# CS 170: Algorithms

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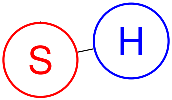
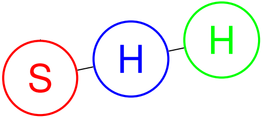
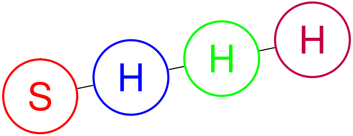


# CS 170: Algorithms



# CS 170: Algorithms



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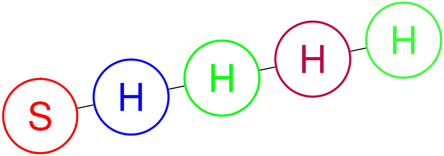

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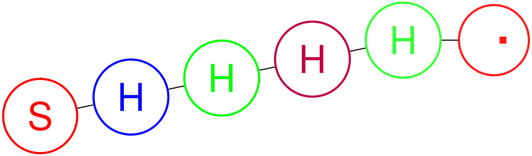

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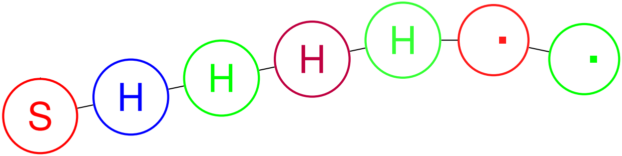

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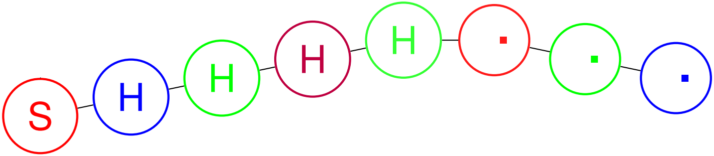

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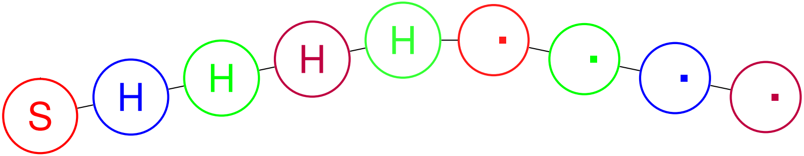

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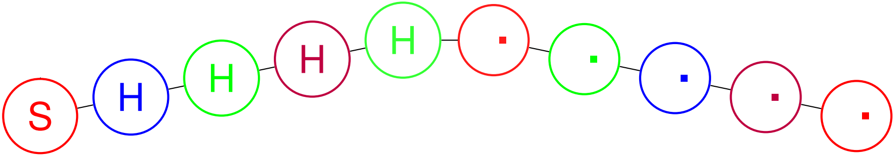

# Lecture in a minute.

FFT Wrapup:

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Evaluate degree $n$ polynomial on $n$ points.

Recursion:

Evaluate Odd/Even polynomials on squares of points.

Which $n = 2^k$ points?

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"squares of $n$th root of unity are $n/2$th roots of unity."

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$T(n) = 2T(n/2) + O(n) = O(n \log n)$.

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$G = (V, E)$, $V$ - vertices, $E$ edges.

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Matrix: $|V|^2$ space, fast check for edges.

Adjacency List: $O(|V| + |E|)$ space. More complicated.

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Procedure: explore($v$).

Explores the graph.

Uses a stack.

Nonrecursive non-loop counting Runtime analysis.

Every little move she makes...

# Fast Fourier Transform.

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where $\omega$ is primitive $n$th root of unity: $\omega^n = 1$.

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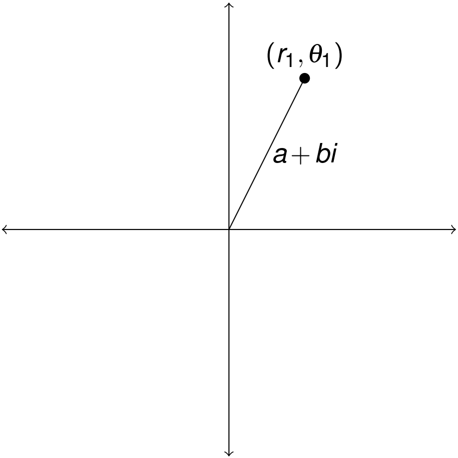

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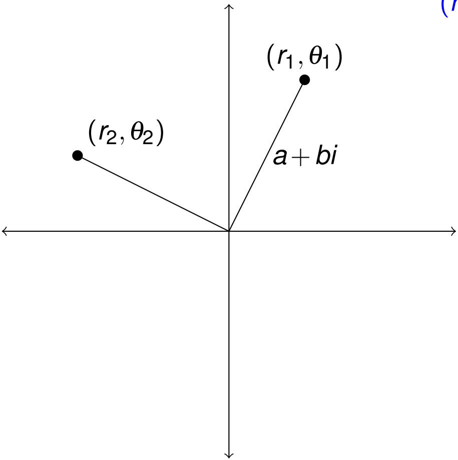
$(r_1,\theta_1)\times(r_2,\theta_2)$



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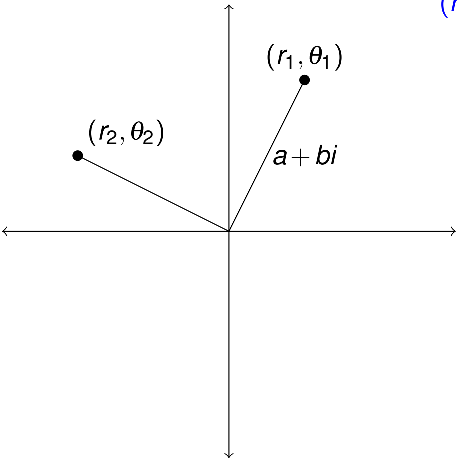


$(r_1, \theta_1) \times (r_2, \theta_2)$

$= r_1 e^{i\theta_1} \times r_2 e^{i\theta_2}$

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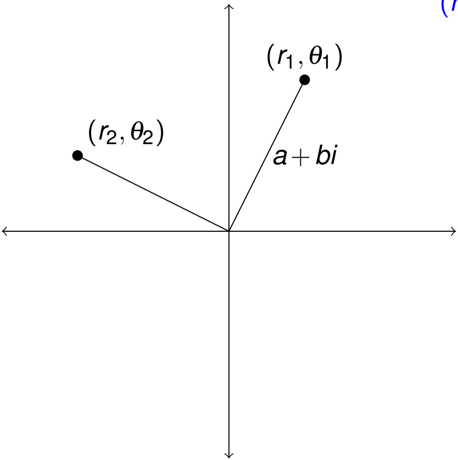


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$$= (r_1 r_2) e^{i(\theta_1 + \theta_2)}$$

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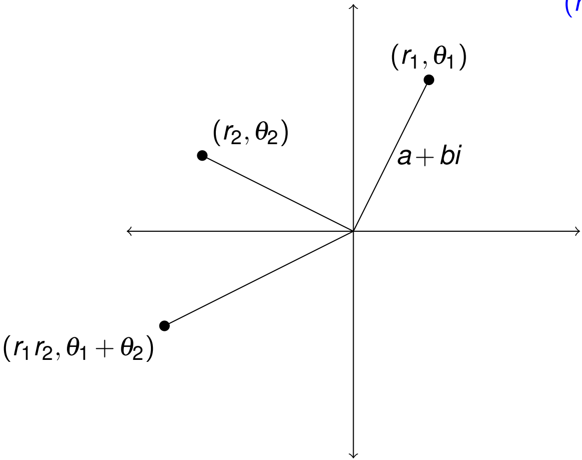


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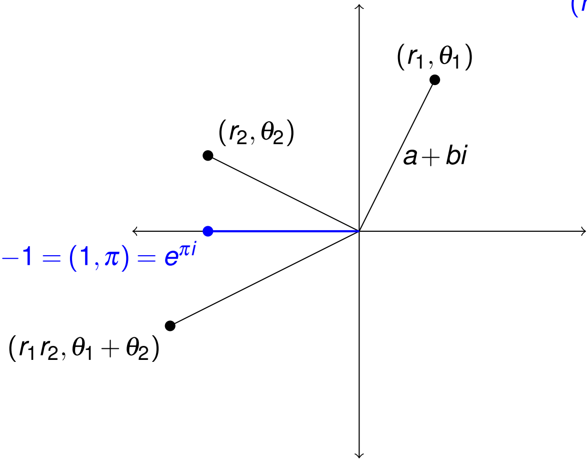


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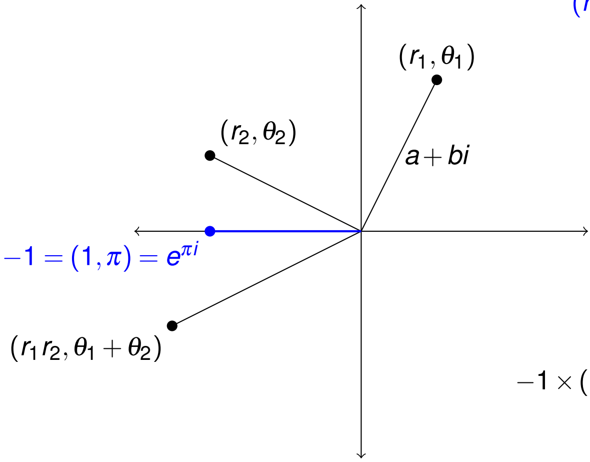


$$-1 \times (r_1, \theta_1) = (r_1, \theta_1 + \pi)$$

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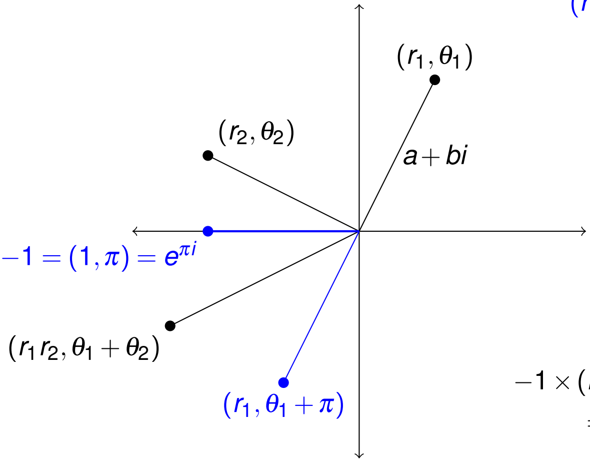
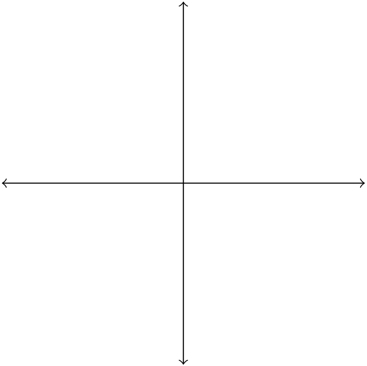
\end{aligned}$$




$$\begin{aligned}
-1 \times (r_1, \theta_1) &= (r_1, \theta_1 + \pi) \\
&= r_1 e^{r_1(\theta_1 + \pi)i}
\end{aligned}$$

# The $n$th complex roots of unity.

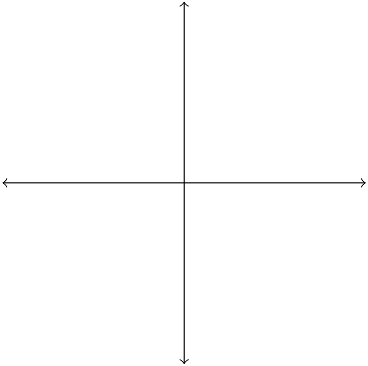
Solutions to $z^n = 1$



# The $n$th complex roots of unity.

Solutions to $z^n = 1$

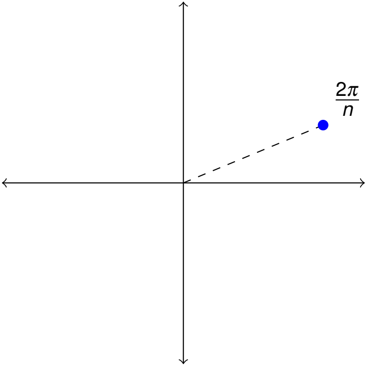
$$(1, \tfrac{2\pi}{n})^n = (1, \tfrac{2\pi}{n} \times n) = (1, 2\pi) = 1!$$



# The $n$th complex roots of unity.

Solutions to $z^n = 1$

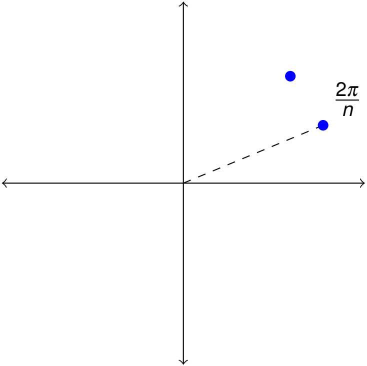
$$(1, \tfrac{2\pi}{n})^n = (1, \tfrac{2\pi}{n} \times n) = (1, 2\pi) = 1!$$



# The $n$th complex roots of unity.

Solutions to $z^n = 1$

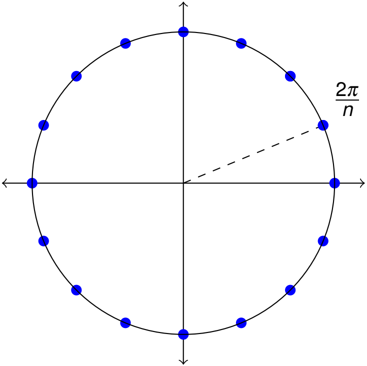
$$(1, \frac{4\pi}{n})^n = (1, \frac{4\pi}{n} \times n) = (1, 4\pi) = 1!$$



# The $n$th complex roots of unity.

Solutions to $z^n = 1$

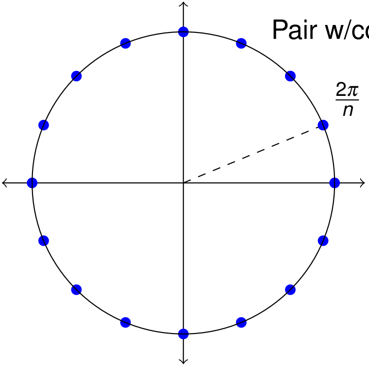
$$(1, \tfrac{2k\pi}{n})^n = (1, \tfrac{2k\pi}{n} \times n) = (1, 2k\pi) = 1!$$



# The $n$th complex roots of unity.

Solutions to $z^n = 1$

$$(1, \theta + \pi)^2 = (1, 2\theta + 2\pi) = \boxed{(1, 2\theta)} = (1, \theta)^2.$$

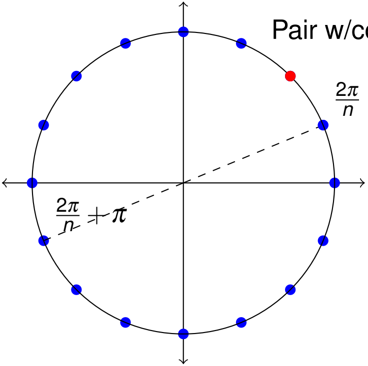



Pair w/common square!

# The $n$th complex roots of unity.

Solutions to $z^n = 1$

$$(1, \theta + \pi)^2 = (1, 2\theta + 2\pi) = \boxed{(1, 2\theta)} = (1, \theta)^2.$$

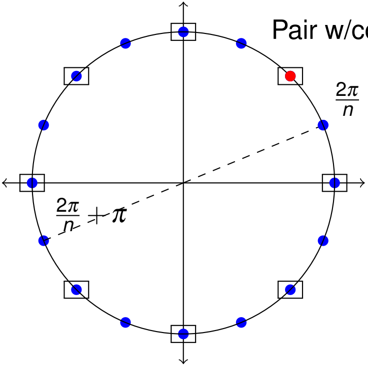



Pair w/common square!

# The $n$th complex roots of unity.

Solutions to $z^n = 1$

$$(1, \theta + \pi)^2 = (1, 2\theta + 2\pi) = \boxed{(1, 2\theta)} = (1, \theta)^2.$$

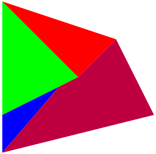


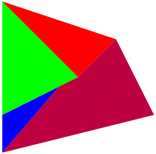

Pair w/common square!

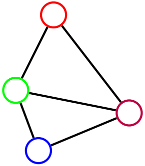
Squares: $\frac{n}{2}$th roots.

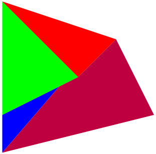
# Next

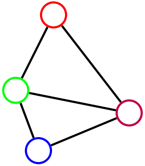
1. Graphs
2. Reachability.



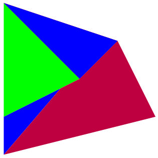
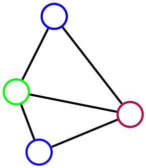




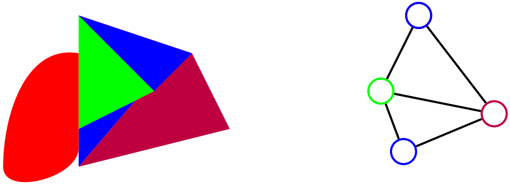


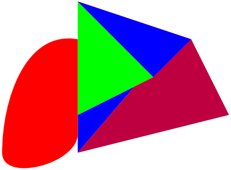


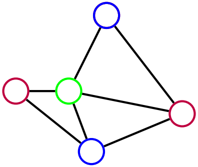
Fewer Colors?

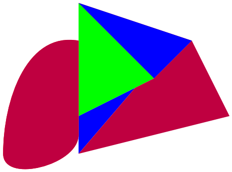
 

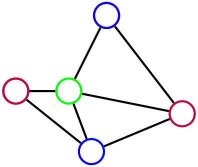
Yes! Three colors.

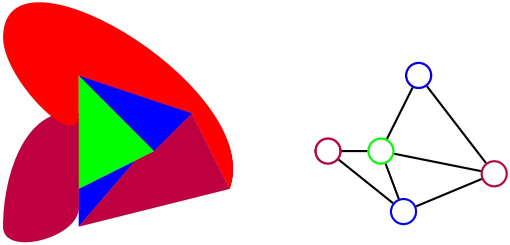


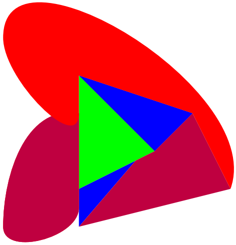


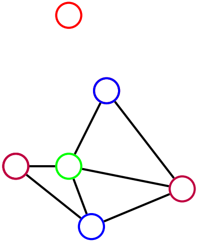


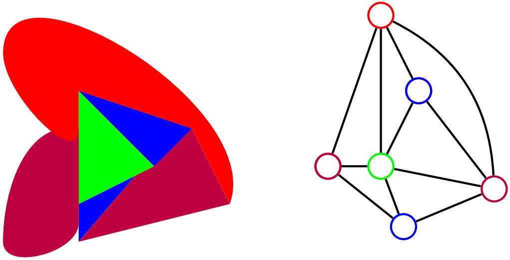


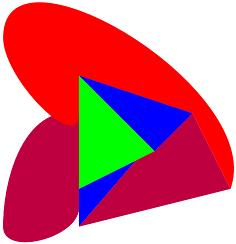


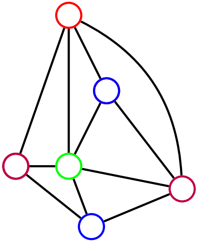




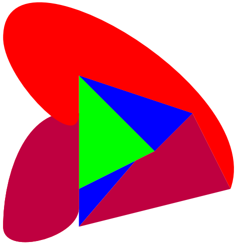


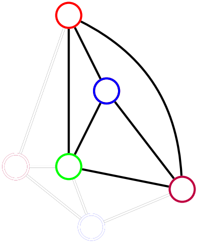


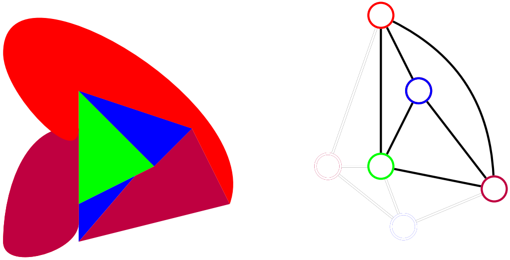




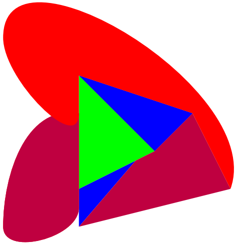
Fewer Colors?

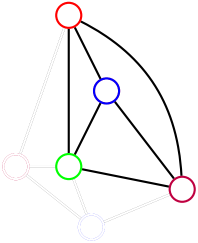






Four colors required!

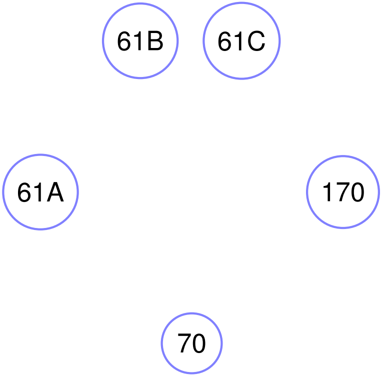




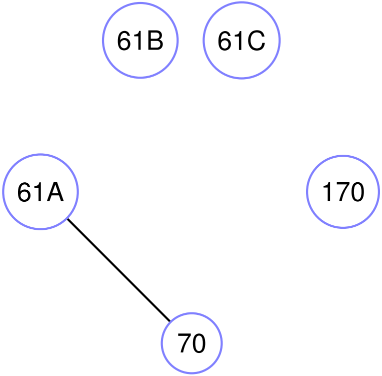
Four colors required!

Theorem: Four colors enough.

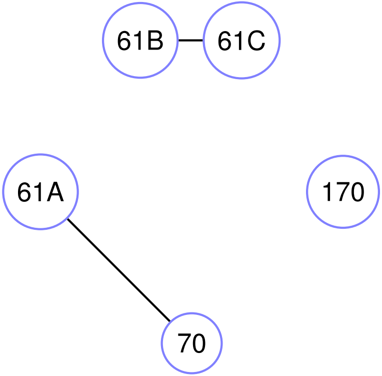
# Scheduling: coloring.



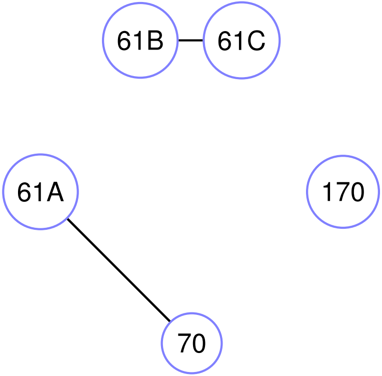
# Scheduling: coloring.



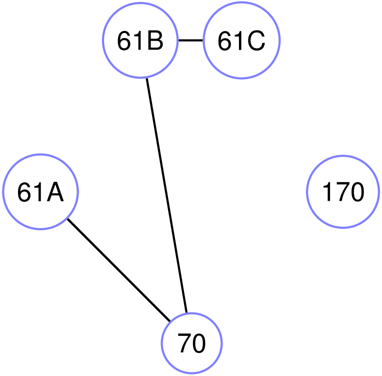
# Scheduling: coloring.



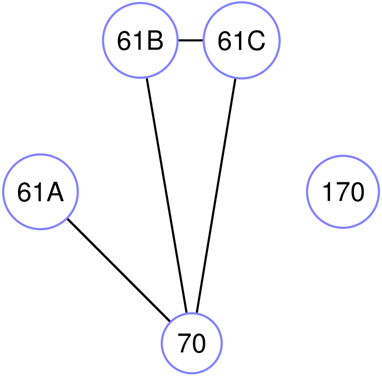
# Scheduling: coloring.



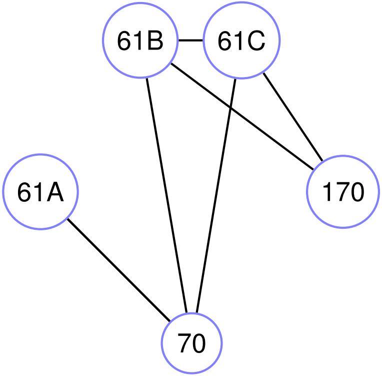
# Scheduling: coloring.



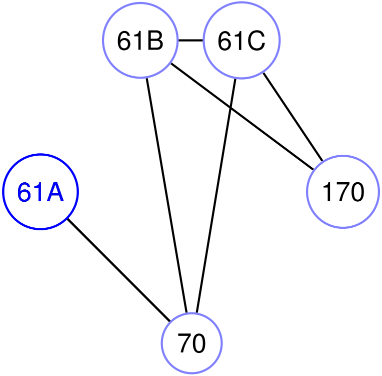
# Scheduling: coloring.



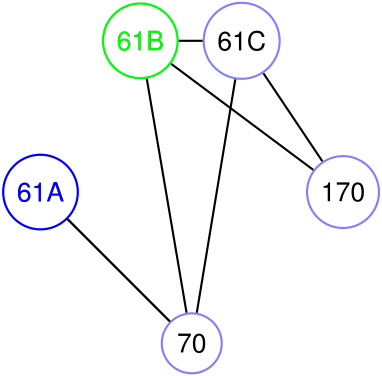
# Scheduling: coloring.



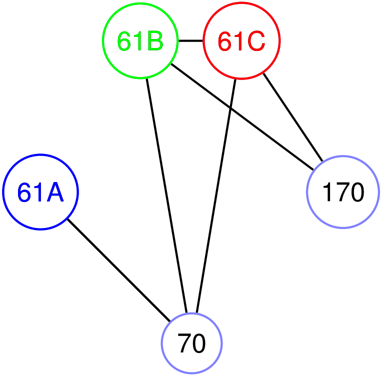
# Scheduling: coloring.



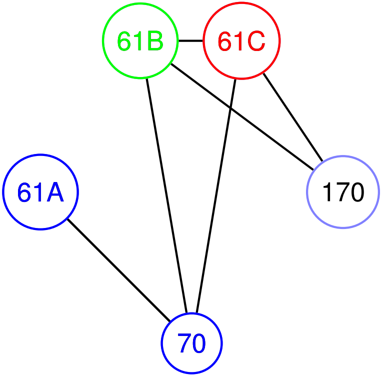
# Scheduling: coloring.



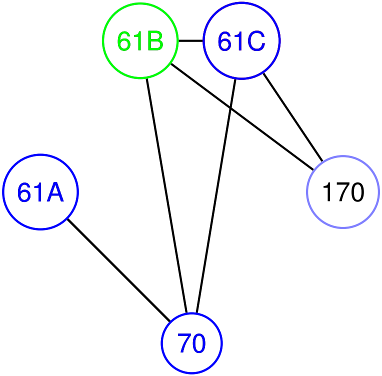
# Scheduling: coloring.



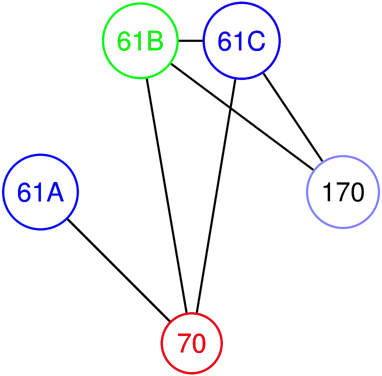
# Scheduling: coloring.



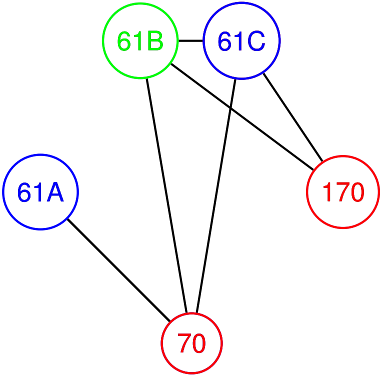
# Scheduling: coloring.



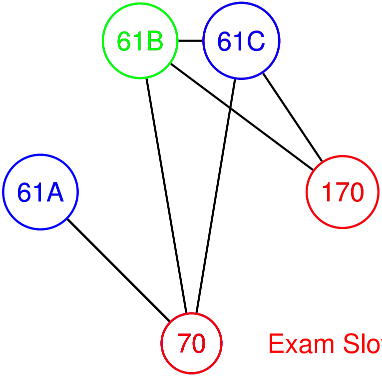
# Scheduling: coloring.



# Scheduling: coloring.



# Scheduling: coloring.



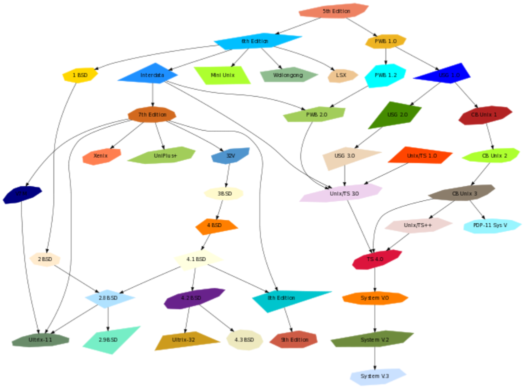

Exam Slot 1.

Exam Slot 2.

Exam Slot 3.

# Directed acyclic graphs.

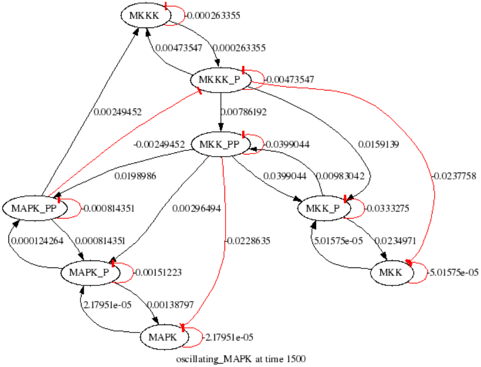
Heritage of Unix.




*Object Oriented Graphs*
*Stephen North, 3/19/93*

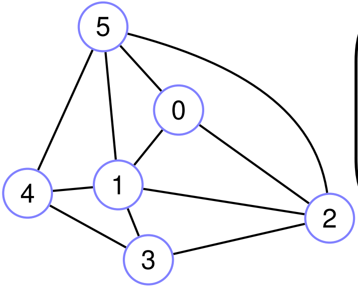
From http://www.graphviz.org/content/crazy.

# Chemical networks.




From http://www.tbi.univie.ac.at/ raim/odeSolver/doc/app.html.

Graph Implementations.



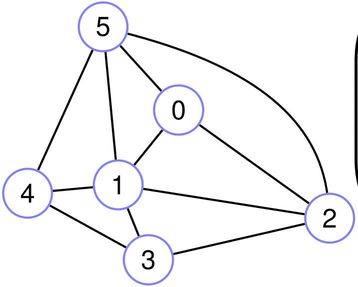

Matrix Representation.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$V = \{0, 1, 2, 3, 4, 5\}$$

$$E = \{(0, 1), (0, 2), (0, 5), (1, 3) \ldots\}$$

Graph Implementations.




Matrix Representation.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

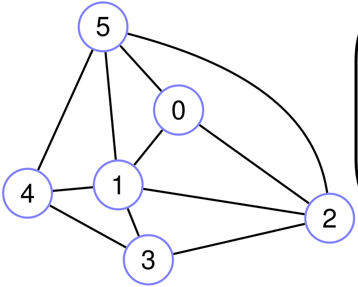
$$V = \{0, 1, 2, 3, 4, 5\}$$

$$E = \{(0, 1), (0, 2), (0, 5), (1, 3) \dots\}$$

Adjacency List

$$\begin{array}{ll} 0: & 1, 2, 5 \\ 1: & 0, 2, 3, 4, 5 \\ 2: & 0, 1, 3 \\ 3: & 1, 2, 4 \\ 4: & 1, 3, 5 \\ 5: & 0, 1, 2, 4 \end{array}$$

Graph Implementations.




Matrix Representation.

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 0 \end{pmatrix}$$

$$V = \{0, 1, 2, 3, 4, 5\}$$

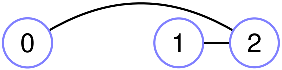
$$E = \{(0,1), (0,2), (0,5), (1,3) \ldots\}$$

Adjacency List

$0 : \quad 1, 2, 5$
$1 : \quad 0, 2, 3, 4, 5$
$2 : \quad 0, 1, 3$
$3 : \quad 1, 2, 4$
$4 : \quad 1, 3, 5$
$5 : \quad 0, 1, 2, 4$

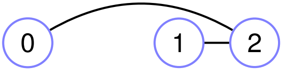
| | Matrix | Adj. List |
|---|---|---|
| Edge $(u, v)$? | $O(1)$ | $O(\|V\|)$ |
| Neighbors of $u$ | $O(\|V\|)$ | $O(d)$ |
| Space | $O(\|V\|^2)$ | $O(\|E\|)$ |

# Test your understanding..




Adjacency list of node 0?

# Test your understanding..



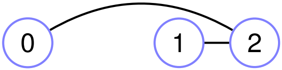

Adjacency list of node 0?

(A) $0 : 1$

(B) $0 : 1, 2$

(C) $0 : 2$

# Test your understanding..




Adjacency list of node 0?

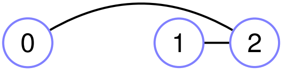
(A) $0 : 1$

(B) $0 : 1, 2$

(C) $0 : 2$

(C)

# Test your understanding..




Adjacency list of node 0?

(A) $0 : 1$

(B) $0 : 1, 2$

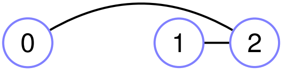
(C) $0 : 2$

(C)

How many edges?

(A) 2

# Test your understanding..




Adjacency list of node 0?

(A) $0 : 1$

(B) $0 : 1, 2$

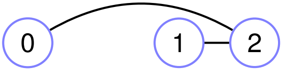
(C) $0 : 2$

(C)
How many edges?

(A) 2

Total length of adacency lists?

# Test your understanding..




Adjacency list of node 0?

(A) $0 : 1$

(B) $0 : 1, 2$

(C) $0 : 2$

(C)
How many edges?

(A) 2

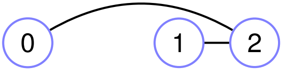
Total length of adacency lists?

(A) 2

(B) 3

(C) 4

# Test your understanding..




Adjacency list of node 0?

(A) $0 : 1$

(B) $0 : 1, 2$

(C) $0 : 2$

(C)

How many edges?

(A) 2

Total length of adacency lists?

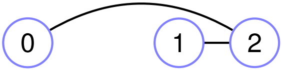
(A) 2

(B) 3

(C) 4

(C)

# Test your understanding..




Adjacency list of node 0?

(A) $0 : 1$

(B) $0 : 1, 2$

(C) $0 : 2$

(C)

How many edges?

(A) 2

Total length of adacency lists?

(A) 2

(B) 3

(C) 4

(C) 2 entries for each edge!

# Exploring a maze.

Theseus: ...

# Exploring a maze.

Theseus: ...gotta kill the minatour

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he’s cute..fortunately

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Gives Theseus Ball of Thread and Chalk!

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he’s cute..fortunately ..she’s smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.
  Look through exit. If marked, next exit.

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.
  Look through exit. If marked, next exit.
  Otherwise go in room unwind thread.

# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.
  Look through exit. If marked, next exit.
  Otherwise go in room unwind thread.
    Explore that room.

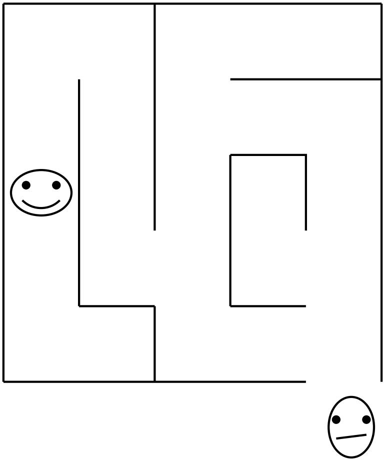
# Exploring a maze.

Theseus: ...gotta kill the minatour ..in the maze
Ariadne: he's cute..fortunately ..she's smart.

Gives Theseus Ball of Thread and Chalk!

Explore a room:
Mark room with chalk.
For each exit.
    Look through exit. If marked, next exit.
    Otherwise go in room unwind thread.
        Explore that room.
Wind thread to go back to "previous" room.

Where is the minatour?



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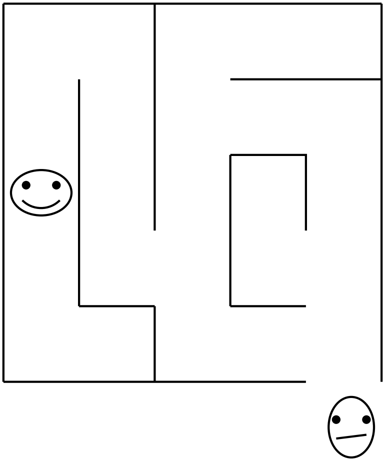

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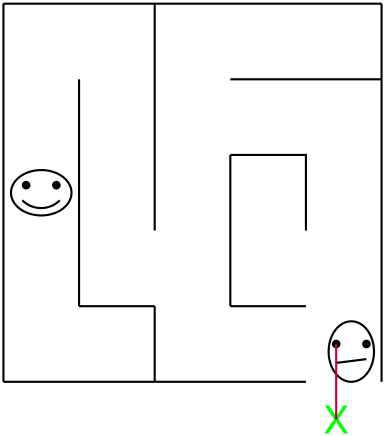

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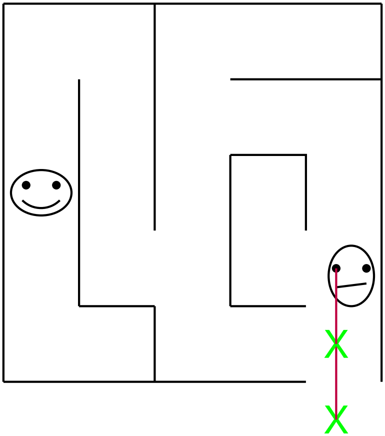

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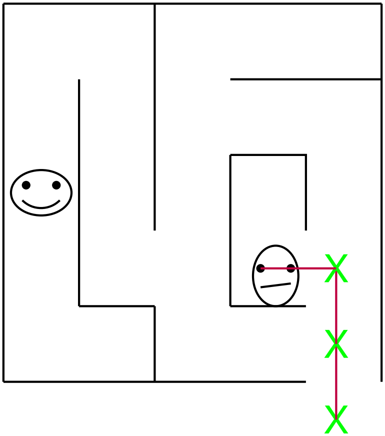

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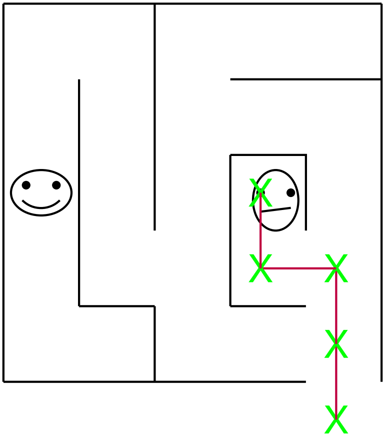

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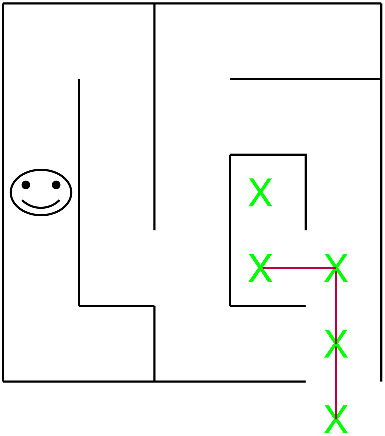

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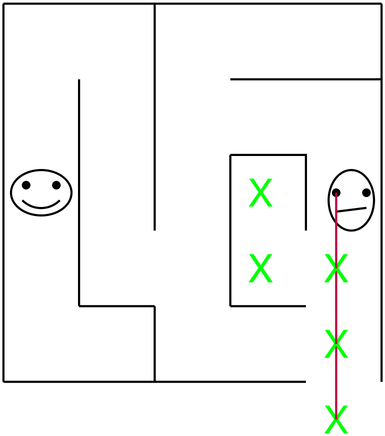

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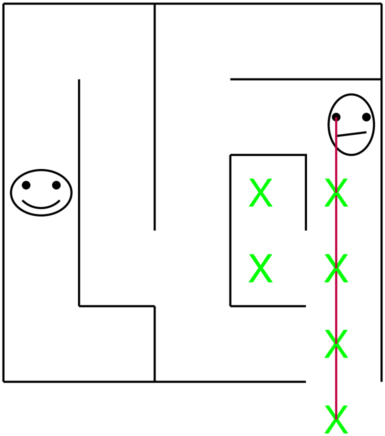

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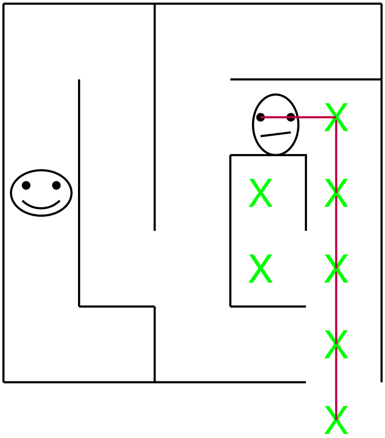

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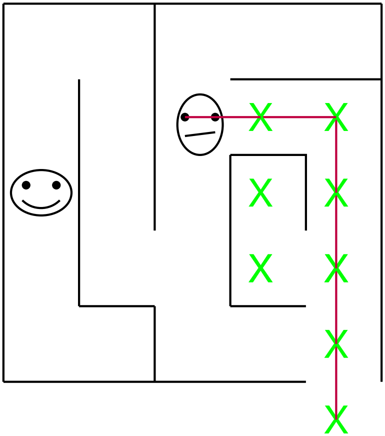

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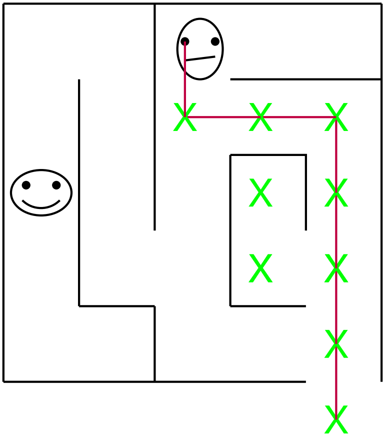

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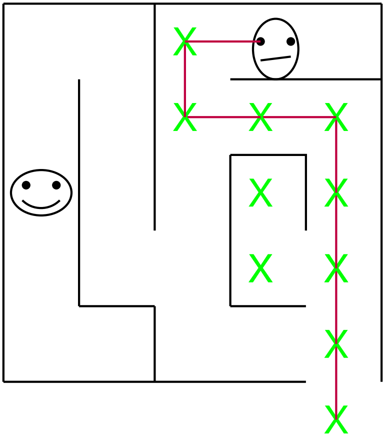

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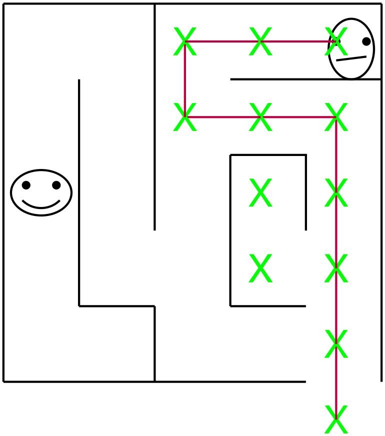

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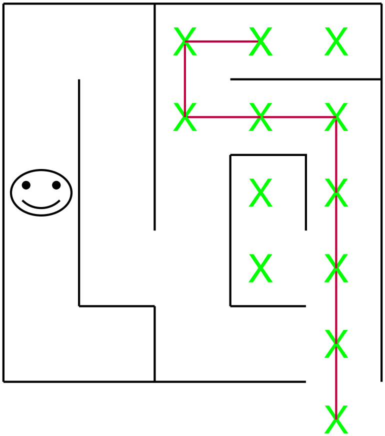

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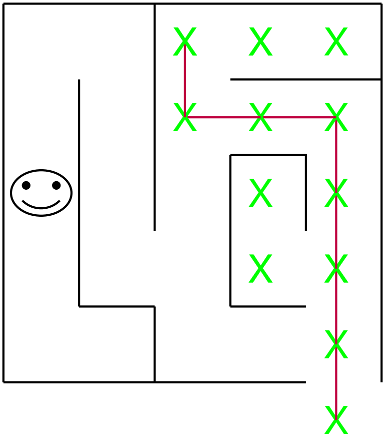

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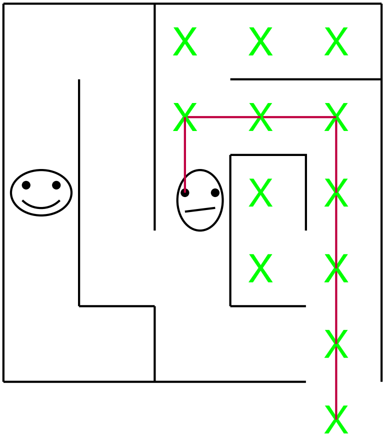

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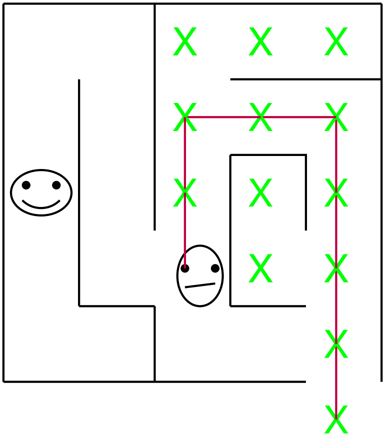

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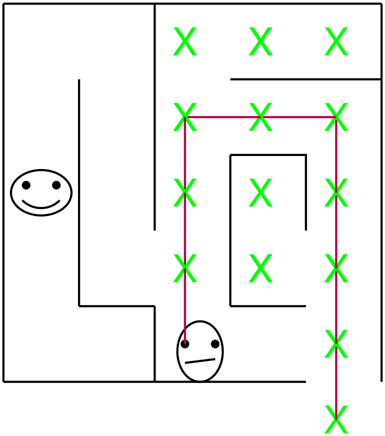
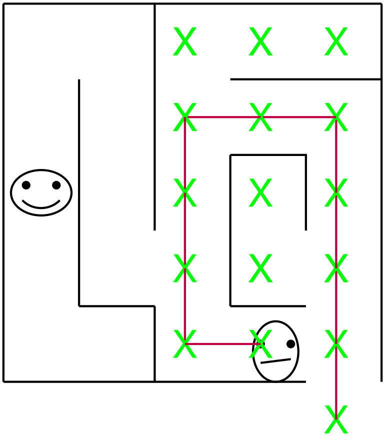


Where is the minatour?



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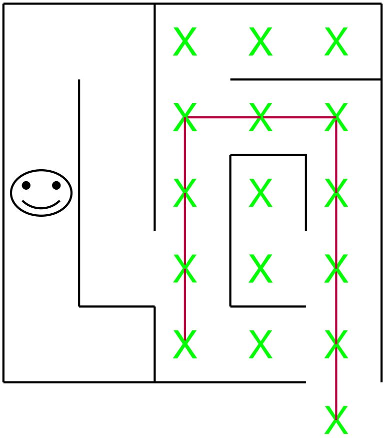

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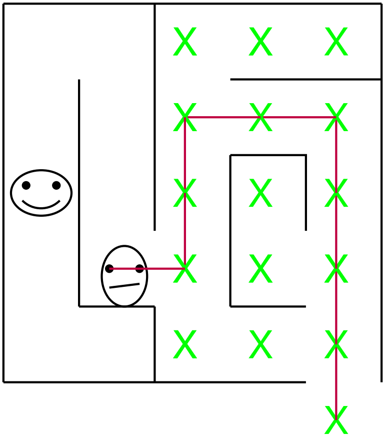

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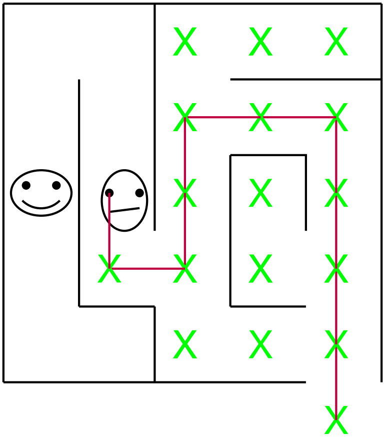

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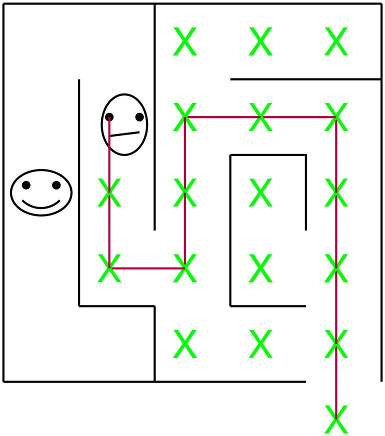

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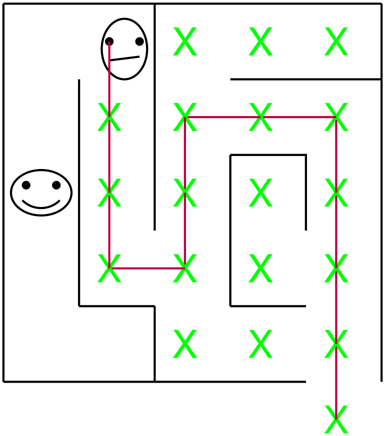

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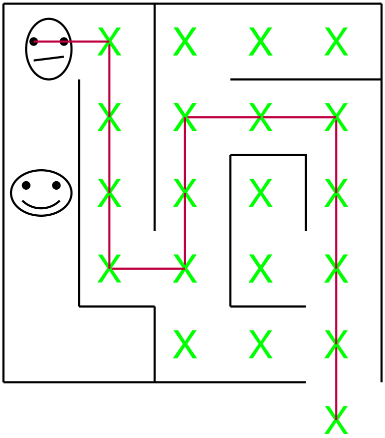

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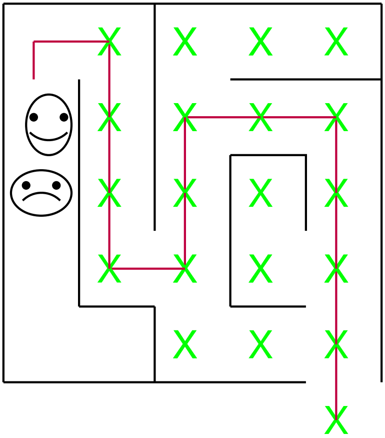

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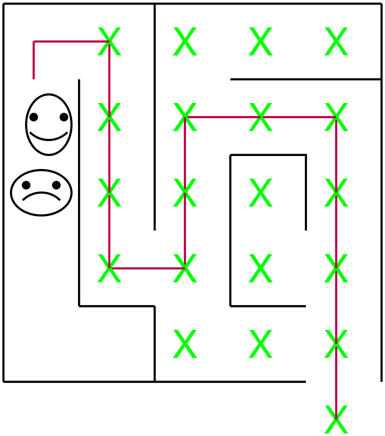

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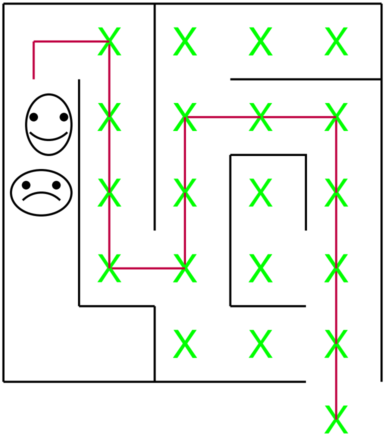

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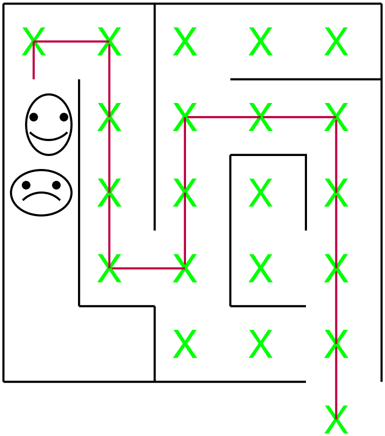


# Searching

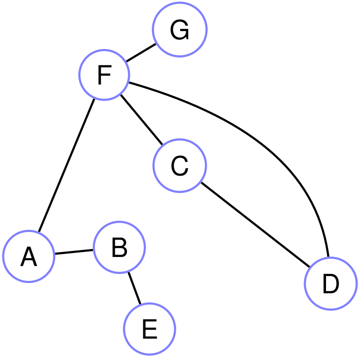
Find a minatour!

# Searching

Find a minatour!

Find out which nodes are reachable from $A$.

# Explore.



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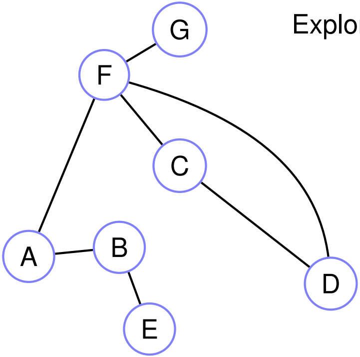
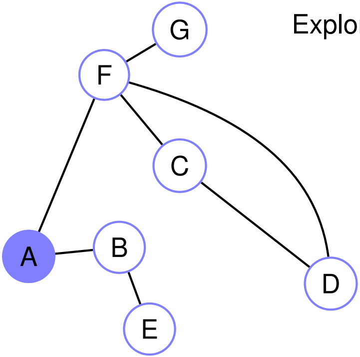



Explore(v):

1. Set visited[v] := true
2. for each edge (v,w) in E
3. if not visited[w]: Explore(w).

Chalk.
Stack is Thread.

# Explore.

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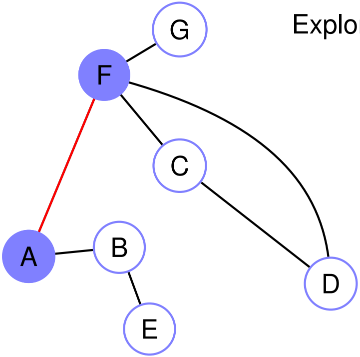


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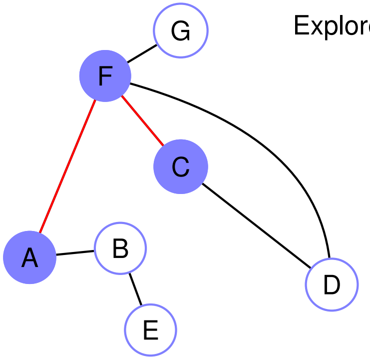


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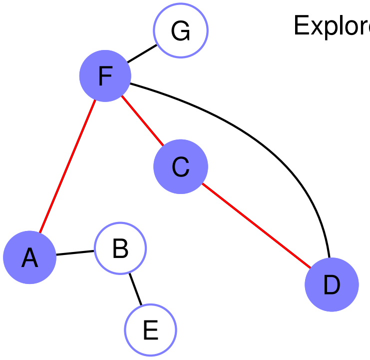


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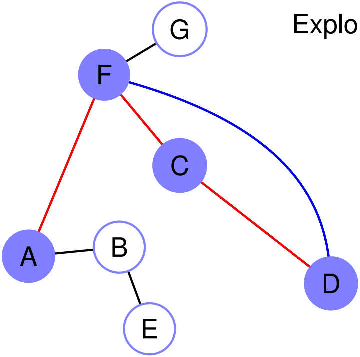


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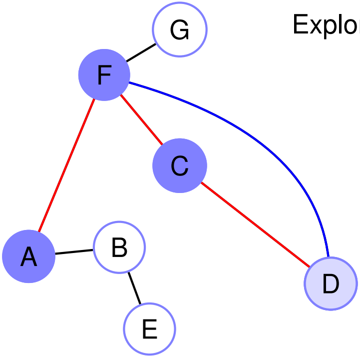


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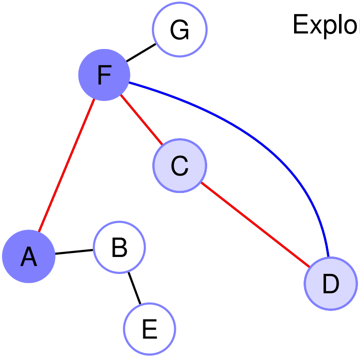


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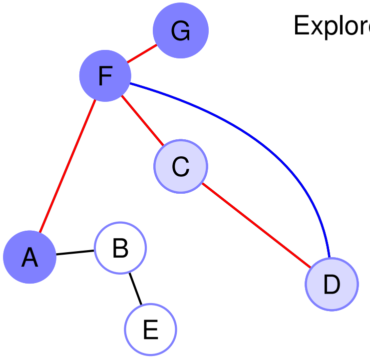


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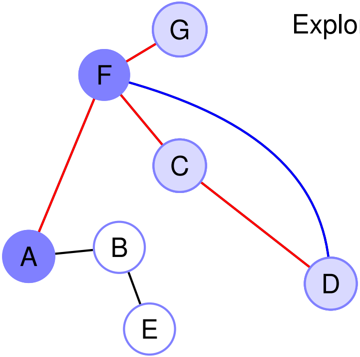


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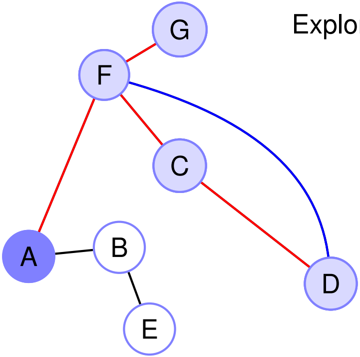


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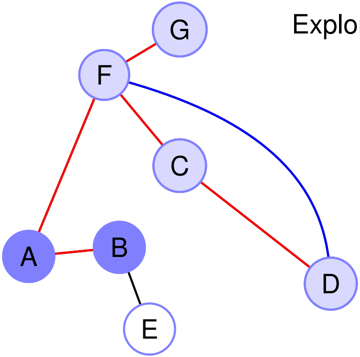


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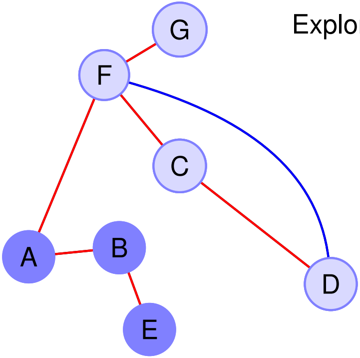


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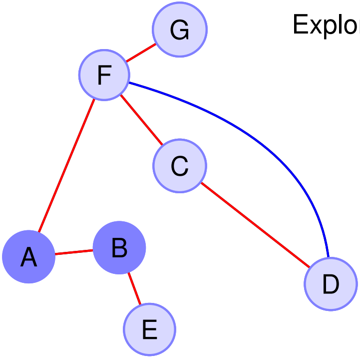


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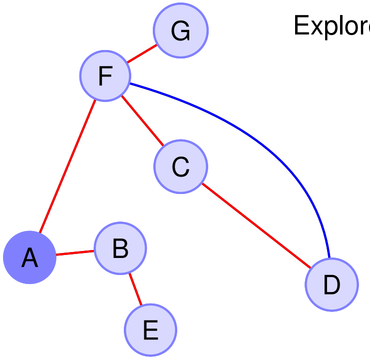


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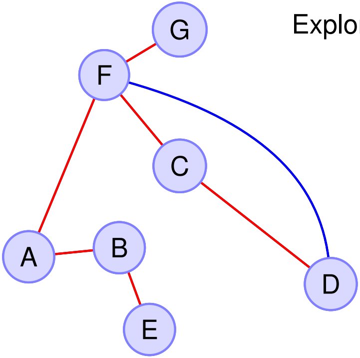


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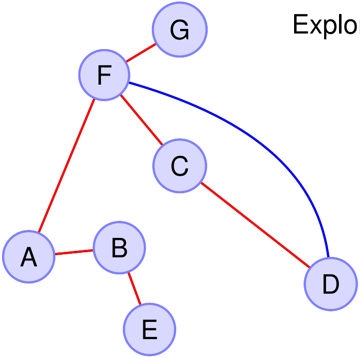


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Chalk.
Stack is Thread.

Explore builds tree.

*Tree* and *back* edges.

# Correctness.

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**Property:**

All and only nodes reachable from $A$ are reached by explore.

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Only: when $u$ visited.

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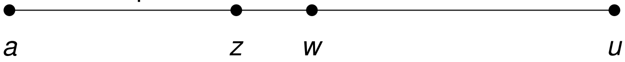

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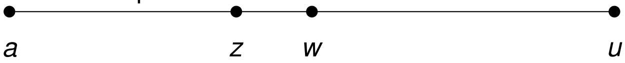


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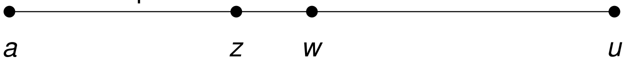


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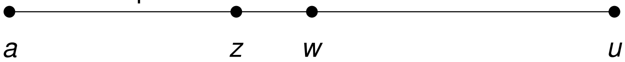


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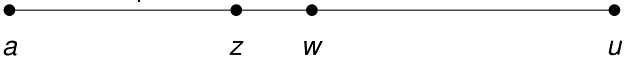
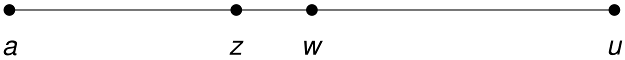



$z$ is explored. $w$ is not!
Explore ($z$) would explore($w$)! Contradiction.

$\square$

# Proof was induction.

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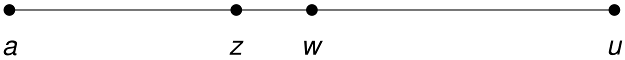


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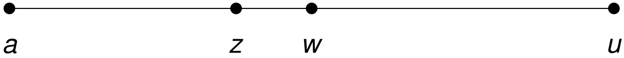


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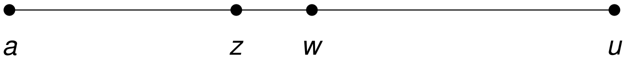


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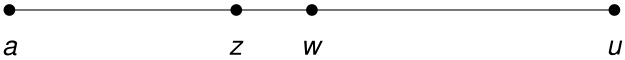


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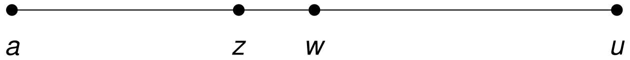


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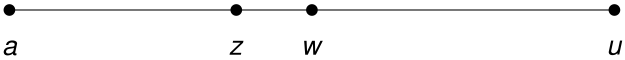


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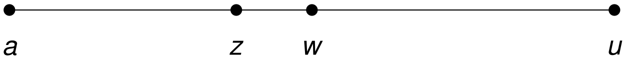


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Total: $O(n+m)$.

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Procedure: explore($v$).

Explores the graph.

Uses a stack.

Nonrecursive non-loop counting Runtime analysis.

Every little move she makes...