EDIT DISTANCE

INPUT: Two strings $x[1..n]$ and $y[1..m]$

GOAL: Minimum # of operations/keystrokes to edit x into y

a) Delete Characters
b) Insert Character
c) Substitute / Replace

x = F A S T $\qquad$ y = CATS

F	A	S	T	_	_	_	_
_	_	_	_	C	A	T	S

4 deletions + 4 insertions

8 operations

F	_	A	_	S	T
_	C	A	T	S	_

deletion, insertion, deletion

2 insertions + 2 deletions

= 4.

DEFINE SUBPROBLEMS:

$$\left\{ \boxed{\begin{array}{cccccc} F & \_ & A & \_ & S & T \\ \_ & C & A & T & S & \_ \end{array}} \right\}$$

Insertion $\boxed{\begin{array}{c} \_ \\ A \end{array}}$

Deletion $\boxed{\begin{array}{c} A \\ \_ \end{array}}$

Substitution $\boxed{\begin{array}{c} A \\ B \end{array}}$

Carry-Over $\boxed{\begin{array}{c} A \\ A \end{array}}$

some prefix of x

F	_	A	_	S	T
_	C	A	T	S	_

some prefix of y

Imagine the optimal solution

SUBPROBLEM

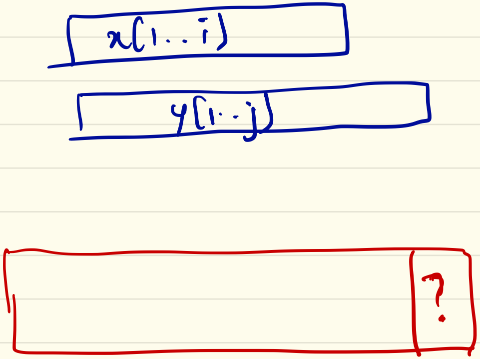
(edit distance)

$E[i, j]$ = # of keystrokes to go from $x[1 \ldots i]$ to $y[1 \ldots j]$

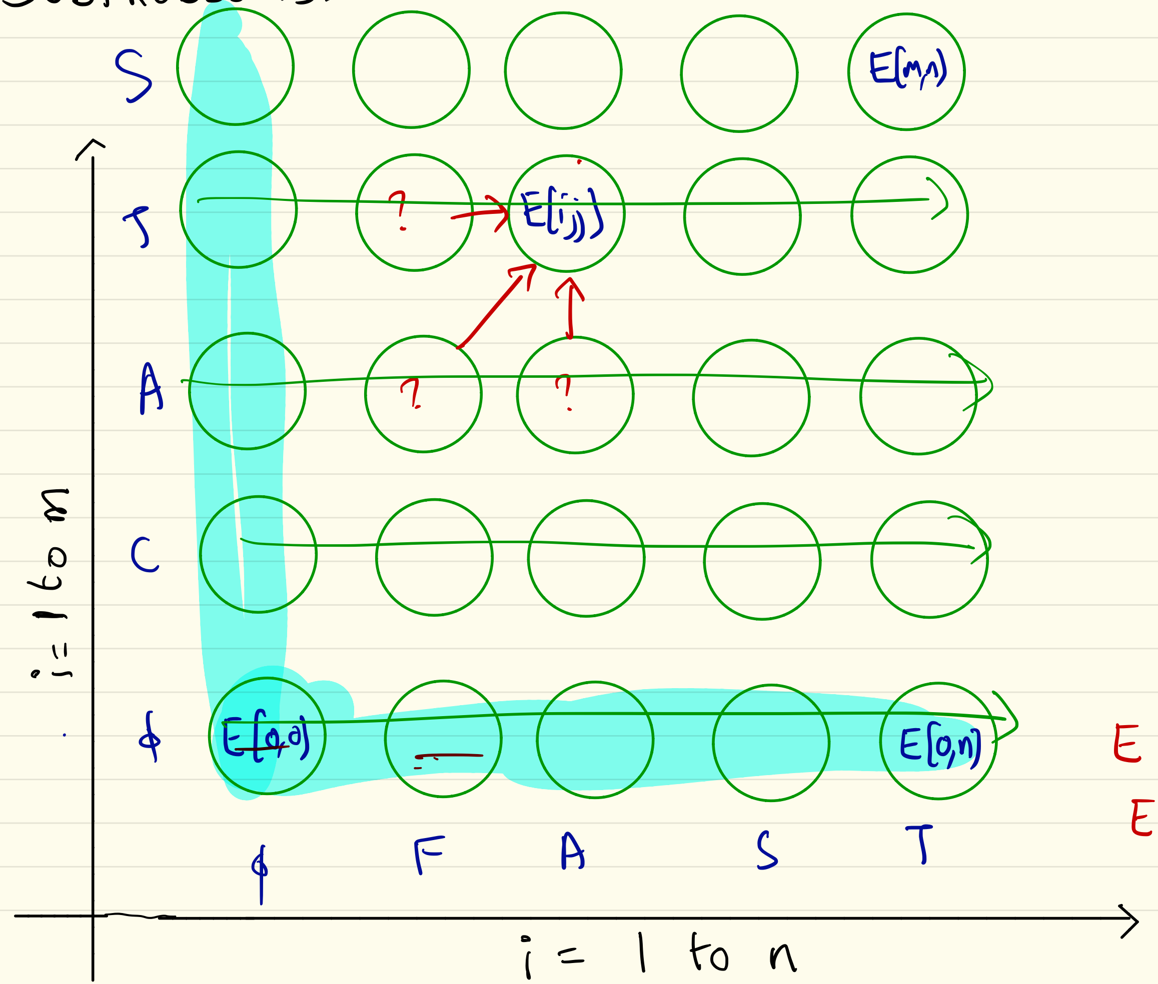
ANSWER: $E[m, n]$

RECURRENCE RELATION:

$$E[i,j] = \min \begin{cases} \boxed{\begin{matrix} - \\ A \end{matrix}} \quad 1 + E[i, j-1] \\[2ex] \boxed{\begin{matrix} A \\ - \end{matrix}} \quad 1 + E[i-1, j] \\[2ex] \boxed{\begin{matrix} A \\ B \end{matrix}} \quad \text{if } (x[i] \neq y[j]) \quad 1 + E[i-1, j-1] \\[2ex] \boxed{\begin{matrix} A \\ A \end{matrix}} \quad \text{if } (x[i] = y[j]) \quad E[i-1, j-1] \end{cases}$$



SUBPROBLEMS:



$E[0,i) = i$

$E[j,0) = j$

$E[i,0] = i \quad \forall \quad i = 1..m$

$E[0,j] = j \quad \forall \quad j = 1..n$

for $i = 1$ to m

    for $j = 1$ to n

        $E[i,j] = \Bigg\{$ recurrence relation

return $E[m,n]$

GAMBLING STRATEGY:

Play $n = 500$ games in a Casino

Game A: w.p $1/2$ earn 2\$

w.p $1/2$ lose 2\$

Game B: w.p $2/3$ earn 5\$

w.p $1/3$ lose 5\$

GOAL: SUCCEED if you win exactly 170\$ after n games.

· COMPUTE OPTIMAL STRATEGY.

STRATEGY = ??

$$\underset{\text{money}}{A[m\,\$},\ \underset{\text{games left}}{l}] = \text{Game A} / \text{Game B}$$

OPTIMAL STRATEGY = Maximizes probability of "SUCCESS" (having exactly 170\$) at end of n games

$P[m\$, \ell] =$ Probability of Success for optimal strategy starting with m\$ and l games left.

$m\$$: money earned

ℓ: games left

$P[0, n]$ ← ANSWER

$$P[m\$, l_{\text{games left}}] = \max \begin{cases} \text{Game A} & \frac{1}{2} P[m+2, l-1] + \frac{1}{2} P[m-2, l-1] \\ \text{Game B} & \frac{2}{3} P[m+5, l-1] + \frac{1}{3} P[m-5, l-1] \end{cases}$$

"Solve subproblems in increasing order of l."

$$P[m, 0] = \begin{cases} 1 & \text{if } m = 170 \\ 0 & \text{otherwise} \end{cases}$$