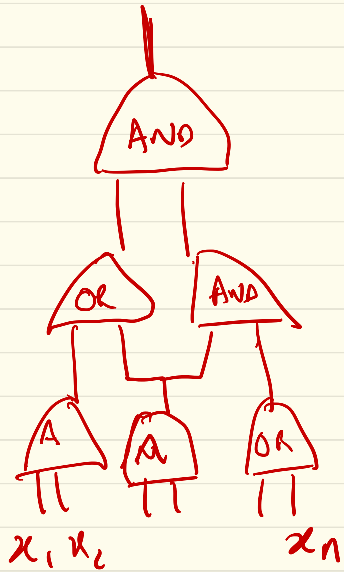
CIRCUIT SAT

INPUT: A boolean circuit C with n boolean inputs $(x_1 .. x_n)$ & one output.

SOL: An input assignment x s.t Output of $C = \underline{1}$.



Thm:

CircuitSAT is NP-complete

Mother of all NP-completeness

Thm: CircuitSAT is NP-complete

Proof: 1) CircuitSAT $\in NP$: (convince)

2) For every problem $A \in NP$

$\Rightarrow \quad A \leq_p$ CircuitSAT

$\quad\quad ||$

Fix $A =$ FACTORIZATION

FACTORIZATION

Input: An n bit number N

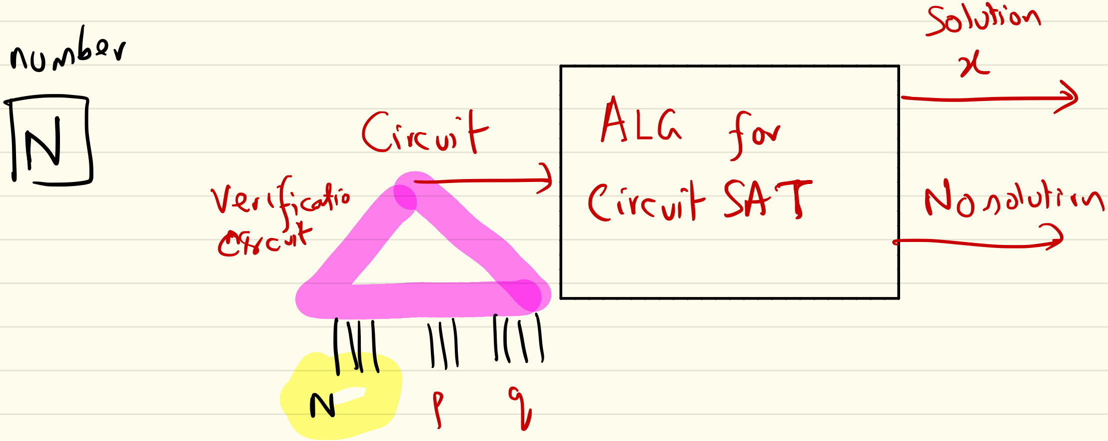
Solution: $p, q > 1$ s.t $p \cdot q = N$

$\leq_P$

CIRCUIT SAT

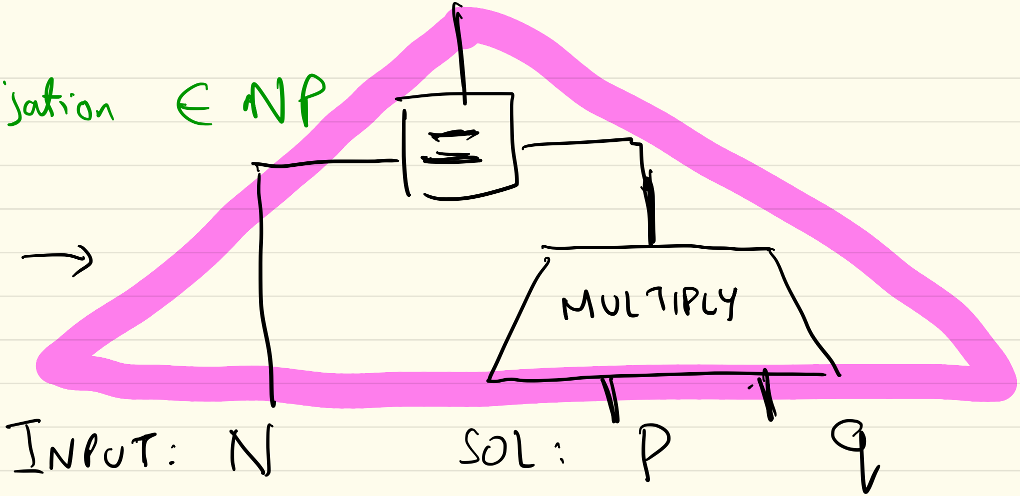
INPUT: Circuit C

Solution: A satisfying assignment



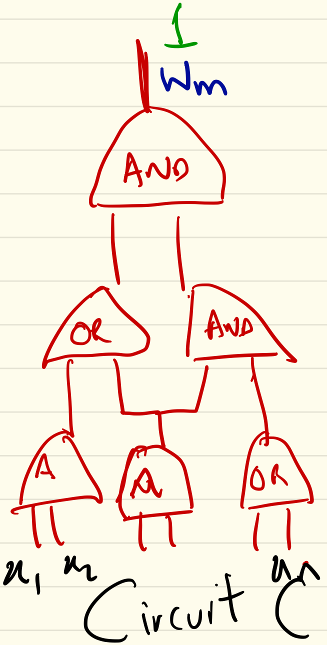
Factorisation $\in$ NP

Verifying circuit $\longrightarrow$



Thm:

CIRCUIT SAT	$\le_P$	3SAT
INPUT: A circuit C		INPUT: A 3SAT formula on $x_1, \dots, x_n$
SOL: A satisfying assignment x		SOL: A satisfying assignment.



3SAT Formula $\longrightarrow$ $\boxed{\text{3SAT Alg}}$

Variables:
$\{x_1 \dots x_n\}$
AND $\{w_1, \dots, w_m\}$ ← internal wires.

Constraints: 1) encode each gate
2) $(w_m \vee w_m \vee w_m)$

3SAT is NP complete

1) $3SAT \in NP$

2) Every problem A in NP

$$A \leq_p 3SAT$$

$\because$ CircuitSAT is NP-complete

$$A \leq_p \boxed{\text{Circuit SAT} \leq_p 3SAT}$$

Prop: $A \leq_p B$ and $B \leq_p C$

then $A \leq_p C$

To prove

A is NP-complete

1) $\left\{\begin{array}{l} \text{3SAT} \\ \text{INDSET} \\ \vdots \\ \vdots \end{array}\right\}$ one of them $\leq_p A$

2) $A \in NP$

Proof: 3SAT is NP-complete.

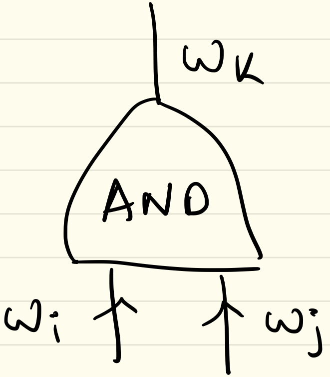
1) If the circuit C has an assignment x

$$C(x) = 1$$

$\implies$ $\exists$ a satisfying assignment to 3SAT formula

2) Given a satisfying assignment to formula

$\implies$ find a satisfying assignment



$$\{ w_k = w_i \wedge w_j \}$$

$$\Updownarrow$$

$$\left\{ \begin{array}{ccc} w_i, & w_j, & w_k \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{array} \right.$$

$$(\overline{w_i} \vee w_j \vee \overline{w_k}) \wedge$$
$$(w_i \vee \overline{w_j} \vee \overline{w_k}) \wedge$$
$$(w_i \vee w_j \vee \overline{w_k}) \wedge$$
$$(\overline{w_i} \vee \overline{w_j} \vee w_k)$$

3SAT: INPUT: A 3-SAT formula on variables $x_1 \ldots x_n \in \{0,1\}$

Formula with m clauses $\{C_1 \ldots \ldots C_m\}$

$(x_1 \vee x_2 \vee x_3) \wedge (x_4 \vee \overline{x_5} \vee x_6) \wedge (\overline{x_i} \vee x_j \vee x_k) \ldots$

clause

Solution:

GOAL: An assignment $\{x_1 \to 0, x_2 \to 1 \ldots \}$ that satisfies all clauses.

3SAT

INPUT: A 3SAT formula

SOL: A satisfying assignment

INDEPENDENT SET

INPUT: Graph $G=(V,E)$, K

P SOL: An independent set of size K

$(x \vee y \vee z) \wedge$

$(\bar{y} \vee \bar{z} \vee w) \wedge$

$(\bar{w} \vee x \vee t) \wedge$

satisfying assignment

$x=1 \quad y=0 \quad z=0$

$w=1 \quad t=0$

3 Δ's