3SAT

INPUT: A 3SAT formula

SOL: A satisfying assignment

INDEPENDENT SET

INPUT: Graph $G=(V,E)$, K

SOL: An independent set of size K

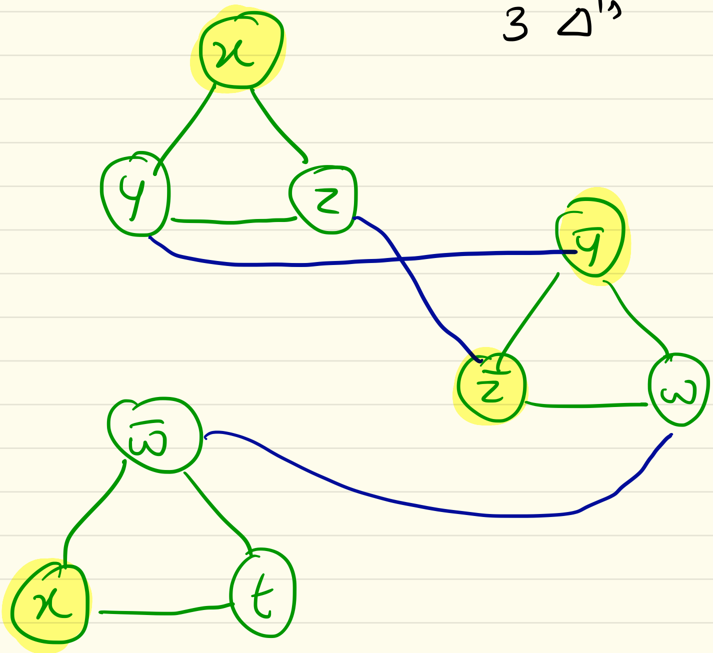
$$(x \vee y \vee z) \wedge$$
$$(\bar{y} \vee \bar{z} \vee w) \wedge$$
$$(\bar{w} \vee x \vee t) \wedge$$

satisfying assignment

$x=1 \quad y=0 \quad z=0$

$w=1 \quad t=0$

3 Δ's



Variable $x_1 \dots x_n$
Literal: $x_1, x_2, \dots x_n, \bar{x}_1, \dots \bar{x}_n$

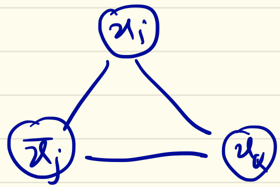
Proof:

REDUCTION: 3SAT formula ϕ with $x_1 \dots x_n$ and clauses $C_1 \dots C_m$

$$C_1 = (x_i \vee \bar{x}_j \vee x_k)$$

$\longrightarrow$

1) $\forall$ clause $(x_i \vee \bar{x}_j \vee x_k)$ introduce a $\triangle$'le



2) $\forall$ pairs of vertices x_i and $\bar{x}_j$ add an edge
3) $K =$ # of clauses

Proof: Suppose $(x_1 \dots x_n)$ is an assignment (satisfying)

In every clause, $\exists$ at least 1 true literal

IndSet = pick any true literal in every $\triangle$'le

Suppose $S \subseteq V$ independent set

$|S| = m$ # of clauses

$\Downarrow$

a satisfying assignment

<u>Obs:</u> S contains 1 vertex from every $\triangle$le

$$x_i \doteq \begin{cases} \text{if } x_i \in S & \text{assign } x_i := \text{TRUE} \\ \text{if } \overline{x_i} \in S & \text{assign } x_i := \text{FALSE} \\ \text{else} & \text{assign } x_i := \text{TRUE / arbitrarily.} \end{cases}$$

INTEGER PROGRAMMING : "Linear programming with variables forced to be integers"

INPUT : A set of linear constraints on $x_1 \ldots x_n$

SOL : A integer assignment satisfying constraints.

$$x_1 + x_2 \geq 5$$

$$x_5 - x_6 \leq 7$$

$$\vdots$$

$$0 \leq x_1 \leq 1$$

INDEPENDENT SET

INPUT: Graph $G=(V,E)$
integer K

SOL: An independent set of size K

$$\text{INDEPENDENT SET} \le_p \text{INTEGER PROGRAMMING}$$

INTEGER PROGRAMMING

INPUT: A set of linear constraints in $\{x_1, \dots, x_n\}$

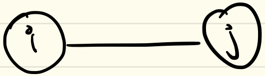
SOL: An integer feasible solution

$$\left\{ \begin{array}{l} \text{Graph } G=(V,E) \\ K \end{array} \right\} \longrightarrow \sum_{i=1}^{n} x_i = K$$

$$0 \le x_i \le 1$$

$x_i \in \{0,1\}$
$\uparrow$
indicate whether $i \in$ IndSet

$$x_i + x_j \le 1 \quad \forall (i,j) \in E$$



CLIQUE :

INPUT: A graph $G = (V, E)$ integer K

SOL :

INDEPENDENT SET

INPUT: Graph $G=(V,E)$
integer K

SOL: An independent set of size K

CLIQUE

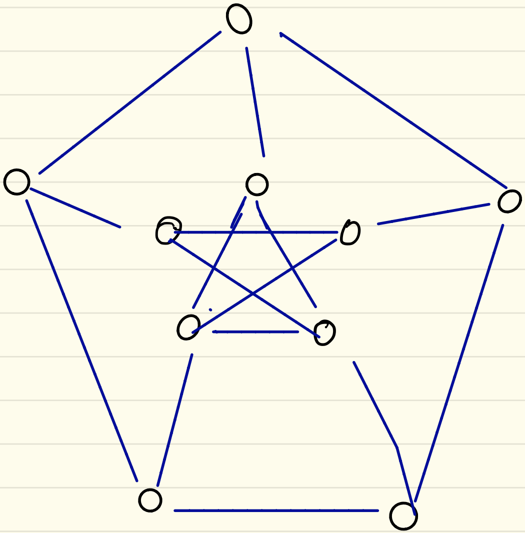
INPUT: Graph $G'=(V',E')$, K'

SOL: A clique $S' \subseteq V'$ of size K'

VERTEX COVER

INPUT :

SOL :



INDEPENDENT SET

INPUT: Graph $G=(V,E)$
integer k

SOL: An independent set of size k

VERTEX COVER

INPUT: Graph $G'=(V',E')$
integer k'

SOL: A vertex cover $S' \subseteq V'$ of size k'

VERTEX COVER	SET COVER
INPUT: Graph $G=(V,E)$, integer k	INPUT: Sets $\{S_1, \dots S_m\} \subseteq U$, size k
SOL: A vertex cover $S \subseteq V$ of size k	SOL: k sets that cover universe U

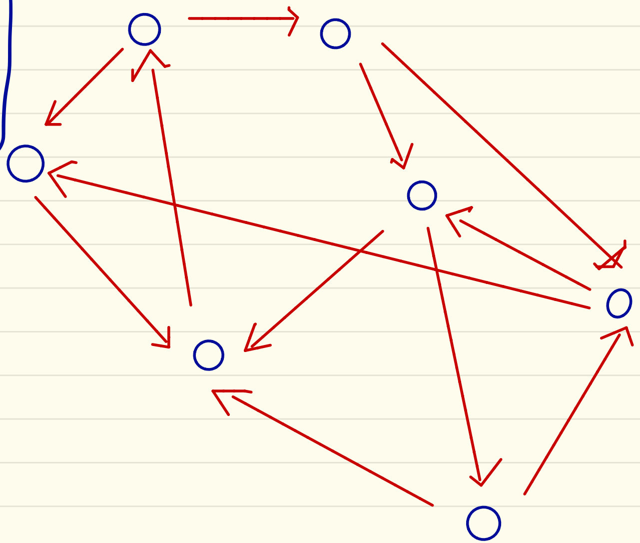
HAMILTONIAN

RUDRATA CYCLE (in directed graph)

INPUT: A directed graph $G=(V,E)$

SOL: A cycle visiting all vertices exactly once

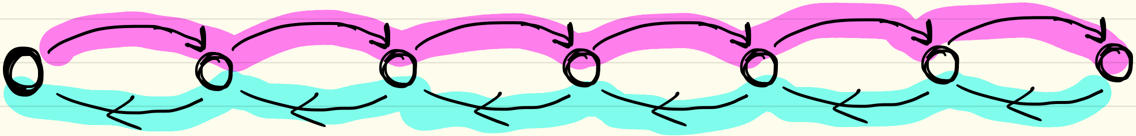
$$\boxed{3SAT \leq_p \text{RUDRATA CYCLE}}$$



$$x_1 \in \{0, 1\}$$

$0 \downarrow$ R to L

$1 \downarrow$ L to R

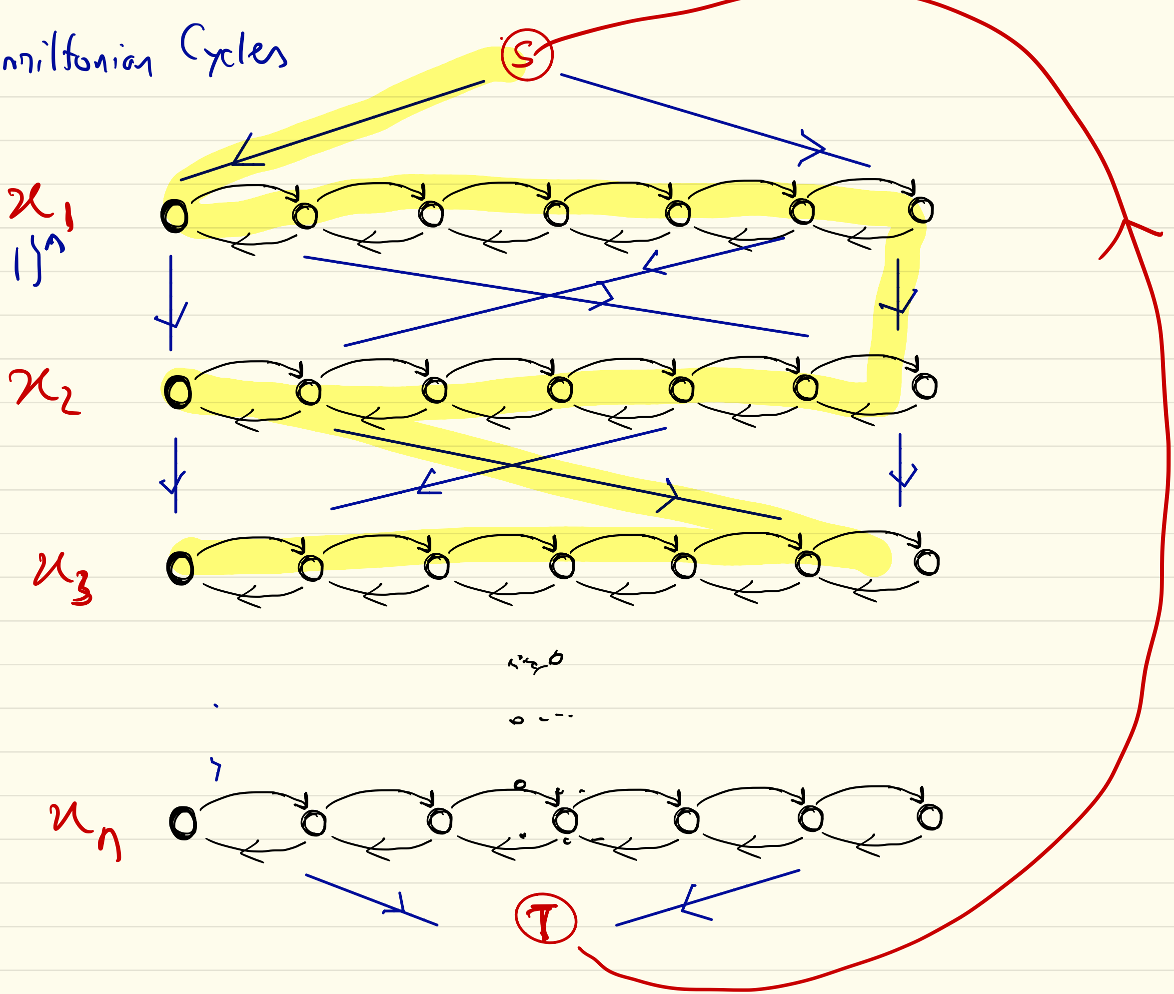


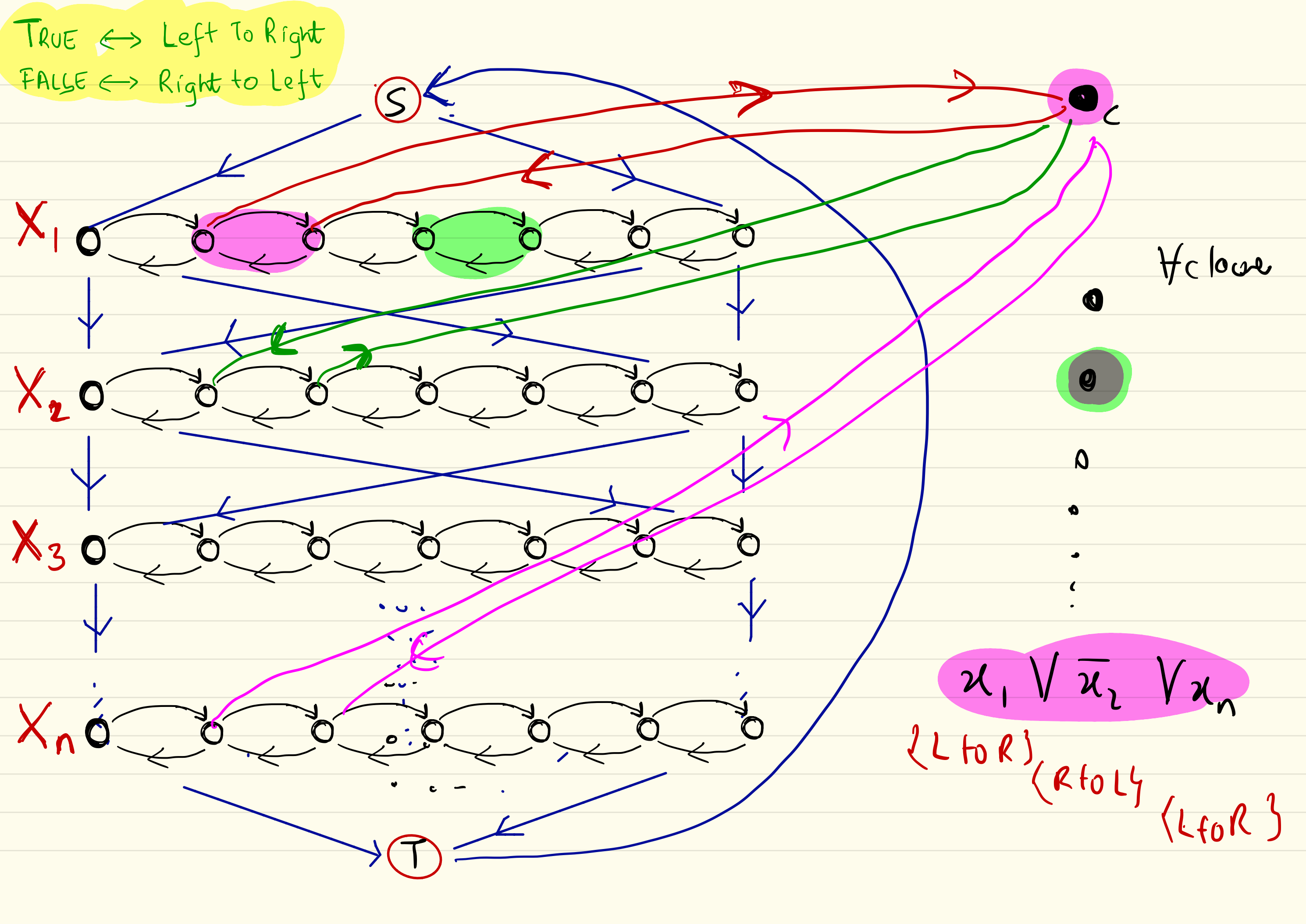
Solutions to 3SAT $\longrightarrow$ Solutions to Rudrata Cycle.

2^n Hamiltonian Cycles

$\Updownarrow$

$x \in \{0,1\}^n$




TRUE ⟷ Left To Right
FALSE ⟷ Right to Left
S
X_1
X_2
X_3
X_n
T
c
∀ clause
$x_1 \vee \bar{x}_2 \vee x_n$
{L to R}
{R to L}
{L to R}