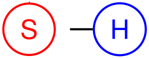
Today:Quantum.

Today:Quantum.

Today:Quantum.

S

Today:Quantum.



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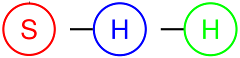

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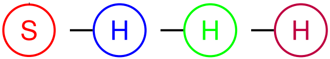

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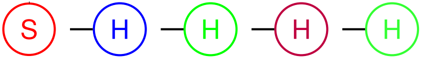

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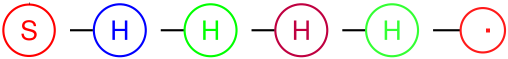

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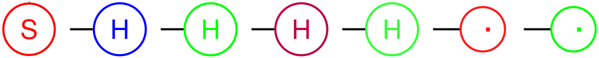

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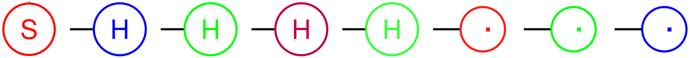

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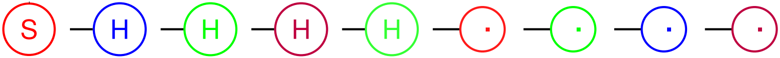

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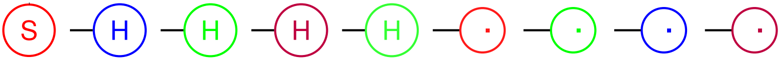

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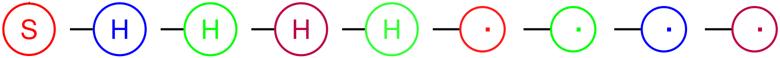

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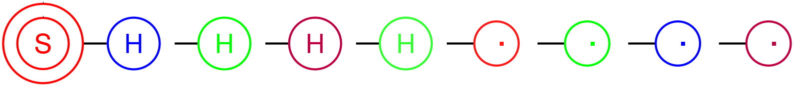

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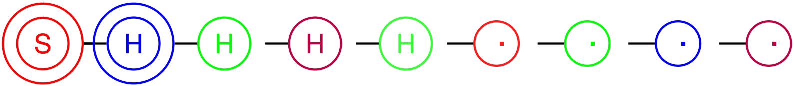

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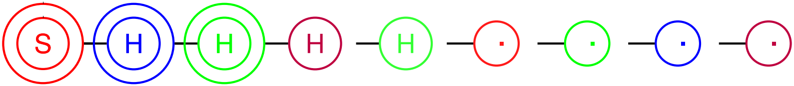

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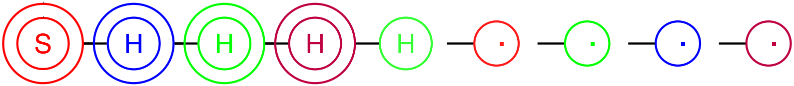

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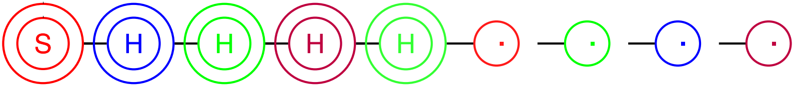

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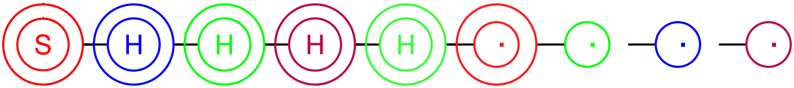

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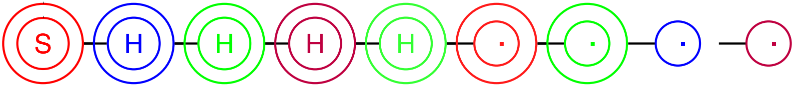

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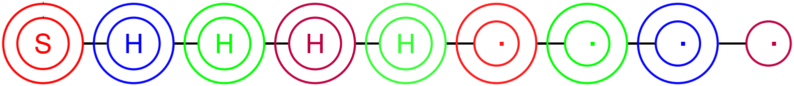

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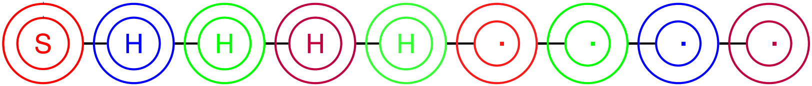
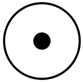


Qubit/electron.

-

Qubit/electron.

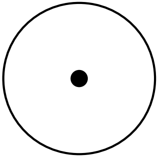
ground state



$|0\rangle$

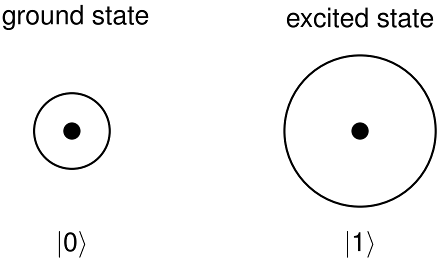
Qubit/electron.

excited state



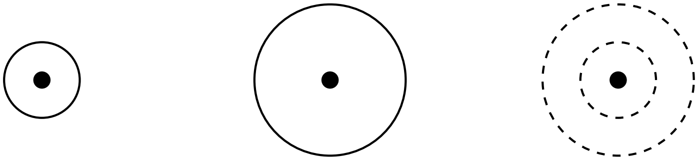
$|1\rangle$

Qubit/electron.



Qubit/electron.

ground state | excited state | Superposition

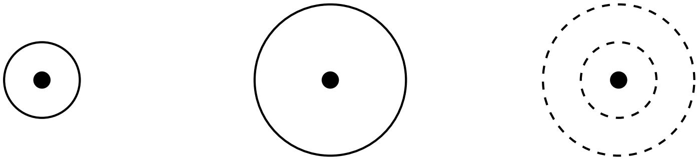


$|0\rangle$ | $|1\rangle$ | $\alpha_0 |0\rangle + \alpha_1 |1\rangle$

Complex numbers α_0 and α_1.

Qubit/electron.

ground state | excited state | Superposition

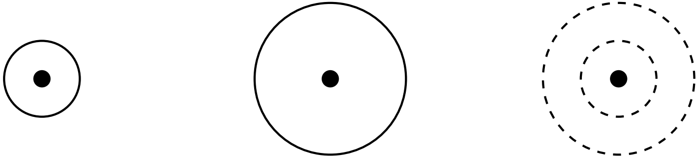


$|0\rangle$ | $|1\rangle$ | $\alpha_0 |0\rangle + \alpha_1 |1\rangle$

Complex numbers α_0 and α_1.
$|\alpha_0|^2 + |\alpha_1|^2 = 1$.

Qubit/electron.

ground state | excited state | Superposition



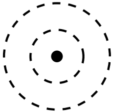
$|0\rangle$ | $|1\rangle$ | $\alpha_0 |0\rangle + \alpha_1 |1\rangle$

Complex numbers α_0 and α_1.

$|\alpha_0|^2 + |\alpha_1|^2 = 1$.

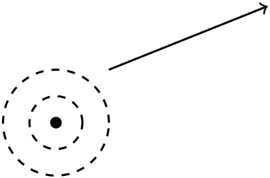
α_0, α_1 are “amplitudes.”

Measurement.



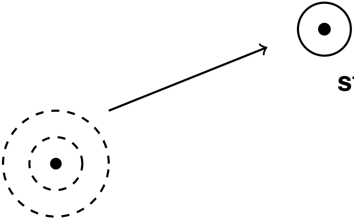
$\alpha_0 |0\rangle + \alpha_1 |1\rangle$

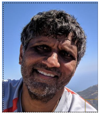
Measurement.



$\alpha_0 |0\rangle + \alpha_1 |1\rangle$

Measurement.

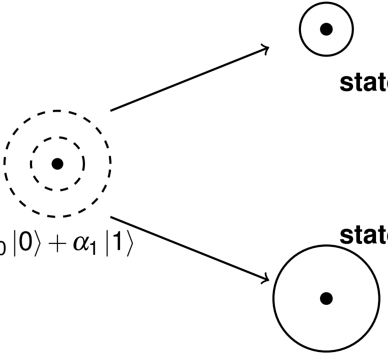


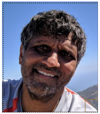


state $|0\rangle$ **with prob** $|\alpha_0|^2$

$\alpha_0 |0\rangle + \alpha_1 |1\rangle$

Measurement.



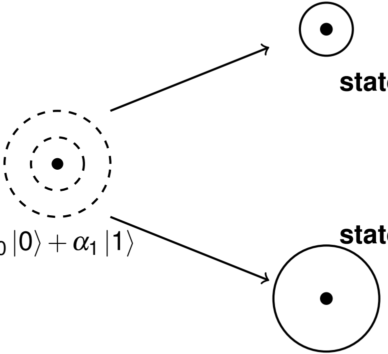


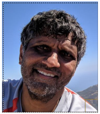
state $|0\rangle$ with prob $|\alpha_0|^2$

$\alpha_0 |0\rangle + \alpha_1 |1\rangle$

state $|1\rangle$ with prob $|\alpha_1|^2$

Measurement.





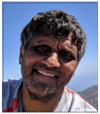
state $|0\rangle$ **with prob** $|\alpha_0|^2$

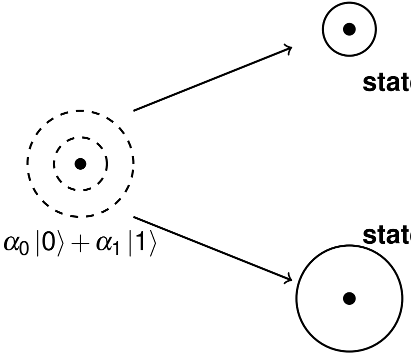
$\alpha_0 |0\rangle + \alpha_1 |1\rangle$

state $|1\rangle$ **with prob** $|\alpha_1|^2$

Remember $|\alpha_0|^2 + |\alpha_1|^2 = 1$.

Measurement.





state $|0\rangle$ **with prob** $|\alpha_0|^2$

state $|1\rangle$ **with prob** $|\alpha_1|^2$

Remember $|\alpha_0|^2 + |\alpha_1|^2 = 1$.

Amplitudes $\rightarrow$ probabilities on measurement!!!

Two qubits..a dollar.

One bit:

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Quantum State:

Two qubits..a dollar.

One bit:
Classic State: 0 or 1.
Quantum State:
Internal:
$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

Two qubits..a dollar.

One bit:
Classic State: 0 or 1.
Quantum State:
Internal:
$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.
Measure : 0 or 1.

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

Measure : 0 or 1.

Two numbers internally,

Two qubits..a dollar.

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Two bits:

Two qubits..a dollar.

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Classic State: 0 or 1.
Quantum State:
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$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.
Measure : 0 or 1.
Two numbers internally, measurement yields one bit.

Two bits:
Classical State: 00, 01, 10, 11.

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Quantum State:

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$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

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Two numbers internally, measurement yields one bit.

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Two qubits..a dollar.

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Quantum State:

Internal:

Two qubits..a dollar.

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Quantum State:
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$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$

Two qubits..a dollar.

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Classic State: 0 or 1.
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Two bits:
Classical State: 00, 01, 10, 11.
Quantum State:
Internal:

$$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$$

$$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$$

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

Measure : 0 or 1.

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Measure : 0 or 1.
Two numbers internally, measurement yields one bit.

Two bits:
Classical State: 00, 01, 10, 11.
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Internal:
$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$
$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$
Measure : 00, 01, 10, 11.
4 internal numbers,

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

Measure : 0 or 1.

Two numbers internally, measurement yields one bit.

Two bits:

Classical State: 00, 01, 10, 11.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$

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Measure : 00, 01, 10, 11.

4 internal numbers, measurement yields two bits.

Two qubits..a dollar.

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Classic State: 0 or 1.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

Measure : 0 or 1.

Two numbers internally, measurement yields one bit.

Ooh!

Two bits:

Classical State: 00, 01, 10, 11.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$

$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$

Measure : 00, 01, 10, 11.

4 internal numbers, measurement yields two bits.

Two qubits..a dollar.

One bit:
Classic State: 0 or 1.
Quantum State:
Internal:
$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.
Measure : 0 or 1.
Two numbers internally, measurement yields one bit.

Ooh! Something new,

Two bits:
Classical State: 00, 01, 10, 11.
Quantum State:
Internal:
$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$
$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$
Measure : 00, 01, 10, 11.
4 internal numbers, measurement yields two bits.

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$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$
$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$
Measure : 00, 01, 10, 11.
4 internal numbers, measurement yields two bits.

Ooh! Something new, with two.

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

Measure : 0 or 1.

Two numbers internally, measurement yields one bit.

Two bits:

Classical State: 00, 01, 10, 11.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$

$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$

Measure : 00, 01, 10, 11.

4 internal numbers, measurement yields two bits.

Ooh! Something new, with two.

Partial Measure: look at one bit.

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

Measure : 0 or 1.

Two numbers internally, measurement yields one bit.

Two bits:

Classical State: 00, 01, 10, 11.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$

$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$

Measure : 00, 01, 10, 11.

4 internal numbers, measurement yields two bits.

Ooh! Something new, with two.

Partial Measure: look at one bit.

Result: 0

Two qubits..a dollar.

One bit:
Classic State: 0 or 1.
Quantum State:
Internal:
$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.
Measure : 0 or 1.
Two numbers internally, measurement yields one bit.

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Internal:
$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$
$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$
Measure : 00, 01, 10, 11.
4 internal numbers, measurement yields two bits.

Ooh! Something new, with two.

Partial Measure: look at one bit.
Result: 0 (with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2$.)

Two qubits..a dollar.

One bit:
Classic State: 0 or 1.
Quantum State:
Internal:
$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.
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$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$
Measure : 00, 01, 10, 11.
4 internal numbers, measurement yields two bits.

Ooh! Something new, with two.

Partial Measure: look at one bit.
Result: 0 (with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2$.)

What is the state of the system if result is 0?

Two qubits..a dollar.

One bit:

Classic State: 0 or 1.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.

Measure : 0 or 1.

Two numbers internally, measurement yields one bit.

Two bits:

Classical State: 00, 01, 10, 11.

Quantum State:

Internal:

$|\alpha\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{01} |10\rangle + \alpha_{11} |11\rangle$

$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$

Measure : 00, 01, 10, 11.

4 internal numbers, measurement yields two bits.

Ooh! Something new, with two.

Partial Measure: look at one bit.

Result: 0 (with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2$.)

What is the state of the system if result is 0?

New Internal state: $\frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}$

Two qubits..a dollar.

One bit:
Classic State: 0 or 1.
Quantum State:
Internal:
$|\alpha\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle$.
Measure : 0 or 1.
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$|\alpha_{00}|^2 + \cdots + |\alpha_{11}|^2 = 1$
Measure : 00, 01, 10, 11.
4 internal numbers, measurement yields two bits.

Ooh! Something new, with two.

Partial Measure: look at one bit.
Result: 0 (with probability $|\alpha_{00}|^2 + |\alpha_{01}|^2$.)

What is the state of the system if result is 0?

New Internal state: $\frac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}$

Scaling to make probabilities add to 1.

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Can all two bit states be decomposed?

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More complicated actually: Bell-CHSH inequalities.

n-qubits.

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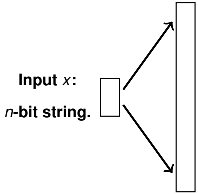
Quantum Computer

Start with n qubits,

Input x**:**

n**-bit string.**

Quantum Computer



Start with n qubits, make superposition,

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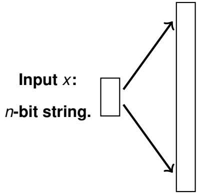

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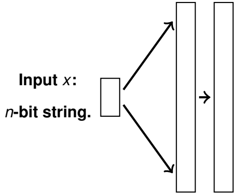

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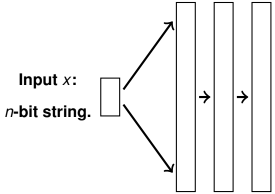

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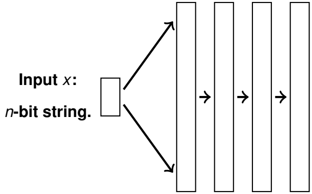

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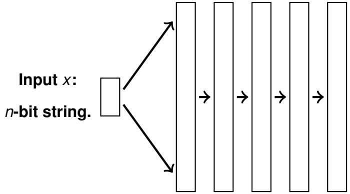

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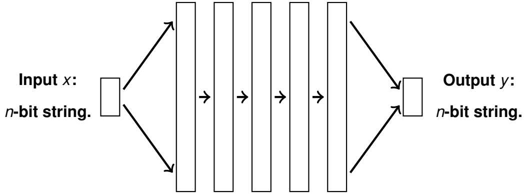

Start with n qubits,
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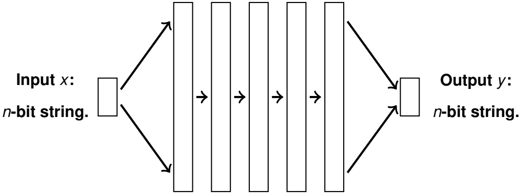

Exponential action

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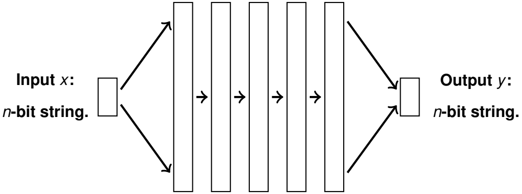

Start with n qubits,
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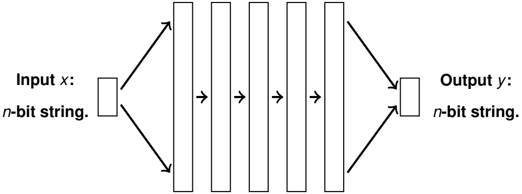
Exponential action $\rightarrow$ Factor in polynomial time!

Quantum Computer



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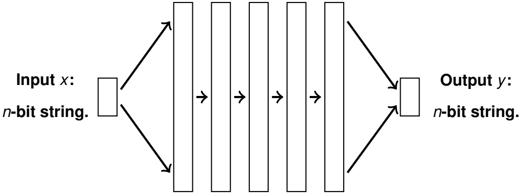

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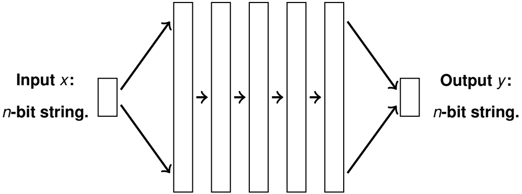

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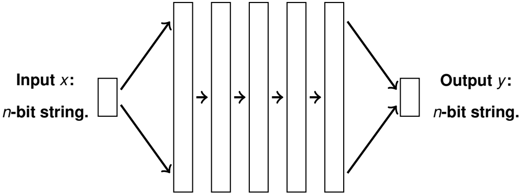

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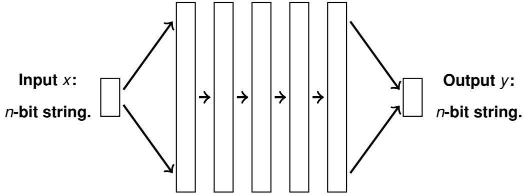

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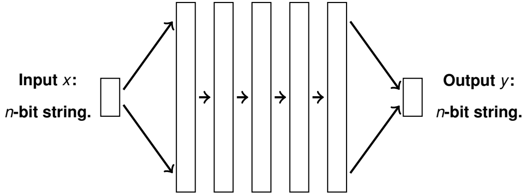

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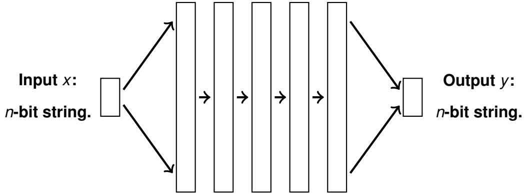

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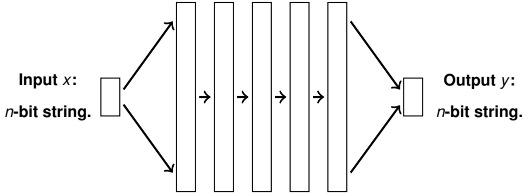

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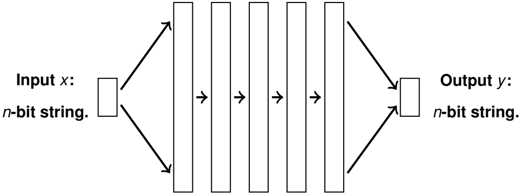

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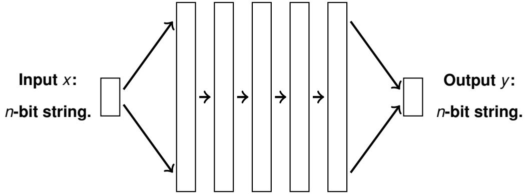

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Not clear how to do it for probability.

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Quantum Fourier Transform Circuit:

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Input: $\sum_{x \in \{0,1\}^n} \alpha_x |x\rangle$.

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Input: $\sum_{x\in\{0,1\}^n} \alpha_x |x\rangle$.
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Where β is Fourier Transform of α.

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Where β is Fourier Transform of α.
Note: n-qubit circuit, computing on 2^n amplitudes!
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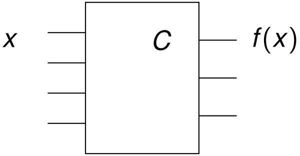
Random computations are fine with this; same α_x.

Classical/Quantum Circuit.



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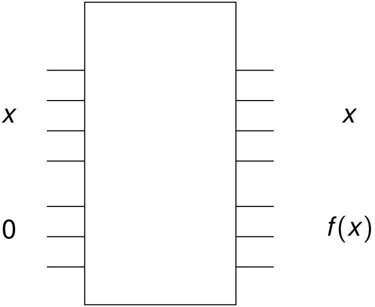

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FFT or multiply by $M(\omega_{2^n})$ finds "period" of periodic input.

Factoring and Roots of Unity

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Example: 15
$4^2 = 1 \pmod{15} \implies 4-1$ or $4+1$ are non-trivial factors of fifteen.

More generally: $x^2 = 1 \pmod{15} \implies x^2 - 1 = (x+1)(x-1) = 0 \pmod{15}$.

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Do several times:
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Result is multiple of $M/(r)$.

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Order of x is r. Check GCD$(N, x^{r/2}+1)$.

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Details: need period r to divide M.

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Need more sophisticated analysis...

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Order of x is r. Check GCD$(N, x^{r/2}+1)$.

Details: need period r to divide M. What is M? 2^n.
Need more sophisticated analysis...but same ideas.

Mini-Conclusion.

Quantum Fourier Transform $\Longrightarrow$ Factoring!

What's a gate look like?

Hadamard Gate.

$$|0\rangle \;\text{—}\boxed{H}\text{—}\!\!\longrightarrow \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \qquad |1\rangle \;\text{—}\boxed{H}\text{—}\!\!\longrightarrow \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

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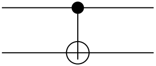
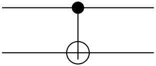
$|00\rangle$  $|00\rangle$ $|10\rangle$  $|11\rangle$

Note:
Operating on $\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$.

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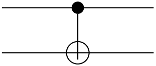
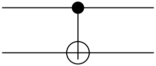
$|00\rangle$  $|00\rangle$ $\qquad$ $|10\rangle$  $|11\rangle$

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Split into odd and even inputs.

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Size: $S(n) = S(n-1) + O(n) = O(n^2)$.

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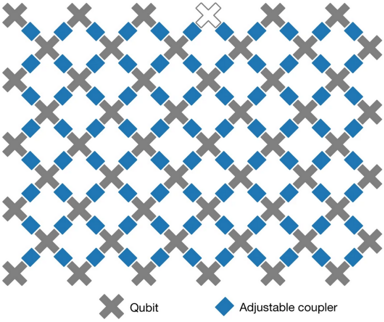

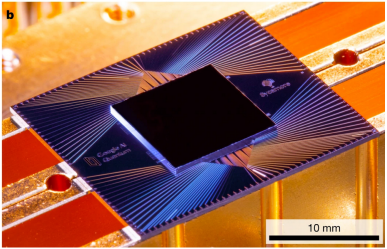

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