CS 170: Algorithms

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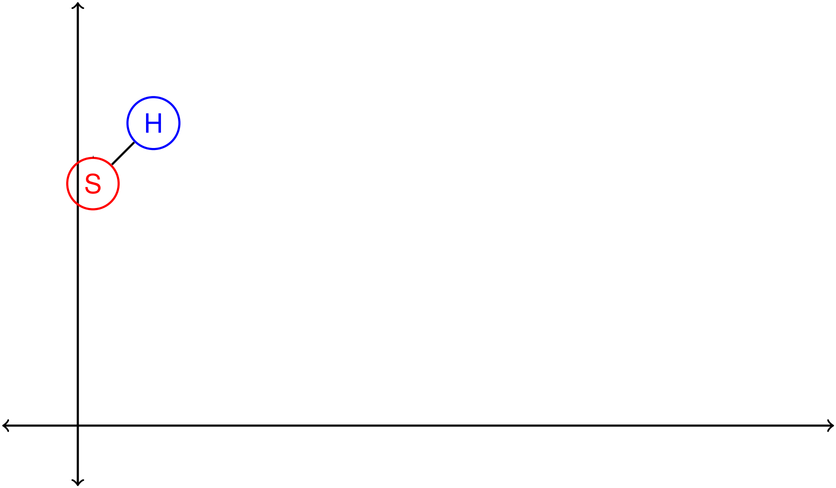

CS 170: Algorithms

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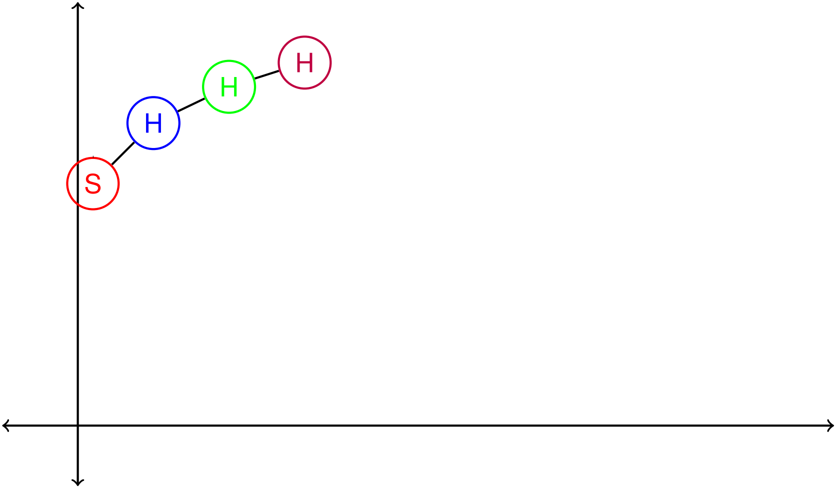
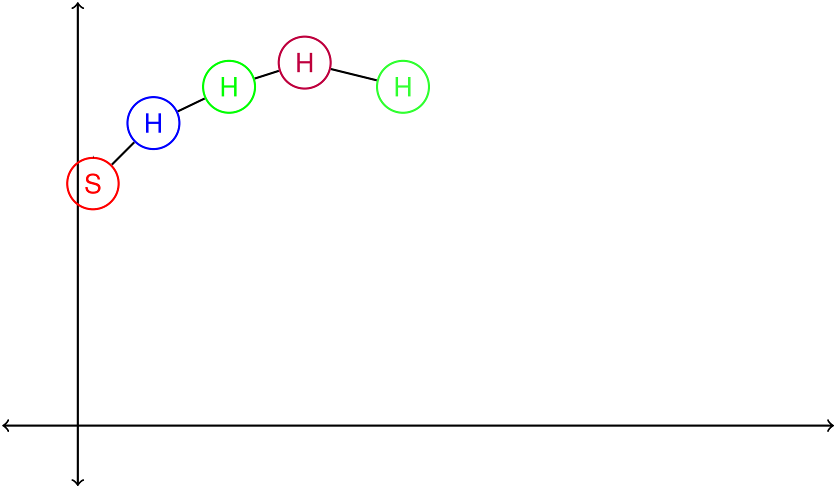
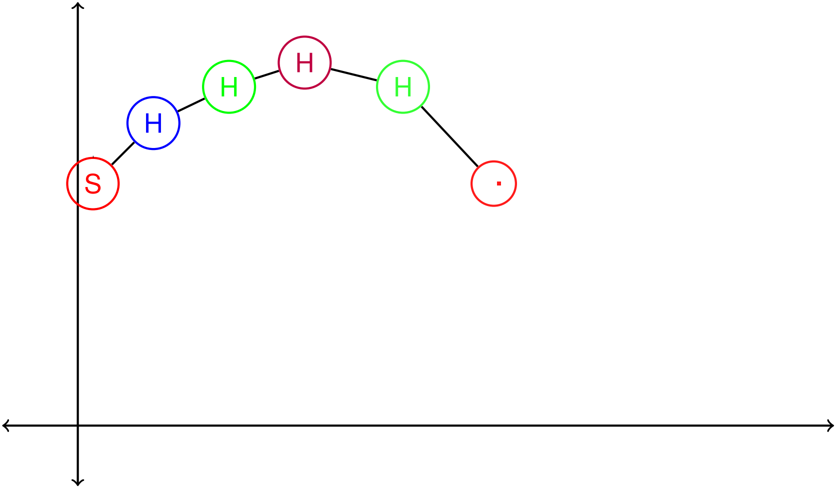


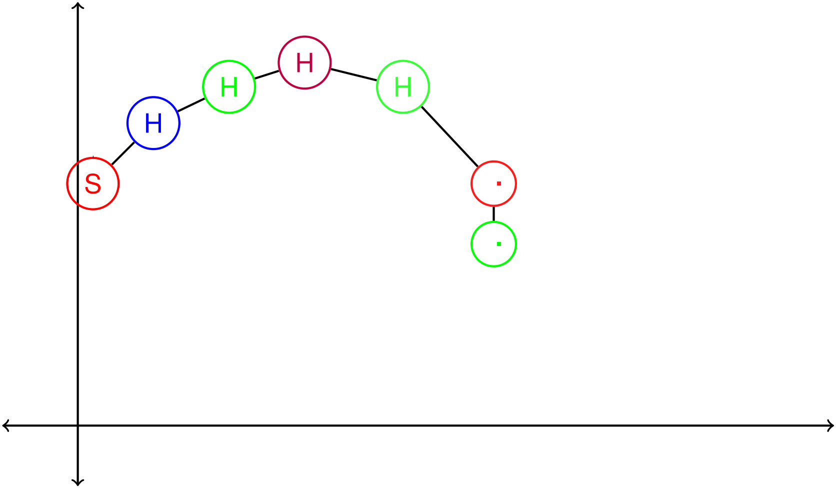
CS 170: Algorithms



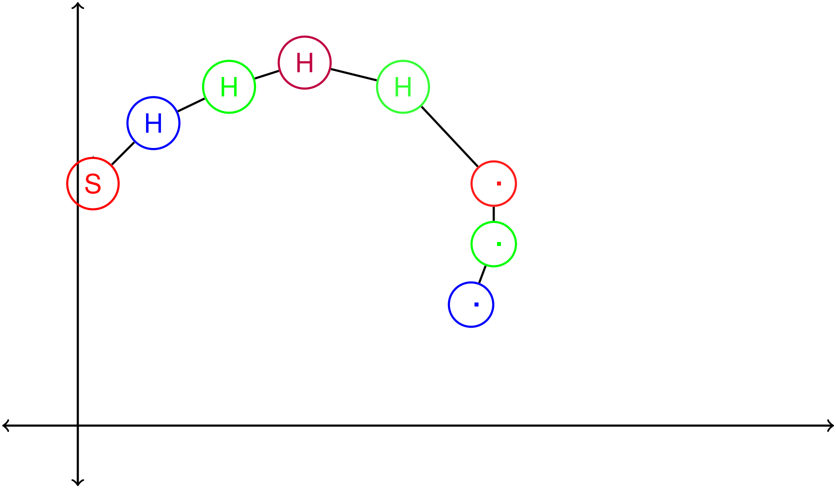
CS 170: Algorithms



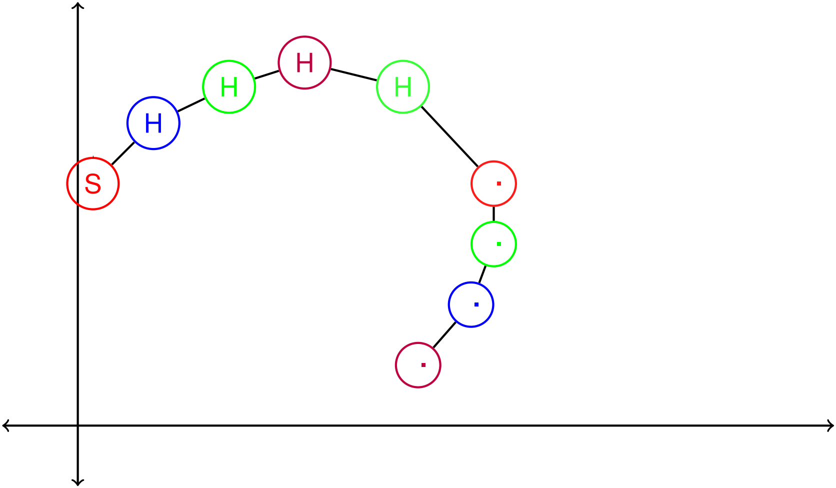
CS 170: Algorithms



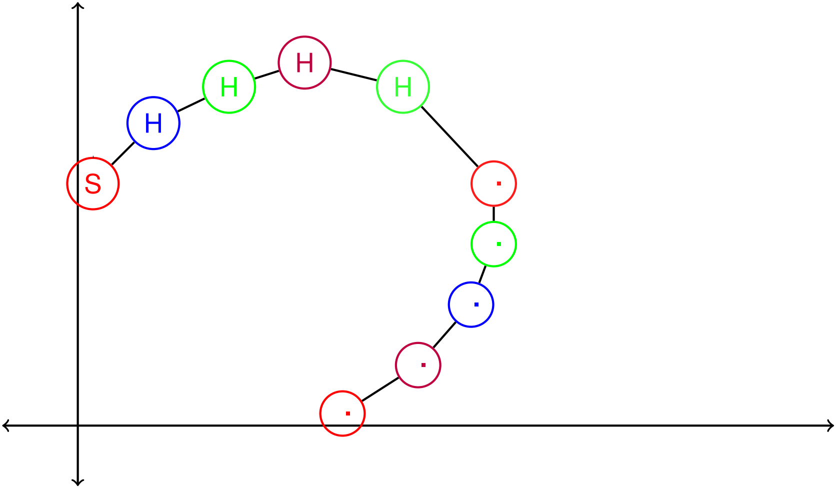
CS 170: Algorithms



CS 170: Algorithms

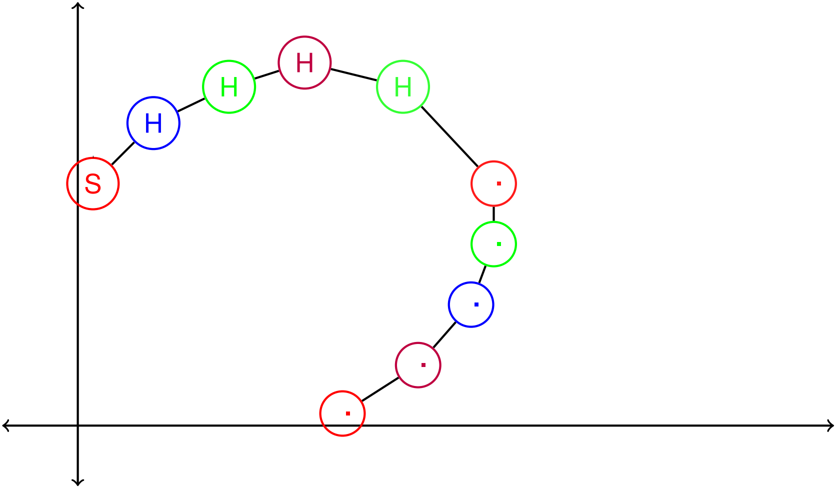


CS 170: Algorithms



Continue Linear Programming. Applications.

CS 170: Algorithms



Continue Linear Programming. Applications.

Lecture in a minute.

What's a linear program?

Lecture in a minute.

What's a linear program?
- Variables.
- Linear inequalities, and a linear objective function.
- Geometrically: a convex region in n.
- Optimal solution at "vertex" of region.
  - Cartoon simplex/duality: move to better vertex, repeatedly.

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Applications:

Lecture in a minute.

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- Applications:
  - Production Planning.
    - Variables hiring/firing/inventory/production.
    - Constraints/Objective encode costs and resource limits.
  - Bandwidth Problem.
    - Variables for routes.

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Lecture in a minute.

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- Linear Programs.

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  Geometrically: a convex region in n.
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Applications:
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Linear Programs.
  Types of constraints: equality.

Lecture in a minute.

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Linear Programs.
- Types of constraints: equality.
- Non-negative versus unrestricted.

Lecture in a minute.

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    - Constraints/Objective encode revenue and resource limits.
- Linear Programs.
  - Types of constraints: equality.
  - Non-negative versus unrestricted.
  - Standard Form.

Lecture in a minute.

What's a linear program?
- Variables.
- Linear inequalities, and a linear objective function.
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- Optimal solution at "vertex" of region.
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Applications:
- Production Planning.
  - Variables hiring/firing/inventory/production.
  - Constraints/Objective encode costs and resource limits.
- Bandwidth Problem.
  - Variables for routes.
  - Constraints/Objective encode revenue and resource limits.

Linear Programs.
- Types of constraints: equality.
- Non-negative versus unrestricted.
- Standard Form.
- Matrix, vector Notation.

Profit maximization.

Plant Carrots or Peas?

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

Profit maximization.

Plant Carrots or Peas?

2\$ bushel of carrots. 4\$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

100 units of water.

Profit maximization.

Plant Carrots or Peas?

2$ bushel of carrots. 4$ for peas.

Carrots take 3 unit of water/bushel.
Peas take 2 units of water/bushel.

100 units of water.

Peas get 3 sq. yards/bushel of sunny land.

Profit maximization.

Plant Carrots or Peas?

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Carrots take 3 unit of water/bushel.
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100 units of water.

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Plant Carrots or Peas?

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Peas take 2 units of water/bushel.

100 units of water.

Peas get 3 sq. yards/bushel of sunny land.
Carrots get 3 sq. yards/bushel of shady land.

Garden has 60 sq. yards of sunny land and 75 sq. yards of shady land.

Profit maximization.

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Profit maximization.

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100 units of water.

Peas get 3 sq. yards/bushel of sunny land.
Carrots get 3 sq. yards/bushel of shady land.

Garden has 60 sq. yards of sunny land and 75 sq. yards of shady land.

To pea or not to pea, that is the question!

To pea or not to pea.

To pea or not to pea.

4$ for peas.

To pea or not to pea.

4$ for peas. 2$ bushel of carrots.

To pea or not to pea.

4$ for peas. 2$ bushel of carrots. x_1- to pea!

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots

Money $4x_1 + 2x_2$

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots

Money $4x_1 + 2x_2$ maximize

To pea or not to pea.

4$ for peas. 2$ bushel of carrots. x_1- to pea! x_2 carrots

Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.

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Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.

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Peas take 3 unit of water/bushel.

Carrot take 2 units of water/bushel. 100 units of water.

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Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.

Peas take 3 unit of water/bushel.

Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

To pea or not to pea.

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Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
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Peas 3 yards/bushel of sunny land. Have 60 sq. yards.

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Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$

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Peas take 3 unit of water/bushel.
Carrot take 2 units of water/bushel. 100 units of water.

$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$
Carrots get 3 yards/bushel of shady land. Have 75 sq. yards.

To pea or not to pea.

4\$ for peas. 2\$ bushel of carrots. x_1- to pea! x_2 carrots
Money $4x_1 + 2x_2$ maximize $\max 4x_1 + 2x_2$.
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$3x_1 + 2x_2 \leq 100$

Peas 3 yards/bushel of sunny land. Have 60 sq. yards.
$3x_1 \leq 60$
Carrots get 3 yards/bushel of shady land. Have 75 sq. yards.
$3x_2 \leq 75$

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$3x_1 + 2x_2 \leq 100$

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$3x_1 \leq 60$

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$3x_2 \leq 75$

Can't make negative!

To pea or not to pea.

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Can't make negative! $x_1, x_2 \geq 0$.

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A linear program.

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A linear program.

$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$

Optimal Point?

$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$

Optimal point?

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Optimal point?

Try every point

Optimal Point?

$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
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Optimal point?

Try every point if we only had time!

Optimal Point?

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How many points?

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How many points?

Real numbers?

Optimal Point?

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3x_1 \leq 60 \\
3x_2 \leq 75 \\
3x_1 + 2x_2 \leq 100 \\
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\end{aligned}$$

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Try every point if we only had time!

How many points?

Real numbers?

Infinite.

Optimal Point?

$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1, x_2 \geq 0 \end{aligned}$$

Optimal point?

Try every point if we only had time!

How many points?

Real numbers?

Infinite. Uncountably infinite!

Where's Waldo?

A linear program.

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A linear program.

$$
\begin{aligned}
\max 4x_1 + 2x_2 \\
3x_1 \le 60 \\
3x_2 \le 75 \\
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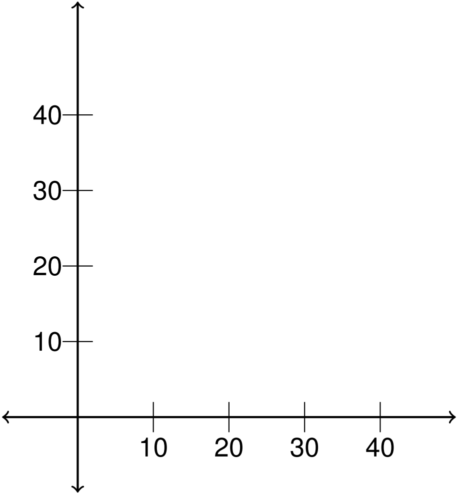

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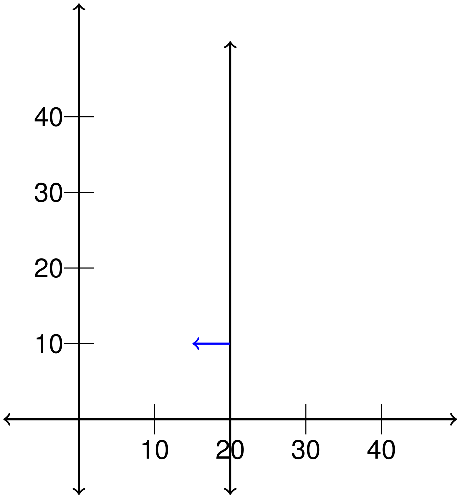

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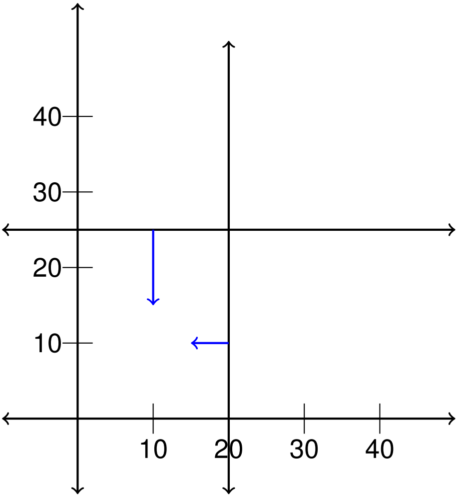

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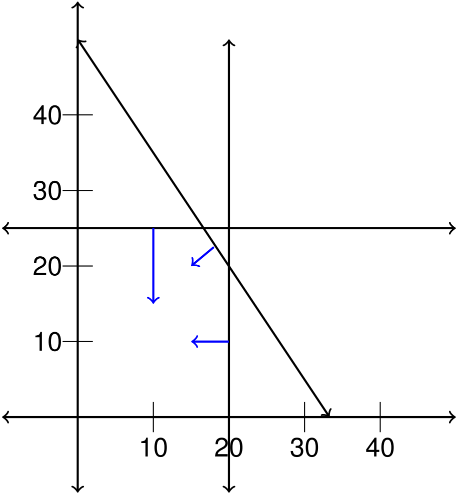

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$$3x_1 + 2x_2 \leq 100$$
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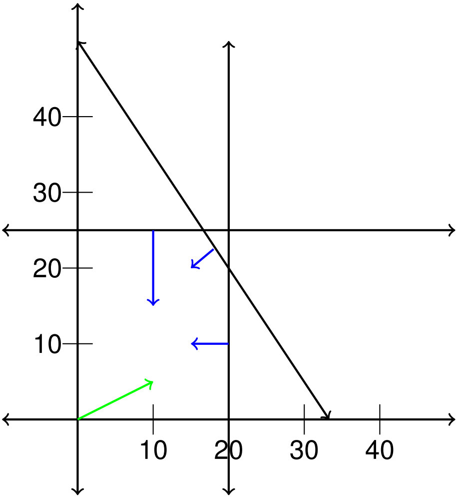
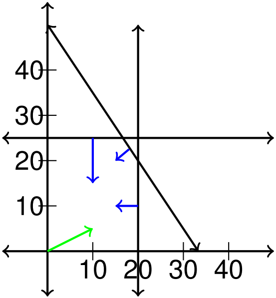
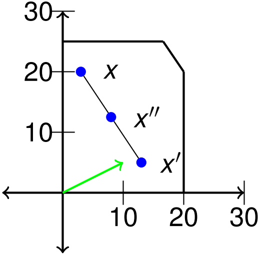


Optimal point?

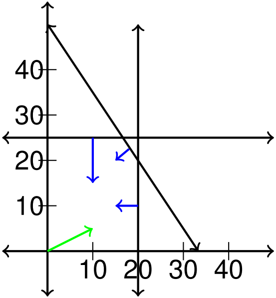
Feasible Region.

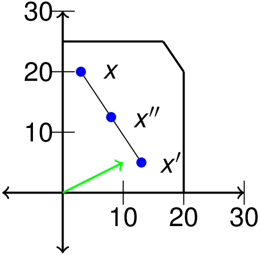


Convex.



Feasible Region.

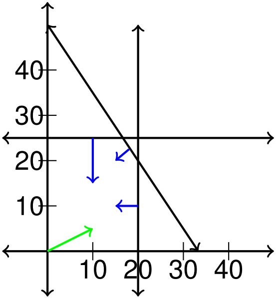


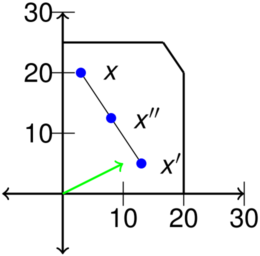


Convex.

Any two points in region connected by a line in region.

Feasible Region.



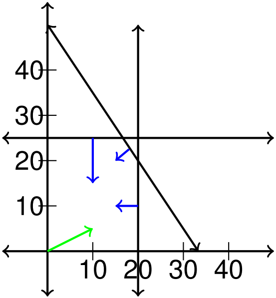


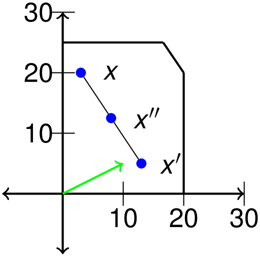
Convex.

Any two points in region connected by a line in region.

Algebraically:

Feasible Region.





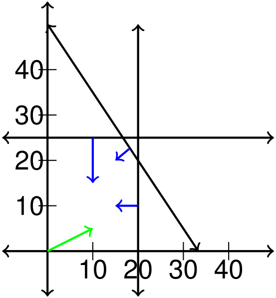
Convex.

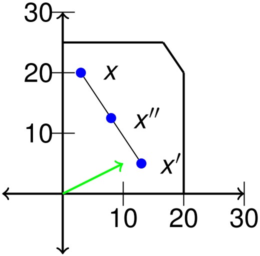
Any two points in region connected by a line in region.

Algebraically:

If x and x' satisfy constraint, so does $x'' = \alpha x + (1-\alpha)x'$

Feasible Region.





Convex.

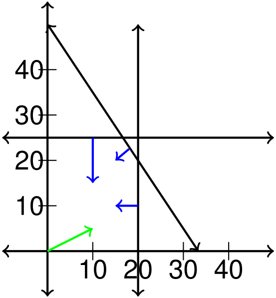
Any two points in region connected by a line in region.

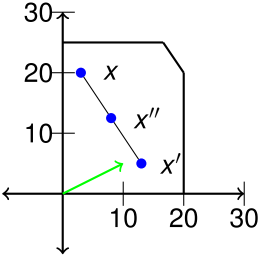
Algebraically:

If x and x' satisfy constraint, so does $x'' = \alpha x + (1-\alpha)x'$

E.g. $3x \leq 60$ and $3x' \leq 60$

Feasible Region.





Convex.

Any two points in region connected by a line in region.

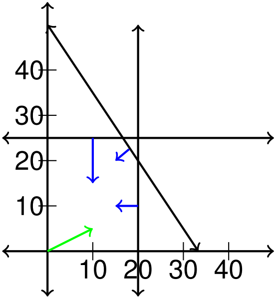
Algebraically:

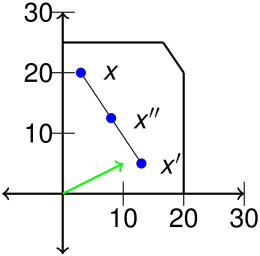
If x and x' satisfy constraint, so does $x'' = \alpha x + (1-\alpha)x'$

E.g. $3x \leq 60$ and $3x' \leq 60$

$\rightarrow 3\alpha x \leq \alpha(60)$

Feasible Region.





Convex.

Any two points in region connected by a line in region.

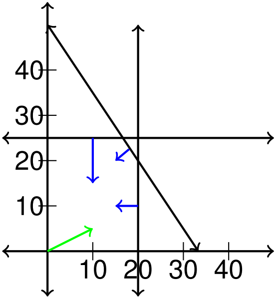
Algebraically:

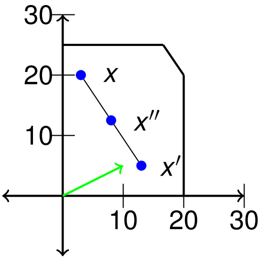
If x and x' satisfy constraint, so does $x'' = \alpha x + (1-\alpha)x'$

E.g. $3x \leq 60$ and $3x' \leq 60$

$\rightarrow 3\alpha x \leq \alpha(60)$ and $3(1-\alpha)x' \leq (1-\alpha)60$

Feasible Region.





Convex.

Any two points in region connected by a line in region.

Algebraically:

If x and x' satisfy constraint, so does $x'' = \alpha x + (1-\alpha)x'$

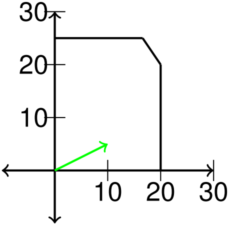
E.g. $3x \le 60$ and $3x' \le 60$

$\rightarrow 3\alpha x \le \alpha(60)$ and $3(1-\alpha)x' \le (1-\alpha)60$

$\rightarrow 3(\alpha(x) + (1-\alpha)x') \le (\alpha + (1-\alpha))60 = 60$

Vertex is a solution.

“Isocline” - all points have same value on hyperplane.



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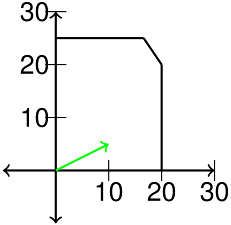
"Isocline" - all points have same value on hyperplane.



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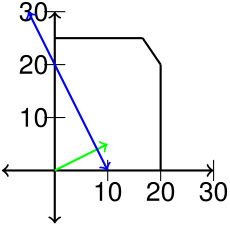

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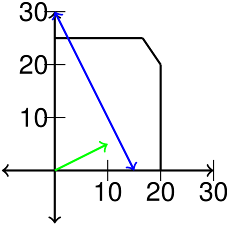

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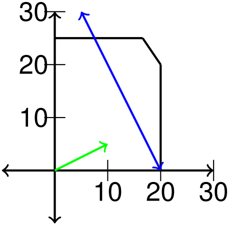
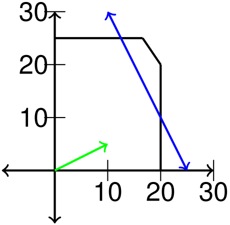


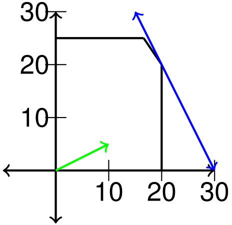
Vertex is a solution.

"Isocline" - all points have same value on hyperplane.



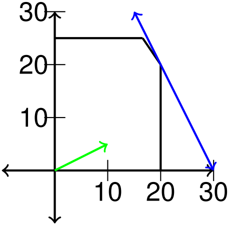
Vertex is a solution.

"Isocline" - all points have same value on hyperplane.



Vertex is a solution.

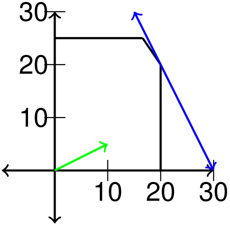
"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!

Vertex is a solution.

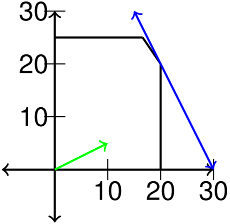
"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.

Vertex is a solution.

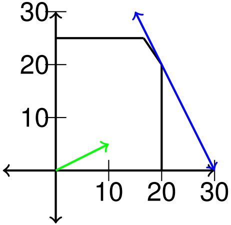
"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines in 2 dimensions!

Vertex is a solution.

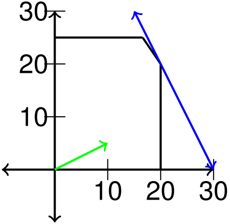
“Isocline” - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines in 2 dimensions!
Try every vertex!

Vertex is a solution.

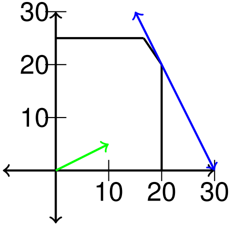
"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines in 2 dimensions!
Try every vertex! Choose best.

Vertex is a solution.

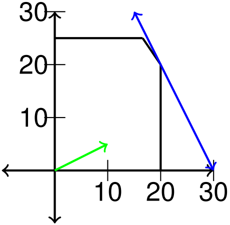
"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines in 2 dimensions!
Try every vertex! Choose best.
$O(m^2)$ if m constraints and 2 variables.

Vertex is a solution.

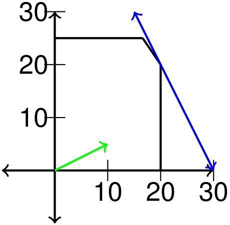
"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines in 2 dimensions!
Try every vertex! Choose best.
$O(m^2)$ if m constraints and 2 variables.
For n variables (dimensions), m constraints, how many?

Vertex is a solution.

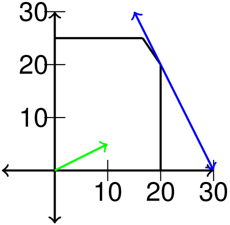
"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines in 2 dimensions!
Try every vertex! Choose best.
$O(m^2)$ if m constraints and 2 variables.
For n variables (dimensions), m constraints, how many?
nm? $\binom{m}{n}$? $n+m$?

Vertex is a solution.

"Isocline" - all points have same value on hyperplane.



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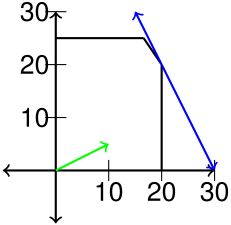
nm? $\binom{m}{n}$? $n+m$?
n constraints define point ....so $\binom{m}{n}$ possible vertices.

Vertex is a solution.

"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines in 2 dimensions!
Try every vertex! Choose best.
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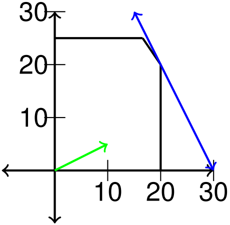
For n variables (dimensions), m constraints, how many?
nm? $\binom{m}{n}$? $n+m$?
n constraints define point ....so $\binom{m}{n}$ possible vertices.
Finite!!!!!!

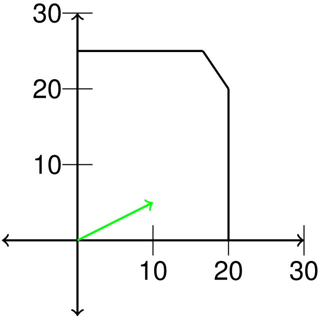
Vertex is a solution.

"Isocline" - all points have same value on hyperplane.



Optimal at pointy part of feasible region!
Vertex of region.
Intersection of two of the constraints! Which are lines in 2 dimensions!
Try every vertex! Choose best.
$O(m^2)$ if m constraints and 2 variables.
For n variables (dimensions), m constraints, how many?
nm? $\binom{m}{n}$? $n+m$?
n constraints define point ....so $\binom{m}{n}$ possible vertices.
Finite!!!!!! But exponential in the number of variables.

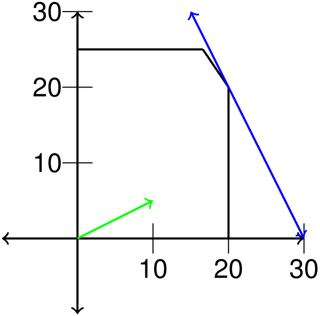
Simplex in 2 dimensions.



Simplex: Start at vertex.

$$\max 4x_1 + 2x_2$$
$$3x_1 \le 60$$
$$3x_2 \le 75$$
$$3x_1 + 2x_2 \le 100$$
$$x_1 \ge 0$$
$$x_2 \ge 0$$

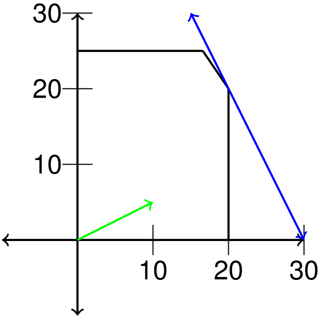
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.

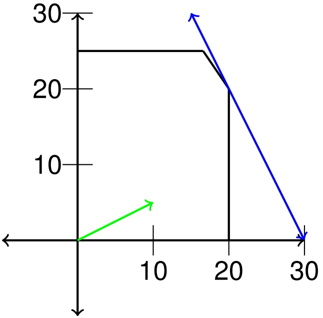
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop.

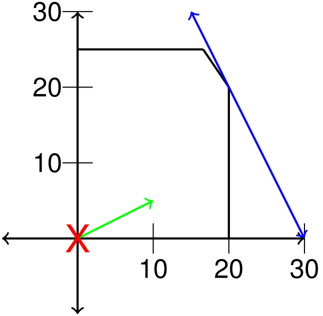
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.

Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
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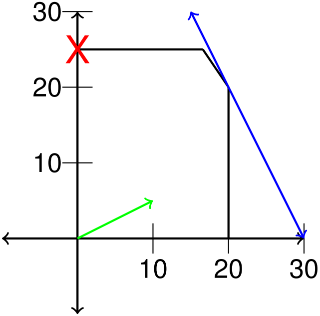
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0.

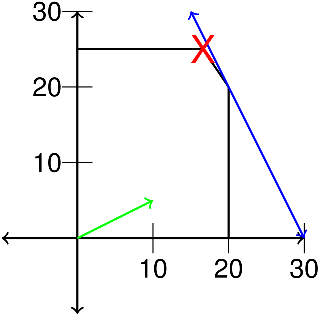
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.

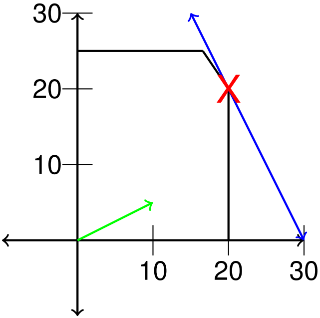
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$

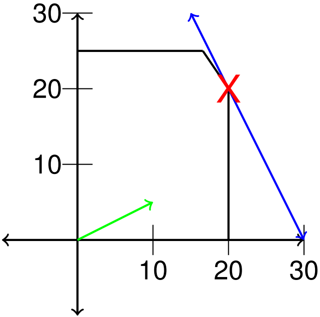
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.

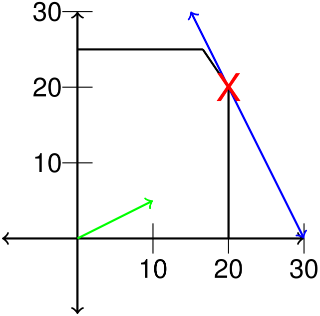
Simplex in 2 dimensions.



$$\begin{aligned} \max 4x_1 + 2x_2 \\ 3x_1 \leq 60 \\ 3x_2 \leq 75 \\ 3x_1 + 2x_2 \leq 100 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{aligned}$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.
Duality:

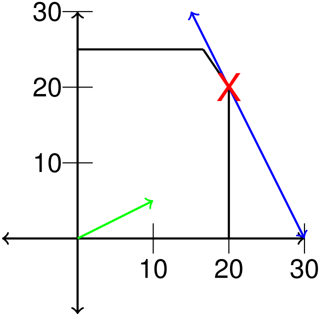
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3},25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.
Duality:
Combine blue equations to upper bound objective function?

Simplex in 2 dimensions.



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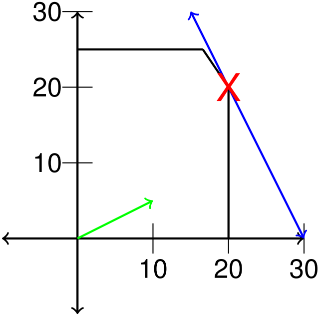
Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.
Duality:
Combine blue equations to upper bound objective function?
$1/3$ times first plus 1 times the third.

Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
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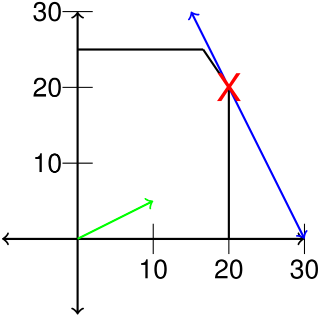
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.
Duality:
Combine blue equations to upper bound objective function?
$1/3$ times first plus 1 times the third.
Get $4x_1 + 2x_2 \leq 120$.

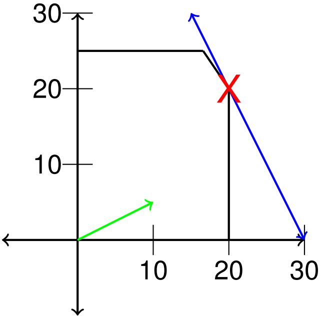
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.
Duality:
Combine blue equations to upper bound objective function?
$1/3$ times first plus 1 times the third.
Get $4x_1 + 2x_2 \leq 120$. Every solution must satisfy this inequality!

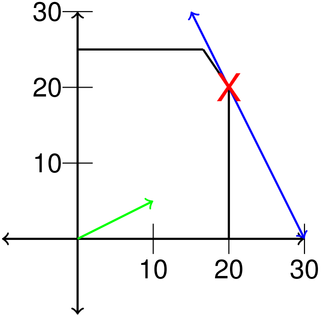
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3},25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.
Duality:
Combine blue equations to upper bound objective function?
$1/3$ times first plus 1 times the third.
Get $4x_1 + 2x_2 \leq 120$. Every solution must satisfy this inequality!
Objective value: 120.
Can we do better?

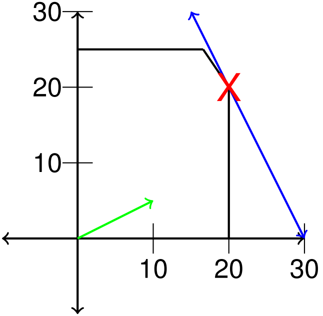
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3},25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.
Duality:
Combine blue equations to upper bound objective function?
$1/3$ times first plus 1 times the third.
Get $4x_1 + 2x_2 \leq 120$. Every solution must satisfy this inequality!
Objective value: 120.
Can we do better? Yes?

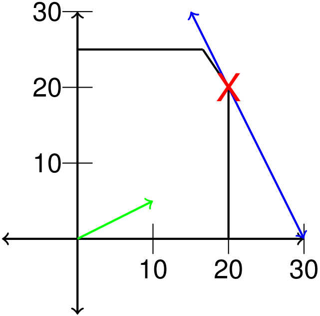
Simplex in 2 dimensions.



$$\max 4x_1 + 2x_2$$
$$3x_1 \leq 60$$
$$3x_2 \leq 75$$
$$3x_1 + 2x_2 \leq 100$$
$$x_1 \geq 0$$
$$x_2 \geq 0$$

Simplex: Start at vertex. Move to better neighboring vertex.
Until you stop. This example.
(0,0) objective 0. $\rightarrow (0,25)$ objective 50.
$\rightarrow (16\frac{2}{3}, 25)$ objective $116\frac{2}{3}$ $\rightarrow (20,20)$ objective 120.
Duality:
Combine blue equations to upper bound objective function?
$1/3$ times first plus 1 times the third.
Get $4x_1 + 2x_2 \leq 120$. Every solution must satisfy this inequality!
Objective value: 120.
Can we do better? Yes? No?

Simplex in 2 dimensions.



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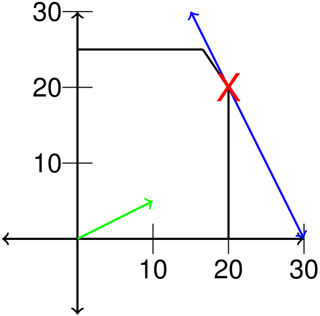

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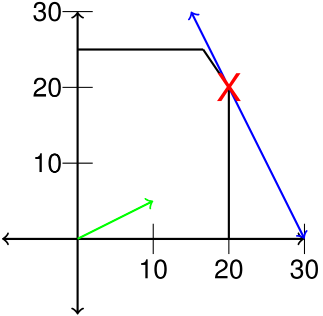

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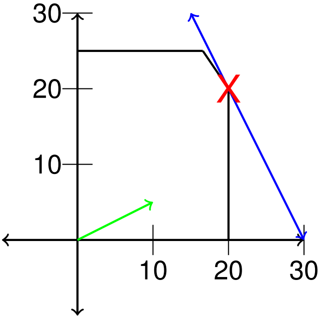
Simplex in 2 dimensions.



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Can we do better? Yes? No? Maybe? No! There is a solution.
Dual problem: add equations to get best upper bound.

More variables.

More vegetables.

More variables.

More vegetables. How about some Kale!

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3$ per bushel.

More variables.

More vegetables. How about some Kale!
3$ per bushel.
2 sq. yards/bushel per sunny or 3 sq. yards/bushel for shady land.

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2 units of water.

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More variables.

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$$\begin{aligned} \max 4x_1 + 2x_2 + 3x_3 + 3x_4 \\ 3x_1 + 2x_3 \leq 60 \\ 3x_2 + 3x_4 \leq 75 \\ 3x_1 + 2x_2 + 2x_3 + 2x_4 \leq 100 \\ x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$

Carpet production planning.

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30 employees. 20 carpets/month. 2000/month.

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Overtime: 80% extra. Also at most 30% for one employee.

Carpet production planning.

Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
30 employees. 20 carpets/month. 2000/month.
Overtime: 80% extra. Also at most 30% for one employee.
Hiring/firing: 320/400.

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30 employees. 20 carpets/month. 2000/month.
Overtime: 80% extra. Also at most 30% for one employee.
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Storage: 8/carpet and no storage at the end of year.

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Variables.

w_i - workers in month i ;

$w_0 = 30$

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Nonnegative:

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Production:

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Nonnegative: $w_i, x_i, o_i, h_i, f_i, s_i \geq 0$
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Employment:

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Employment: $w_i = w_{i-1} + h_i - f_i$

Carpet production planning.

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Regulations: $o_i \leq 6w_i$

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Demands: $d_1, d_2, \ldots, d_{12}$, range: $440 - 920$
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Regulations: $o_i \leq 6w_i$
Objective:

$$\min \quad 2000 \sum_i w_i$$

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Regulations: $o_i \leq 6w_i$
Objective:

$$\min \quad 2000 \textstyle\sum_i w_i + 320 \sum_i h_i$$

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Variables.

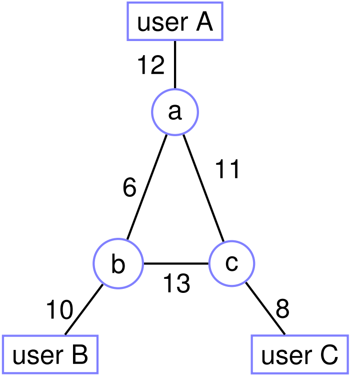
w_i - workers in month i ;
$w_0 = 30$
x_i - carpets made in month i
o_i - overtime carpets in month i

h_i, f_i - hired/fired in month i
s_i - stored at end of month i ;
$s_{12} = 0$

Nonnegative: $w_i, x_i, o_i, h_i, f_i, s_i \geq 0$
Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \leq 6w_i$
Objective:

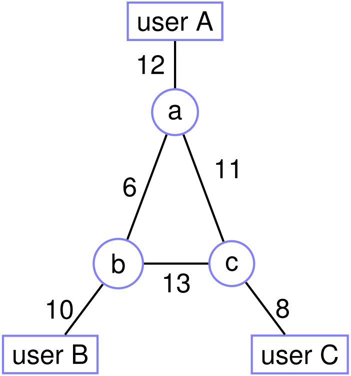
$$\min \quad 2000 \sum_i w_i + 320 \sum_i h_i + 400 \sum f_i + 8 \sum s_i + 180 \sum_i o_i.$$

Bandwidth.



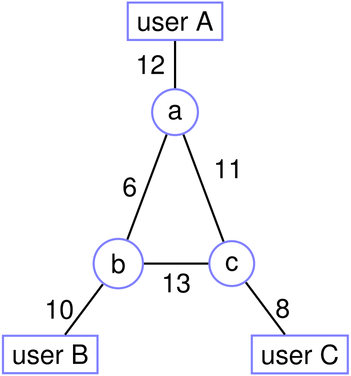
Problem:

Bandwidth.



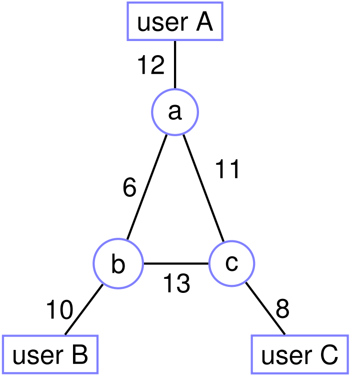
Problem:
$A - B$ pays 3\$ per unit,
$A - C$ pays 2\$ per unit,
$B - C$ pays 4\$ per unit.

Bandwidth.



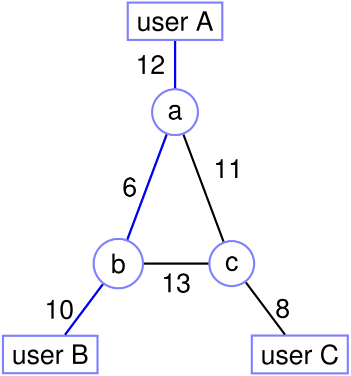
Problem:
$A-B$ pays 3\$ per unit,
$A-C$ pays 2\$ per unit,
$B-C$ pays 4\$ per unit.
Every pair gets 2 units.

Bandwidth.



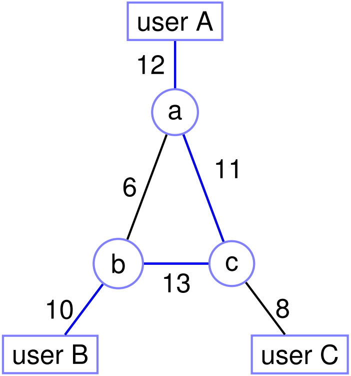
Problem:
$A - B$ pays 3$ per unit,
$A - C$ pays 2$ per unit,
$B - C$ pays 4$ per unit.
Every pair gets 2 units.
Linear Program Variables/Constraints:

Bandwidth.



Problem:
$A - B$ pays 3\$ per unit,
$A - C$ pays 2\$ per unit,
$B - C$ pays 4\$ per unit.
Every pair gets 2 units.
Linear Program Variables/Constraints:
X_{AB} - flow along $A - a - b - B$.

Bandwidth.



Problem:
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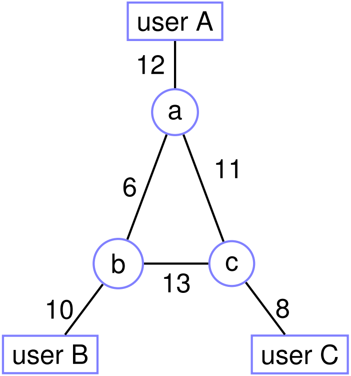
$B - C$ pays 4\$ per unit.
Every pair gets 2 units.
Linear Program Variables/Constraints:
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$

Bandwidth.



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Linear Program Variables/Constraints:
X_{AB} - flow along $A - a - b - B$.
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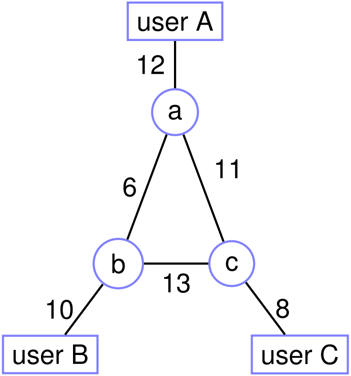
Capacity constraint on edge (a, b):

Bandwidth.



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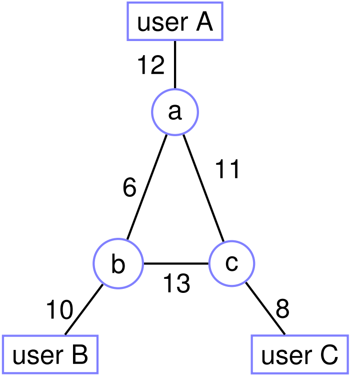
Linear Program Variables/Constraints:
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):

$$X_{AB} + X'_{BC} + X'_{AC}$$

Bandwidth.



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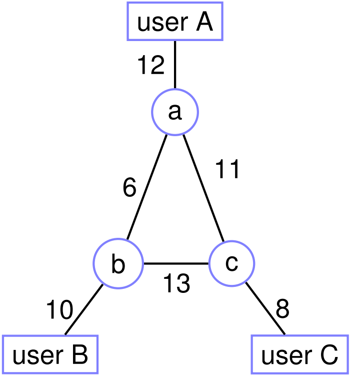
Every pair gets 2 units.
Linear Program Variables/Constraints:
X_{AB} - flow along $A-a-b-B$.
X'_{AB} is flow along path $A-a-c-b-B$
Capacity constraint on edge (a,b):
$X_{AB}+X'_{BC}+X'_{AC}\leq 6$
Bandwidth constraint:

Bandwidth.



Problem:
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Linear Program Variables/Constraints:
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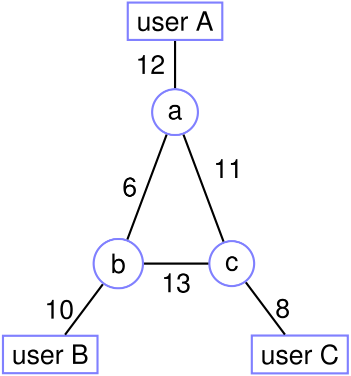
Capacity constraint on edge (a,b):
$X_{AB}+X'_{BC}+X'_{AC} \leq 6$
Bandwidth constraint:
$X_{AB}+X'_{AB}$

Bandwidth.



Problem:

$A-B$ pays 3\$ per unit,

$A-C$ pays 2\$ per unit,

$B-C$ pays 4\$ per unit.

Every pair gets 2 units.

Linear Program Variables/Constraints:

X_{AB} - flow along $A-a-b-B$.

X'_{AB} is flow along path $A-a-c-b-B$

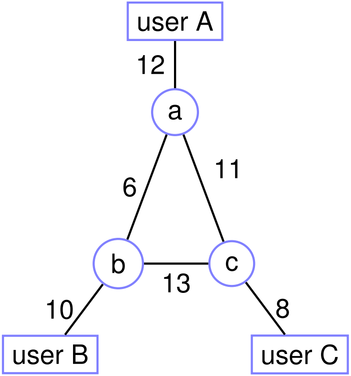
Capacity constraint on edge (a,b):

$$X_{AB}+X'_{BC}+X'_{AC} \leq 6$$

Bandwidth constraint:

$$X_{AB}+X'_{AB} \geq 2$$

Bandwidth.



How many edge constraints?

Problem:
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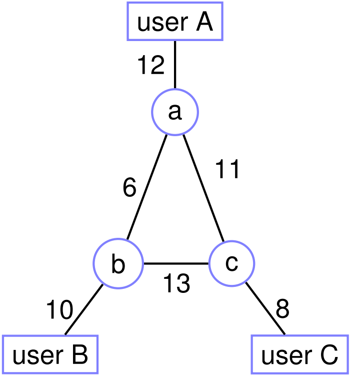
$B - C$ pays 4$ per unit.
Every pair gets 2 units.
Linear Program Variables/Constraints:
X_{AB} - flow along $A - a - b - B$.
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):

$$X_{AB} + X'_{BC} + X'_{AC} \leq 6$$

Bandwidth constraint:

$$X_{AB} + X'_{AB} \geq 2$$

Bandwidth.



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Capacity constraint on edge (a,b):
$X_{AB} + X'_{BC} + X'_{AC} \leq 6$
Bandwidth constraint:
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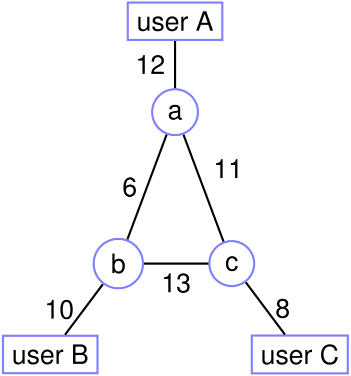
How many edge constraints? 6.

Bandwidth.



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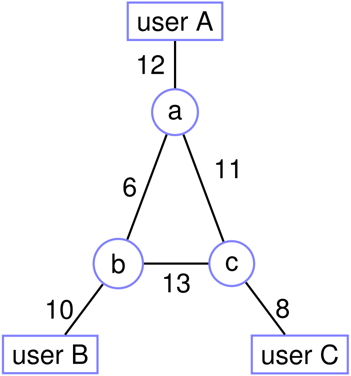
How many edge constraints? 6.
How many bandwidth constraints?

Bandwidth.



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Capacity constraint on edge (a,b):

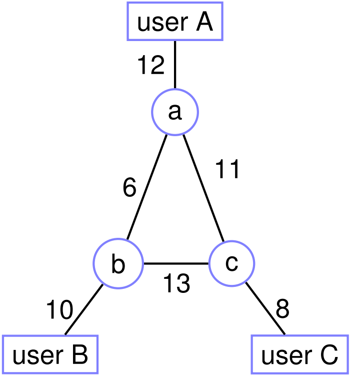
$$X_{AB}+X'_{BC}+X'_{AC}\le 6$$

Bandwidth constraint:

$$X_{AB}+X'_{AB}\ge 2$$

How many edge constraints? 6.
How many bandwidth constraints? 3.

Bandwidth.



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Bandwidth constraint:
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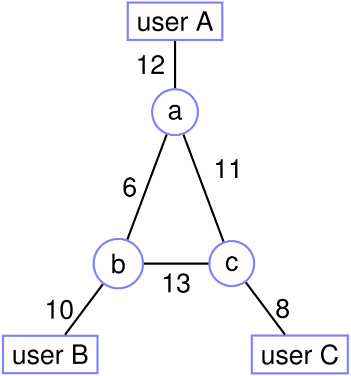
How many edge constraints? 6.
How many bandwidth constraints? 3.
Objective function?

Bandwidth.



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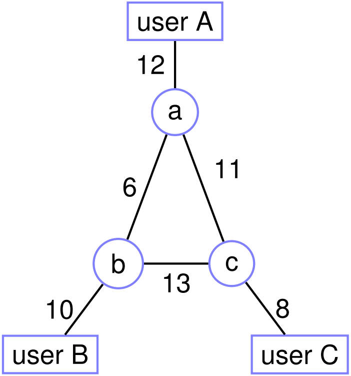
X'_{AB} is flow along path $A - a - c - b - B$
Capacity constraint on edge (a, b):
$X_{AB} + X'_{BC} + X'_{AC} \leq 6$
Bandwidth constraint:
$X_{AB} + X'_{AB} \geq 2$

How many edge constraints? 6.
How many bandwidth constraints? 3.
Objective function?

$$3(X_{AB} + X'_{AB})$$

Bandwidth.



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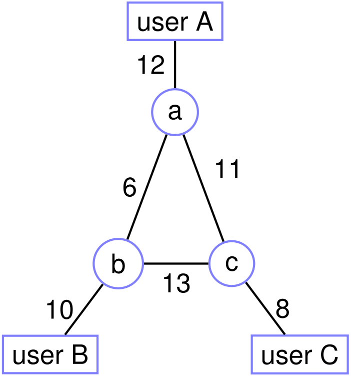
Capacity constraint on edge (a,b):
$X_{AB}+X'_{BC}+X'_{AC} \leq 6$
Bandwidth constraint:
$X_{AB}+X'_{AB} \geq 2$

How many edge constraints? 6.
How many bandwidth constraints? 3.
Objective function?

$$3(X_{AB}+X'_{AB})+4(X_{BC}+X'_{BC})$$

Bandwidth.



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$$X_{AB}+X'_{BC}+X'_{AC} \leq 6$$

Bandwidth constraint:

$$X_{AB}+X'_{AB} \geq 2$$

How many edge constraints? 6.
How many bandwidth constraints? 3.
Objective function?

$$3(X_{AB}+X'_{AB})+4(X_{BC}+X'_{BC})+2(X_{AC}+X'_{AC})$$

Again with carpets!

Production:

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment:

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory:

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations:

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \leq 6w_i$

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \le 6w_i$

$$\min 2000\sum_i w_i + 320\sum_i h_i + 400\sum f_i + 8\sum s_i + 180\sum_i o_i.$$

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \leq 6w_i$

$$\min 2000\sum_i w_i + 320\sum_i h_i + 400\sum f_i + 8\sum s_i + 180\sum_i o_i.$$

Different form!

Again with carpets!

Production: $x_i = 20w_i + o_i$
Employment: $w_i = w_{i-1} + h_i - f_i$
Inventory: $s_i = s_{i-1} + x_i - d_i$
Regulations: $o_i \le 6w_i$

$$\min 2000\sum_i w_i + 320\sum_i h_i + 400\sum f_i + 8\sum s_i + 180\sum_i o_i.$$

Different form!

Not for example: $x_1 + x_2 \le 7$.

Variants of linear programs.

1. Maximization or minimization.
2. Equations or inequalities.
3. Non-negative variables or unrestricted variables.

Translations/Reductions.

1. Maximization to minimization?

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?
   Multiply both sides by (-1) again!

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?
   Multiply both sides by (-1) again!
   Example: $4 \geq 3$

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?
   Multiply both sides by (-1) again!
   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?
   Multiply both sides by (-1) again!
   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
3. Inequalities and equalities.
   (a) $\sum_i a_i x_i \leq b$ into equality?

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?
   Multiply both sides by (-1) again!
   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
3. Inequalities and equalities.
   (a) $\sum_i a_i x_i \leq b$ into equality?
   $\sum_i a_i x_i + s = b$

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.

2. Less than inequalities into greater than?
   Multiply both sides by (-1) again!
   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.

3. Inequalities and equalities.

   (a) $\sum_i a_i x_i \leq b$ into equality?
   $\sum_i a_i x_i + s = b$ and $s \geq 0$.

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?
   Multiply both sides by (-1) again!
   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
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Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
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   $\sum_i a_i x_i \leq b$ and $\sum_i a_i x_i \geq b$

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
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   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
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   $\sum_i a_i x_i \leq b$ and $\sum_i a_i x_i \geq b$
4. Simulate unrestricted variable x with positive variable

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
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1. Maximization to minimization?
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   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
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   $\sum_i a_i x_i \leq b$ and $\sum_i a_i x_i \geq b$
4. Simulate unrestricted variable x with positive variables.
   - Introduce x_+, and x_-.

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?
   Multiply both sides by (-1) again!
   Example: $4 \geq 3$ to $(-1)4 \leq (-1)3$.
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1. Maximization to minimization?
   Multiply objective function by -1.
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   - Replace x by $(x_+ - x_-)$.

Translations/Reductions.

1. Maximization to minimization?
   Multiply objective function by -1.
2. Less than inequalities into greater than?
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4. Simulate unrestricted variable x with positive variables.
   - Introduce x_+, and x_-.
   - Replace x by $(x_+ - x_-)$.

   $(x_+ - x_-)$ could be any real number!

Standard Form.

Standard form.

Standard Form.

Standard form.
Minimization,

Standard Form.

Standard form.
Minimization, positive variables,

Standard Form.

Standard form.
Minimization, positive variables, and “greater than” inequalities.

Standard Form.

Standard form.
Minimization, positive variables, and “greater than” inequalities.

Peas and carrots.

Standard Form.

Standard form.
Minimization, positive variables, and "greater than" inequalities.

Peas and carrots.

$$
\begin{aligned}
\max 4x_1 + 2x_2 \\
2x_1 \leq 60 \\
3x_2 \leq 75 \\
3x_1 + 2x_2 \leq 100 \\
x_1, x_2 \geq 0
\end{aligned}
$$

Standard Form.

Standard form.
Minimization, positive variables, and “greater than” inequalities.

Peas and carrots.

	Standard Form.
$\max 4x_1 + 2x_2$	$\min -4x_1 - 2x_2$
$2x_1 \leq 60$	$-2x_1 \geq -60$
$3x_2 \leq 75$	$-3x_2 \geq -75$
$3x_1 + 2x_2 \leq 100$	$-3x_1 - 2x_2 \geq -100$
$x_1, x_2 \geq 0$	$x_1, x_2 \geq 0$

Matrix Form.

Recall Linear equations: $Ax = b$?

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Can do that here, too!

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$$\min[-4, -2] \cdot [x_1, x_2]$$

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$m \times n$ matrix A;

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$$\min cx$$

$$Ax \geq b$$

Linear Program Problem

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   100

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   100 ,200 ,300 ... no limit!
   Unbounded.

Lecture in a minute.

What's a linear program?

Lecture in a minute.

What's a linear program?

- Variables.
- Linear inequalities, and a linear objective function.
- Geometrically: a convex region in n.
- Optimal solution at "vertex" of region.
  - Cartoon simplex/duality: move to better vertex, repeatedly.

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Applications:

- Production Planning.
  - Variables hiring/firing/inventory/production.
  - Constraints/Objective encode costs and resource limits.
- Bandwidth Problem.
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- Matrix, vector Notation.