CS 170: Algorithms

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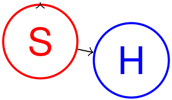
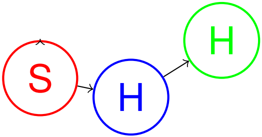
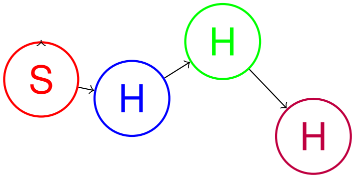


CS 170: Algorithms

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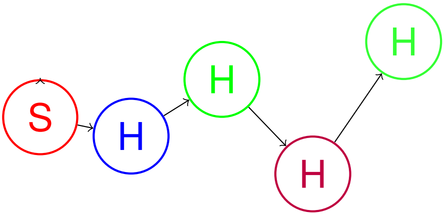
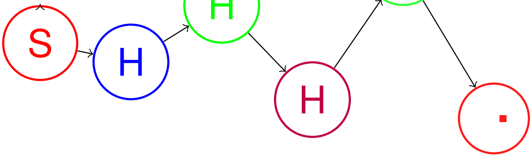
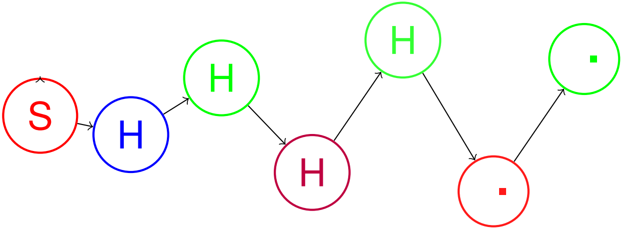
CS 170: Algorithms



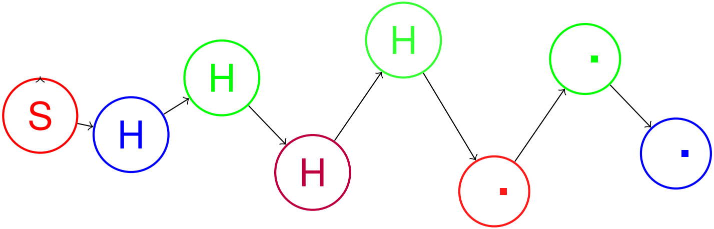
CS 170: Algorithms



CS 170: Algorithms



CS 170: Algorithms



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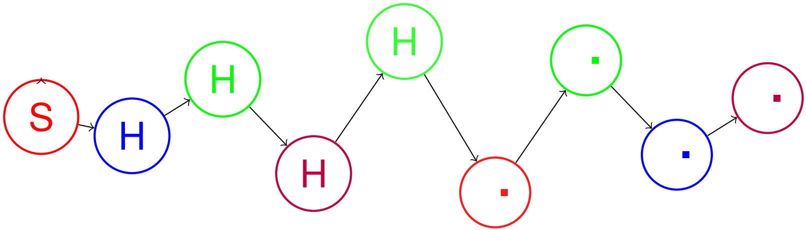
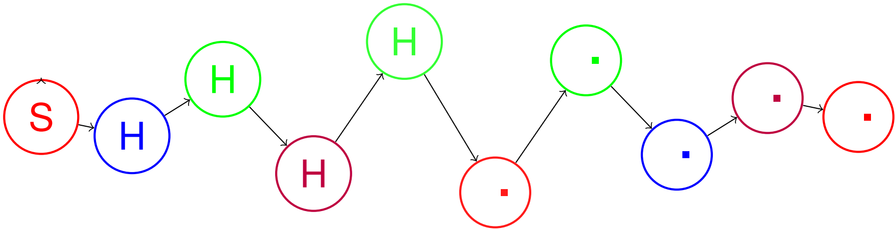


CS 170: Algorithms



Lecture in a Minute

Depth First Search.
   Call explore until explore the whole graph.
   Connected Components.
   Tree/back edges.
   Back edge $\iff$ cycle

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- Interval of time "on stack".
- Quick cycle test.

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Directed Graphs.
- Tree/Back/Forward/Cross edges.
- From pre/post!
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Topological Sort.
  Alg 1:Inverse Post order number.

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  Alg 1:Inverse Post order number.
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Directed Graphs.
  Tree/Back/Forward/Cross edges.
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Topological Sort.
  Alg 1:Inverse Post order number.
    Inverse order of "stack" pop.
  Alg 2: Peeling off sources.

Explore.

Graph $G = (V, E)$.

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Size of adjacency list representation?

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Size of adjacency list representation? $O(|V| + |E|)$.

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Reachable means there is a path,

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Runtime: $O(|V| + |E|)$.

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Runtime: $O(|V| + |E|)$.

The time is proportional to total size of the adjacency lists.

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Process whole graph.

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DFS(G)

1: For each node u,

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For each node u,
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Depth first search.

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Running time: $O(|V| + |E|)$.

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Running time: $O(|V| + |E|)$.

Intuitively: tree for each “connected component”.

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Intuitively: tree for each "connected component".
Several trees

Depth first search.

Process whole graph.

DFS(G)

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Intuitively: tree for each “connected component”.
Several trees or Forest!

Depth first search.

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Intuitively: tree for each “connected component”.
Several trees or Forest! Output connected components?

DFS and connected components.

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Change explore a bit:

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Each node will be labelled with connected component number.
Runtime: $O(|V| + |E|)$.

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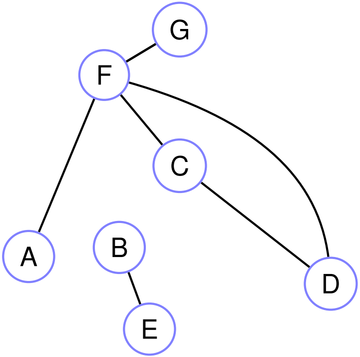

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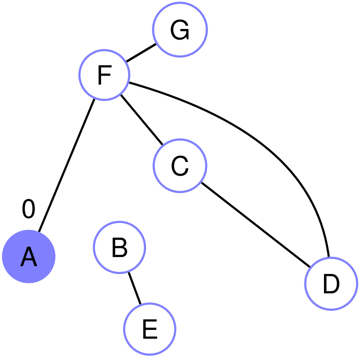

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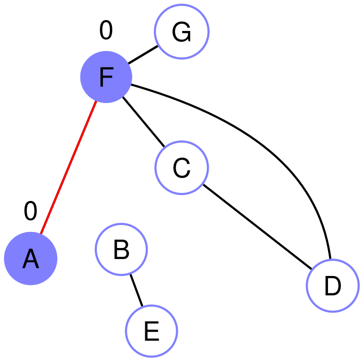

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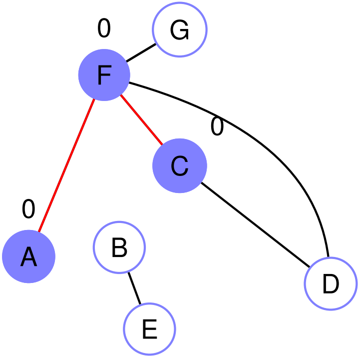

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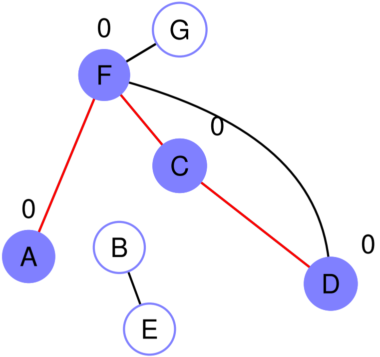

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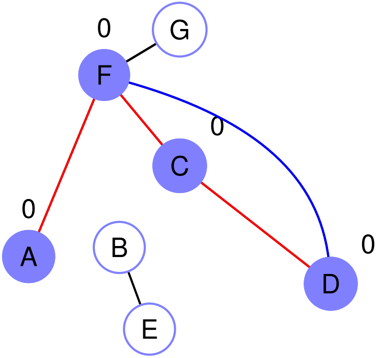

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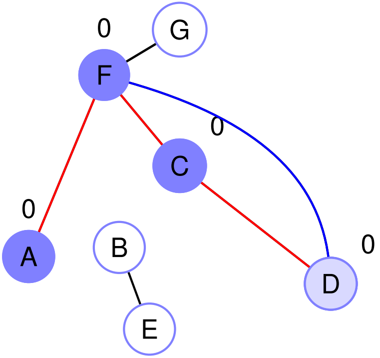

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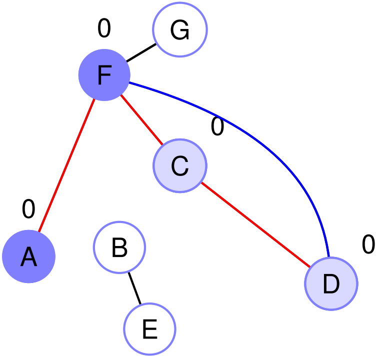

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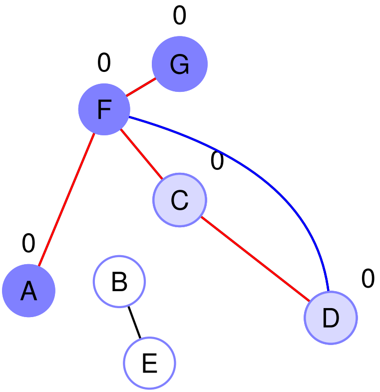

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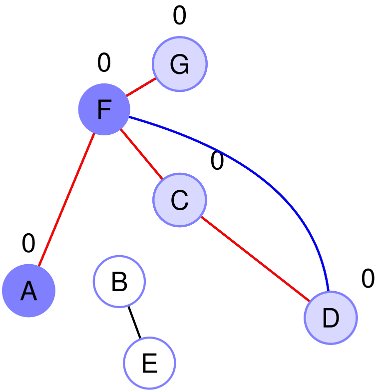

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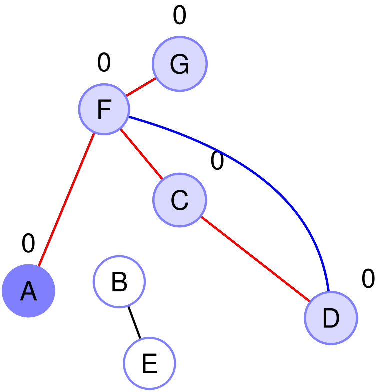

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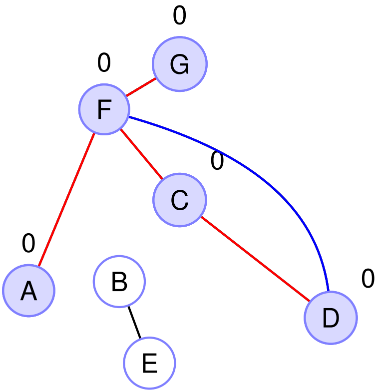

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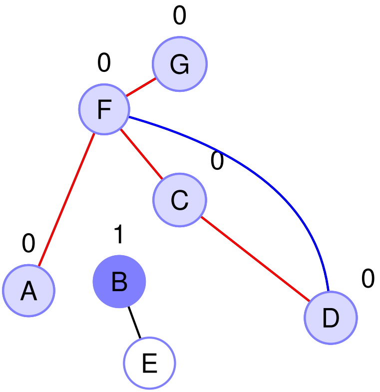

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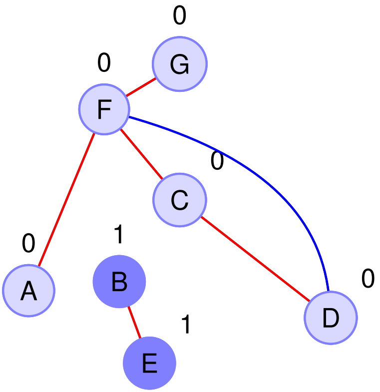

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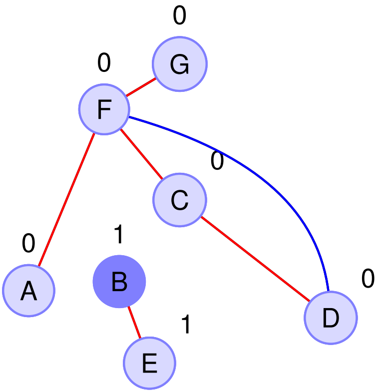

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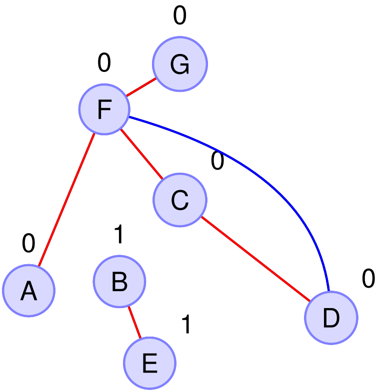
1. Set cc[v] := ccnum.

DFS(G): 0. Set cc := 0.

1. for each v in V:
2.   if not visited[v]:
3.   explore(v)
4.   ccnum = ccnum+1



Connected Components.

explore(v):

1. Set visited[v] := **true**.
2. previsit(v)
3. For each edge (v,w) in E
4.   if not visited[w]: explore(w).
5. postvisit(v)

Previsit(v):

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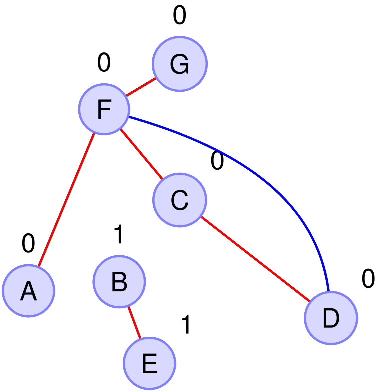

Introspection: pre/post.

Introspection: pre/post.

Previsit(v):

1. Set pre[v] := clock.
2. clock := clock+1

Introspection: pre/post.

Previsit(v):

1. Set pre[v] := clock.
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DFS(G):

0. Set clock := 0.

. . .

Postvisit(v):

1. Set post[v] := clock.
2. clock := clock+1

Introspection: pre/post.

Previsit(v):

1. Set pre[v] := clock.
2. clock := clock+1

DFS(G):

0. Set clock := 0.

$\ldots$

Clock: goes up to

Postvisit(v):

1. Set post[v] := clock.
2. clock := clock+1

Introspection: pre/post.

Previsit(v):

1. Set pre[v] := clock.
2. clock := clock+1

DFS(G):

0. Set clock := 0.

   . . .

Postvisit(v):

1. Set post[v] := clock.
2. clock := clock+1

Clock: goes up to 2 times number of tree edges.

Introspection: pre/post.

Previsit(v):

1. Set pre[v] := clock.
2. clock := clock+1

DFS(G):

0. Set clock := 0.

   ...

Postvisit(v):

1. Set post[v] := clock.
2. clock := clock+1

Clock: goes up to 2 times number of tree edges.

First pre:

Introspection: pre/post.

Previsit(v):

1. Set pre[v] := clock.
2. clock := clock+1

DFS(G):

0. Set clock := 0.

 . . .

Postvisit(v):

1. Set post[v] := clock.
2. clock := clock+1

Clock: goes up to 2 times number of tree edges.
First pre: 0

Introspection: pre/post.

Previsit(v):

1. Set pre[v] := clock.
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0. Set clock := 0.

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1. Set post[v] := clock.
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DFS(G):

0. Set clock := 0.

$\ldots$

Clock: goes up to 2 times number of tree edges.
First pre: 0

Property: For any two nodes, u and v, $[pre(u), post(u)]$ and $[pre(v), post(v)]$ are either disjoint or one is contained in other.

Introspection: pre/post.

Previsit(v):

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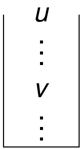
Interval is "clock interval on stack."



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Interval is “clock interval on stack.”

$$\begin{array}{|c|} u \\ \vdots \\ v \\ \vdots \\ \hline \end{array}$$

Either both on stack at some point (contained) or not (disjoint.)

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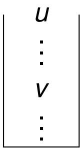
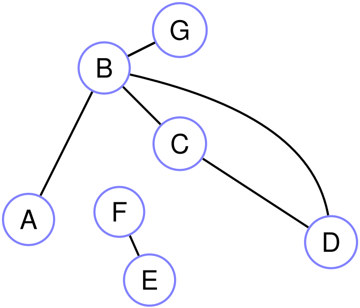


Either both on stack at some point (contained) or not (disjoint.)

Let's just watch it work!

Example: Pre/Post numbering.



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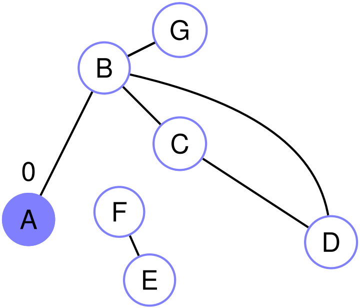

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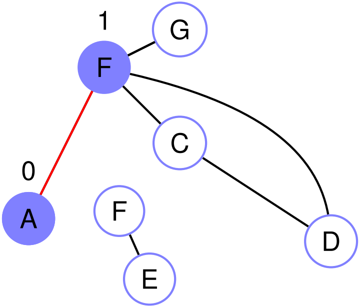
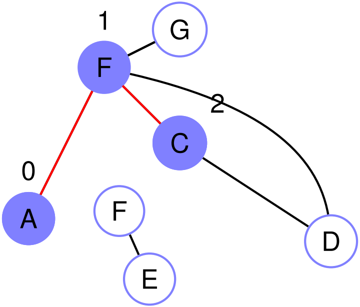


Example: Pre/Post numbering.



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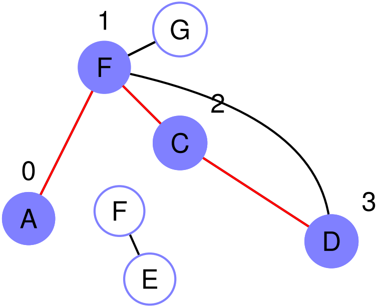

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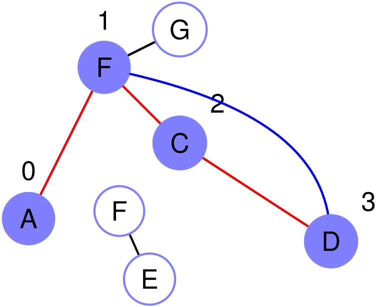

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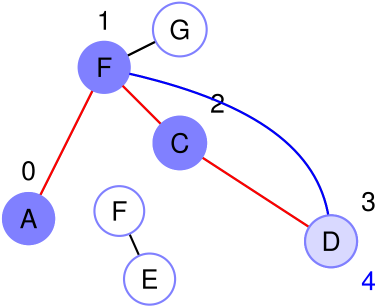

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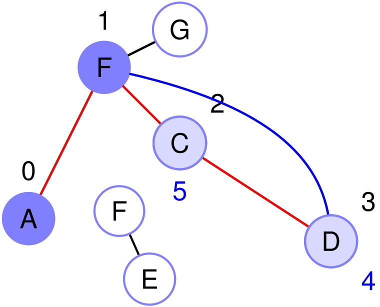

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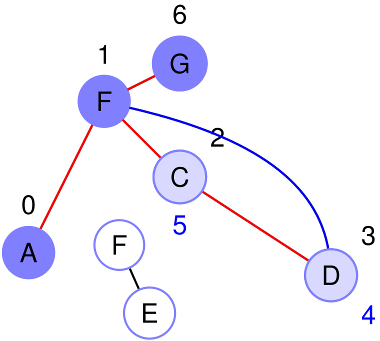

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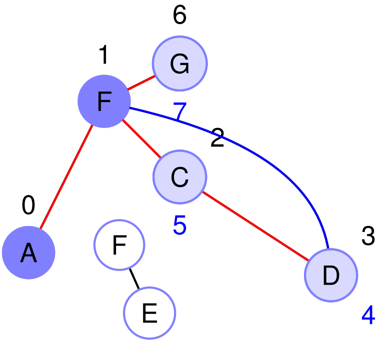

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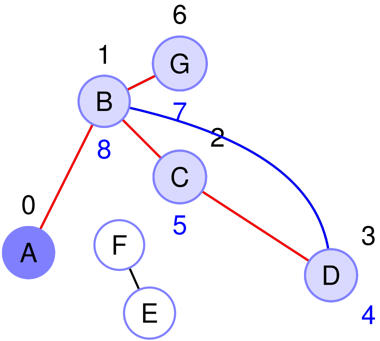

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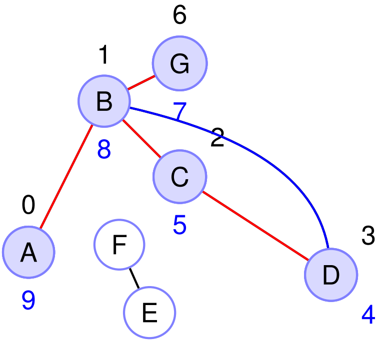

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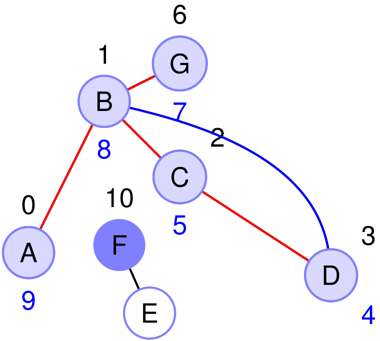

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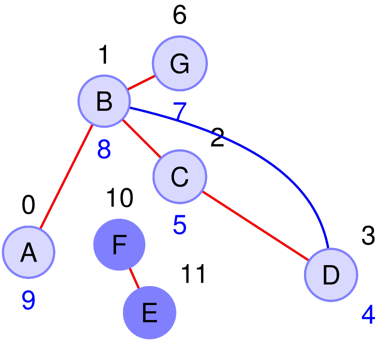

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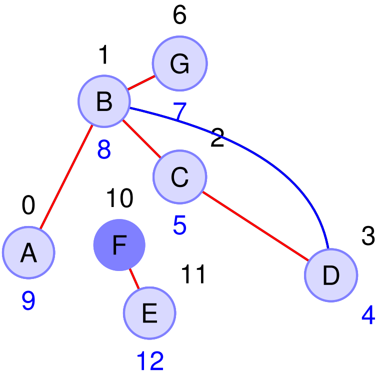

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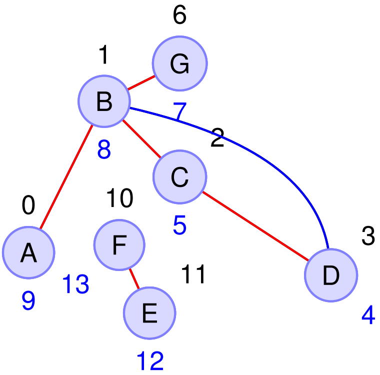
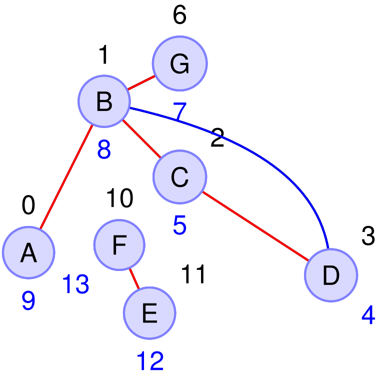


Example: Pre/Post numbering.



Explored edge (u, v) first from u.

Tree edge iff $[pre[v], post[v]] \in [pre[u], post[u]]$.
u on stack before v.

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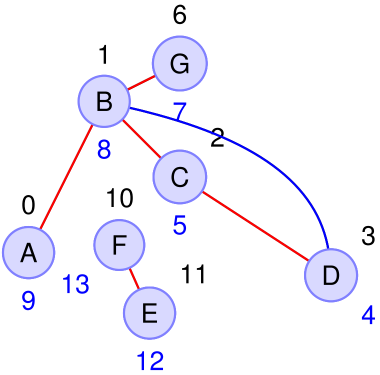

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v on stack when v on stack. Path from v to u! Cycle!

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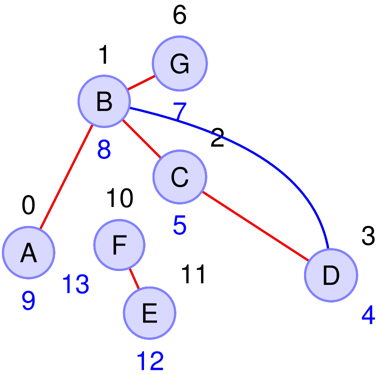

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v on stack when v on stack. Path from v to u! Cycle!

No edge between u and v if disjoint intervals.

Directed graphs.

$G = (V, E)$

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From u to v.
Tail – u

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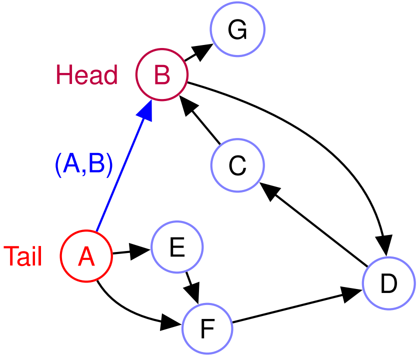

DFS on directed graphs.

Terminology:

DFS on directed graphs.

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Root: Starting point.

DFS on directed graphs.

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v is ancestor of u:

DFS on directed graphs.

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v is ancestor of u:
v on path from/to root.

DFS on directed graphs.

Terminology:

Root: Starting point.

v is ancestor of u:
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v is descendant of u:

DFS on directed graphs.

Terminology:

Root: Starting point.

v is ancestor of u:
v on path from/to root.

v is descendant of u:
u is an ancestor of v.

DFS on directed graphs.

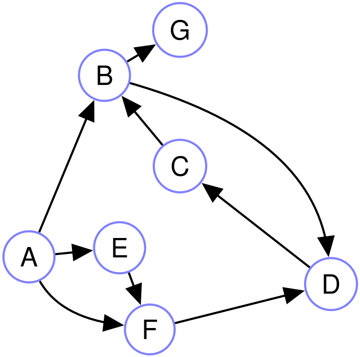
Terminology:

Root: Starting point.

v *is ancestor of* u*:*
v on path from/to root.

v *is descendant of* u*:*
u is an ancestor of v.

Depth first search: directed.



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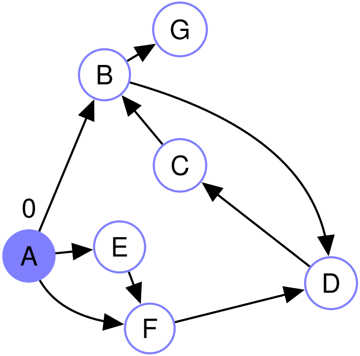

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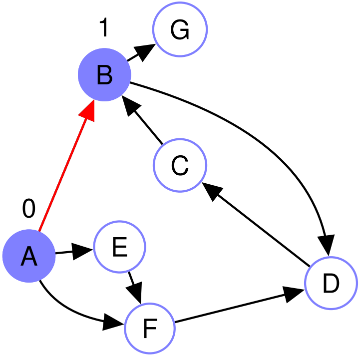

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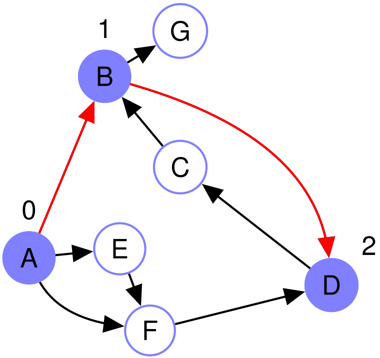

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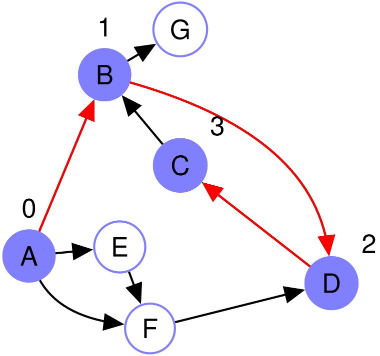

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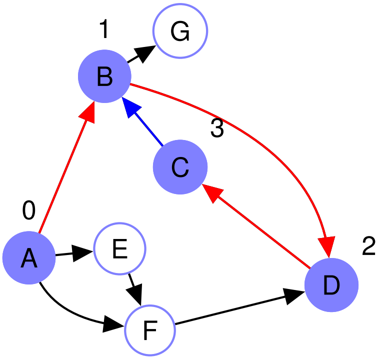

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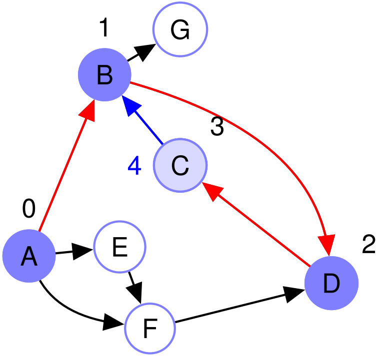

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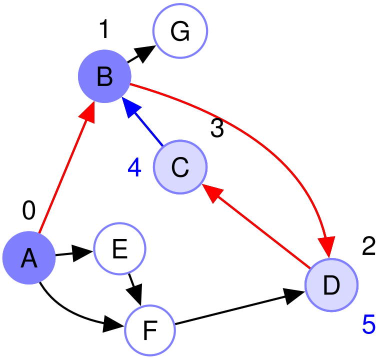

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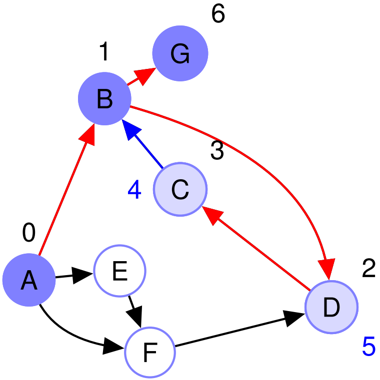

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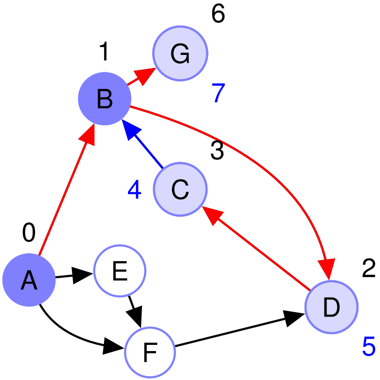

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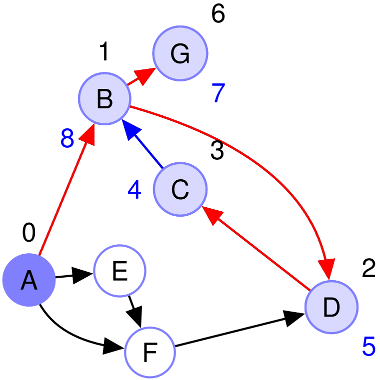

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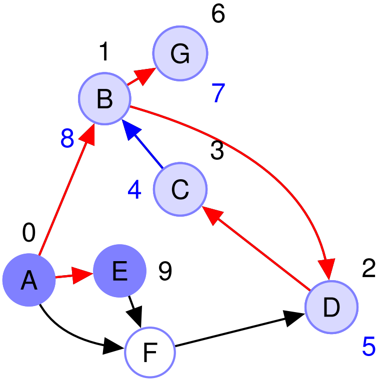

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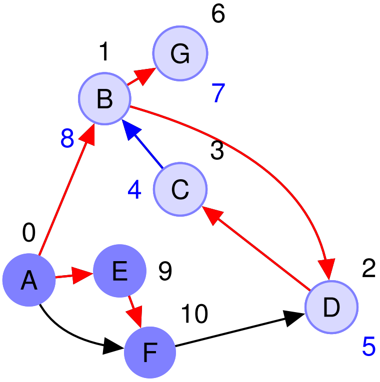

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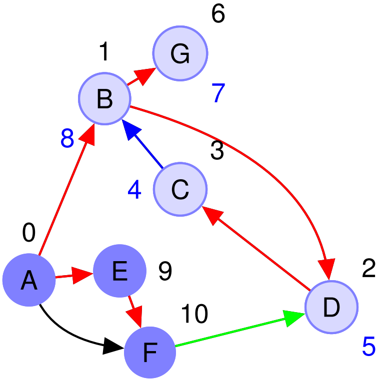

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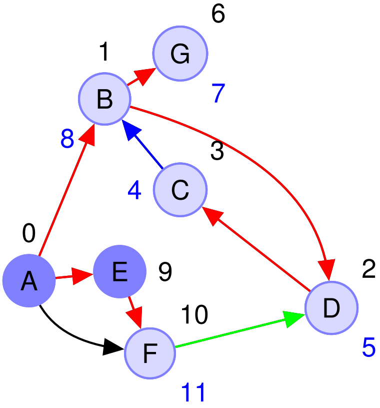

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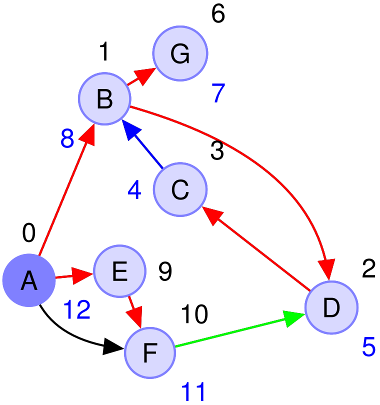
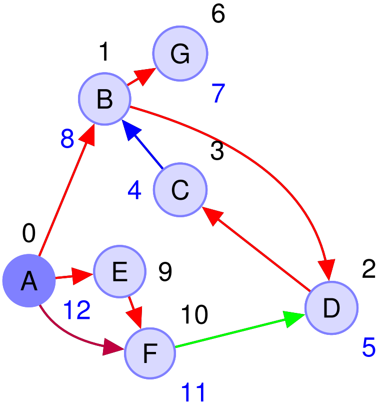


Depth first search: directed.



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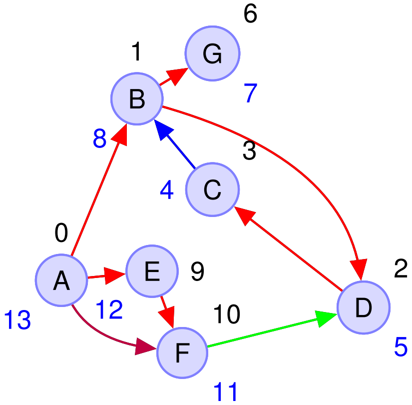

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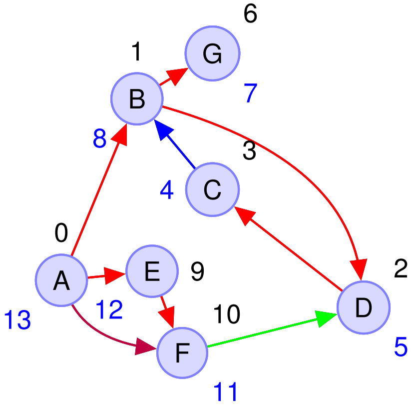
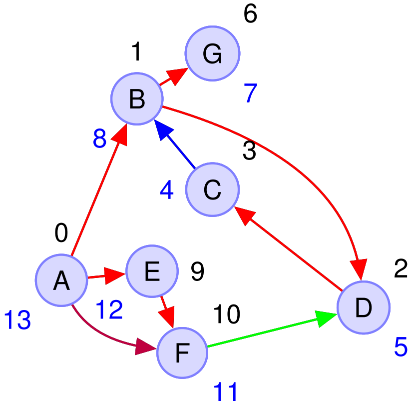


Tree/forward edge (u, v): int(v) in int(u).

Depth first search: directed.



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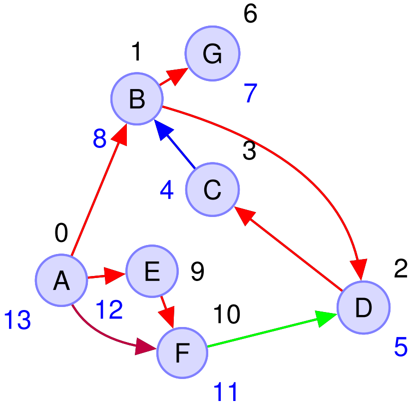
Forward (A, F): [10,11] in [0,13] or [0,[10,11],13]

Depth first search: directed.



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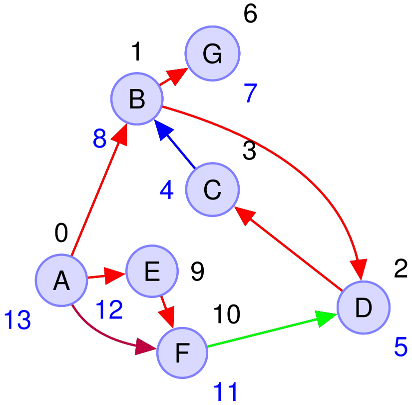

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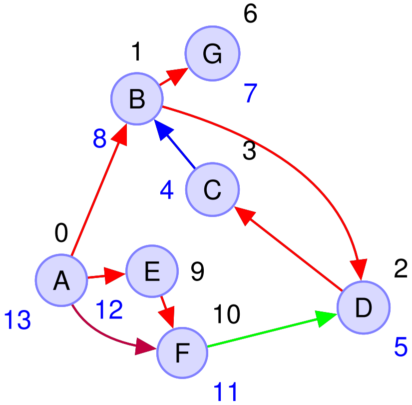
(C,B): [3,4] in [1,8] or [1, [3, 4], 8]

Depth first search: directed.



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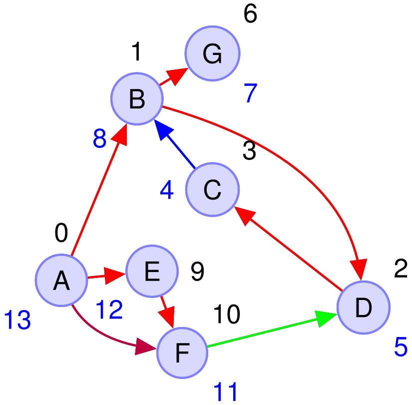
Back edge (u, v): int(v) contains int(u).

(C, B): [3,4] in [1,8] or [1, [3, 4], 8]

Cross edge (u, v): int(v) before int(u).

Depth first search: directed.



Tree/forward edge (u, v): int(v) in int(u).

Forward (A, F): [10,11] in [0,13] or [0,[10,11],13]

Back edge (u, v): int(v) contains int(u).

(C, B): [3,4] in [1,8] or [1, [3, 4], 8]

Cross edge (u, v): int(v) before int(u).

(F, D): [2,5] before [10,11]

Directed Acyclic Graphs: Depth First Search

Edge: (u, v)

Directed Acyclic Graphs: Depth First Search

Edge: (u, v)
From u to v.

Directed Acyclic Graphs: Depth First Search

Edge: (u, v)

From u to v.

Tail – u

Directed Acyclic Graphs: Depth First Search

Edge: (u, v)
  From u to v.
    Tail – u
    Head – v

Directed Acyclic Graphs: Depth First Search

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From u to v.

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Head – v

Tree edge – “Direct call tree of explore.”

Directed Acyclic Graphs: Depth First Search

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Forward edge – "Edge to descendant (not in tree)"

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    v already explored before u is visited.

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Directed Acyclic Graph

Directed Graph ...

Directed Acyclic Graph

Directed Graph ...without cycles.

Directed Acyclic Graph

Directed Graph ...without cycles. Cycle: $v_0 \rightarrow v_1 \rightarrow \ldots v_k \rightarrow v_0$.

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Why?

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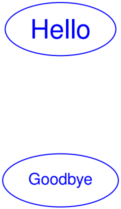

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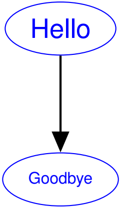

“Hello” before “Goodbye”

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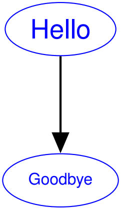
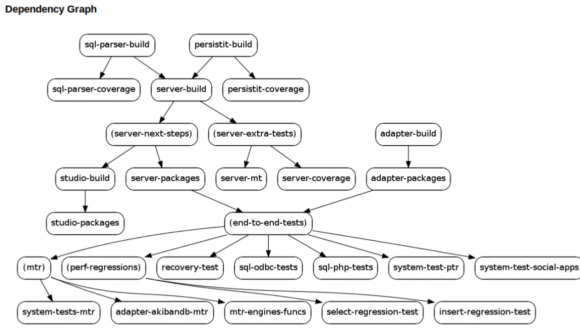


"Hello" before "Goodbye"



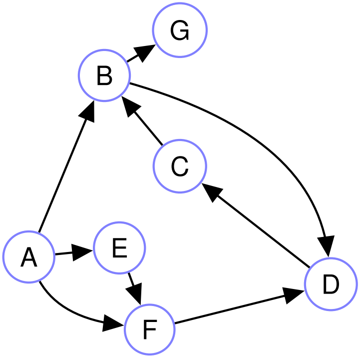
Example.

Example.

Acyclic Graph?

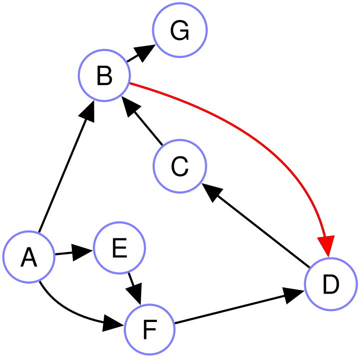
Example.

Acyclic Graph?



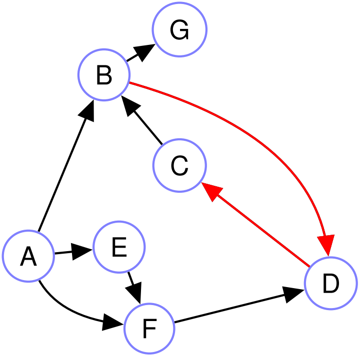
Example.

Acyclic Graph?



Example.

Acyclic Graph?



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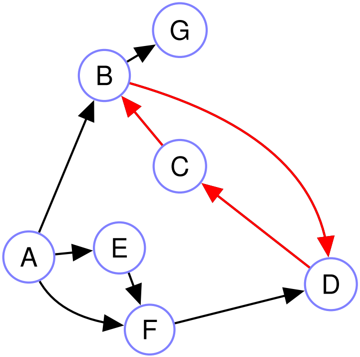

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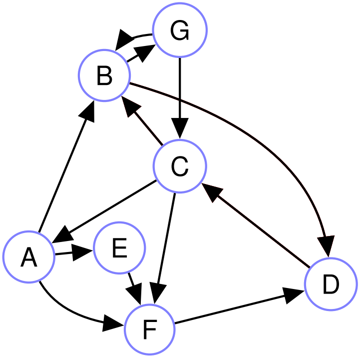
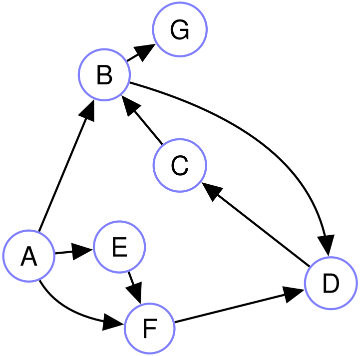


Depth first search: directed.



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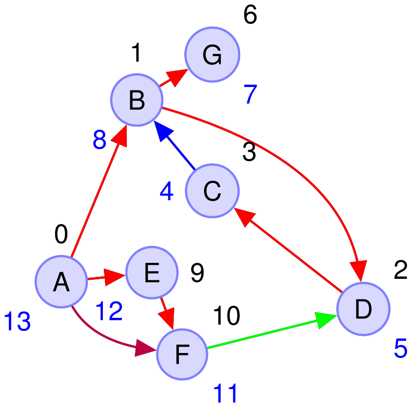

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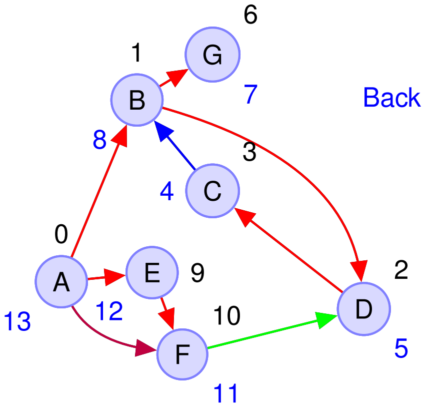

Back edge (u, v): int(v) contains int(u).

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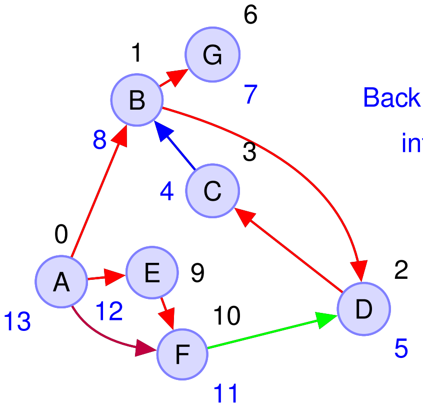
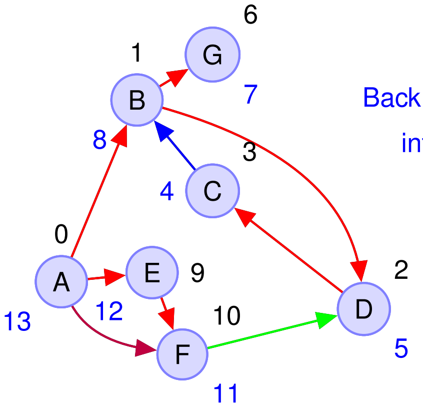


Back edge (u, v): int(v) contains int(u).

int(C) $= [3, 4]$ and int(B) $= [1, 8]$.

Depth first search: directed.



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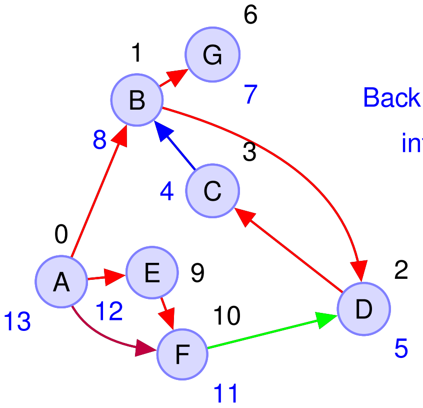
Back edge (u, v)

Depth first search: directed.



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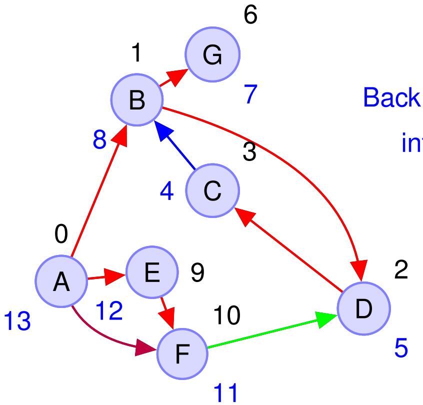
Back edge (u, v)
....edge to ancestor

Depth first search: directed.



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Back edge (u, v)
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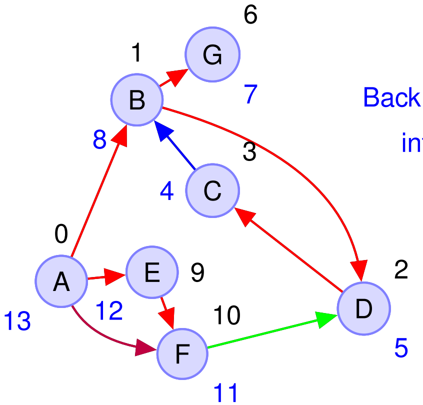
tree edges from v to u.

Depth first search: directed.



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Back edge (u, v)
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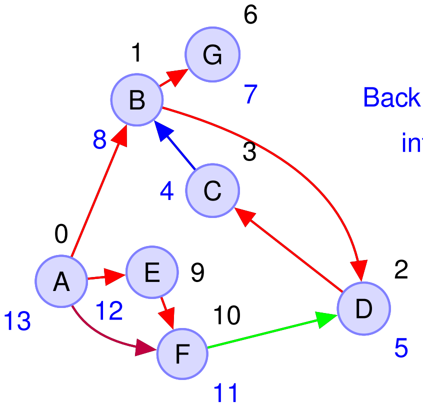
tree edges from v to u.
Back edge means cycle!

Depth first search: directed.



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int$(C) = [3, 4]$ and int$(B) = [1, 8]$.

Back edge (u, v)
....edge to ancestor
tree edges from v to u.
Back edge means cycle! $\implies$ not acyclic!

Testing for cycle.

Thm: A graph has a cycle if and only if there is back edge.

Testing for cycle.

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Proof:

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Back edge $\implies$ cycle!

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There is a cycle

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There is a cycle

$v_0 \to v_1$

Testing for cycle.

Thm: A graph has a cycle if and only if there is back edge.

Proof:
Back edge $\implies$ cycle!

There is a cycle

$v_0 \rightarrow v_1 \rightarrow v_2$

Testing for cycle.

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Back edge $\implies$ cycle!

There is a cycle

$v_0 \to v_1 \to v_2 \cdots \to v_k \to v_0$

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There is a cycle

$v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k \rightarrow v_0$

Assume that v_0 is the first node explored.
(without loss of generality since can renumber vertices.)

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All nodes on cycle explored when **explore**(v_0) returns

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For each v_i: int[v_i] $\in$ int[v_0]!

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$v_0 \rightarrow v_1 \rightarrow v_2 \cdots \rightarrow v_k \rightarrow v_0$

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For each v_i: int$[v_i] \in$ int$[v_0]$!

$\implies (v_k, v_0)$ is a back edge.

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Proof:
Back edge $\implies$ cycle!

There is a cycle

$v_0 \to v_1 \to v_2 \cdots \to v_k \to v_0$

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All nodes on cycle explored when **explore**(v_0) returns

For each v_i: int[v_i] $\in$ int[v_0]!

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Cycle $\implies$ back edge!

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Proof:
Back edge $\implies$ cycle!

There is a cycle

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For each v_i: int$[v_i] \in$ int$[v_0]$!

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Cycle $\implies$ back edge! $\square$

Fast checking algorithm.

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Run DFS.

Fast checking algorithm.

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Run DFS. $O(|V| + |E|)$ time.

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Run DFS. $O(|V| + |E|)$ time.
For each edge (u, v): is int(u) in int(v).

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Run DFS. $O(|V|+|E|)$ time.
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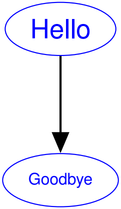
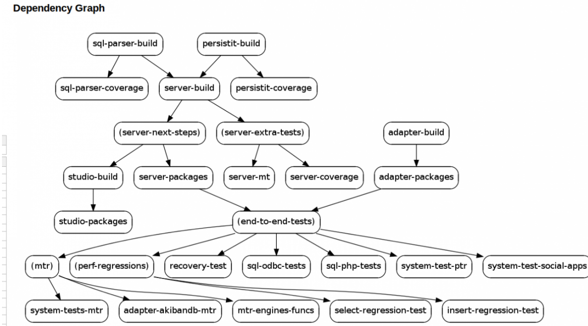
$O(|V|+|E|)$ time algorithm for checking if graph is acyclic.

Directed Acyclic Graph



“Hello” before “Goodbye”



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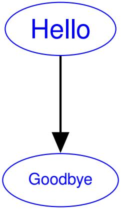
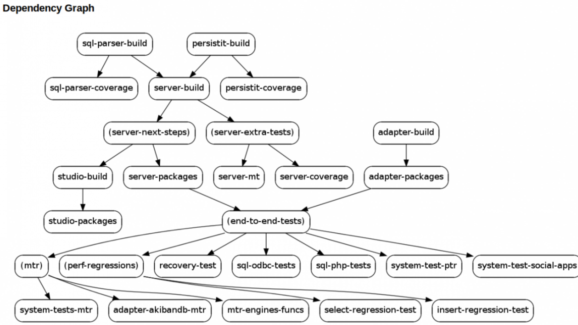
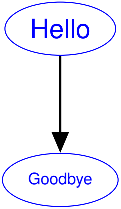


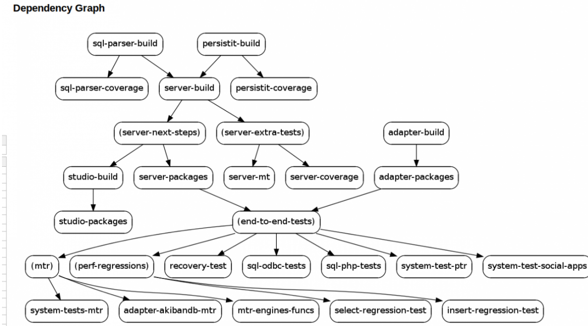


“Hello” before “Goodbye”

No cycles!

Directed Acyclic Graph

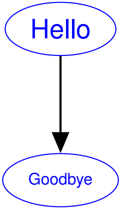




“Hello” before “Goodbye”

No cycles! Can tell in linear time!

Directed Acyclic Graph



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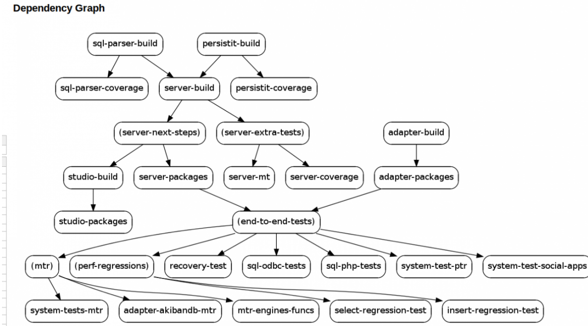
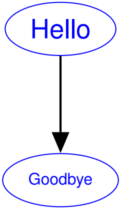


No cycles! Can tell in linear time!

Ohhh...

Directed Acyclic Graph



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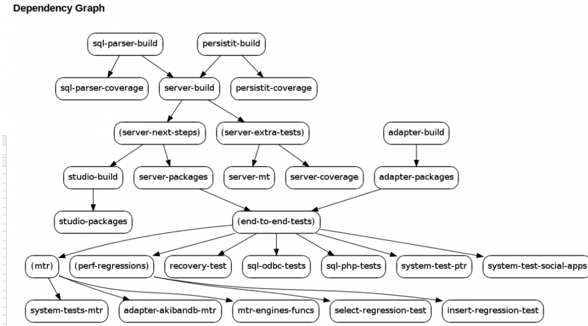

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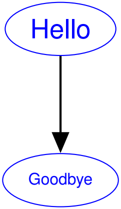

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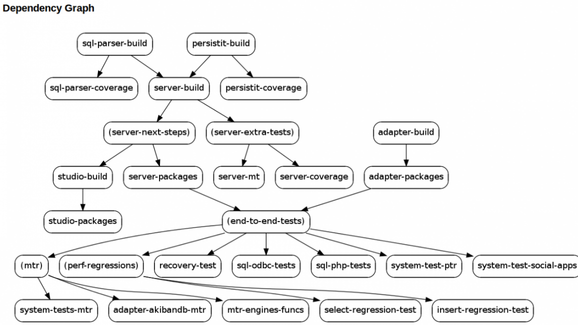

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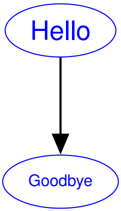
Ohhh...Kayyyy...
Really want to find ordering for build!

Directed Acyclic Graph



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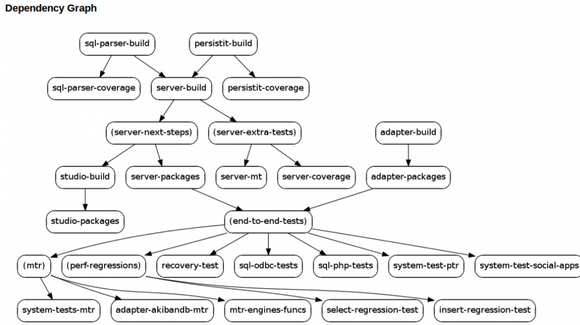


No cycles! Can tell in linear time!

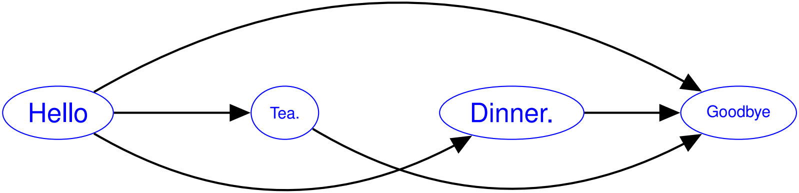
Ohhh...Kayyyy...
Really want to find ordering for build!
Where things are cool!

Linearize.

Topological Sort: For $G = (V, E)$, find ordering where each edge goes from earlier vertex to later in acyclic graph.

Linearize.

Topological Sort: For $G = (V, E)$, find ordering where each edge goes from earlier vertex to later in acyclic graph.



Linearize.

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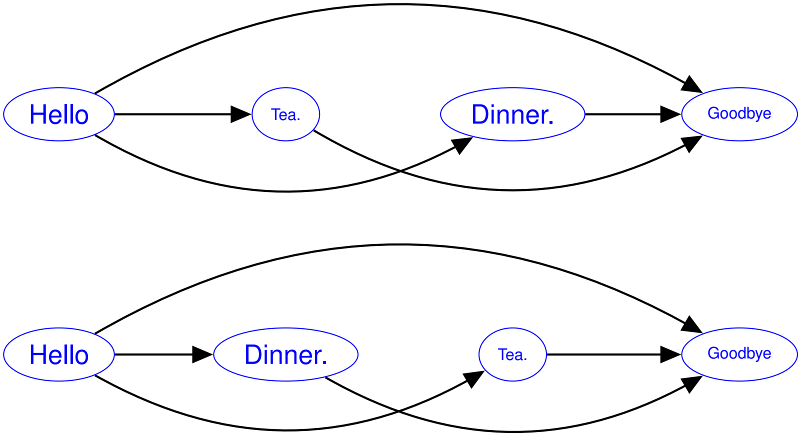
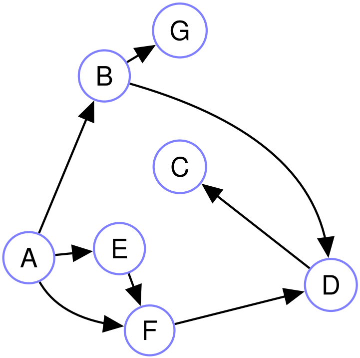


Topological Sort Example.



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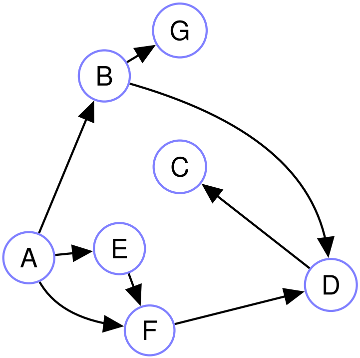
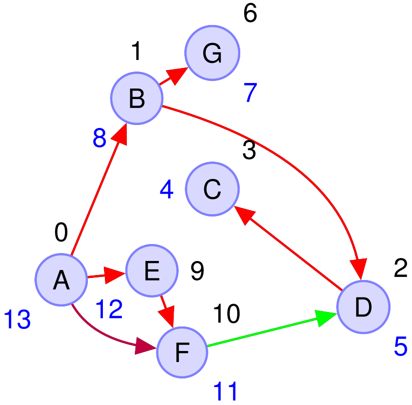


A linear order:

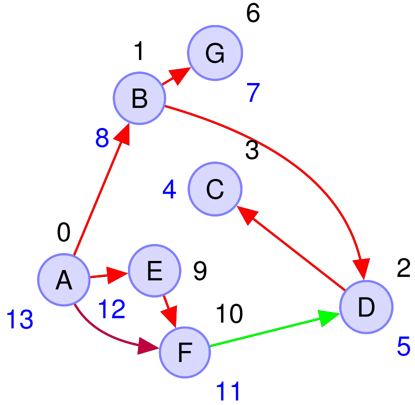
Topological Sort Example.



A linear order:

A, F, E, B, G, D, C?

Topological Sort Example.



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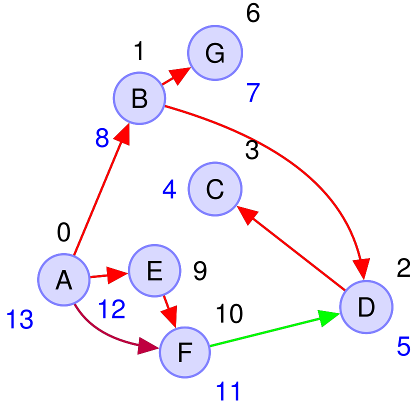

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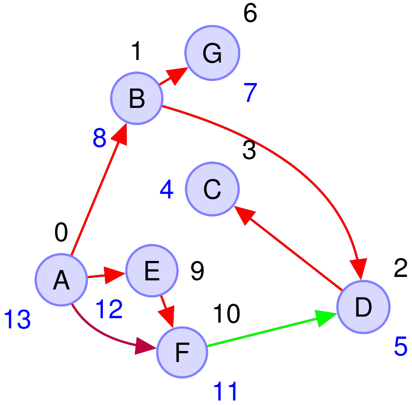

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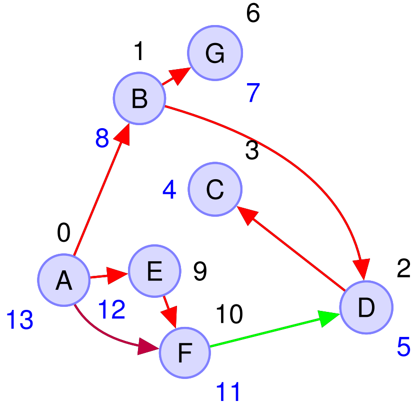
In DFS: When is A popped off stack?

Topological Sort Example.



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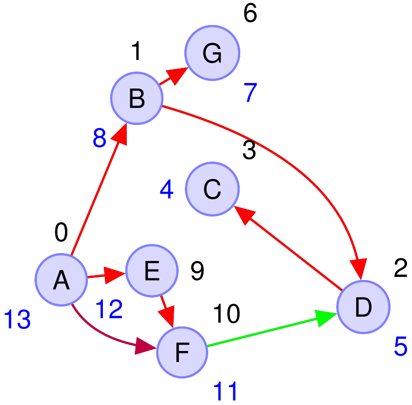
A, E, F, B, G, D, C

In DFS: When is A popped off stack?

Last!

Topological Sort Example.



A linear order:

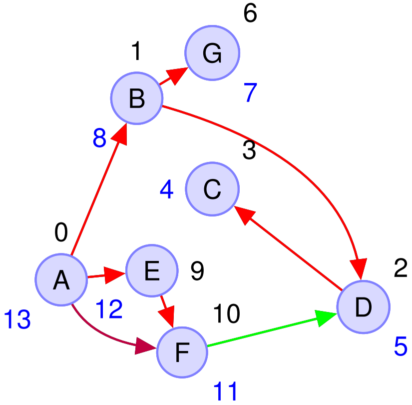
A, F, E, B, G, D, C? Nope.

A, E, F, B, G, D, C

In DFS: When is A popped off stack?

Last! E

Topological Sort Example.



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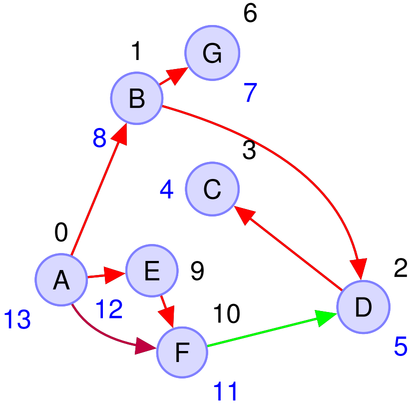

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Useful on Monday.

Lecture in a Minute

Depth First Search.

Call explore until explore the whole graph.

Connected Components.

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Back edge $\iff$ cycle

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- Tree/Back/Forward/Cross edges.
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Topological Sort.
  Alg 1:Inverse Post order number.
    Inverse order of "stack" pop.
  Alg 2: Peeling off sources.