CS 170: Algorithms

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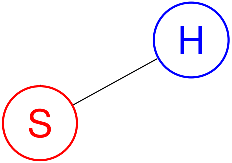
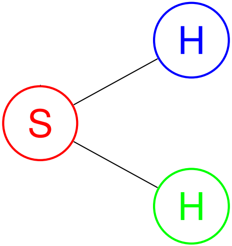
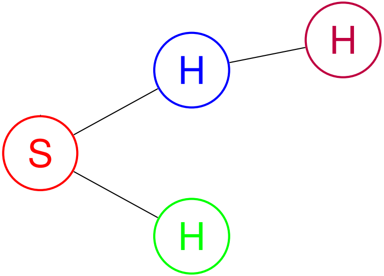


CS 170: Algorithms

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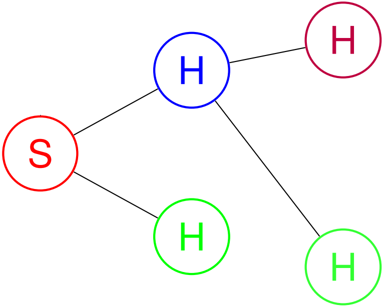
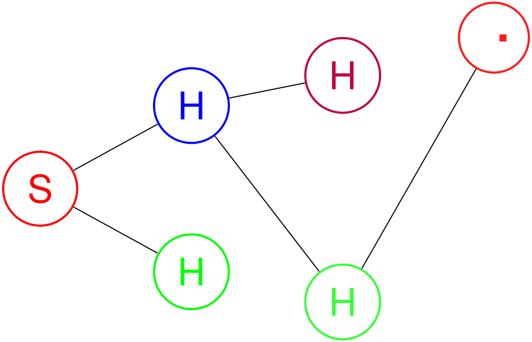
CS 170: Algorithms

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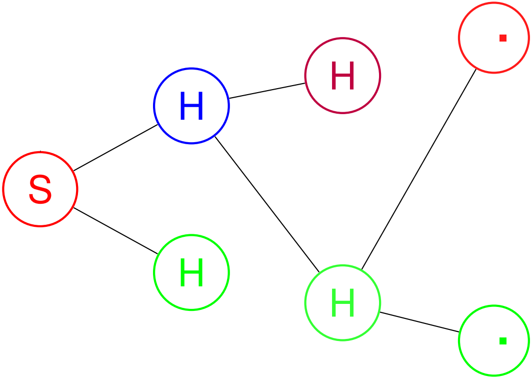
CS 170: Algorithms



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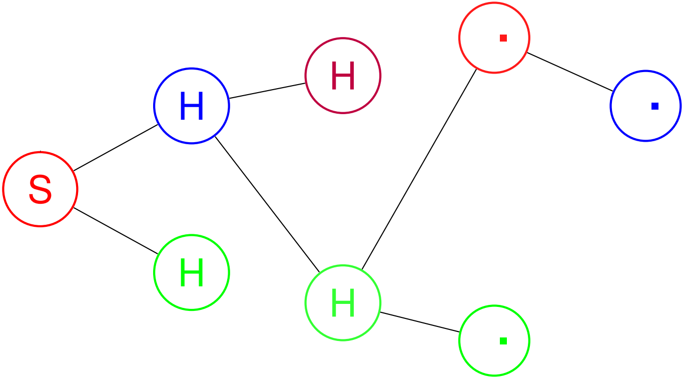

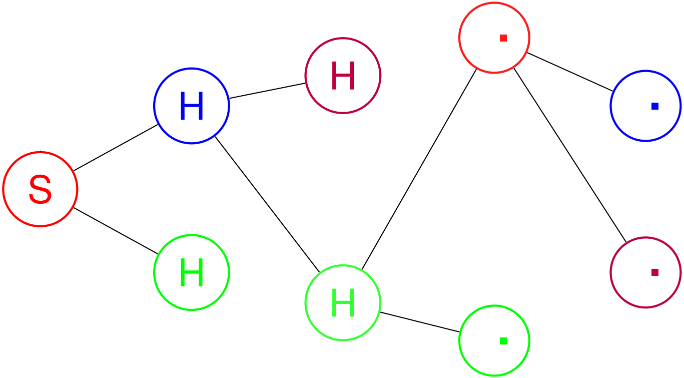
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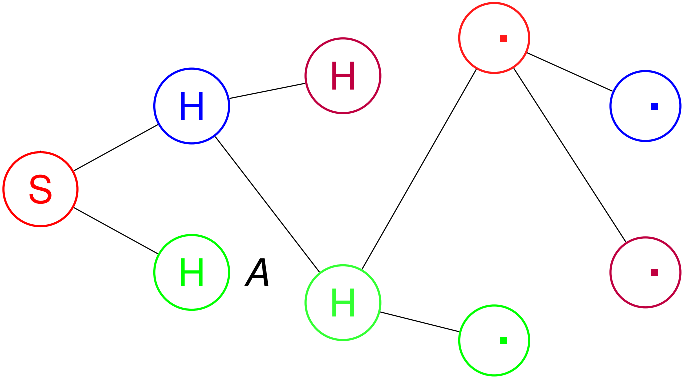

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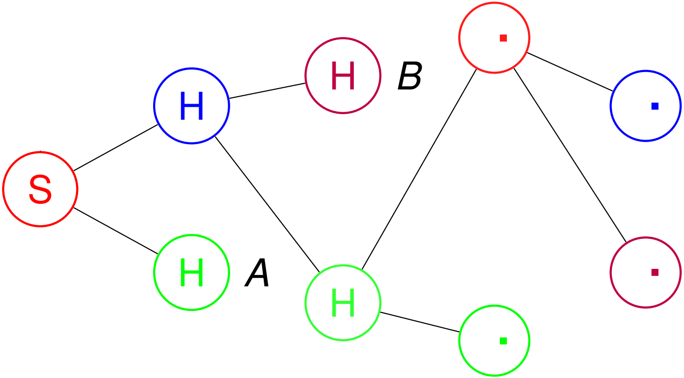

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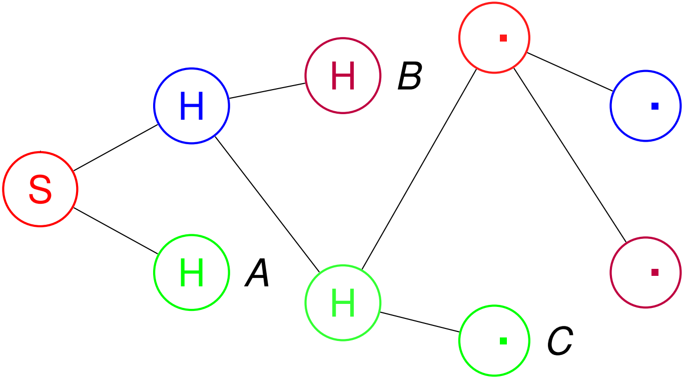

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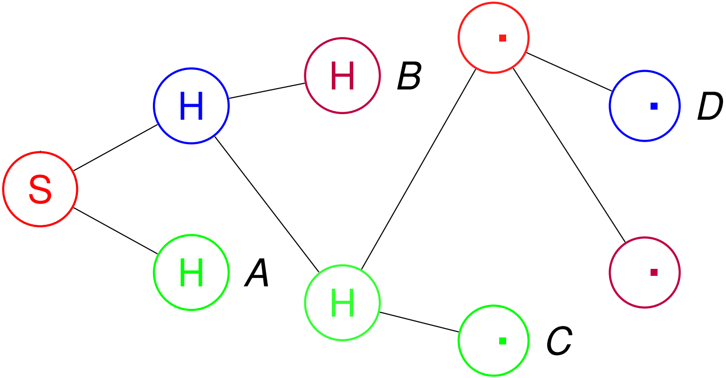

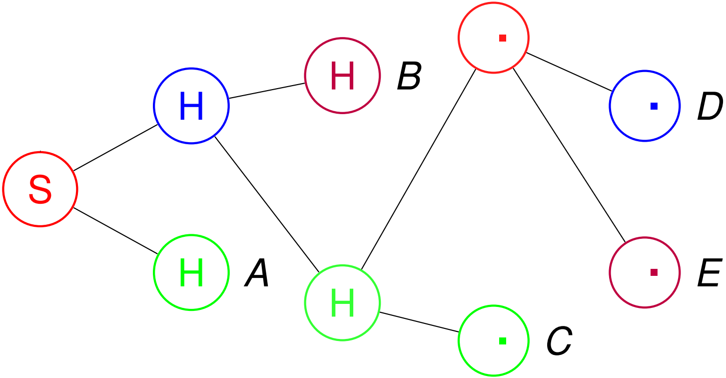
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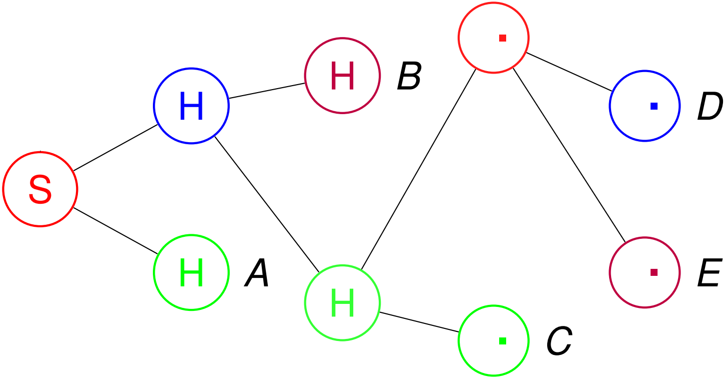
CS 170: Algorithms



Prims and Huffman Coding.

Lecture in a minute.

Cut Property: MST.

Exists MST that uses minimum weight edge across cut.

Exchange argument.   Prim: $S = \{s\}$

Add cheapest edge (u, v) across $(S, V - S)$

$S = S + v$.

Repeat.

Use priority queue: $O((|V| + |E|) \log |V|)$.

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Huffman Coding.

Symbols, s, with frequencies.

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Symbols, s, with frequencies.
Prefix-Free code.
$\equiv$ binary tree with symbols at leaves.
Cost: sum of depth(s)$\times$$freq(s)$.
Cost2: sum of frequencies of internal nodes.

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Huffman Coding.
  Symbols, s, with frequencies.
  Prefix-Free code.
   $\equiv$ binary tree with symbols at leaves.
  Cost: sum of depth(s)$\times$*freq*(s).
  Cost2: sum of frequencies of internal nodes.
Algorithm: merge lowest fequency symbols, recurse.
  Exchange Argument $\implies$
    exists optimal tree with this structure.

What is a Cut?

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What is a cut in an undirected graph, $G = (V, E)$?

(A) A set of edges whose removal disconnects a graph.

(B) For partition of V, $(S, V - S)$, set of edges across it; $E \cap (S \times V - S)$.

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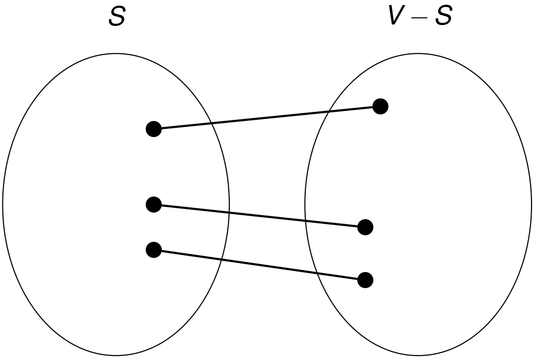

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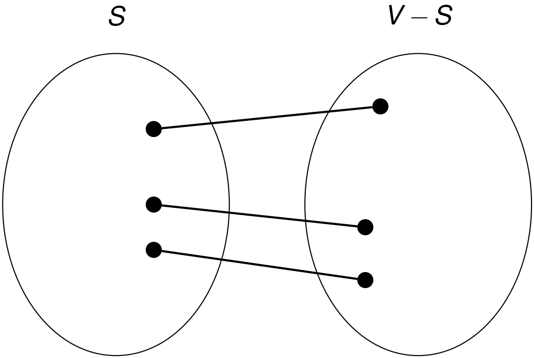

Note:

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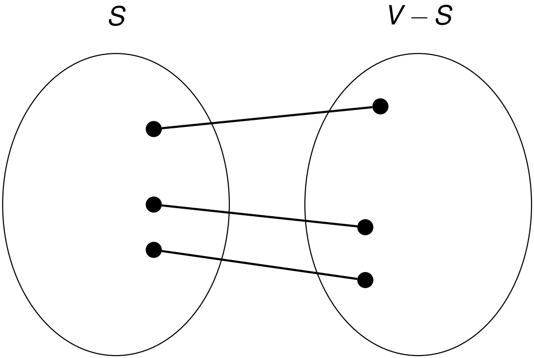

Note:sometimes specified as $(S, V - S)$

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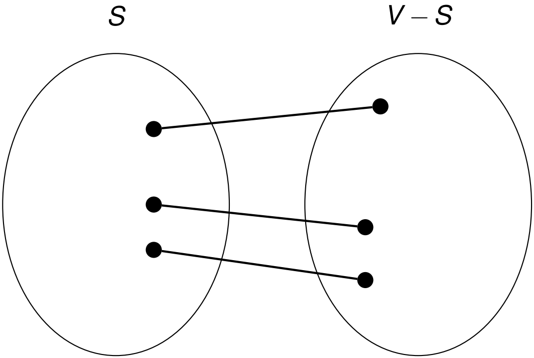

Note:sometimes specified as $(S, V - S)$
..... sometimes explicitly as subset of edges E'.

What is the cut property for MSTs?

(A) Any edges in a cut is in some mst.

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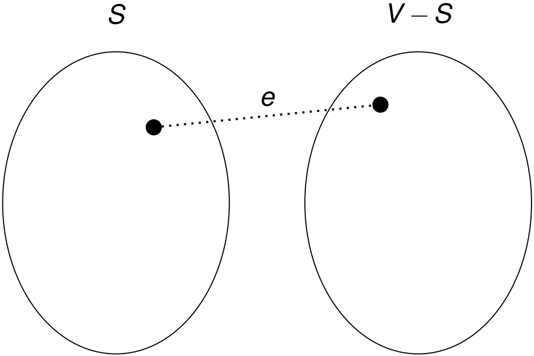

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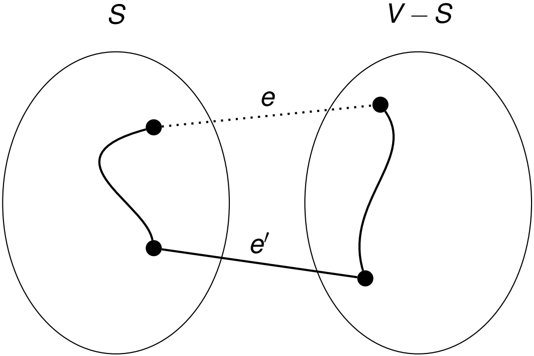

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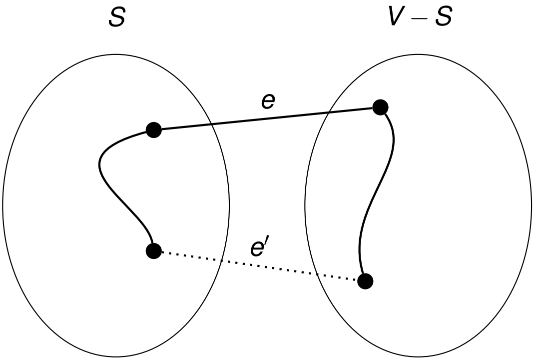

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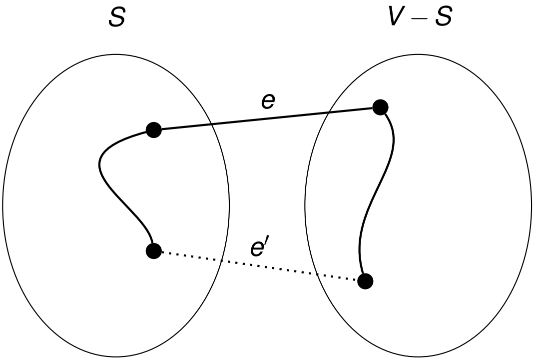

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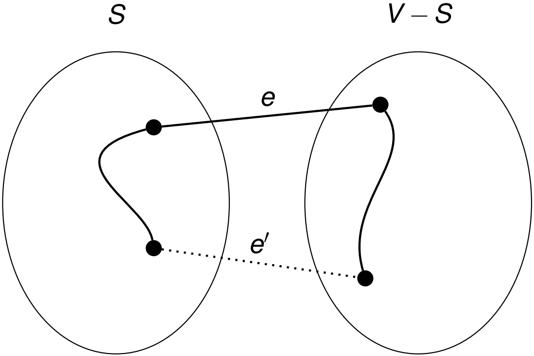

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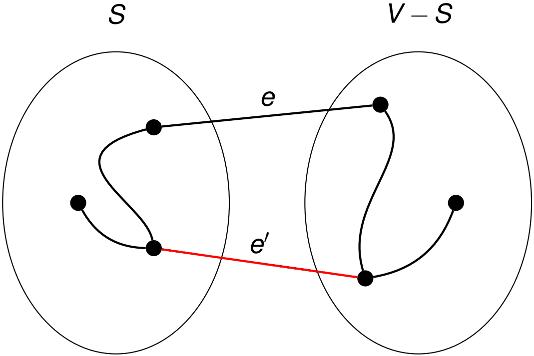

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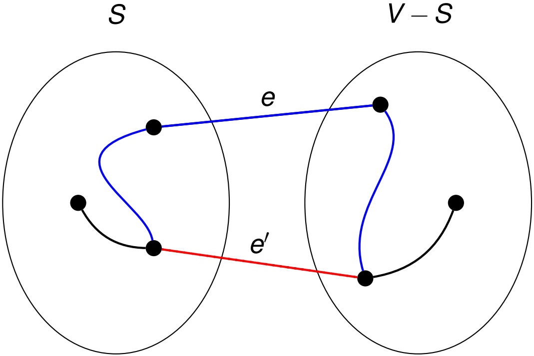

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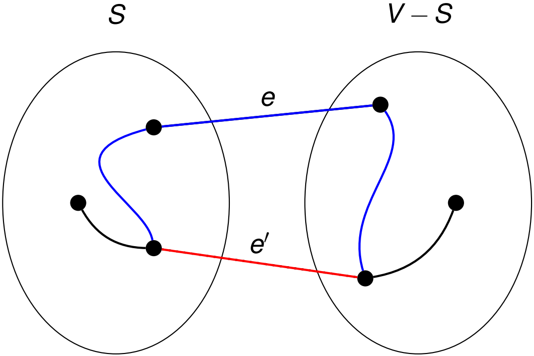

Cut Property: Any edge of minimal weight in a cut is in some MST.
(If unique, it must be in MST.)
Proof: replace, $n-1$ edges still connected and cheaper.

Prim's algorithm

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Generic MST:

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While not connected.

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Prim's algorithm

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Correctness: cut property with unique weight edges.

Break ties for smallest edge according to lowest neighbors.

Prim's Algorithm.

Generic MST:

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Start with $S = v$:

Prim's Algorithm.

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Start with $S = v$:

Find smallest edge (x, y) in crossing $(S, V - S)$

let $S = S \cup \{y\}$

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Implementation?

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Implementation?

Find smallest edge (x, y)?

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Find smallest edge (x, y)? $O(m)$ time.

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$O(nm)$ time.

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Look at same edges over and over again!

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Actually...

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Find smallest edge (x, y)? $O(m)$ time.

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Look at same edges over and over again!

Use a priority queue to keep edges!

Actually... use a priority queue to keep "closest" node.

Prim's Algorithm

Prim(G,s)

foreach $v \in V$: $c(v) = \infty, prev(v) = nil$

c(s) = 0, prev(s) = s

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Dijkstras:

      if $c(v)+w(v,w) < c(w))$:
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Runtime?

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Runtime? $\Theta(mn)$? $\Theta((m+n)\log n)$? $\Theta(m+n\log n)$?

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if $c(v) + w(v,w) < c(w))$:
$c(w) = c(v) + w(v,w), prev(w) = v$

Runtime? $\Theta(mn)$? $\Theta((m+n)\log n)$? $\Theta(m+n\log n)$? $O((m+n)\log n)$

Prim's Algorithm

Prim(G,s)
foreach $v \in V$: $c(v) = \infty, prev(v) = nil$
c(s) = 0, prev(s) = s

H = make_pqueue(V,c)
while (v = deletemin(H)):
foreach (v,w):
if $(w(v,w) < c(w))$:
$c(w) = w(v,w), prev(w) = v$
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With Fibonacci Heaps: $O(m+n\log n)$.

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Can decode!

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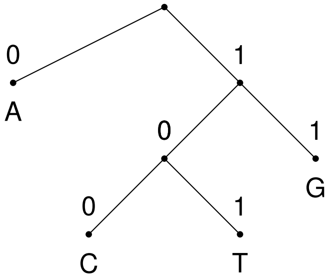

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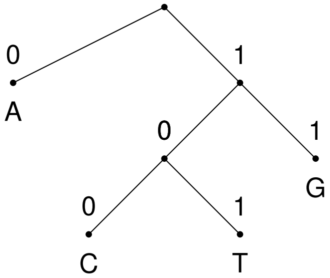

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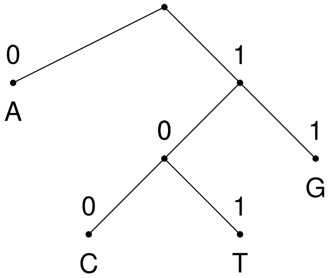

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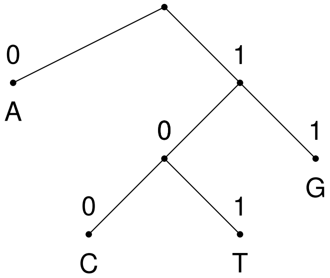

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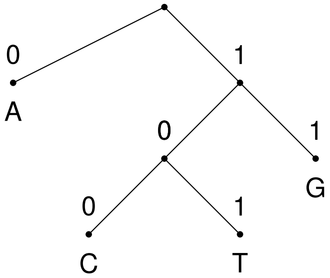

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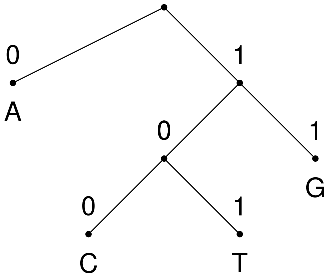

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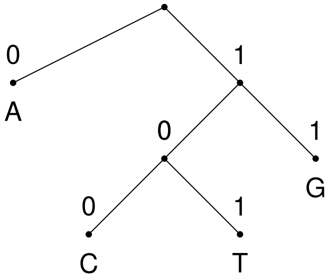

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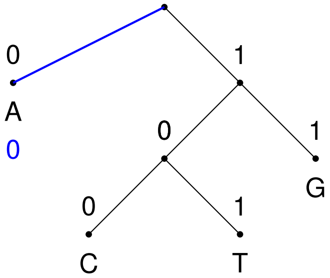

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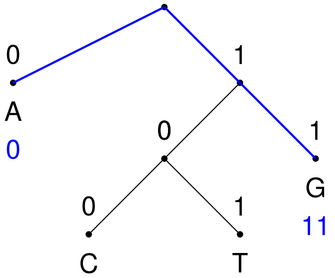

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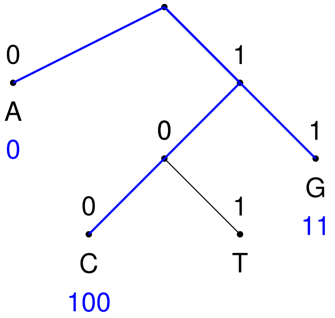

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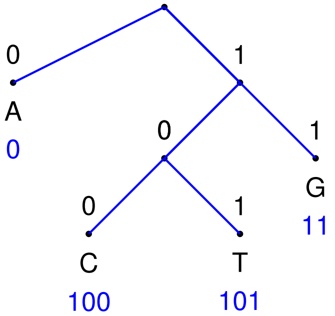

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A G C T

Tree from Prefix-Free Code.

Given prefix free code:
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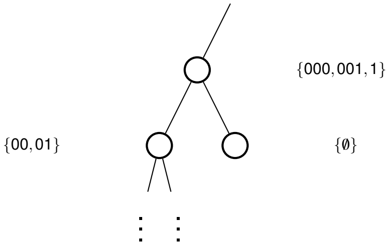

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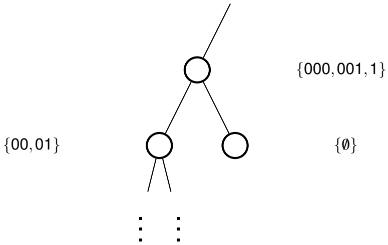

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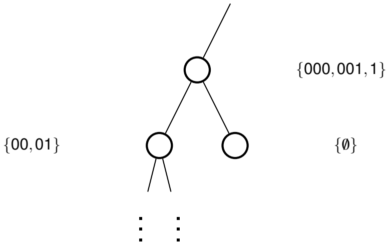

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Let S_p be subset corresponding to node at “path” p in tree.

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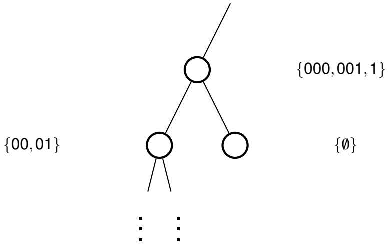

Correctness: Every codeword/symbol corresponds to leaf.

Let S_p be subset corresponding to node at "path" p in tree.

Corresponds to strings where p is prefix.

Tree from Prefix-Free Code.

Given prefix free code:
$S = \{s_1, s_2, \ldots, s_n\}$ for symbols $\{c_1, \ldots, c_n\}$.

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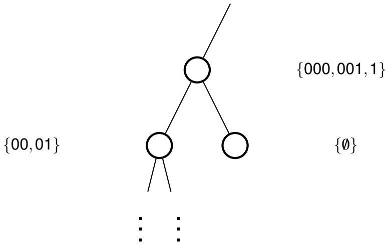

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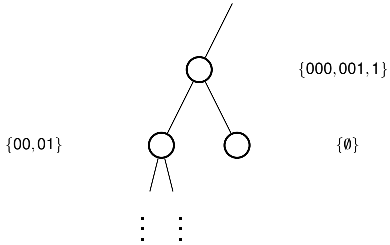

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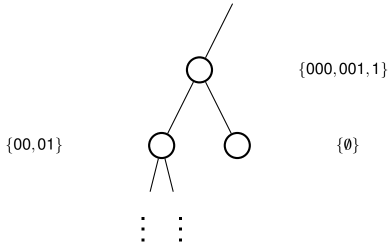

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Contradiction.

Huffman Coding.

Given symbol frequencies $f_1, \ldots, f_n$, find "best" prefix code.

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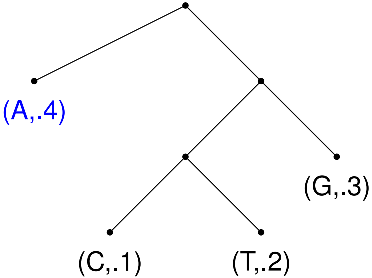

Cost: $.4 * 1$

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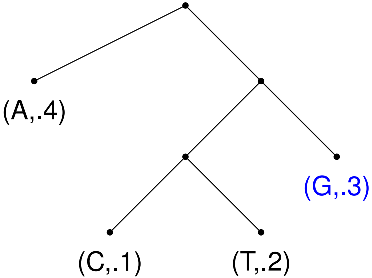

Cost: $.4 * 1 + .3 * 2$

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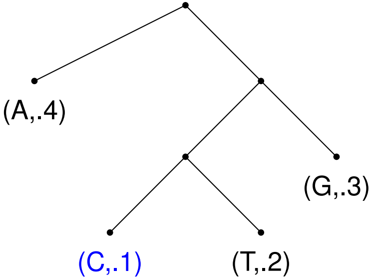

Cost: $.4 * 1 + .3 * 2 + .1 * 3$

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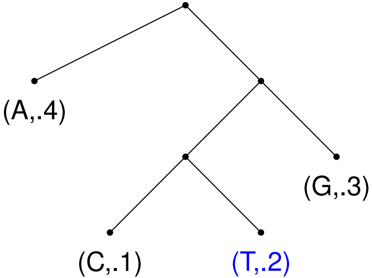

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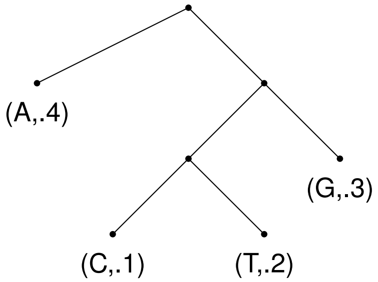
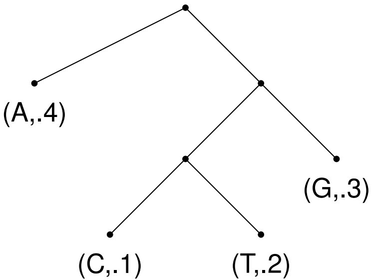


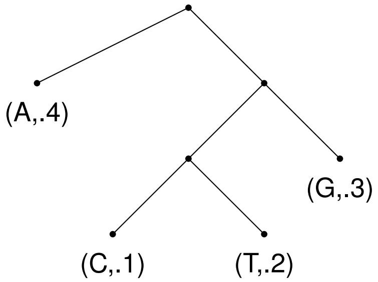
Cost: $.4*1+.3*2+.1*3+.2*3=1.9$

Another view of cost.



Sum over all nodes, except root, of their frequency.

Another view of cost.



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.4

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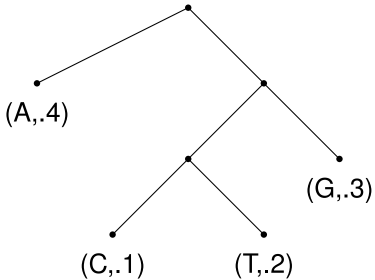

Sum over all nodes, except root, of their frequency.
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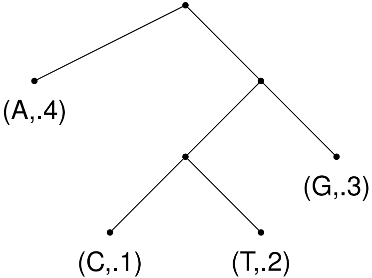

Sum over all nodes, except root, of their frequency.

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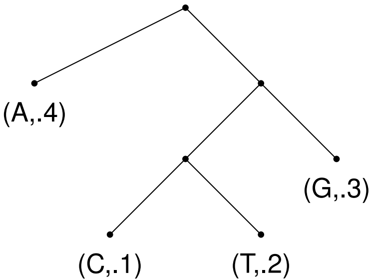
Another view of cost.



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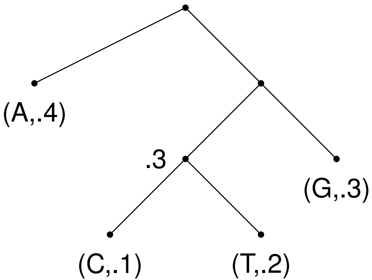

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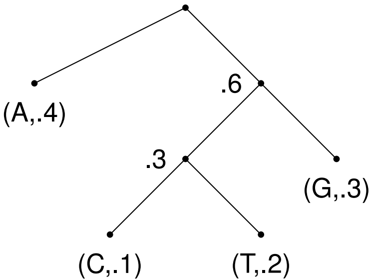

Sum over all nodes, except root, of their frequency.
$.4 + .1 + .2 + .3 + .3 + .6$

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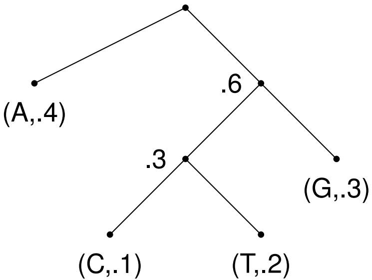

Sum over all nodes, except root, of their frequency.
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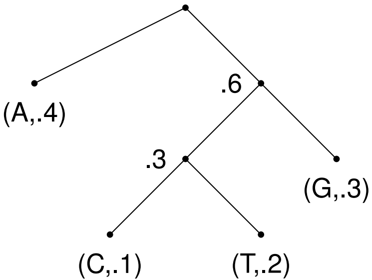

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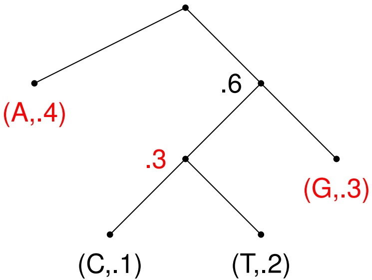

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Optimal Tree should be optimal above any subtree.
E.g., Optimal tree on $\{(.4, A), (.3, \{C, T\}), (.3, G)\}$.

Greedy Algorithm.

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Recursive View:

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$(\{A, \{\{C, T\}, G\}\}, 1)$

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Cost2: Sum over all nodes, except root, of their “frequency”.

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(A,.4)

(G,.3)

(C,.1)   (T,.2)

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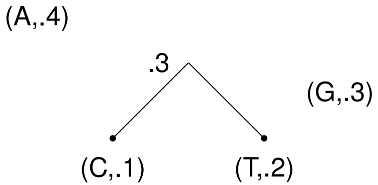

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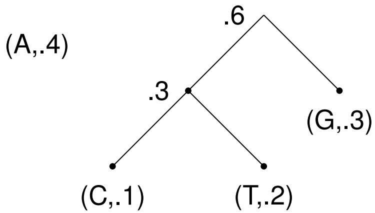

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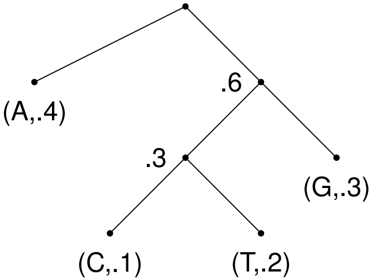

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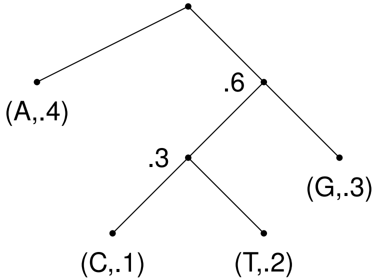

Implementation: priority queue to get lowest frequency trees.

Correctness..

Recall MST: added a “could have” edge in tree every time.

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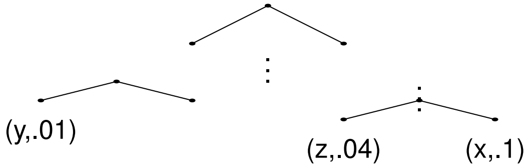

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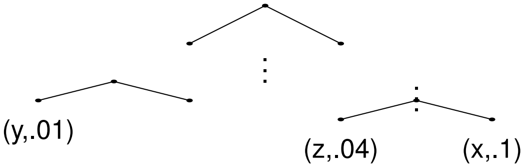

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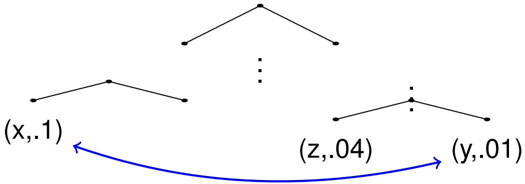

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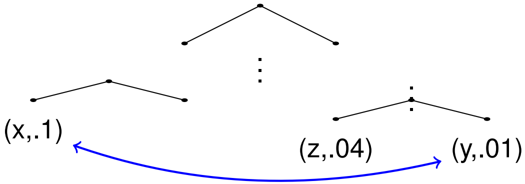
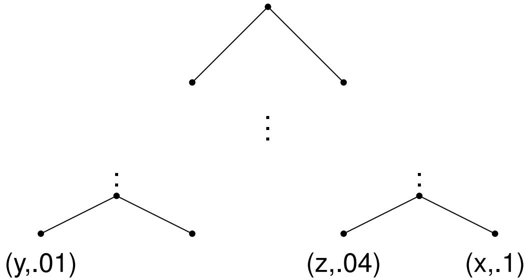


Cost gets better with this switch.
Lowest frequency pair are now siblings!

Lowest frequency at same depth.

What about same depth?



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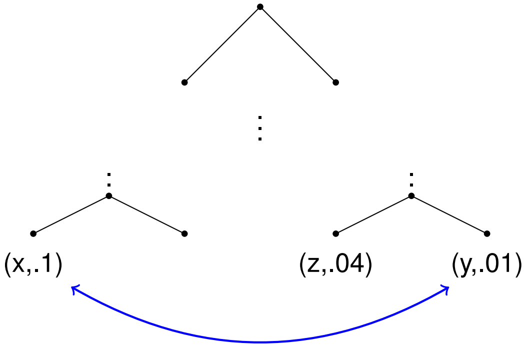
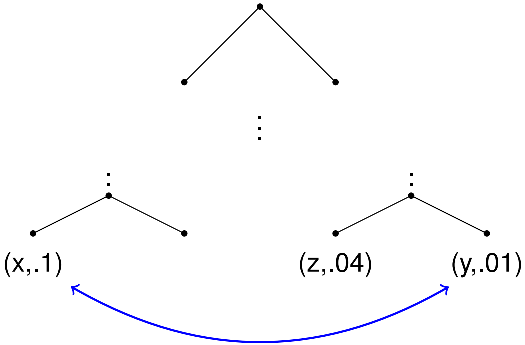


Cost stays the same,

Lowest frequency at same depth.

What about same depth?



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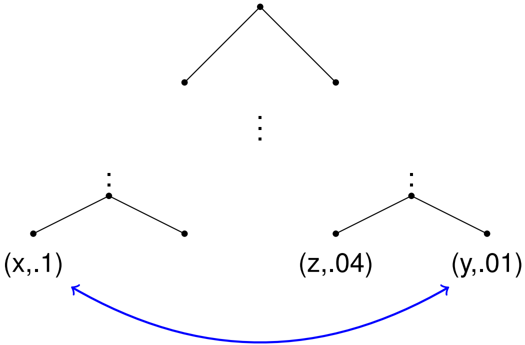

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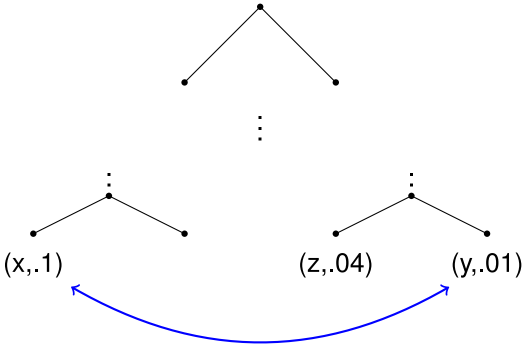

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"Algorithm: merge lowest frequency pair, and recurse."

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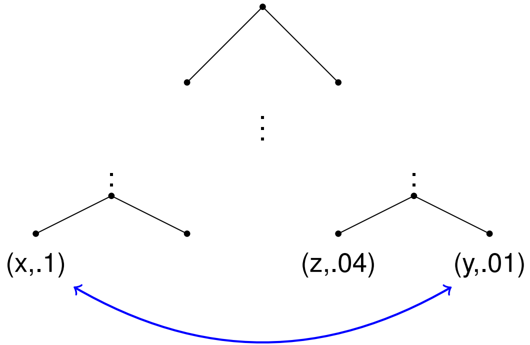

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"Algorithm: merge lowest frequency pair, and recurse."

Produces optimal tree.

Lecture in a minute.

Cut Property: MST.

Exists MST that uses minimum weight edge across cut.

Exchange argument. Prim: $S = \{s\}$

Add cheapest edge (u, v) across $(S, V - S)$

$S = S + v$.

Repeat.

Use priority queue: $O((|V| + |E|)\log|V|)$.

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Symbols, s, with frequencies.

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- Prefix-Free code.
- $\equiv$ binary tree with symbols at leaves.
- Cost: sum of depth(s)$\times$*freq*(s).
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$\equiv$ binary tree with symbols at leaves.
Cost: sum of depth(s)$\times$*freq*(s).
Cost2: sum of frequencies of internal nodes.
Algorithm: merge lowest fequency symbols, recurse.
Exchange Argument $\implies$
exists optimal tree with this structure.