ALL-PAIR SHORTEST PATHS

$|V| = n$ $\{+ve/-ve\}$

INPUT: Graph $G = (V, E)$ with edge weights $\{w_e\}$

GOAL: Compute shortest paths / distances between all pairs.

(+ve)

Naive: Run Djikstra from every node i

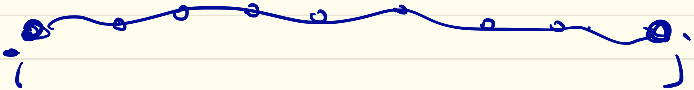
→ n - Djikstras

→ $|V| \cdot (|E| \log|V|) = |V| \cdot |E| \log|V|$.

Subproblem:

$D[i, j, k]$ = length of the shortest path from $i \rightsquigarrow j$ which only uses $\{1..k\}$ as intermediate vertices

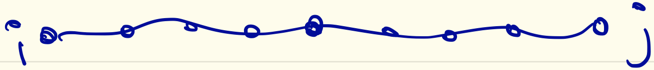
$k \in \{1..k\}$



$D[i, j, 0]$ = length of shortest path with NO intermediate vertices

$= w_{ij}$

$D[i, j, n]$ = length of shortest path $i \rightsquigarrow j$

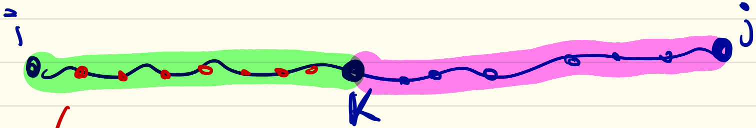


$K \notin$ Shortest path $i \sim j$

every intermediate vertex $\in \{1 \sim K-1\}$

$D[i,j,K-1]$

$$D[i,j,K] = \min$$



$K \in$ Shortestpath $i \sim j$ using $\{1..K\}$.

$D[i,K,K-1] + D[K,j,K-1]$

$$D[i,j,K] = \min\left\{D[i,j,K-1],\ D[i,K,K-1] + D[K,j,K-1]\right\}$$

PSEUDO CODE

```
for i = 1 to n
    for j = 1 to n
        D[i,j,0] = w_ij

for k = 0 to n
    for i = 1 to n
        for j = 1 to n
            D[i,j,k] = min { D[i,j,k-1],
                             D[i,k,k-1] + D[k,j,k-1] }
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LINEAR PROGRAMMING - family of optimization problems

Variables: $x_1 \dots x_n \in \mathbb{R}$

Max / Min linear function $(x_1 \dots x_n)$

Subject to

linear constraints

COFFEE SHOP:

	COFFEE	MILK
x = 1 unit of CAPPUCINO	$3x$ +	$1 \cdot x$ +
y = 1 unit of LATTE	$2y$	$4y$
	≤ 6	≤ 8

AVAILABLE COFFEE = 6

AVAILABLE MILK = 8

Maximum # of drinks = $\underline{x+y}$

X = # of units of Cappucino

y = # of units of Latte

$\in$ positive real numbers.

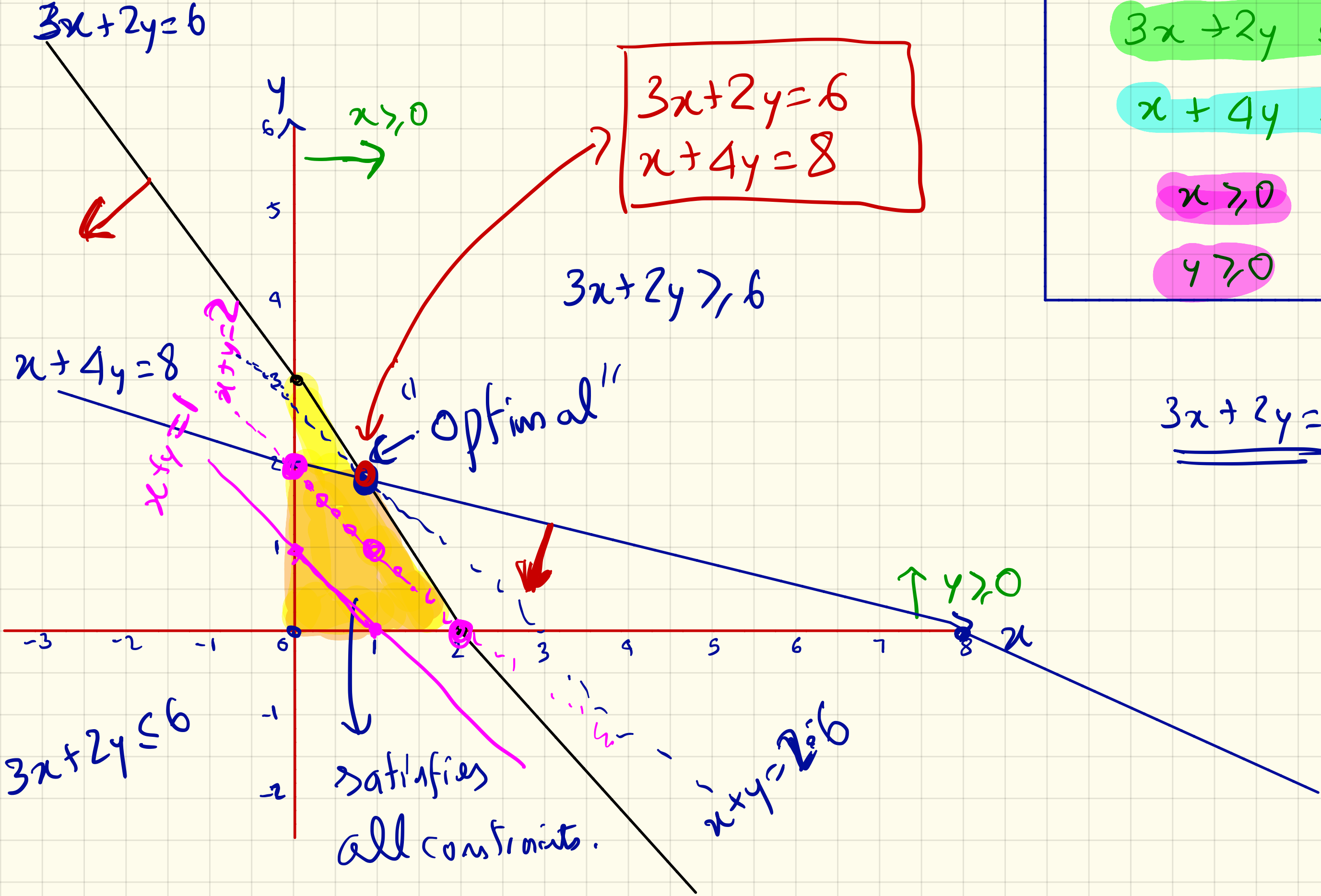
Max $x + y$

$3x + 2y \leq 6$

$x + 4y \leq 8$

$x \geq 0$

$y \geq 0$



$3x + 2y = 6$

Feasible = satisfy all constraints

Feasible region = range of variables that satisfy all constraints.

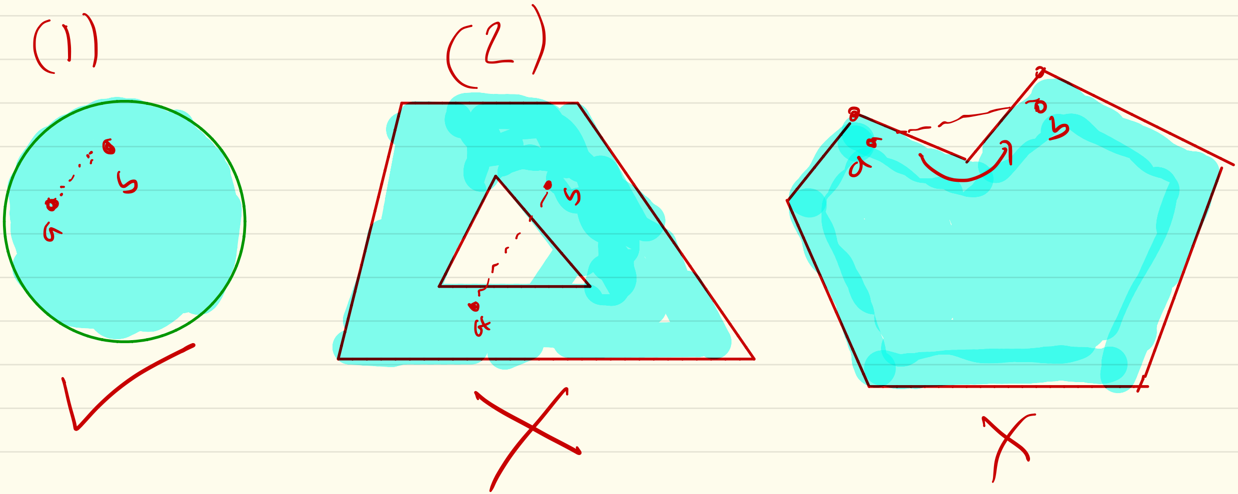
FACT 1: Feasible region of a linear program is a CONVEX POLYGON/Polyhedron.

FACT 2: The optima of a linear program occurs at vertex of a polygon/polyhedron.

FACT 3: Corner/Vertex of polygon is given by the intersection of constraints

CONVEX: A subset of points $S \subseteq \mathbb{R}^d$ is convex if $\forall\ a, b \in S$

(Line joining a, b) $\in S$.



Maximize $x + y$

OPT = "2.6"

$$(0.3)\cdot(3x + 2y \leq 6)$$
$$(0.1)\cdot(x + 4y \leq 8)$$
$$0\cdot(-x \leq 0)$$
$$0\cdot(-y \leq 0)$$

$$\boxed{x + y \leq 2.6}$$