CS 170: Algorithms

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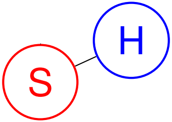
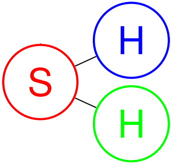
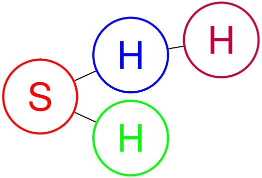


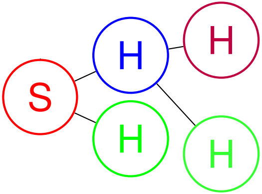
CS 170: Algorithms



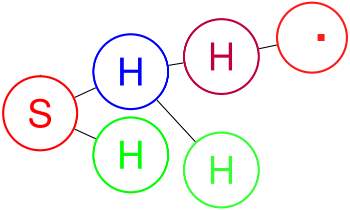
CS 170: Algorithms



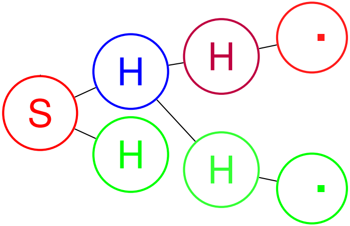
CS 170: Algorithms



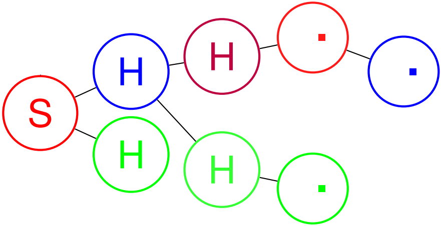
CS 170: Algorithms



CS 170: Algorithms



CS 170: Algorithms



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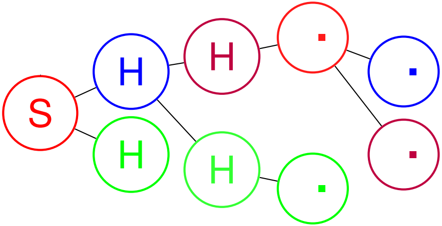

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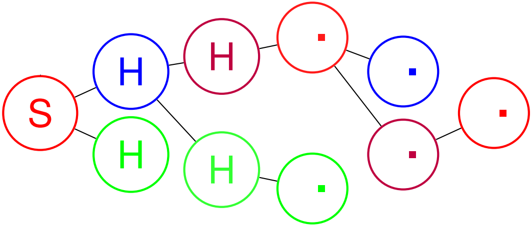

Breadth First Search/Dijkstra.

Lecture in a minute.

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Breadth First Search of graph:
Search with queue instead of stack.
Get “distances” from source.
Proof idea: queue has distance 0, then level 1, . . .

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Djikstra: shortest paths in weighted graph:
  Replace weights by paths + BFS
  Implement using priority queue.
    Idea: ignores “new” nodes.
  Runtime: $|V|$ extracts, $|E|$ reduce-key for p-queue.

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Priority Queue:
  Implementation: degree d tree.

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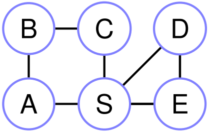
Definition:
Distance$(s) = 0$, Distance$(v) = \min_{N(v)} d(v) + 1$



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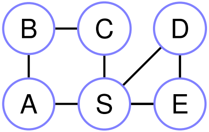

Try depth first search.

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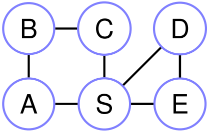

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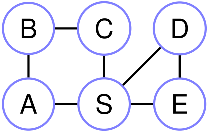

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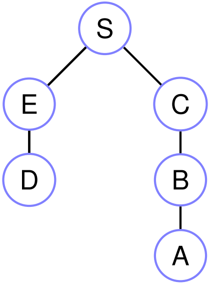

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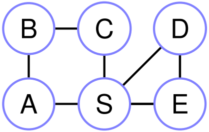

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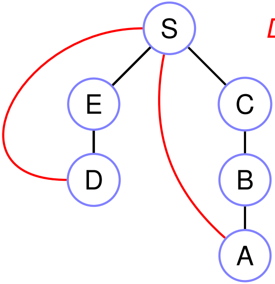

D and A at distance 1.

Algorithm 2: Pick it up!

$G = (V, E)$.

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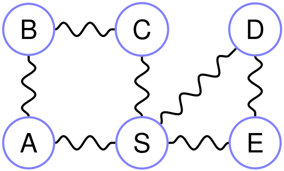
Distance - length of shortest path between u and v.



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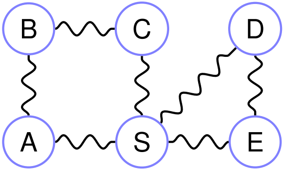

Pick it up by s!

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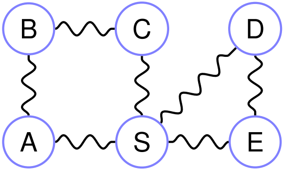

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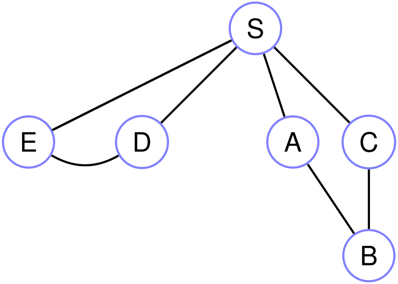
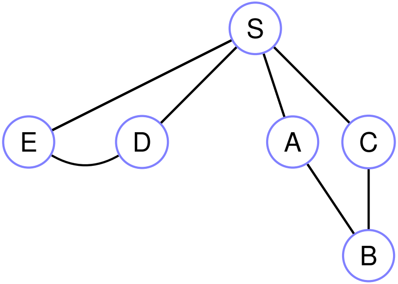
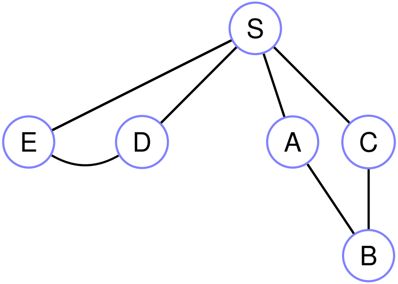




Distance 0.

Distance 1.

Distance 2.

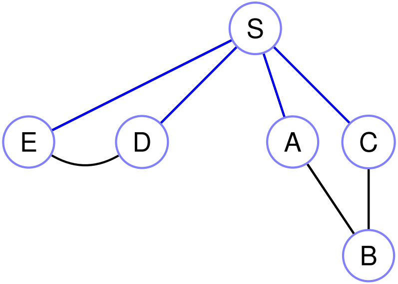


Distance 0.

Distance 1.

Distance 2.

So..proceed layer by layer.
Distance 0.



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So..proceed layer by layer.
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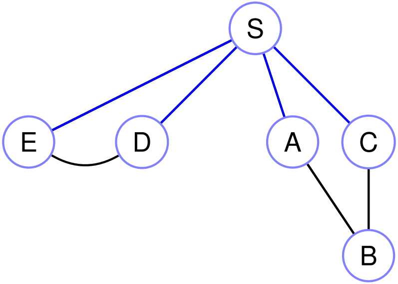
Neighbors of “0” node are distance 1 nodes.



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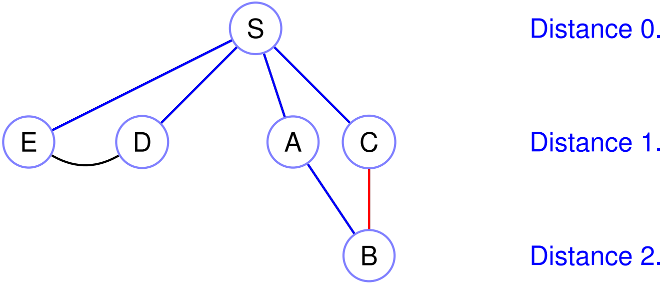

So..proceed layer by layer.
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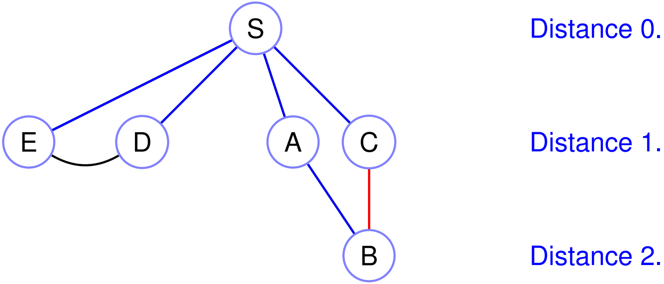
⋮
Untouched Neighbors of d nodes are $d + 1$ nodes.



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Distance 0.
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Untouched Neighbors of d nodes are $d+1$ nodes.
What data structure should we use to organize this?

Queue: Breadth First Search

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d(s) = 0;

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Nodes “explored” in order of distance from s.

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Has **no** $l > d+1$ **nodes** since only $\leq d$ level nodes explored.

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In action!

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  foreach (u,v):
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      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: S
0

In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

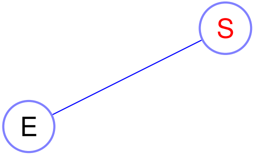
Queue:

In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: E
1

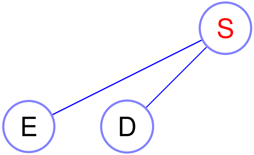


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: E, D
$1, 1$

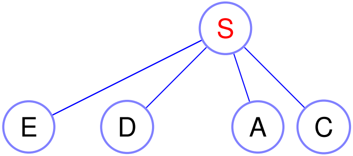


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: E, D, A, C
$1, 1, 1, 1$

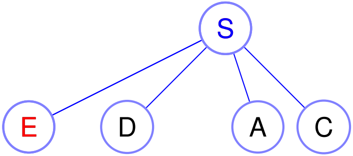


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: D, A, C
$1, 1, 1$

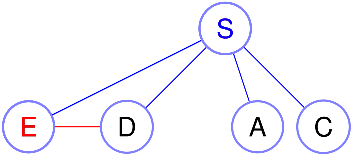


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: D, A, C
$1, 1, 1$

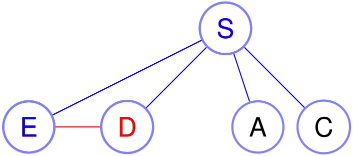


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: A, C
$1, 1$

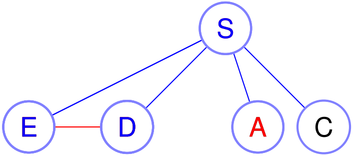


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: C
1

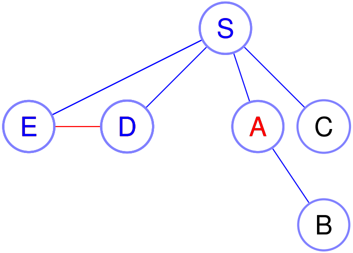


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: C, B
$1, 2$

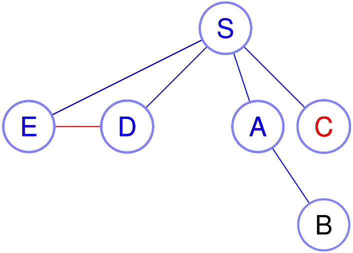


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: B
2



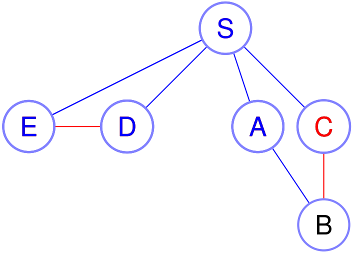
In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue: B

2

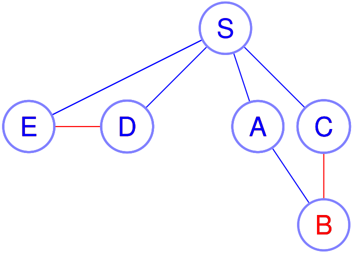


In action!

Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

Queue:

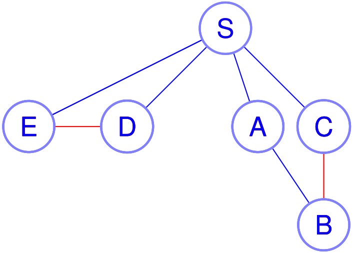


In action!

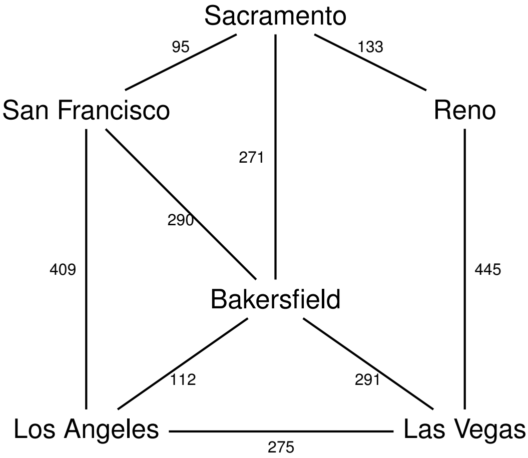
Breadth First Search:

d(s) = 0
visited[s] = **true**
put S in Q
While $u = Q$:
  foreach (u,v):
    if (visited[v] == **false**):
      visited[v] = **true**
      d[v] = d[u]+1
      put v in Q

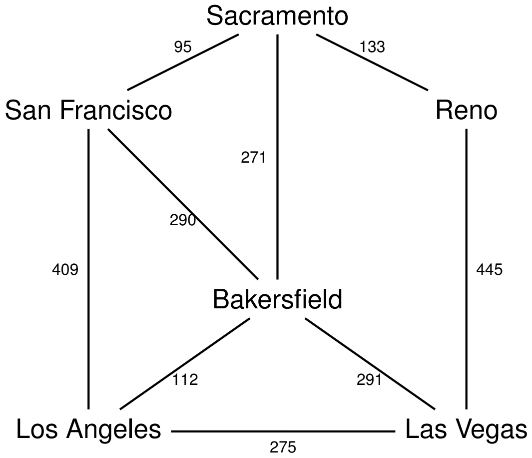
Queue:



Edge lengths: BFS?

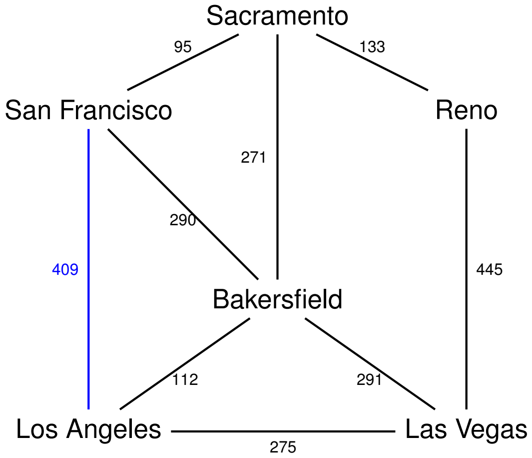


Edge lengths: BFS?



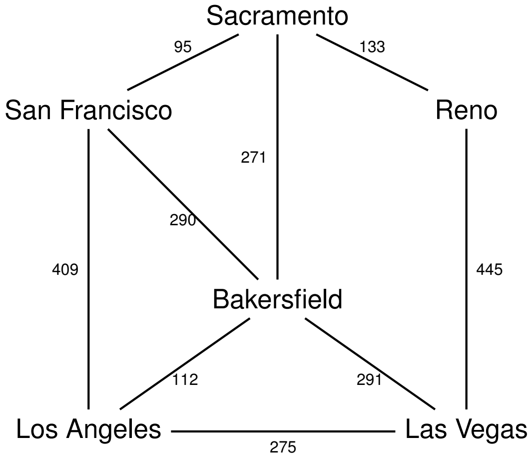
S.F. to L. A.:

Edge lengths: BFS?



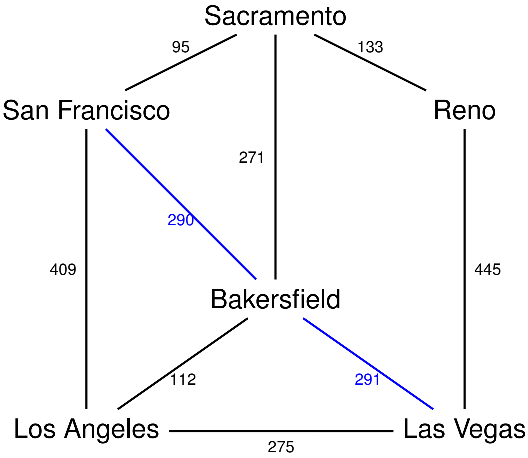
S.F. to L. A.: One hop.

Edge lengths: BFS?



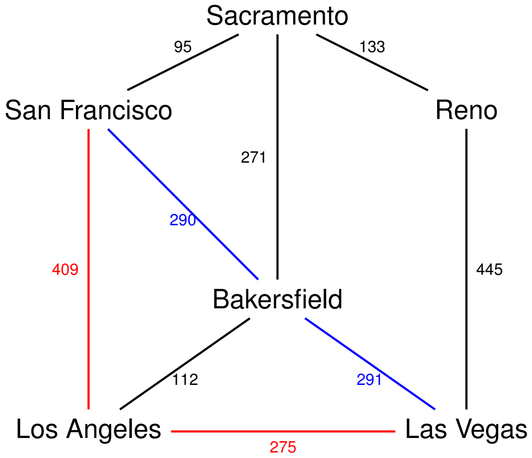
S.F. to L. A.: One hop.
S.F. to Vegas:

Edge lengths: BFS?



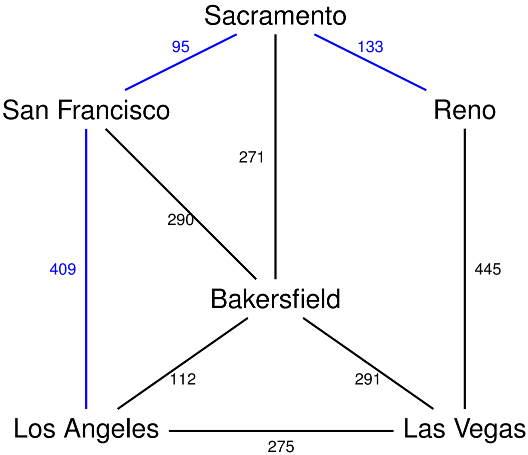
S.F. to L. A.: One hop.
S.F. to Vegas: Two hops

Edge lengths: BFS?



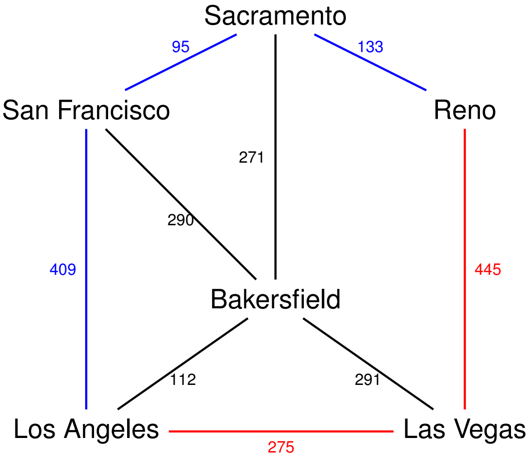
S.F. to L. A.: One hop.
S.F. to Vegas: Two hops (which two).

Edge lengths: BFS?



S.F. to L. A.: One hop.
S.F. to Vegas: Two hops (which two).
Reno to L.A.: Three hops.

Edge lengths: BFS?



S.F. to L. A.: One hop.
S.F. to Vegas: Two hops (which two).
Reno to L.A.: Three hops. not two!

BFS with edge lengths?

Graph: $G = (V, E)$.

BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

Find shortest paths from s.

BFS with edge lengths?

Graph: $G = (V, E)$.
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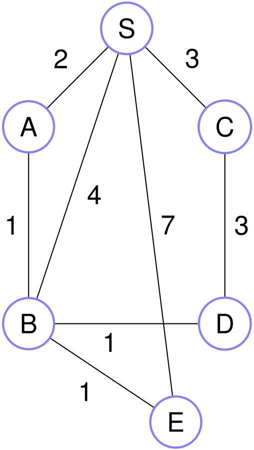
Find shortest paths from s.

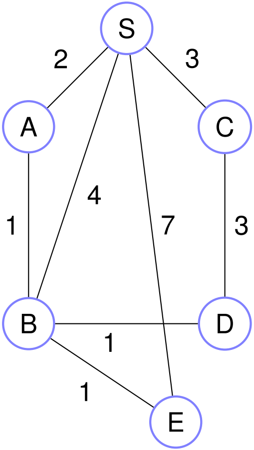


Make G' from G.

BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

Find shortest paths from s.

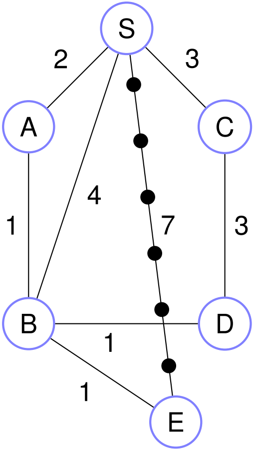


Make G' from G.
 For each edge e
  Replace w/len l_e path.

BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

Find shortest paths from s.



Make G' from G.
For each edge e
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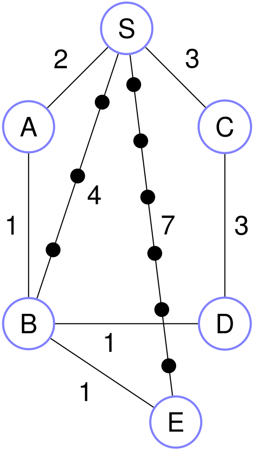
BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

Find shortest paths from s.

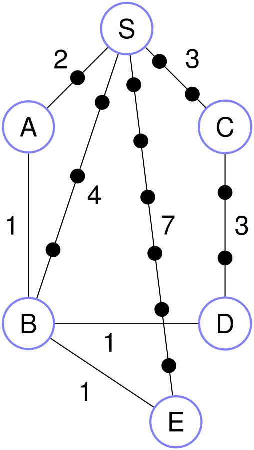


Make G' from G.
For each edge e
Replace w/len l_e path.

BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

Find shortest paths from s.

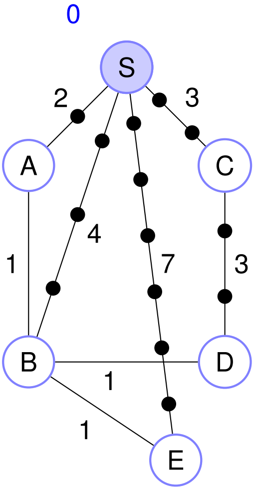


Make G' from G.
For each edge e
Replace w/len l_e path.

BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

Find shortest paths from s.



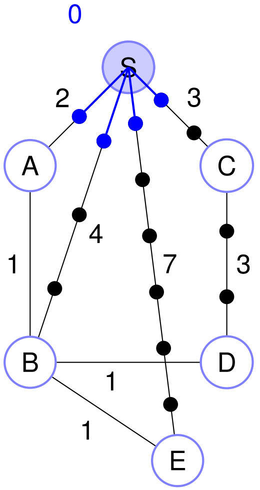
Make G' from G.
For each edge e
Replace w/len l_e path.

Run BFS on G'.

BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

Find shortest paths from s.



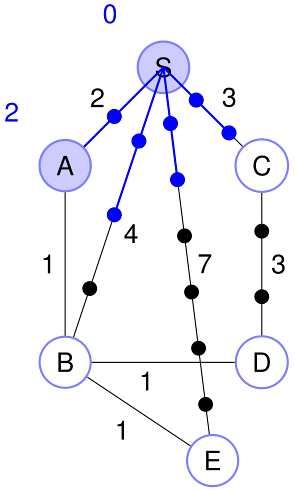
Make G' from G.
For each edge e
Replace w/len l_e path.

Run BFS on G'.

BFS with edge lengths?

Graph: $G = (V, E)$.
Length of e: l_e.

Find shortest paths from s.



Make G' from G.
For each edge e
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Run BFS on G'.

BFS with edge lengths?

Graph: $G = (V, E)$.
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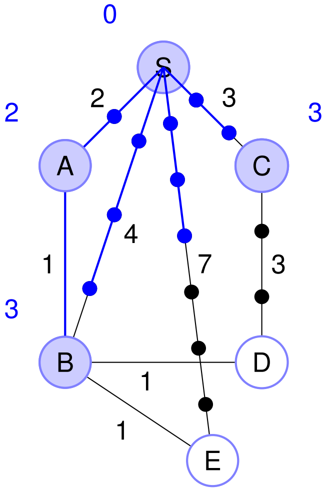
Find shortest paths from s.



Make G' from G.
For each edge e
Replace w/len l_e path.

Run BFS on G'.

BFS with edge lengths?

Graph: $G = (V, E)$.
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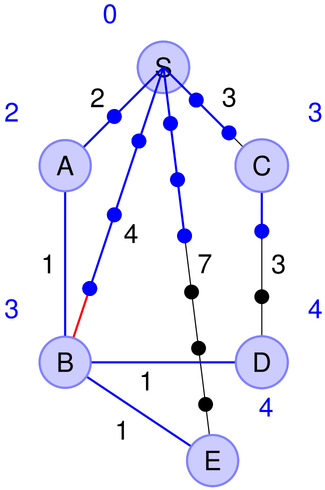
Find shortest paths from s.



Make G' from G.
For each edge e
Replace w/len l_e path.

Run BFS on G'.

BFS in exploded graph.

Correct: follows from the correctness of BFS.

BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time?

BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

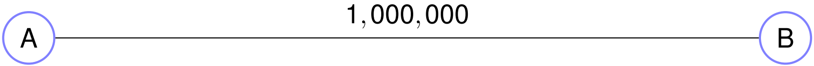
BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

Compared to size of problem.



Size of representation: 6 digits plus one edge.

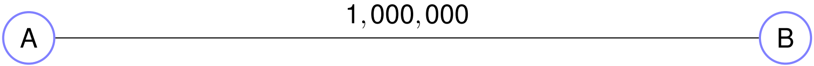
BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

Compared to size of problem.



Size of representation: 6 digits plus one edge. 10 ish.

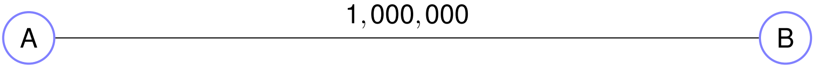
BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

Compared to size of problem.



Size of representation: 6 digits plus one edge. 10 ish.

Time: 1,000,000.

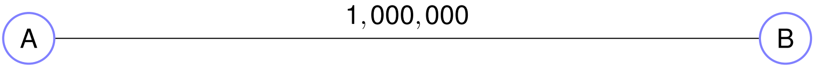
BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

Compared to size of problem.



Size of representation: 6 digits plus one edge. 10 ish.

Time: 1,000,000. ...ish.

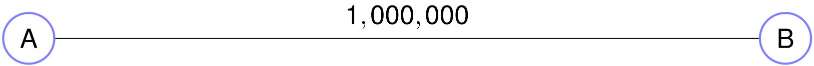
BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

Compared to size of problem.



Size of representation: 6 digits plus one edge. 10 ish.

Time: 1,000,000. ...ish.

Hmmm...

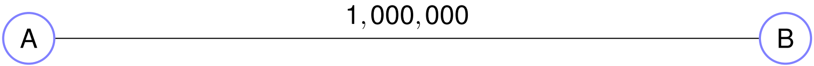
BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

Compared to size of problem.



Size of representation: 6 digits plus one edge. 10 ish.

Time: 1,000,000. ...ish.

Hmmm...

The distance is

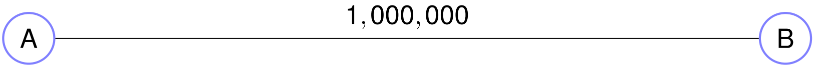
BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

Compared to size of problem.



Size of representation: 6 digits plus one edge. 10 ish.

Time: 1,000,000. ...ish.

Hmmm...

The distance is ..obviously 1,000,000.

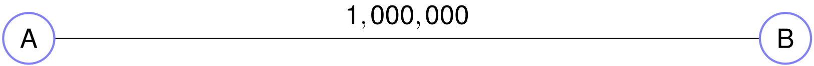
BFS in exploded graph.

Correct: follows from the correctness of BFS.

Time? $O(\sum_e l_e)$.

Very very large.

Compared to size of problem.



Size of representation: 6 digits plus one edge. 10 ish.

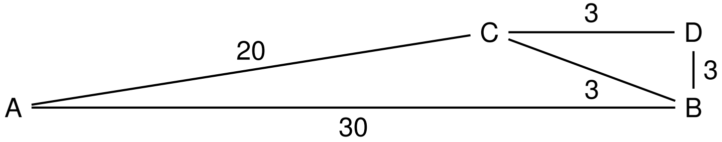
Time: 1,000,000. ...ish.

Hmmm...

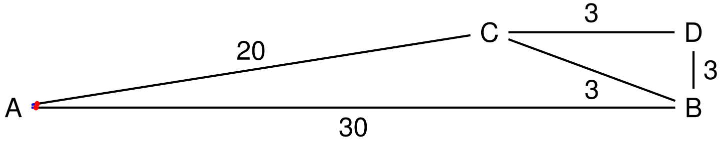
The distance is ..obviously 1,000,000.

Could it be easier?

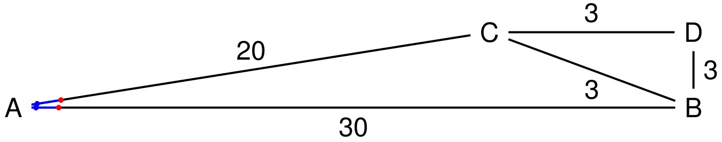
Looking again.



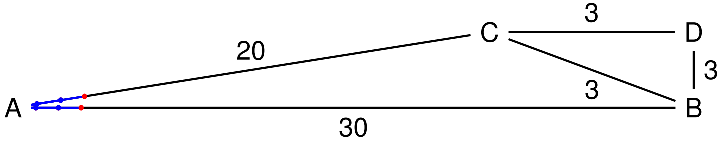
Looking again.



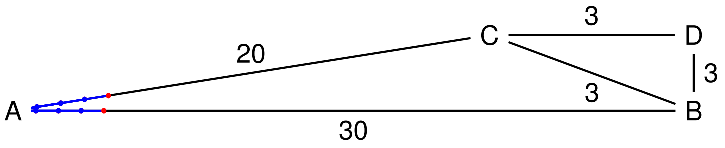
Looking again.



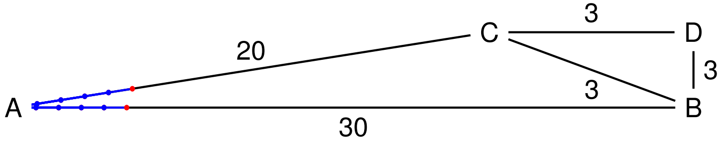
Looking again.



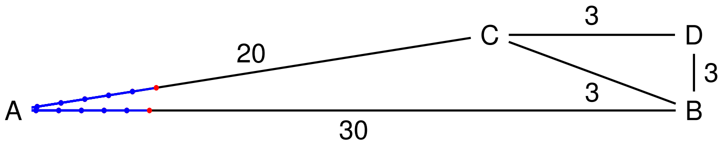
Looking again.



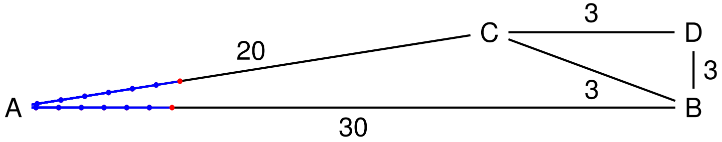
Looking again.



Looking again.

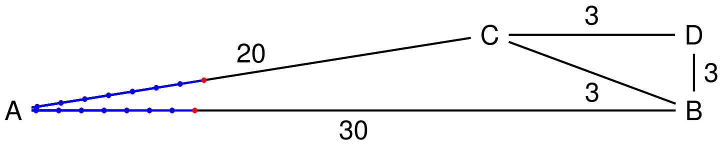


Looking again.



Queue next node on long edges again and again...

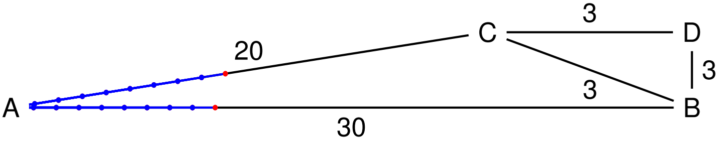
Looking again.



Queue next node on long edges again and again...

Nothing interesting until 20 steps.

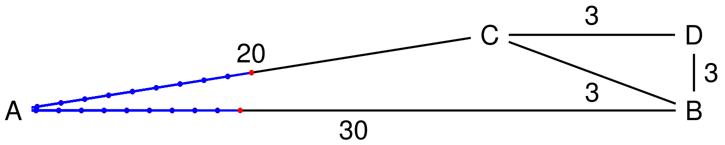
Looking again.



Queue next node on long edges again and again...

Nothing interesting until 20 steps.

Looking again.

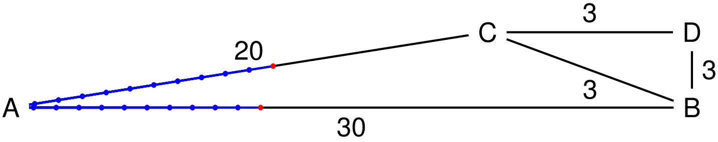


Queue next node on long edges again and again...

Nothing interesting until 20 steps.

Wake me then!

Looking again.

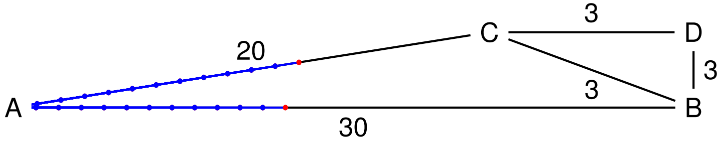


Queue next node on long edges again and again...

Nothing interesting until 20 steps.

Wake me then!

Looking again.

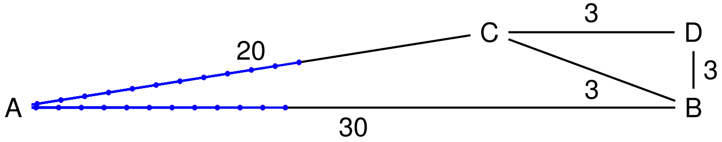


Queue next node on long edges again and again...

Nothing interesting until 20 steps.

Wake me then!

Looking again.



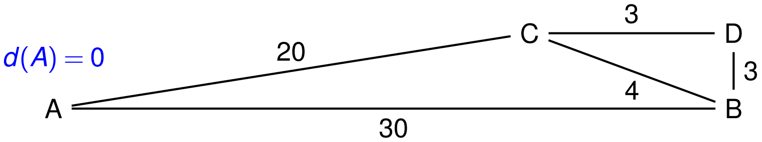
Queue next node on long edges again and again...

Nothing interesting until 20 steps.

Wake me then!

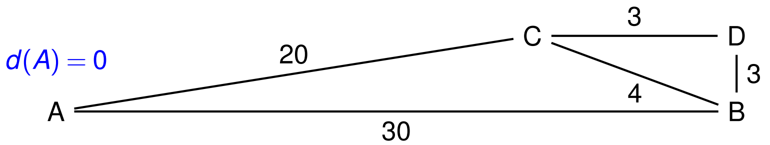
Alarms.

Alarms.



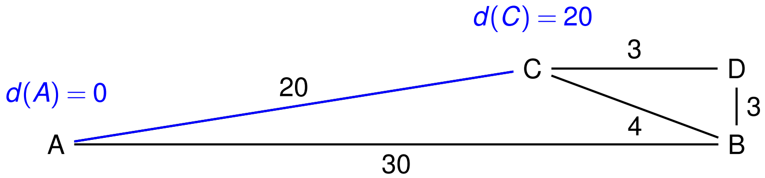
Process A: $d(A) = 0$

Alarms.



Process A: $d(A) = 0$
For Edge (A, C):

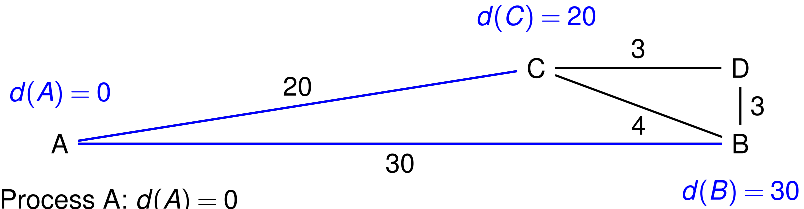
Alarms.



Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

Alarms.

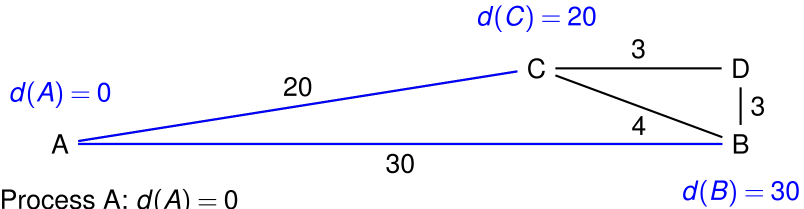


Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B):

Alarms.

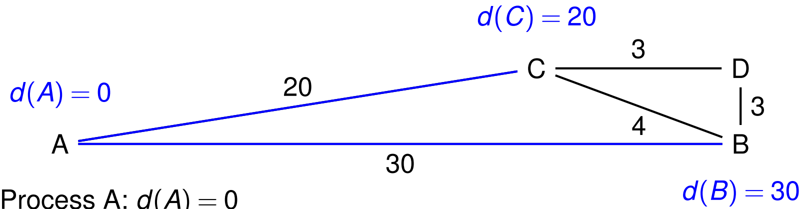


Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

Alarms.



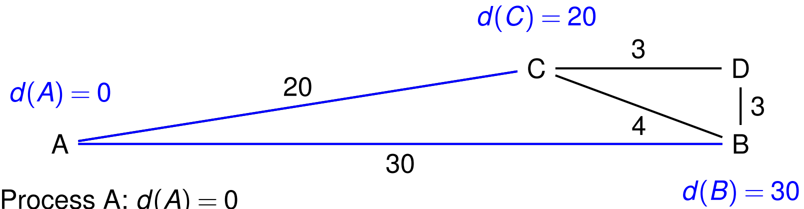
Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:

Alarms.



Process A: $d(A) = 0$

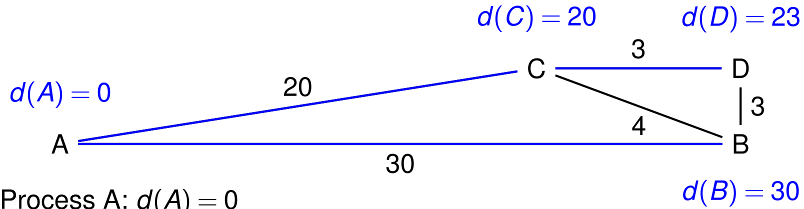
For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:

Process C.

Alarms.



Process A: $d(A) = 0$

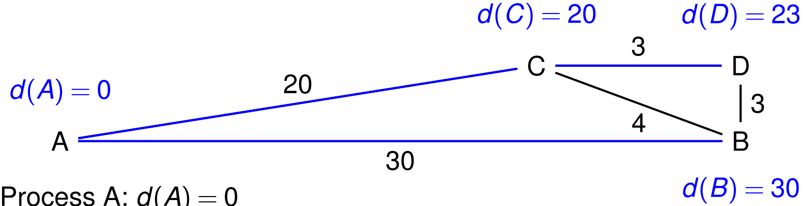
For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:

Process C.

Alarms.



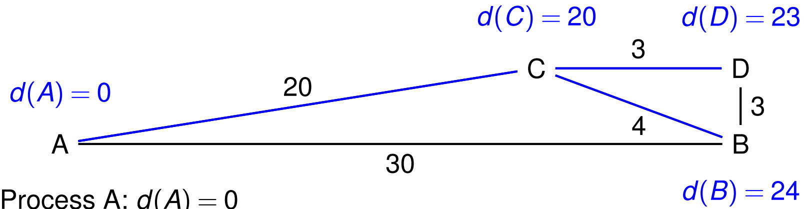
Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.
For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:
Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.

Alarms.



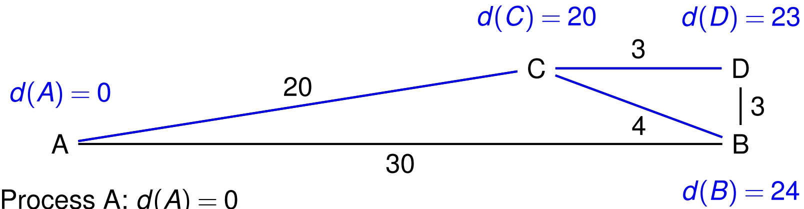
Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.
For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:
Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.
For Edge (C, B):

Alarms.



Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

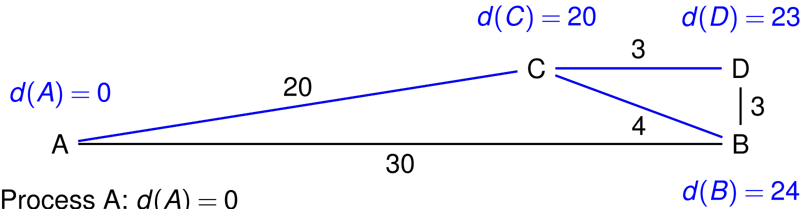
At time 20:

Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.

For Edge (C, B): Reset $d(B) = 24$ and alarm for 24.

Alarms.



Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:

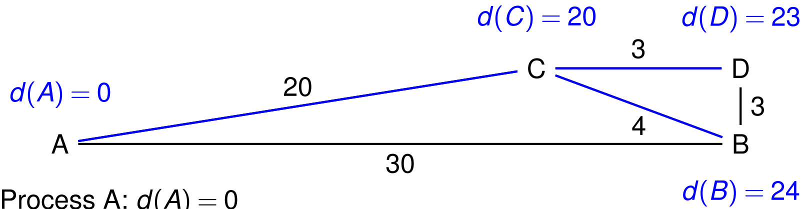
Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.

For Edge (C, B): Reset $d(B) = 24$ and alarm for 24.

At time 23:

Alarms.



Process A: $d(A) = 0$

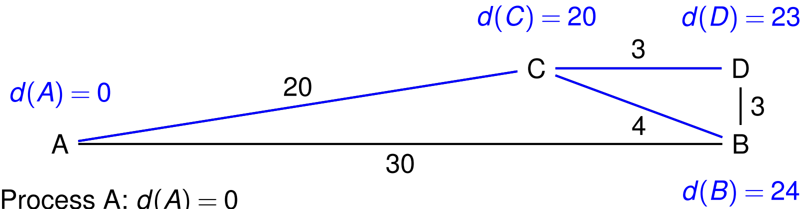
For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.
For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:
Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.
For Edge (C, B): Reset $d(B) = 24$ and alarm for 24.

At time 23:
Process D.

Alarms.



Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:

Process C.

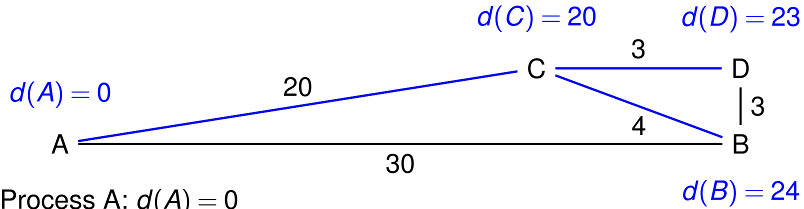
For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.

For Edge (C, B): Reset $d(B) = 24$ and alarm for 24.

At time 23:

Process D. Set $d(D) = 23$.

Alarms.



Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:

Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.

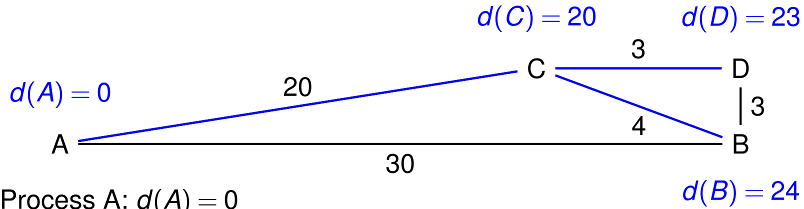
For Edge (C, B): Reset $d(B) = 24$ and alarm for 24.

At time 23:

Process D. Set $d(D) = 23$.

For Edge (D, B):

Alarms.



Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:

Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.

For Edge (C, B): Reset $d(B) = 24$ and alarm for 24.

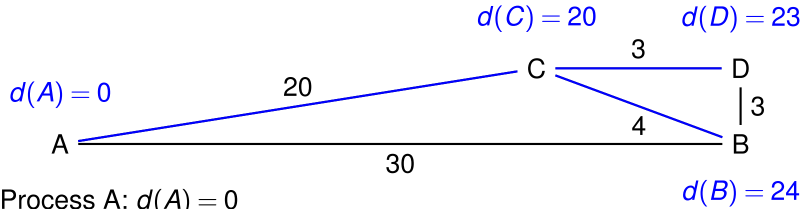
At time 23:

Process D. Set $d(D) = 23$.

For Edge (D, B):

$d(D) = 24$ which is less than $d(D) + 3 = 26$ so leave it.

Alarms.



Process A: $d(A) = 0$

For Edge (A, C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.
For Edge (A, B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:
Process C.

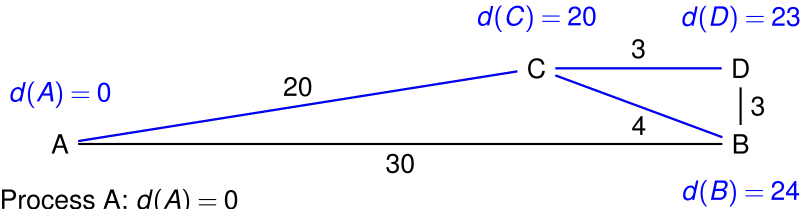
For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.
For Edge (C, B): Reset $d(B) = 24$ and alarm for 24.

At time 23:
Process D. Set $d(D) = 23$.

For Edge (D, B):
$d(D) = 24$ which is less than $d(D) + 3 = 26$ so leave it.

At time 24: Process B.

Alarms.



Process A: $d(A) = 0$

For Edge (A,C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.

For Edge (A,B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:

Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.

For Edge (C,B): Reset $d(B) = 24$ and alarm for 24.

At time 23:

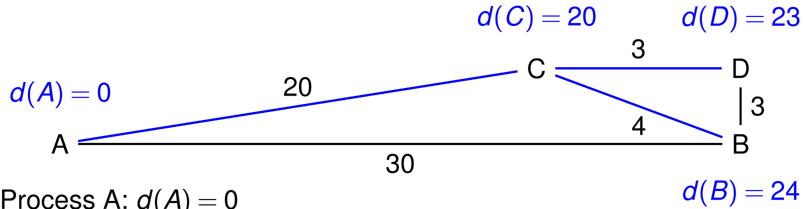
Process D. Set $d(D) = 23$.

For Edge (D,B):

$d(D) = 24$ which is less than $d(D) + 3 = 26$ so leave it.

At time 24: Process B.

Alarms.



Process A: $d(A) = 0$

For Edge (A,C): Set $d(C) = d(A) + 20 = 20$ and alarm for 20.
For Edge (A,B): Set $d(B) = d(A) + 30 = 30$ and alarm for 30.

At time 20:
Process C.

For Edge (C,D): Set $d(D) = d(C) + 3 = 23$ and alarm for 23.
For Edge (C,B): Reset $d(B) = 24$ and alarm for 24.

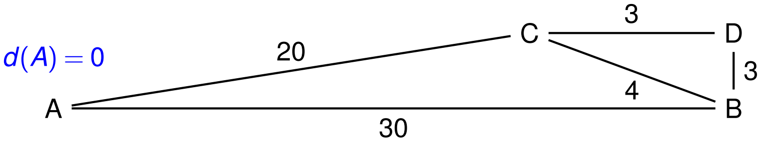
At time 23:
Process D. Set $d(D) = 23$.

For Edge (D,B):
$d(D) = 24$ which is less than $d(D) + 3 = 26$ so leave it.

At time 24: Process B. Done.

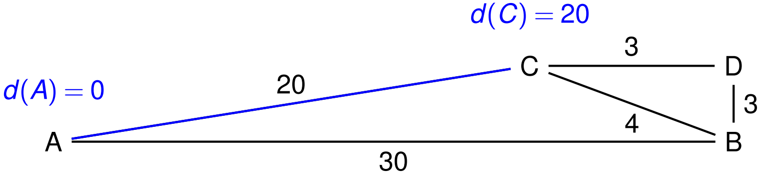
Hmmm...

..what needed to be done?



Hmmm...

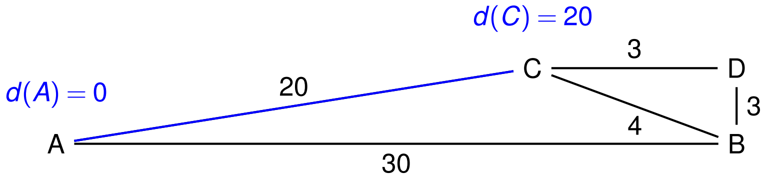
..what needed to be done?



Set a distance.

Hmmm...

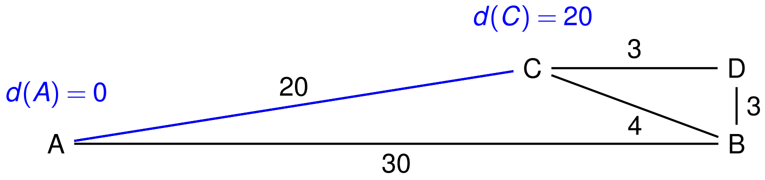
..what needed to be done?



Set a distance. Set an alarm.

Hmmm...

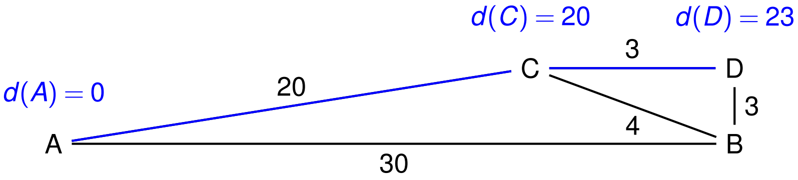
..what needed to be done?



Set a distance. Set an alarm.
Find next alarm.

Hmmm...

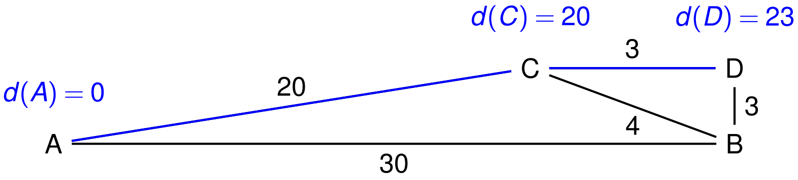
..what needed to be done?



Set a distance. Set an alarm.
Find next alarm. Set another distance.

Hmmm...

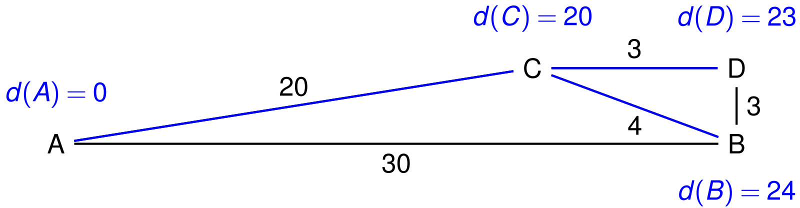
..what needed to be done?



Set a distance. Set an alarm.
Find next alarm. Set another distance. And set alarm.

Hmmm...

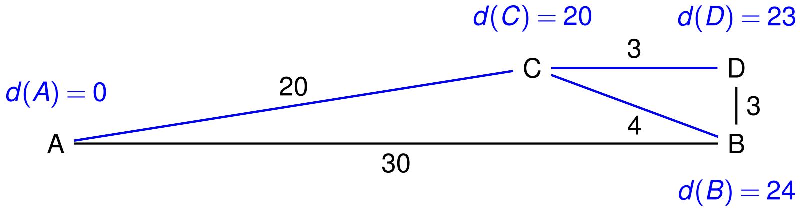
..what needed to be done?



Set a distance. Set an alarm.
Find next alarm. Set another distance. And set alarm. Reset distance.

Hmmm...

..what needed to be done?



Set a distance. Set an alarm.
Find next alarm. Set another distance. And set alarm. Reset distance. Reset Alarm.

Alarm Algorithm.

Set an alarm clock for node s at time 0.

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:
– The distance from s to u is T.

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:
  - If no alarm for v, set alarm for $T + l(u, v)$.

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node *u*. Then:

- The distance from *s* to *u* is T.
- For each neighbor *v* of *u* in G:
  - If no alarm for *v*, set alarm for $T + l(u, v)$.
  - If *v*'s alarm is $\geq T + l(u, v)$

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:
  * If no alarm for v, set alarm for $T + l(u, v)$.
  * If v's alarm is $\geq T + l(u, v)$
    then reset it $T + l(u, v)$.

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:
  * If no alarm for v, set alarm for $T + l(u, v)$.
  * If v's alarm is $\geq T + l(u, v)$
    then reset it $T + l(u, v)$.

Implementation:

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:
  - If no alarm for v, set alarm for $T + l(u, v)$.
  - If v's alarm is $\geq T + l(u, v)$ then reset it $T + l(u, v)$.

Implementation:
Need to maintain alarm for each node.

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:
  * If no alarm for v, set alarm for $T + l(u, v)$.
  * If v's alarm is $\geq T + l(u, v)$
    then reset it $T + l(u, v)$.

Implementation:
Need to maintain alarm for each node.
Possibly need to decrease alarm for a node.
Find next alarm time.

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:
  - If no alarm for v, set alarm for $T + l(u, v)$.
  - If v's alarm is $\geq T + l(u, v)$
    then reset it $T + l(u, v)$.

Implementation:
Need to maintain alarm for each node.
Possibly need to decrease alarm for a node.
Find next alarm time.

Insert: (v, key)

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:
  * If no alarm for v, set alarm for $T + l(u, v)$.
  * If v's alarm is $\geq T + l(u, v)$ then reset it $T + l(u, v)$.

Implementation:
Need to maintain alarm for each node.
Possibly need to decrease alarm for a node.
Find next alarm time.

Insert: (v, key)
DecreaseKey: $(v, newkey)$

Alarm Algorithm.

Set an alarm clock for node s at time 0.
Repeat until there are no more alarms:
Next alarm goes off at time T, for node u. Then:

- The distance from s to u is T.
- For each neighbor v of u in G:
  - If no alarm for v, set alarm for $T + l(u, v)$.
  - If v's alarm is $\geq T + l(u, v)$
    then reset it $T + l(u, v)$.

Implementation:
Need to maintain alarm for each node.
Possibly need to decrease alarm for a node.
Find next alarm time.

Insert: (v, key)
DecreaseKey: $(v, newkey)$
DeleteMin: Q **returns** v with min. key

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.
$d(s) = 0$.

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.
$d(s) = 0$.
Q.Insert(s,0)

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.
$d(s) = 0$.
Q.Insert(s,0)
While u = Q.DeleteMin():
  foreach edge (u, v):

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.
$d(s) = 0$.
Q.Insert(s,0)
While u = Q.DeleteMin():
   foreach edge (u,v):
      if $d(v) > d(u) + l(u,v)$:
         $d(v) = d(u) + l(u,v)$
         Q.InsertOrDecreaseKey(v,d(v))

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.
$d(s) = 0$.
Q.Insert(s,0)
While u = Q.DeleteMin():
  foreach edge (u, v):
    if $d(v) > d(u) + l(u, v)$:
      $d(v) = d(u) + l(u, v)$
      Q.InsertOrDecreaseKey(v,d(v))

Runtime:

Dijkstra's Algorithm.

foreach v: $d(v) = \infty$.
$d(s) = 0$.
Q.Insert(s,0)
While u = Q.DeleteMin():
  foreach edge (u, v):
    if $d(v) > d(u) + l(u, v)$:
      $d(v) = d(u) + l(u, v)$
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$|V|$ DeleteMins.

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Binary heap: $O((|V| + |E|) \log |V|)$

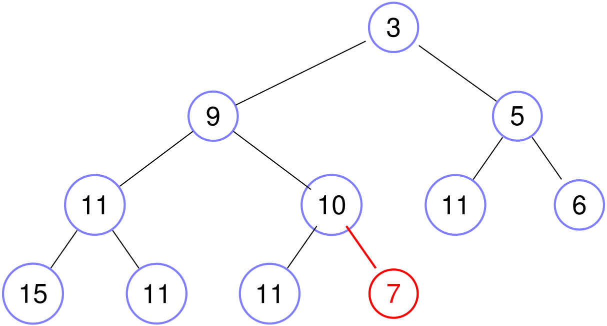
Binary Heap.

Heap[1]: bigger children.

[1] values only

Binary Heap.

Heap[1]: bigger children.
$\Longrightarrow$ smallest at root.



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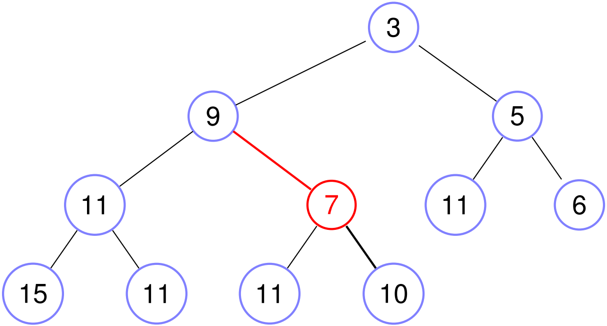

Insert(7):

[1]values only

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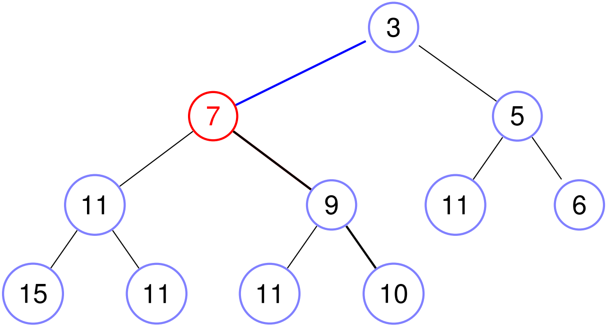

Insert(7): Bubble up: check parent.

[1] values only

Binary Heap.

Heap[1]: bigger children.
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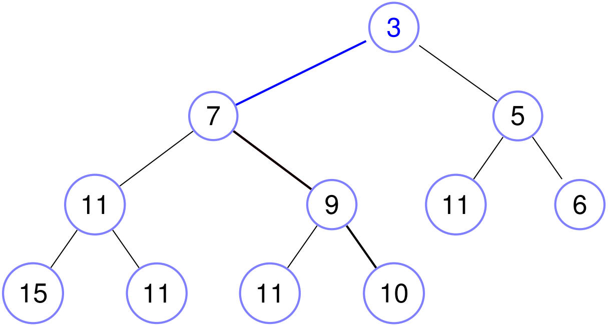

Insert(7): Bubble up: check parent. . **depth** comp.

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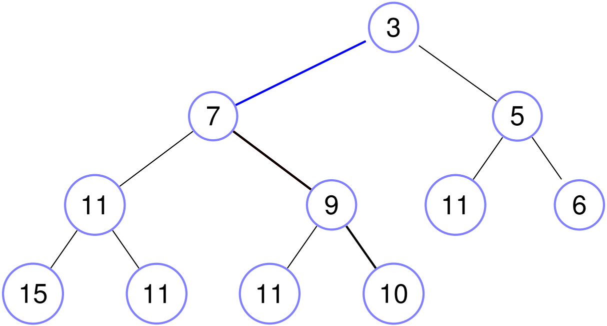

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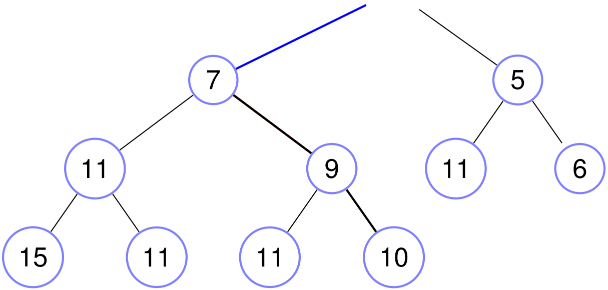

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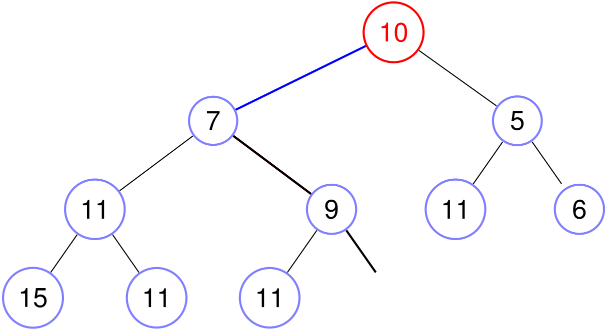

Insert(7): Bubble up: check parent. . **depth** comp.
DeleteMin: Replace.

[1] values only

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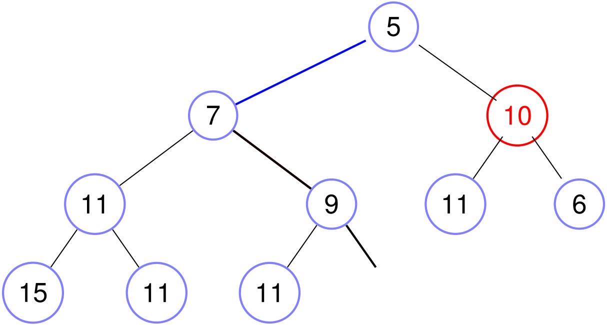

Insert(7): Bubble up: check parent. . **depth** comp.
DeleteMin: Replace. Bubble down: check **both** children..

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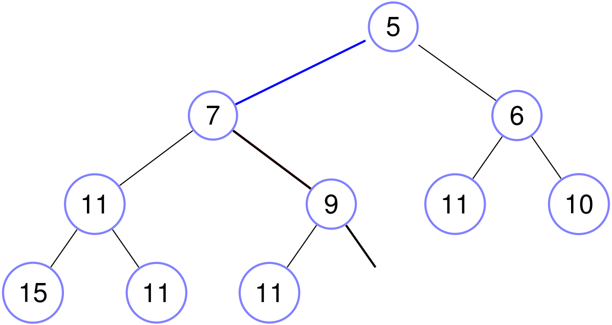

Insert(7): Bubble up: check parent. . **depth** comp.
DeleteMin: Replace. Bubble down: check **both** children..
2$\times$ **depth** – comparisons.

[1]values only

d-ary heap

Degree – d, Depth – $\log_d n$.

d-ary heap

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Insert/DecreaseKey – $\log n / \log d$.

d-ary heap

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Insert/DecreaseKey – $\log n / \log d$.
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d-ary heap

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Dijkstra:

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

Dijkstra:
$O(|V|)$ deletemins. $O(d \log n / \log d)$ each.

d-ary heap

Degree – d, Depth – $\log_d n$.
Insert/DecreaseKey – $\log n / \log d$.
DeleteMin – $d \log n / \log d$. (Check all children.)

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$O(|V| d \log n / \log d + |E| \log n / \log d)$.

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Optimal Choice:

d-ary heap

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For dense graphs it approaches linear.

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Fibonacci Heaps:

d-ary heap

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Fibonacci Heaps:
$O(\log n)$ per delete.

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Fibonacci Heaps:
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Fibonacci Heaps:
$O(\log n)$ per delete.
$O(1)$ average decrease-key.

$O(|V| \log |V| + |E|)$.
Linear for moderately dense graphs!

Lecture in a minute.

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Breadth First Search of graph:
  Search with queue instead of stack.
  Get “distances” from source.
    Proof idea: queue has distance 0, then level 1, ...

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  Replace weights by paths + BFS
  Implement using priority queue.
    Idea: ignores "new" nodes.
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