CS170: Algorithms

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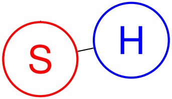
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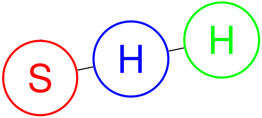

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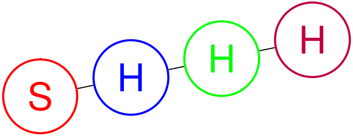

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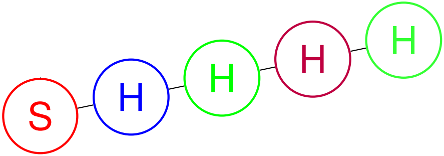

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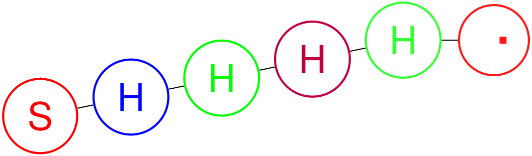

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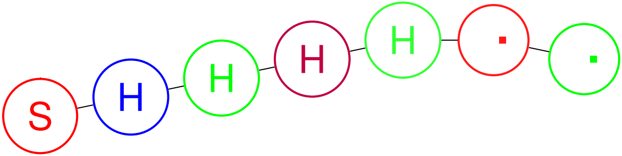

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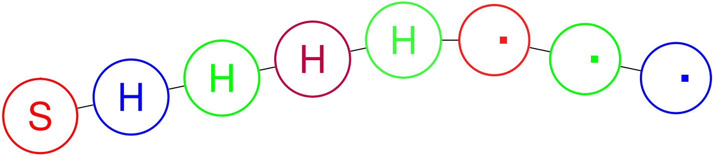

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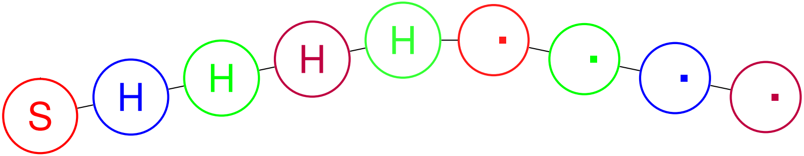

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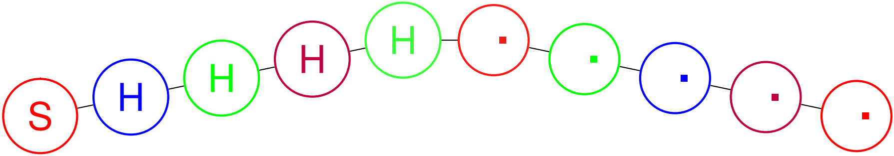

Lecture in a minute.

Horn Formula:

Implications of positive literals with ANDs on one side.

Plus ORs of negatives.

Negative clauses problem only with true literals.

Greedy: only set true if have to.

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Set Cover:

Given subsets of some elements.

Find: min number of sets that contains every element.

Greedy: choose largest set w.r.t. remaining elements.

$O(\log n)$ approximate solution.

Proof Idea:optimal of size $k \implies$ Cover $1/k$ of the remaining elts.

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Path Compression:
  $O(m \log^* n)$ time for m finds.
  Some finds expensive but cheap on average.
  Idea: group ranks into $log^* n$ sets.
    Small number of pointers across sets in any find.
    Total movement inside sets $O(n)$.
     Idea: from not more than 2^k nodes of rank k.

Taint Analysis

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$$a = http.read\_response();$$

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$\vdots$

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$$a = http.read\_response();$$

$$\vdots$$

$$b = a + c;$$

$$\vdots$$

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"*a* is tainted."

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Satisfiable?

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Satisfiable?

Not in this case.

Horn SAT

Implications:

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$$\implies A,\ A \implies B,\ \overline{B}.$$

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$$\implies A, A \implies B, \overline{B}.$$

Is this satisfiable?

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Is this satisfiable?

True is the problem.

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Taint Example:

$$\implies A, A \implies B, \overline{B}.$$

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True is the problem.
If every literal is False:

Horn SAT

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All $\wedge$ implication statements are good.

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except for implication: $\implies A$.

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Negative clauses:
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$\implies A$, $A \implies B$, $\overline{B}$.

Is this satisfiable?

True is the problem.
If every literal is False:
All $\wedge$ implication statements are good.
All $\vee$ statements are true.
except for implication: $\implies A$.

This forces a true literal.

Horn Sat: another view.

$$
\begin{aligned}
x_1 \wedge x_2 &\implies x_4 \\
x_3 &\implies x_2 \\
x_1 &\implies x_3 \\
x_5 \wedge x_1 &\implies x_3 \\
x_2 \wedge x_6 &\implies x_5 \\
&\implies x_1
\end{aligned}
$$

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Problem: Find consistent assignment with fewest “True” literals.

Horn Sat: another view.

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x_1 \wedge x_2 &\implies x_4 \\
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Problem: Find consistent assignment with fewest "True" literals.

Greedy algorithm:

Horn Sat: another view.

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Problem: Find consistent assignment with fewest “True” literals.

Greedy algorithm: Only set literals to True if you have to.

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Example:
x_1 must be True

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$$\begin{aligned} x_1 \wedge x_2 &\implies x_4 \\ x_3 &\implies x_2 \\ x_1 &\implies x_3 \\ x_5 \wedge x_1 &\implies x_3 \\ x_2 \wedge x_6 &\implies x_5 \\ &\implies x_1 \end{aligned}$$

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x_1 &\Longrightarrow x_3 \\
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Problem: Find consistent assignment with fewest “True” literals.

Greedy algorithm: Only set literals to True if you have to.

Example:
x_1 must be True so x_3 must be True

Horn Sat: another view.

$$\begin{aligned}
x_1 \wedge x_2 &\implies x_4 \\
x_3 &\implies x_2 \\
x_1 &\implies x_3 \\
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Problem: Find consistent assignment with fewest "True" literals.

Greedy algorithm: Only set literals to True if you have to.

Example:
x_1 must be True so x_3 must be True
so x_2 must be True

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Problem: Find consistent assignment with fewest "True" literals.

Greedy algorithm: Only set literals to True if you have to.

Example:
x_1 must be True so x_3 must be True
so x_2 must be True so x_4 must be True
Solution: $\{x_1, x_2, x_3, x_4\}$ are True

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Could also set x_5 to true, or both x_5 and x_6 to true...

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Same as horn sat!

Why same as HornSAT?

Horn SAT had negative clauses.

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By induction. First k set to true...

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The $k+1$ set variable set to true

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        so it must be true.

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Negative clauses only problem for true variables.

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The $k+1$ set variable set to true
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Horn has negative clauses.

Negative clauses only problem for true variables.

Any variable that is true must be true.

So if a negative clause is false, it must be.

Any SAT formula?

$$x_1 \implies x_2 \vee x_3.$$

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x_1 being true may mean nothing for x_3?

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More later...in the course.

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Analysis:
n_t elements remain at time t (after using t sets.)

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In more detail...

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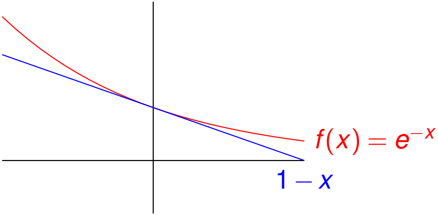

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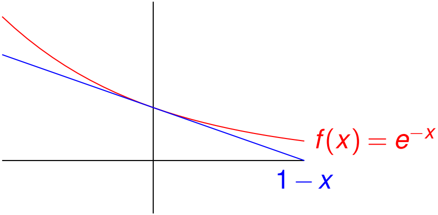

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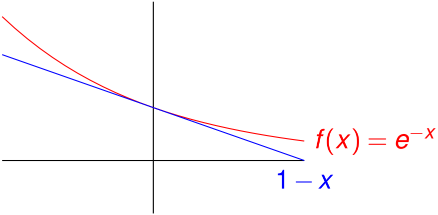

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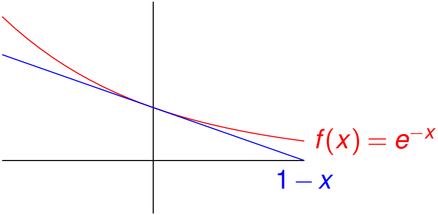

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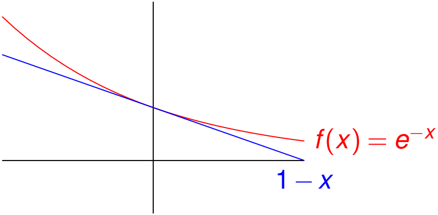

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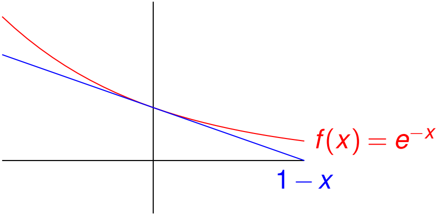

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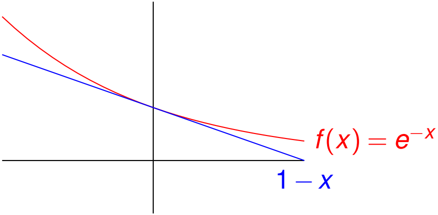

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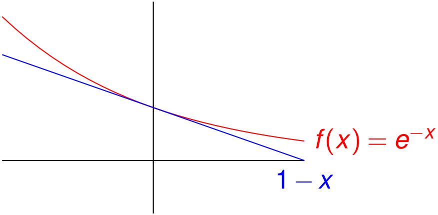

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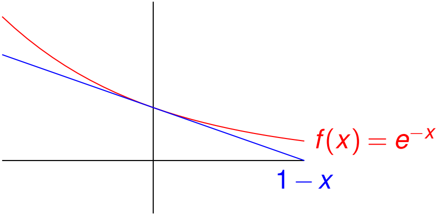

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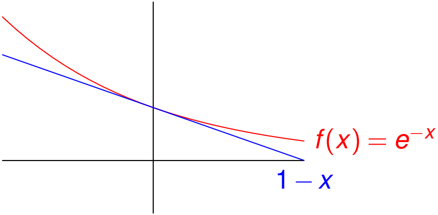

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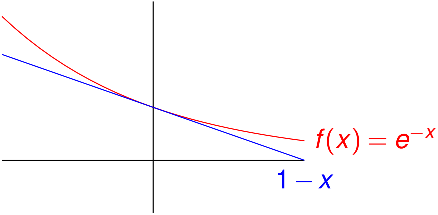

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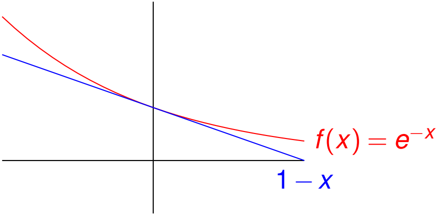

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Within $\ln n$ of k, which is the best possible!

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More later in the course.

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union(x,y)
  r_x = find(x)
  r_y = find(y)
  if rank(r_x) $<$ rank(r_y):
    $\pi(r_x) = r_y$
  else:
    $\pi(r_y) = r_x$
    if rank(r_x) == rank(r_y):
      rank(r_x) $+= 1$

Disjoint Set Data Structure

Maintain pointers: $\pi(x)$ for each x. Initially: rank(x) $= 0$.

makeset(x) $\pi(x) = x$.

find(x)
  if $\pi(x) == x$
    return x
  else
    find($\pi(x)$)

union(x,y)
  r_x = find(x)
  r_y = find(y)
  if rank(r_x) $<$ rank(r_y):
    $\pi(r_x) = r_y$
  else:
    $\pi(r_y) = r_x$
    if rank(r_x) == rank(r_y):
      rank(r_x) $+= 1$

Properties:

Disjoint Set Data Structure

Maintain pointers: $\pi(x)$ for each x. Initially: rank(x) $= 0$.

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Properties:
(1) Parent has a strictly higher rank.

Disjoint Set Data Structure

Maintain pointers: $\pi(x)$ for each x. Initially: rank(x) $= 0$.

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  else:
    $\pi(r_y) = r_x$
    if rank(r_x) == rank(r_y):
      rank(r_x) $+= 1$

Properties:
(1) Parent has a strictly higher rank.
(2) Rank doesn't change for internal nodes.

Disjoint Set Data Structure

Maintain pointers: $\pi(x)$ for each x. Initially: rank(x) $= 0$.

makeset(x) $\pi(x) = x$.

find(x)
  if $\pi(x) == x$
    return x
  else
    find($\pi(x)$)

union(x,y)
  r_x = find(x)
  r_y = find(y)
  if rank(r_x) $<$ rank(r_y):
    $\pi(r_x) = r_y$
  else:
    $\pi(r_y) = r_x$
    if rank(r_x) == rank(r_y):
      rank(r_x) $+= 1$

Properties:
(1) Parent has a strictly higher rank.
(2) Rank doesn't change for internal nodes.
(3) $rank(x) = rank(y) = k$

Disjoint Set Data Structure

Maintain pointers: $\pi(x)$ for each x. Initially: rank(x) $= 0$.

makeset(x) $\pi(x) = x$.

find(x)
    if $\pi(x) == x$
        return x
    else
        find($\pi(x)$)

union(x,y)
    r_x = find(x)
    r_y = find(y)
    if rank(r_x) $<$ rank(r_y):
        $\pi(r_x) = r_y$
    else:
        $\pi(r_y) = r_x$
        if rank(r_x) == rank(r_y):
            rank(r_x) $+= 1$

Properties:
(1) Parent has a strictly higher rank.
(2) Rank doesn't change for internal nodes.
(3) $rank(x) = rank(y) = k$
  (i) Each have $\geq 2^k$ vertices in sets

Disjoint Set Data Structure

Maintain pointers: $\pi(x)$ for each x. Initially: rank(x) $= 0$.

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union(x,y)
  r_x = find(x)
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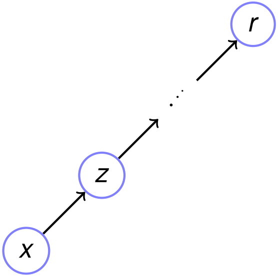
    if rank(r_x) == rank(r_y):
      rank(r_x) $+= 1$

Properties:
(1) Parent has a strictly higher rank.
(2) Rank doesn't change for internal nodes.
(3) $rank(x) = rank(y) = k$
  (i) Each have $\geq 2^k$ vertices in sets
  (ii) and the sets are disjoint.

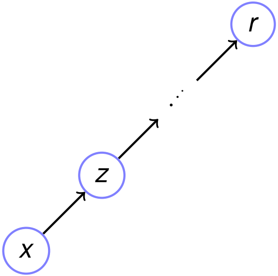
Path Compression

find(x)
  if $\pi(x) == x$
    return x
  else
    find($\pi(x)$)



Path Compression

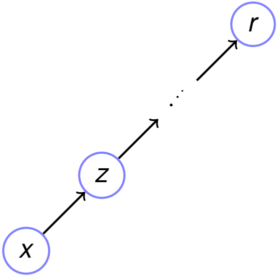
find(x)
  if $\pi(x) == x$
    return x
  else
    find($\pi(x)$)



What happens if we find(x) again?

Path Compression

find(x)
  if $\pi(x) == x$
    return x
  else
    find($\pi(x)$)

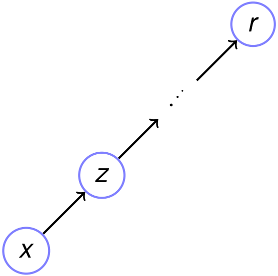


What happens if we find(x) again?

Chase again!

Path Compression

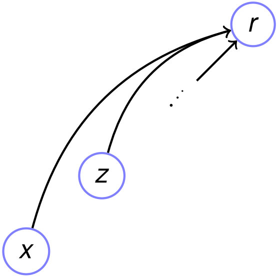
find(x)
  if $\pi(x) == x$
    return x
  else
    find($\pi(x)$)



What happens if we find(x) again?

Chase again!

find(x)
  if $\pi(x) == x$
    return x
  else
    $\pi(x) = \text{find}(\pi(x))$
    return $\pi(x)$



Is this better..

..asymptotically?

Is this better..

..asymptotically?

Take a deep breath.

Is this better..

..asymptotically?

Take a deep breath.

Fancy stuff..next!

Is this better..

..asymptotically?

Take a deep breath.

Fancy stuff..next!

Don’t worry.

Is this better..

..asymptotically?

Take a deep breath.

Fancy stuff..next!

Don’t worry.

...do try..

Is this better..

..asymptotically?

Take a deep breath.

Fancy stuff..next!

Don’t worry.

...do try..you’ll get smarter!

Path Compression Analysis

Union is same.

Path Compression Analysis

Union is same. Only affects root nodes.

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.
    rank to parent is higher

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.
rank to parent is higher and $\geq 2^k$ node in rank k

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.
rank to parent is higher and $\geq 2^k$ node in rank k

Every find is asymptotically faster?

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.
rank to parent is higher and $\geq 2^k$ node in rank k

Every find is asymptotically faster?

No. Can make a find take $\Theta(\log n)$ time.

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.
rank to parent is higher and $\geq 2^k$ node in rank k

Every find is asymptotically faster?

No. Can make a find take $\Theta(\log n)$ time.

Do you see how?

(A) Yes

(B) No

Path Compression Analysis

Union is same. Only affects root nodes.

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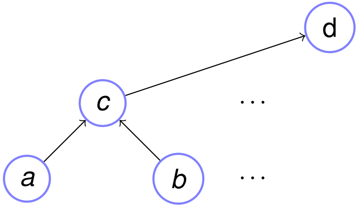

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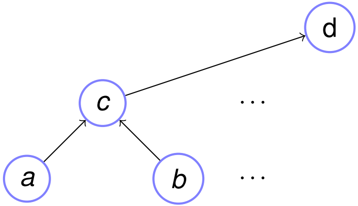

union(a,c)

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.
    rank to parent is higher and $\geq 2^k$ node in rank k

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Do you see how?

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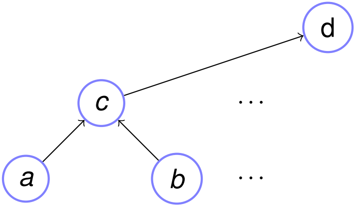
(B) No



union(a,c) union(b,c)

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.

rank to parent is higher and $\geq 2^k$ node in rank k

Every find is asymptotically faster?

No. Can make a find take $\Theta(\log n)$ time.

Do you see how?

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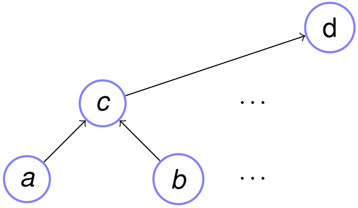

union(a,c) union(b,c)

. . .

Path Compression Analysis

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Rank properties still hold.
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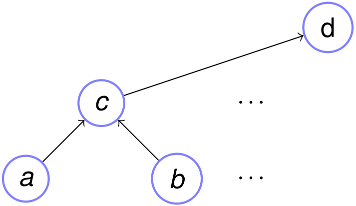

union(a,c) union(b,c)
$\cdots$ union(c,d)

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.

rank to parent is higher and $\geq 2^k$ node in rank k

Every find is asymptotically faster?

No. Can make a find take $\Theta(\log n)$ time.

Do you see how?

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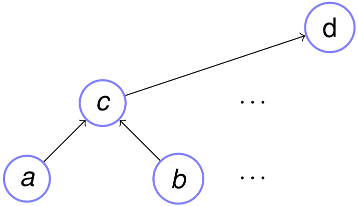


union(a,c) union(b,c)

$\cdots$ union(c,d)

union subtree roots to build tree

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.
  rank to parent is higher and $\geq 2^k$ node in rank k

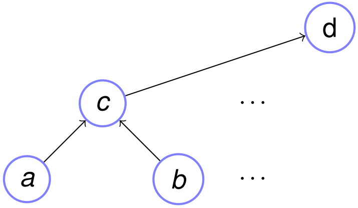
Every find is asymptotically faster?

No. Can make a find take $\Theta(\log n)$ time.

Do you see how?

(A) Yes

(B) No



union(a,c) union(b,c)
  $\cdots$ union(c,d)
union subtree roots to build tree
find(a)

Path Compression Analysis

Union is same. Only affects root nodes.

Rank properties still hold.
    rank to parent is higher and $\geq 2^k$ node in rank k

Every find is asymptotically faster?

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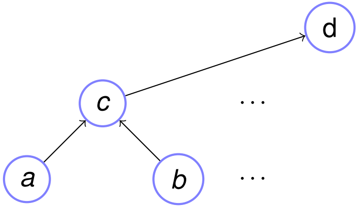
(A) Yes

(B) No



union(a,c) union(b,c)
    $\cdots$ union(c,d)
union subtree roots to build tree
find(a)
$\Theta(\log n)$ time for this find.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C. $\log 16 = 4$,

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C. $\log 16 = 4$, $\log 4 = 2$, $\log 2 = 1$.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C. $\log 16 = 4$, $\log 4 = 2$, $\log 2 = 1$. 3 times.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C. $\log 16 = 4$, $\log 4 = 2$, $\log 2 = 1$. 3 times.

Also $2^{2^2} = 16$.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

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C. $\log 16 = 4$, $\log 4 = 2$, $\log 2 = 1$. 3 times.

Also $2^{2^2} = 16$. height of powers of two!

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

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Also $2^{2^2} = 16$. height of powers of two!

$\log 1,000,000$ versus $\log^* 1,000,000$?

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C. $\log 16 = 4$, $\log 4 = 2$, $\log 2 = 1$. 3 times.

Also $2^{2^2} = 16$. height of powers of two!

$\log 1,000,000$ versus $\log^* 1,000,000$?

20 versus 5.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C. $\log 16 = 4$, $\log 4 = 2$, $\log 2 = 1$. 3 times.

Also $2^{2^2} = 16$. height of powers of two!

$\log 1,000,000$ versus $\log^* 1,000,000$?

20 versus 5.

$\log 1,000,000^{1,000,000}$ versus $\log^* 1,000,000^{1,000,000}$?

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C. $\log 16 = 4$, $\log 4 = 2$, $\log 2 = 1$. 3 times.

Also $2^{2^2} = 16$. height of powers of two!

$\log 1,000,000$ versus $\log^* 1,000,000$?

20 versus 5.

$\log 1,000,000^{1,000,000}$ versus $\log^* 1,000,000^{1,000,000}$?

$20,000,000$ versus 6.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time.

$O(\log^* n)$ time on average!

$\log^* n$ is number of times one takes $\log$ to get to 1.

$\log^*(16)$?

(A) 4

(B) 2

(C) 3

C. $\log 16 = 4$, $\log 4 = 2$, $\log 2 = 1$. 3 times.

Also $2^{2^2} = 16$. height of powers of two!

$\log 1,000,000$ versus $\log^* 1,000,000$?

20 versus 5.

$\log 1,000,000^{1,000,000}$ versus $\log^* 1,000,000^{1,000,000}$?

$20,000,000$ versus 6.

Grows very slowly.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time in total.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time in total.

$O(\log^* n)$ time on average!

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time in total.

$O(\log^* n)$ time on average!

Amortize cost = average over many operations.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time in total.

$O(\log^* n)$ time on average!

Amortize cost = average over many operations.

Who else amortizes?

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time in total.

$O(\log^* n)$ time on average!

Amortize cost = average over many operations.

Who else amortizes?

Bankers!

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time in total.

$O(\log^* n)$ time on average!

Amortize cost = average over many operations.

Who else amortizes?

Bankers!

Hand out some money

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time in total.

$O(\log^* n)$ time on average!

Amortize cost = average over many operations.

Who else amortizes?

Bankers!

Hand out some money
..... use it to pay for each pointer change.

Amortized Analysis.

Show that m finds take $O(m \log^* n)$ time in total.

$O(\log^* n)$ time on average!

Amortize cost = average over many operations.

Who else amortizes?

Bankers!

Hand out some money
..... use it to pay for each pointer change.

Only hand out $O(m \log^* n)$ dollars.

Handing out dollars.

Will hand out money to internal nodes
.....to pay for them changing pointers in find.

Handing out dollars.

Will hand out money to internal nodes
.....to pay for them changing pointers in find.

Notice: When a node becomes an internal node.

Handing out dollars.

Will hand out money to internal nodes
.....to pay for them changing pointers in find.

Notice: When a node becomes an internal node.
rank will no longer change!

Handing out dollars.

Will hand out money to internal nodes
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Notice: When a node becomes an internal node.
rank will no longer change!

Divide non-zero ranks into levels.

Handing out dollars.

Will hand out money to internal nodes
.....to pay for them changing pointers in find.

Notice: When a node becomes an internal node.
rank will no longer change!

Divide non-zero ranks into levels.

$\{1\}, \{2,3,4\}, \{5,\ldots,16\} \cdots \{k+1,\ldots 2^k\} \cdots$

Handing out dollars.

Will hand out money to internal nodes
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Notice: When a node becomes an internal node.
rank will no longer change!

Divide non-zero ranks into levels.

$\{1\},\{2,3,4\},\{5,\ldots,16\}\cdots\{k+1,\ldots 2^k\}\cdots$

How many groups of ranks?

(A) $\Theta(\log n)$

(B) $\Theta(\log^* n)$

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How many groups of ranks?

(A) $\Theta(\log n)$

(B) $\Theta(\log^* n)$

B.

Handing out dollars.

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Notice: When a node becomes an internal node.
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Divide non-zero ranks into levels.

$\{1\}, \{2,3,4\}, \{5,\ldots,16\} \cdots \{k+1, \ldots 2^k\} \cdots$

How many groups of ranks?

(A) $\Theta(\log n)$

(B) $\Theta(\log^* n)$

B. Each group grows by powering two!

Handing out dollars.

Will hand out money to internal nodes
.....to pay for them changing pointers in find.

Notice: When a node becomes an internal node.
rank will no longer change!

Divide non-zero ranks into levels.

$\{1\}, \{2,3,4\}, \{5,\ldots,16\} \cdots \{k+1, \ldots 2^k\} \cdots$

How many groups of ranks?

(A) $\Theta(\log n)$

(B) $\Theta(\log^* n)$

B. Each group grows by powering two!

How many internal nodes ever get rank r?

(A) $O(n/2^r)$

(B) $\Theta(n)$

Handing out dollars.

Will hand out money to internal nodes
.....to pay for them changing pointers in find.

Notice: When a node becomes an internal node.
rank will no longer change!

Divide non-zero ranks into levels.

$\{1\}, \{2,3,4\}, \{5,\ldots,16\} \cdots \{k+1,\ldots 2^k\} \cdots$

How many groups of ranks?

(A) $\Theta(\log n)$

(B) $\Theta(\log^* n)$

B. Each group grows by powering two!

How many internal nodes ever get rank r?

(A) $O(n/2^r)$

(B) $\Theta(n)$

A.

Handing out dollars.

Will hand out money to internal nodes
.....to pay for them changing pointers in find.

Notice: When a node becomes an internal node.
rank will no longer change!

Divide non-zero ranks into levels.

$\{1\}, \{2,3,4\}, \{5,\ldots,16\} \cdots \{k+1,\ldots 2^k\} \cdots$

How many groups of ranks?

(A) $\Theta(\log n)$

(B) $\Theta(\log^* n)$

B. Each group grows by powering two!

How many internal nodes ever get rank r?

(A) $O(n/2^r)$

(B) $\Theta(n)$

A. Each contained $\geq 2^r$ nodes when root.

Handing out dollars.

Will hand out money to internal nodes
.....to pay for them changing pointers in find.

Notice: When a node becomes an internal node.
rank will no longer change!

Divide non-zero ranks into levels.

$\{1\},\{2,3,4\},\{5,\ldots,16\}\cdots\{k+1,\ldots 2^k\}\cdots$

How many groups of ranks?

(A) $\Theta(\log n)$

(B) $\Theta(\log^* n)$

B. Each group grows by powering two!

How many internal nodes ever get rank r?

(A) $O(n/2^r)$

(B) $\Theta(n)$

A. Each contained $\geq 2^r$ nodes when root. Separate nodes.

Handing out money!

Will hand out money to internal nodes

Handing out money!

Will hand out money to internal nodes
.....since they change pointers in find.

Handing out money!

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Bound cost of find operation.

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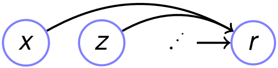

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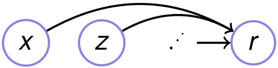

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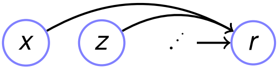

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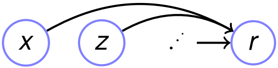

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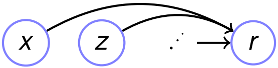

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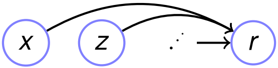

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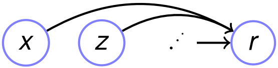

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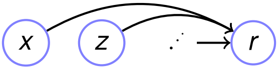

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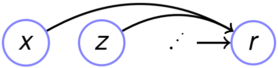

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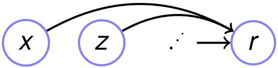

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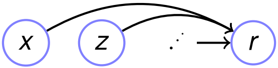

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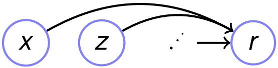

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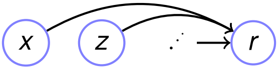

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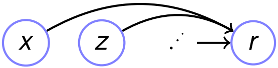

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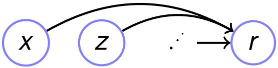

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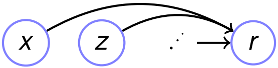

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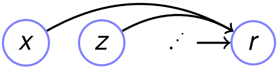

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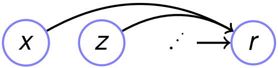

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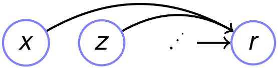

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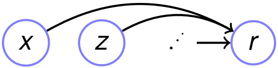

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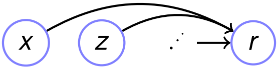

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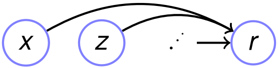

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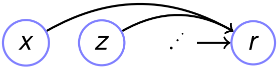

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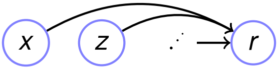

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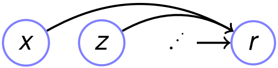

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fancy business.

Ackerman’s function.

Can we do better?

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$f(k)$ is $2^{2^{2^{\cdot^{\cdot^{\cdot}}}}}$ of height k.

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Grows fast.

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There is MST algorithm that runs in $O(m\alpha(m,n))$ where $\alpha(m,n)$ is inverse Ackerman's function.

Lecture in a minute.

Horn Formula:

- Implications of positive literals with ANDs on one side.
- Plus ORs of negatives.

Negative clauses problem only with true literals.

- Greedy: only set true if have to.

Lecture in a minute.

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Given subsets of some elements.

Find: min number of sets that contains every element.

Greedy: choose largest set w.r.t. remaining elements.

$O(\log n)$ approximate solution.

Proof Idea:optimal of size $k \implies$ Cover $1/k$ of the remaining elts.

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Path Compression:
$O(m \log^* n)$ time for m finds.
Some finds expensive but cheap on average.
Idea: group ranks into $log^* n$ sets.
Small number of pointers across sets in any find.
Total movement inside sets $O(n)$.
Idea: from not more than 2^k nodes of rank k.