium: Row: $(0, \frac{1}{3}, \frac{1}{6}, \frac{1}{2})$. Column: $(\frac{1}{3}, \frac{1}{2}, \frac{1}{6})$.

Payoff? Remember: weighted average of pure strategies.

Row Player.

Strategy 1: $\frac{1}{3} \times 0 + \frac{1}{2} \times -1 + \frac{1}{6} \times 1 = -\frac{1}{3}$
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Both only play optimal strategies! Complementary slackness.
Why play more than one? Limit opponent payoff!

Equilibrium: always?

Equilibrium pair: (x^*, y^*)?

$$p(x,y) = (x^*)^T A y^* = \min_y (x^*)^T A y = \max_x \; x^T A y^*.$$

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Does an equilibrium pair:, (x^*, y^*), exist?

Best Response

Column goes first:

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Find y, where best row is not too high..

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Example: Roshambo.

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Example: Roshambo. Value of R?

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Example: Roshambo. Value of R?

Row goes first:
Find x, where best column is not low.

Best Response

Column goes first:
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Example: Roshambo. Value of R?

Row goes first:
Find x, where best column is not low.

$$C = \max_x \min_y (x^T A y).$$

Best Response

Column goes first:
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Duality.

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Doesn't matter who plays first!

Proof of Equilibrium.

Sort of.

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Aproximate equilibrium ...

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Equilibrium: (x, y)

$R(y) = C(x)$

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$R(y) = C(x) \rightarrow R(y) - C(x) = 0.$

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Approximate Equilibrium: $R(y) - C(x) \leq \varepsilon.$

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With $R(y) > C(x)$ (weak duality)

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$\to$ "Response y to x is within ε of best response"

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Proof of approximate equilibrium.

How?

(A) Using geometry.

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How?

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(B) Using a fixed point theorem.

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How?

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(C) Using multiplicative weights.

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(C)
Not hard.

Proof of approximate equilibrium.

How?

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(B) Using a fixed point theorem.

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(C)
Not hard. Even easy.

Proof of approximate equilibrium.

How?

(A) Using geometry.

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(C)
Not hard. Even easy. Still, head scratching happens.

Games and experts

Again: find (x^*, y^*), such that

Games and experts

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Experts Framework:
n Experts,

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Multiplicative Weights Method yields loss L where

$L \leq (1+\varepsilon)L^* + \frac{\log n}{\varepsilon}$

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Assume: A has payoffs in $[0, 1]$.

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Choose row of A that maximizes column's expected loss.

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Let y_t be distribution (column strategy) on day t.

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Choose row of A that maximizes column's expected loss.
Let x_t be indicator vector for this row.

Picture of Algorithm.

x-player (row)

$t = 1$ $t = 2$ $t = 3$ $t = 4$ $\cdots$

y-player (col)

$\propto w^t$ mult. weights

y_1

y_2

$\vdots$

y_m

Picture of Algorithm.

x-player (row)

		$t=1$	$t=2$	$t=3$	$t=4$	$\cdots$
	y_1	1				
y-player (col)	y_2	0				
$\propto w^t$ mult. weights	$\vdots$	$\vdots$				
	y_m	0				

Picture of Algorithm.

x-player (row)

		$t=1$	$t=2$	$t=3$	$t=4$	$\cdots$
	y_1	1	0			
y-player (col)	y_2	0	0			
$\propto w^t$ mult. weights	$\vdots$	$\vdots$	$\vdots$			
	y_m	0	1			

Picture of Algorithm.

x-player (row)

y-player (col)

$\propto w^t$ mult. weights

	$t=1$	$t=2$	$t=3$	$t=4$	$\cdots$
y_1	1	0	0		
y_2	0	0	1		
$\vdots$	$\vdots$	$\vdots$	$\vdots$		
y_m	0	1	0		

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	y_1	1	0	0	1	
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	y_1	1	0	0	1	$\cdots$
y-player (col)	y_2	0	0	1	0	$\cdots$
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	y_m	0	1	0	0	$\cdots$

Approximate Equilibrium!

Experts: y_t is MW strategy on day t, x_t is best row against y_t.

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Claim: (x^*, y^*) are 2ε-optimal for matrix A for $T = \frac{\ln n}{\varepsilon^2}$.

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Thus, algorithm loss, L, is $\geq T \times R(y^*)$.

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Let $x^* = \frac{1}{T}\sum_t x_t$ and $y^* = \text{argmin}_{y_t} x_t A y_t$.

Claim: (x^*, y^*) are 2ε-optimal for matrix A for $T = \frac{\ln n}{\varepsilon^2}$.

Row payoff: $R(y^*) = \max_x xAy^*$.
Loss on day t, $x_t A y_t \geq R(y^*)$ by the choice of y^* .
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$\square$

Wrapping up duality theorem.

For any ε, there exists an ε-Approximate Equilibrium.

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Does an equilibrium exist?

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$T = \frac{\ln n}{\varepsilon^2} \to O(nm\frac{\log n}{\varepsilon^2})$.

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Versus Linear Programming: $O(n^3 m)$

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"In practice."

Boosting.

This is for fun. Not testable.

Boosting...

Get labelled dataset.

Boosting...

Get labelled dataset.

Learn model 1.

Boosting...

Get labelled dataset.

Learn model 1.

Take points previous models don’t learn well.

Boosting...

Get labelled dataset.

Learn model 1.

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Learn model 2.

Boosting...

Get labelled dataset.

Learn model 1.

Take points previous models don't learn well.

Learn model 2.

Repeat.

Boosting...

Get labelled dataset.

Learn model 1.

Take points previous models don't learn well.

Learn model 2.

Repeat.

Combine models $1, \ldots n$, for better predictor?

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Get labelled dataset.

Learn model 1.

Take points previous models don't learn well.

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Repeat.

Combine models $1, \ldots n$, for better predictor?

How?

Boosting...

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Learn model 1.

Take points previous models don't learn well.

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Repeat.

Combine models $1, \ldots n$, for better predictor?

How? Majority Vote.

Boosting...

Get labelled dataset.

Learn model 1.

Take points previous models don't learn well.

Learn model 2.

Repeat.

Combine models $1, \ldots n$, for better predictor?

How? Majority Vote.

How to analyse?

Boosting Example.

Learning just a bit.

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Example: set of labelled points, find hyperplane that separates.

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Learning just a bit.

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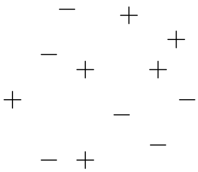

Looks hard.

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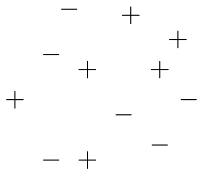

Looks hard.

$1/2$ of them?

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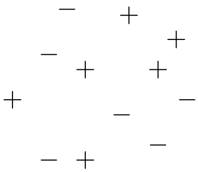

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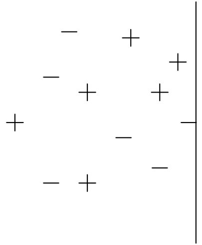

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Arbitrary line.

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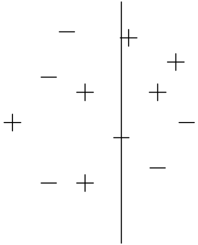

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Arbitrary line. And Scan.

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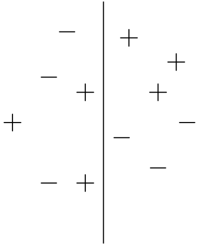

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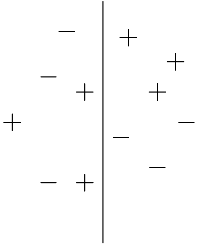

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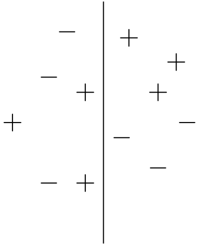

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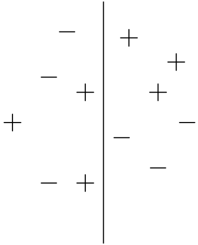

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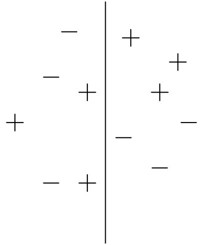

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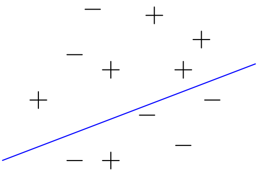
Not really important but ...

Boosting Example.

Learning just a bit.

Example: set of labelled points, find hyperplane that separates.



Looks hard.

$1/2$ of them? Easy.
Arbitrary line. And Scan.

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Weak Learner/Strong Learner

Input: n labelled points.

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produce hypothesis correctly classifies $\frac{1}{2} + \varepsilon$ fraction

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Same thing?

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Can one use weak learning to produce strong learner?

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Same thing?

Can one use weak learning to produce strong learner?

Boosting: use a weak learner to produce strong learner.

Poll.

Given a weak learning method (produce ok hypotheses.)

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Can we do this?

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Can we do this?

(A) Yes

(B) No

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If yes.

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Multiplicative Weights!

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Multiplicative Weights!

The endpoint to a line of research.

Boosting/MW Framework

Experts are points.

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Experts are points. ‘Adversary” is weak learner.

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Strong learner algorithm will come from adversary.

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Really? Proof?

Adaboost proof.

Claim: $h(x)$ is correct on $1-\mu$ of the points

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Adaboost proof.

Claim: $h(x)$ is correct on $1 - \mu$ of the points !

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majority of $h_t(x)$ are wrong for $x \in S_{bad}$.

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Let S_{bad} be the set of points where $h(x)$ is incorrect.

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Consider $W(t) = \sum_i w_i^t$.

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$W(t+1) \leq (\frac{1}{2} - \gamma) W(t) + (\frac{1}{2} + \gamma)(1-\varepsilon) W(t)$

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Combining

$|S_{bad}|(1-\varepsilon)^{T/2} \leq W(T) \leq n e^{-\varepsilon(\frac{1}{2}+\gamma)T}$

Calculation..

$$|S_{bad}|(1-\varepsilon)^{T/2} \leq ne^{-\varepsilon(\frac{1}{2}+\gamma)T}$$

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Set $\varepsilon = \gamma$, take logs.

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Set $\varepsilon = \gamma$, take logs.

$$\ln\left(\frac{|S_{bad}|}{n}\right) + \frac{T}{2}\ln(1-\gamma) \leq -\gamma T(\frac{1}{2}+\gamma)$$

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Again, $-\gamma - \gamma^2 \leq \ln(1-\gamma)$,

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And $T = \frac{2}{\gamma^2}\ln\mu$,

$\rightarrow \ln\left(\frac{|S_{bad}|}{n}\right) \leq \ln\mu \rightarrow \frac{|S_{bad}|}{n} \leq \mu.$

The misclassified set is at most μ fraction of all the points.

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The hypothesis correctly classifies $1-\mu$ fraction of the points

Calculation..

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Conclusion.

Standard method in practice for machine learning for combining repeated base learning algorithms.